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On the Görtler Instability of Boundary Layers*

J.M. Floryan*¹

ABSTRACT

Görtler instability occurs due to the action of centrifugal forces in boundary layers over curved surfaces, where it most commonly results in the generation of steady streamwise vortices. This study provides a summary of the current knowledge of this phenomenon. All existing experimental evidence is discussed. Experimental facts that bear on the theoretical modelling of the instability are pointed out. Linear theory is discussed in detail and compared critically with available experiments. Understanding of the nonlinear evolution of the vortices and their effect on the laminar-turbulent transition process are reviewed. The available nonlinear theories and direct numerical simulations are critically compared with the experiments. Centrifugal instability of turbulent boundary layers is discussed in detail. Effects of the instability on heat transfer and the associated buoyancy forces on the instability are discussed. A summary of the open questions is given.

Keywords: Boundary layers, Laminar-turbulent transition, Hydrodynamic stability, Görtler instability

概 要

ゲルトラー不安定は曲面上の境界層に対する遠心力の作用で起り、それによって定常な縦渦が発生する。この現象について現在までに得られた知識を総括し、未解決な問題点を明確にする。はじめに、既存の実験的事実を網羅し、不安定の理論的記述に重要と思われる諸点を摘出するとともに、線形安定理論を詳細に議論して、関連する実験結果との比較検討を行う。つぎに、縦渦の非線形発達段階と層流から乱流への遷移機構に与える影響を理解するための各種試みについて考察を加え、既存の非線形理論や直接数値シミュレーション結果と実験の詳細な比較検討を行う。最後に乱流境界層の遠心力不安定、不安定の熱伝達に対する効果、浮力の不安定に及ぼす影響等について議論し、今後の研究に指針を与える。

NOTATION

b : half width of wall jet

$G = (U_{\infty} \delta / \nu)(\delta / R)^{1/2}$

: Görtler number

p' : dimensionless pressure disturbance

R : radius of curvature of the wall

$Re = U_{\infty} \delta / \nu$

: Reynolds number

U_{∞} : free stream velocity, reference velocity

u, v, w : dimensionless components of the total velocity vector in the x, y, z directions

u', v', w' : dimensionless components of the disturbance velocity vector in the x, y, z directions

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- U, V : dimensionless basic-state velocity components in the x, y directions
- x, y, z ($\bar{x}, \bar{y}, \bar{z}$) : dimensionless (dimensional) streamwise, normal-to-the-wall and spanwise coordinates
- α : dimensionless wavenumber
- β : dimensionless spatial amplification
- $\delta_r = (v \bar{x} / U_\infty)^{1/2}$: boundary layer reference length scale
- δ : boundary layer thickness ($U = 0.995$)
- $\Lambda = (U_\infty \lambda / \nu)(\lambda / R)^{1/2}$: dimensionless wavelength parameter
- λ : dimensional vortex wavelength
- ν : kinematic viscosity

1. Introduction

The special feature of flows along curved walls is the occurrence of the centrifugal force associated with the change of direction of fluid motion. In the boundary layer on concave walls it decreases from the outer margin of the boundary layer towards the wall. If in the resulting force field a particle is deflected by a disturbance from its equilibrium position towards the inner and outer edge of the boundary layer, then it reaches zones of lesser or greater centrifugal force and in each case its movement is reinforced. Taylor [155] in 1923 was the first to study the stability of such a flow both theoretically and experimentally using the example of concentrically rotating cylinders. In so doing, he found that starting with a certain number characteristic for the problem, the Couette flow in the gap between the two cylinders became unstable with respect to disturbances in the form of counter-rotating vortex pairs with axes in the circumferential direction.

Görtler [63] showed in 1940 on the basis of theoretical analysis that a similar instability would occur in boundary layers over curved walls provided that a parameter defined as

$$G = (U_\infty \delta_r / \nu)(\delta_r / R)^{1/2}$$

and later referred to as Görtler number, exceeds a certain critical value. Here, U_∞ denotes free-

stream velocity, $\delta_r = (v \bar{x} / U_\infty)^{1/2}$ stands for boundary layer thickness, ν is kinematic viscosity, $\bar{x}$ stands for distance from the leading edge and R denotes radius of curvature of the wall. In the analysis, the flow was assumed to consist of a superposition of boundary layer and counter-rotating vortex pairs whose axes were parallel to the direction of the external stream, as shown in Fig. 1. Before the theoretical work of Görtler [63], Clauser and Clauser [27] noted in 1937 that the laminar boundary layer on a concave wall becomes turbulent at smaller Reynolds numbers than on flat and convex walls. Liepmann [107, 108] in 1943 attributed this effect to the occurrence of Görtler instability and showed that the Görtler number gives a good description of the stability conditions. The first direct experimental evidence of the existence of Görtler vortices has been provided by Gregory and Walker [64] in 1956 on the basis of the china-clay flow visualization method and by Tani [152] in 1962 on the basis of direct in-flow measurements. Since then numerous theoretical and experimental studies have been carried out in order to improve the understanding of this phenomenon and to assess its influence on the laminar-turbulent transition process in boundary layers.

This paper reviews the existing state of knowledge of the Görtler instability. Section 2 discusses the instability mechanism, Section 3 gives a description of the linear range of the instability, Section 4 deals with the nonlinear effects, Section 5 looks at turbulent boundary layers, Section 6 discusses heat transfer effects and Section 7 gives concluding remarks.

2. Mechanism of the Instability

The instability mechanism can be discussed on the basis of inviscid theory. Since the instability occurs owing to the dynamical effect of rotation, viscosity acts as a source of dissipation and does not contribute to the driving mechanism itself.

It has been shown by Rayleigh [134] that in

Fig. 1 Vortex-like disturbances in a boundary-layer flow over a curved wall.

the case of two-dimensional, circularly symmetric flows an increase of the magnitude of angular momentum with increasing radius provides a sufficient condition for stability. Bayly [15] showed that this remains true for arbitrary noncircular closed convex streamlines. The above proofs do not apply, strictly speaking, to boundary layers where streamlines are neither circular nor closed.

Floryan [56] considered inviscid stability of boundary layers and showed, on the basis of parallel flow approximation, that the necessary and sufficient condition for the flow to be stable is that

- (i) $d(U^2)/dy < 0$ in the case of concave wall, and
- (ii) $d(U^2)/dy > 0$ in the case of convex wall.

In the above, $U(y)$ denotes mean flow velocity component in the streamwise direction and y is

the distance from the wall. The stability properties of flows depend on whether the velocity distribution is monotonic or not. In the former case, the velocity gradient maintains a constant sign across the boundary layer and, since it is positive, the flow is stable in the case of convex walls and unstable in the case of concave walls. In the latter case, the velocity gradient changes sign inside the boundary layer and the flow may be unstable for both convex as well as concave walls. Figure 2 illustrates these conclusions. The fact that Görtler instability can occur over convex walls is generally overlooked and, as a matter of fact, all of the existing analyses, with the exception of [56], deal exclusively with flows over concave walls.

Jallade et al. [88] extended the inviscid analysis to compressible flows but have not formulated any general criteria.

Fig. 2 Shear layers and their stability according to the inviscid stability criterion. Cases A and B —monotonic velocity distribution. Cases C and D—nonmonotonic velocity distribution (wall jet). ... —inviscid stability criterion violated, - - - —inviscid stability criterion satisfied. Cases A, C, D are potentially unstable.

3. Linear Stages of the Instability

It is of principal interest to determine critical conditions leading to the initiation of the instability, the character of the disturbance motion emerging from the instability and evolution of the unstable flow in the streamwise direction.

3.1 Experimental Evidence

Flow visualizations are the primary techniques used in investigations of the Görtler instability, owing mainly to its three dimensional character.

Surface signature of the flow has been obtained in [33, 34, 62, 64, 102, 104, 115, 116]. The solid surface is covered with a thin layer of substance whose rate of sublimation is proportional to the surface shear stress. Instability increases shear stress at locations corresponding to the downwash induced by the vortex (section c in Fig. 3) and decreases at locations corresponding to the upwash (section a in Fig. 3). Figure 4 shows the resulting pattern of streamwise streaks. In this particular

case [34] white biphenyl has been sprayed over a black surface, thus black streaks correspond to the downward motion induced by the vortex. Spanwise distance between two neighbouring streaks of the same colour corresponds to one pair of counterrotating vortices and defines the spanwise wavelength of the instability. The form of the streaks demonstrates that the wavelength remains constant in the streamwise direction.

Detailed information regarding the character of the three-dimensional motion induced by the instability has been gained by direct visualization and supporting measurements in the interior of the flow field. Tellurium [10, 87, 165-167], hydrogen bubbles [8, 21, 22, 106, 138, 157, 158, 164], dye [130, 162] and smoke [1, 4-6, 8, 79, 83-86, 100, 147-151, 154] visualizations have been combined with hot wire [1, 3-6, 84, 100, 147-151, 154] and more recently with laser [29-31, 33, 34, 62, 106, 115, 116, 130, 138, 162, 163] measurements. The character of the motion will be discussed in the context of visualization experiments

Fig. 3 Modification of the streamwise velocity distribution in the spanwise cross-section induced by the instability. Section a—vortex induced upwash; section b—centre of the vortex; section c—vortex induced downwash.

performed by Ito [83, 85].

Ito's work represents a typical experiment in Görtler instability, where a boundary layer is generated and directed to flow above a plate of constant streamwise curvature. A schematic diagram of the experimental apparatus is shown in Fig. 5. In this particular experiment the boundary layer begins at the beginning of the curved wall. The flow is visualized by introducing spanwise lines of tracers (smoke). If the instability does occur, these tracers are expected to converge at locations corresponding to the upwash induced by the vortices, as shown schematically in Fig. 6. Experimental results shown in Figs. 7-9 demonstrate that the instability evolves spatially, i.e. it has a convective character. Presence of the instability (i.e. slight convergence of the tracers) can be first detected at a distance of 400 mm from the leading edge (Fig. 7). Maximum convergence, which corresponds to rotation of the vortices by 90° , occurs at approximately 680 mm. First signs

of breakdown of the laminar flow can be seen at 800 mm. Distributions of the streamwise velocity component u displayed in Fig. 8 show no detectable sign of the instability at 200 mm. The form of the velocity distribution in the y -direction suggests linear growth at 400 mm and 500 mm. Nonlinear distortions appear at 600 mm and are clearly seen at 700 mm. Disturbances begin to grow near the surface of the wall and expand into the outer part of the boundary layer further downstream. Movements of smoke in the spanwise direction are shown in Fig. 9. Instability can be detected at 400 mm. Lifting up of the smoke between 600 mm and 700 mm corresponds to the appearance of nonlinear distortions in the mean flow (see Fig. 8). The characteristic bell-shape form appears at 800 mm.

The above results demonstrate that the flow reacts very slowly to the curvature. It becomes unstable according to the linear theory at $G = 0.4638$ [51], but no disturbances can be

Fig. 6 Motion of tracers in the spanwise plane induced by the vortices; ...—initial location, - - -—location after 90° rotation.

detected until $G = 10.1$ (400 mm). The vortices rotate very slowly and they turn by little more than 90° before causing transition. About half of the growth in terms of streamwise distance can be considered linear and the amplification rate is noticeably small. The instability is obviously weak and could be dominated by other types of instability if such were simultaneously present. It can be deduced from Fig. 7 that the normal-to-the-wall (v') and spanwise (w') disturbance velocity components, which form the rotational motion, may be difficult to measure due to their small magnitudes. In contrast, the streamwise (u') disturbance velocity component, which is only indirectly produced by the instability, is quite easily recognizable; it arises as a defect in the mean flow that is generated by the vortex motion and, as a such, its characteristic scale is similar to that of the mean flow velocity scale. Because of the large disparities in the magnitudes of u' and v' and w' , most of the information about the instability comes from measurements of the streamwise velocity component u only and, as a matter of fact, the appearance of spanwise periodicity in the streamwise distribution of u is considered to be the most characteristic sign of the instability (Fig. 8).

The interpretation of the instability motion shown schematically in Fig. 1 should be re-examined in view of the experimental evidence shown in Fig. 7. Figure 1 suggests existence of rotating vortices inside the boundary layer and this

is intuitively acceptable based on analogy with the Taylor problem. Various visualization experiments have clearly shown existence of such vortices in the flow between concentric rotating cylinders. There, a fluid particle is continuously affected by the instability forces and, as a result, it follows a swirling trajectory. In contrast, in the case of a boundary layer, the fluid particles remain in the unstable part of the flow for only a limited period of time. This short residency period results in a small deflection of the particles from their original trajectories, as shown in Fig. 7, rather than in formation of a true vortex motion, as implied by Fig. 1. The location where the deflections begin to take place may be difficult to determine experimentally. Since measurements are, in effect, focused on the amplitude of deflections and the growth of deflections in streamwise direction is slow, the instability may be detectable only far downstream from its initiation and measurements could be very sensitive to the initial deflections, i.e., to the amplitude of disturbances present in the oncoming stream.

The questions of existence and determination of the critical stability conditions have not been conclusively settled. It is possible that either (i) the flow is always linearly unstable or (ii) that critical stability conditions do exist. The resolution of this question requires experimental determination of conditions corresponding to zero spatial amplification ($\beta = 0$) of disturbances. The resulting neutral curve defines the marginal stability

Fig. 7 Visualization of the Görtler instability (from [83-86]).

Diagram of the wind tunnel is shown in Fig. 5. $U_{\infty} = 2.5$ m/sec, $R = 1$ m.

conditions, i.e., it separates stable and unstable regions in the parameter space. The existing experimental evidence does not allow any of the options listed above to be excluded. The difficulties in sorting out of this question have to do with the weakness of the instability. Determination of the neutral curve requires introduction of small disturbances of a specified wavelength into the flow and subsequent measurements of their streamwise evolution. This technique is very effective for the vortices of large wavenumbers which are rapidly amplified when the neutral curve is crossed. The character of the critical conditions is however determined by the vortices with wavenumber $\alpha = 0$ (1) and smaller; these vortices are very weakly amplified and this combined with the overall weakness of the instability may preclude their early and accurate identification.

Bippes [22] and Bippes and Görtler [21] introduced spanwise periodic disturbances by heating longitudinally oriented wires placed on the surface of the plate with a specified spanwise periodicity. The instability was detected by observing changes in the spanwise distribution of hydrogen bubbles.

Since disturbances had to be strong in order to observe them, Bippes [22] judged this technique not to be suitable for large vortices. Aihara et al. [8], Tomita [157] and Tomita et al. [158] used a set of thin plates located upstream of the leading edge as disturbance generators and, in an experiment similar to Bippes', reproduced his results and extended the neutral curve to wavenumbers $\alpha = 0$ (10^{-2}). It is convenient to begin discussion of these results with the large wavenumber segment of the neutral curve. Different experiments [8, 21, 22, 157, 158] produced essentially identical results and, therefore, existence of this part of the neutral curve can be considered as well documented. The location of this curve, however, does not agree with the results obtained on the basis of linear theory (see Fig. 10). It is possible that this disagreement is caused by nonlinear effects since, according to the measurements of the disturbance velocity field provided by Bippes [22], u' was of the order of 10% of the mean flow. Accuracy of the experimental determination of the remaining part of the neutral curve is uncertain. Since large

Fig. 8 Measurements of the streamwise velocity component u (from [83-86]). Experimental conditions as in Fig. 7. A—normal-to-the-wall distribution, B—spanwise distributions

Fig. 9 Visualization of the instability in the spanwise plane (from [83-86]).
Experimental conditions as in Fig. 7.

vortices are very weak, the strength of initial disturbances used in the experiments could critically alter their behaviour. This strength has been neither controlled nor varied [8, 157, 158]. In the absence of direct measurements of the disturbance velocity field, the agreement with Bippes [22] obtained for large wavenumbers suggests that the initial disturbances might have been too strong. The form of the neutral curve reported there [8, 157, 158] for $\alpha = 0$ (1) and smaller could therefore be an artifact of the experimental procedure. Also, since disturbances had been introduced upstream of the leading edge, it is possible that they might have undergone a selective amplification in the stagnation region thus blurring the initial stages of the instability. The strong effects of the initial disturbances have been underscored by Wortmann [165, 166], who was unable to locate the neutral curve in his early experiments, because its location varied with the form and intensity of the initial disturbances. He used a set of small airfoils as disturbance generators and varies their angle of attack and position upstream of the leading edge. Aihara [9] suggested that the way around these problems might be to focus on the measurements of the amplification rates in the linear regime and then to locate the neutral curve by extrapolating upstream.

Attempts to measure critical wavenumber by working with naturally occurring disturbances were unsuccessful and showed the results to depend on the form of the oncoming disturbances [22, 30] and on the edge effects of a particular experimental apparatus [152, 153, 163]. Kottke [104] and Kottke and Mpourdis [103] measured the effects of intensity of the initial disturbances on the instability by varying distance between screens that were acting as a source of disturbances and the plate. The presence of vortices was identified by surface signature of the flow based on molecular mass transfer technique. The results showed no detectable sign of the instability when the screens were placed sufficiently far upstream. Bippes [23] similarly observed in his original

Fig. 10 Location of the neutral curve for the Blasius boundary layer (from [51]).

Experimental points due to Bippes [22]: $\circ$ -attenuated disturbances, $\square$ -amplified disturbances

experiments [22] (without publishing this information) that if the oncoming flow was free of disturbances, no instability could have been detected. This opens a possibility that the Görtler problem is not an instability problem; the flow merely selectively amplifies disturbances already present in the oncoming stream and no secondary motion develops unless these disturbances have a sufficient magnitude and an appropriate form, as is the case with the stagnation flow [26]. This is not likely to be the case here, since the stagnation flow has been shown to be linearly stable [161] while the boundary layer is not [51]. Moreover, the existence of the large wavenumber segment of the neutral curve has already been

documented experimentally. Kottke [104] and Kottke and Mpourdis [103] experiments probably show that because of a very low level of initial disturbances, the instability motion has not yet evolved to the point where it could have been detected. This interpretation is consistent with the overall picture of the weakness of the instability. Kottke's experiments are very intriguing and it would be very desirable to have them repeated in other facilities in order to assess their facility dependence and to quantify the role of the oncoming disturbances.

The above discussion shows that based on the existing experimental evidence it is not possible to determine whether the critical conditions occur

at zero or at a finite Görtler number. This question may not be amenable to experimental investigations at all. Linear stability analysis shows (Section 3.2) that disturbances are influenced by the effects of boundary layer growth which have to be modelled by the higher-order boundary layer approximations. Such small effects cannot be effectively controlled in experiments. Since these effects depend on the overall geometry of the experimental apparatus rather than on the local wall curvature, variation of results between different experimental facilities might occur. The first step towards settling the issue of critical conditions might therefore involve precise definition of the geometry subject to experimentation and its standardization among different experimental facilities. Additional difficulties may arise due to the fact that once the vortices do set up, they may extend upstream [9]. Also, the neutral conditions occur close to the leading edge [9] where the state of the mean flow may not be well described by the boundary-layer equations (see discussion of the linear stability results in Section 3.2).

The demonstrated coupling between the oncoming disturbances and the observable development of the instability suggests that a very strong receptivity mechanism is present. Kottke and Mpourdis [103] and Kottke [104] have shown that by changing the form and intensity of the oncoming disturbances different vortices or no vortices at all could be observed in a nominally unstable boundary layer. Bippes [22] was able to bypass the receptivity problem by introducing an isotropic field of disturbances into the oncoming flow. The observed vortices corresponded then to the most amplified disturbances according to the linear theory [51, 53] (see Fig. 11). The existence of the wavelength selection mechanism based on the fastest growing mode has been confirmed by measurements carried out in a low turbulence wind tunnel at NASA Langley Research Centre under a variety of free stream conditions [33, 34, 115, 116]. The receptivity problem has probably been

bypassed in these experiments either because the test section was sufficiently close to the screens in the settling chamber or because of the presence of a uniform surface roughness associated with the surface flow visualization technique. The wavelength selection mechanism has been shown to be weak and to be easily affected by results of the receptivity process [22, 30, 103, 104, 148, 149, 152, 153, 163]. The mechanics of the receptivity process by itself is not understood. It has been shown that the receptivity could be manipulated by applying suction to the wind tunnel side walls [126], but again, the mechanics of the coupling is not understood.

The initial growth of the disturbances is qualitatively well represented in Figs. 7-9. Bippes [22] measured all components of the disturbance velocity field as well as streamwise growth of u' . His results agree only qualitatively with linear theory [50, 51, 53]. The measured disturbances were very strong, e.g., $u' = 0.1 u$, and it is likely that they were affected by nonlinear effects. The character of the differences is consistent with the possible presence of nonlinearities. Extensive measurements of the streamwise disturbance velocity component at different streamwise locations reported in [33, 34, 115, 116] show agreement with the predictions based on linear theory. No other measurements of the disturbance velocity field and its streamwise evolution are available.

The process of damping of vortices has attracted very little attention so far. One might have a concave wall with a Blasius-type boundary layer which is then followed by a flat or a convex wall, where centrifugal forces either do not exist or contribute only to stability. The mechanics governing evolution of the vortices might be completely different there. It has been observed so far [93] that the existing vortices may lift up, another layer of vortices may form underneath, and a half wavelength shift may occur in the spanwise direction. Enlargement and lifting up of the vortices is another possibility [130]. Further experimental observations are required.

Fig. 11 Linear stability diagrams for the Blasius boundary layer (from [51]).

A—Görtler number versus wavenumber, B—Görtler number versus wavelength parameter.

Experimental points due to Tani and Sakagami [153] with naturally occurring vortices:

○— $U_\infty = 11$ m/sec, $R = 10$ m; □— $U_\infty = 7$ m/sec, $R = 3$ m; ◇— $U_\infty = 7$ m/sec, $R = 3$ m;

△— $U_\infty = 16$ m/sec, $R = 10$ m; ▽— $U_\infty = 7$ m/sec, $R = 1$ m; Experimental points due to

Bippes [22] with isotropic field of disturbances in the oncoming flow: ▷1— $U_\infty = 0.3$ m/sec,

$R = 5$ m; ▷2— $U_\infty = 0.075$ m/sec, $R = 0.5$ m. ▷3— $U_\infty = 0.075$ m/sec, $R = 1$ m; Curves of

maximum amplification identify maximum amplification for a given Görtler number.

Behaviour of Görtler vortices in three-dimensional flows has only recently attracted attention. Kohama [100] investigated flow on a swept wing and observed that for zero or very small sweep angle Görtler instability was the primary destabilizing mechanism. An increase of the sweep angle resulted in an occurrence of the cross-flow instability and transformation of the counter-rotating Görtler vortices into the co-rotating crossflow vortices.

Görtler vortices have also been observed under a variety of other flow configurations. Transient boundary layers were simulated by suddenly stopping a liquid-filled rotating cylindrical vessel. Vortices would then develop in a growing boundary layer on the inner wall of the outer cylinder [46, 105, 156]. Similar vortices have been observed on inner walls of supersonic nozzles at Mach numbers 5 [16] and 3.5 [17]. All these results are qualitative and provide no details that could be used for verification of the theoretical predictions.

3.2 Theoretical Modelling

Accurate representation of the centrifugal force field requires knowledge of the local velocity as well as local streamline curvature. Both these factors depend on the geometry of the wall. While the shape of the wall must be consistent with the boundary-layer approximations, there are still several parameters that are involved in its characterization, as illustrated in Fig. 12. Wall curvature is acknowledged to be important. One may inquire, however, for example (i) what is the effect of the streamwise extent D of the curved wall (case A, Fig. 12), or (ii) what is the effect of the length L of the flat plate upstream of the curved wall (case B), or (iii) is the total turning angle γ more important than the curvature of the wall (case C), or (iv) how to capture effects of the total turning angle (case D), and (v) downstream shape of the model contributes to the higher-order boundary layer theory but does it affect the instability (case E)? The question arises as how

to identify all of the relevant parameters and how to capture their effects.

The configuration that has been studied most extensively, both theoretically and experimentally, is shown in Fig. 12a. Here, the wall has a constant streamwise curvature whose radius and streamwise extent are large compared to the boundary-layer thickness. The wall is placed tangentially to the oncoming stream at the leading edge. The beginning of the curvature coincides with the beginning of the boundary layer. The curvature is sufficiently small so that pressure remains constant in the streamwise direction. Equivalent configurations have been realized in many experiments using both wind and water tunnels. It was a standard practice to extend the curved segment of the wall sufficiently far downstream so that the finite streamwise extent of the curvature had no perceived effect on the instability. The flow was adjusted by various means in order to maintain constant pressure.

It is worth noting that studies based on the above configuration do not capture effects of all of the relevant parameters. It has been observed, for example, that in the case of a model consisting of a straight wall followed by a curved wall, vortices existed already above the flat wall [33, 34, 93].

Analysis of the instability in the context of the configuration shown in Fig. 12a has been given by Floryan and Saric [47, 51]. It is useful to review the main issues that have been addressed in the analysis since they often seem to be misunderstood. The first problem involves the reference coordinate system. The usual boundary-layer approximation involves local body-oriented coordinates while the outer flow is analyzed in terms of global coordinates. Arbitrariness in matching both parts of the solutions in the case of curved walls has been removed by implementing the concept of optimal coordinates. Such coordinates provide means for construction of a uniformly valid boundary-layer approximation and are indispensable when studying behaviour of

Fig. 12 Examples of geometrical factors that might influence the instability.

large vortices. The second problem deals with the determination of the basic state, e.g., with the question of the correct form of the boundary-layer approximation in the case of curved walls. Since in the classical boundary-layer theory the leading-order approximation is not affected by the curvature, one may suspect that its use in representing the mean flow may lead to erroneous modelling of the centrifugal effects. This has been shown not to be the case. The third problem deals with the modelling of the disturbance motion. The

essential features of the physics of the motion, as discussed in Section 3.1, have been captured by introducing two distinct scales for different components of the disturbance velocity vector. The streamwise component, which appears as a defect in the mean flow due to the vortex motion, is scaled using the same scale as is used for the mean flow, i.e., U_∞ . The weakness of the vortex motion is captured by introducing an order-of-magnitude smaller velocity scale for v' and w' , i.e., U_∞/Re . Since pressure disturbance is associated with the

vortex, it is scaled on the vortex velocity scale. Normal-to-the-wall and spanwise directions are scaled using boundary layer thickness δ , as a reference scale, in agreement with the experimental observations of the size of the vortices. Slow streamwise evolution of disturbances is captured by using boundary layer evolution scale δ, Re for the streamwise direction, which is again consistent with the experimental observations. Only spatial stability theory is considered owing to the fact that disturbances evolve in a spatial rather than temporal manner (see Figs. 7-9). The resulting disturbance equations in the leading order of approximation assume the following form:

$$\frac{\partial u'}{\partial x} + \frac{\partial v'}{\partial y} + \frac{\partial w'}{\partial z} = 0 \quad (1)$$

$$\frac{\partial U}{\partial x} u' + U \frac{\partial u'}{\partial x} + \frac{\partial U}{\partial y} v' + V \frac{\partial u'}{\partial y} = \frac{\partial^2 u'}{\partial y^2} + \frac{\partial^2 u'}{\partial z^2} \quad (2)$$

$$U \frac{\partial v'}{\partial x} + \frac{\partial V}{\partial x} u' + \frac{\partial V}{\partial y} v' + V \frac{\partial v'}{\partial y} + 2G^2 U u' = -\frac{\partial p'}{\partial y} + \frac{\partial^2 v'}{\partial y^2} + \frac{\partial^2 v'}{\partial z^2} \quad (3)$$

$$U \frac{\partial w'}{\partial x} + V \frac{\partial w'}{\partial y} = -\frac{\partial p'}{\partial z} + \frac{\partial^2 w'}{\partial y^2} + \frac{\partial^2 w'}{\partial z^2} \quad (4)$$

In the above, (U, V) denote the dimensionless mean flow velocity components in the (x, y) directions, (u', v', w') stand for the dimensionless disturbance velocity components in the (x, y, z) directions, p' denotes the pressure disturbance and G stands for the Görtler number. The above equations display the same features as the disturbance motion. Only the normal-to-the-wall y and spanwise z momentum transport equations, which are directly associated with the vortex motion, contain pressure disturbance. Weak motion ($O(Re^{-1})$) in the y and z directions (Eqs. 3 and 4) induces large changes ($O(1)$) in the streamwise x momentum transport (Eq. 2).

Model equations (1)-(4) were proposed for the first time in 1979 by Floryan and Saric [47]. New models subsequently proposed by Ragab and Nayfeh [132] and Hall [65] are identical to the above one.

Linear disturbance equations (1)-(4) demonstrate that the parallel flow approximation is not a tenable concept in the Görtler instability and, therefore, no analogies exist with the Tollmien-Schlichting instability. Indeed, breaking away from the concepts based on the Tollmien-Schlichting instability proved to be the biggest stumbling block in achieving progress in this problem. Early works that started with the parallel flow approximation led to incomplete disturbance equations (see [80] for a review). Initial attempts to include the effects of boundary layer growth are discussed in [47, 51].

Extraction of the relevant stability information from the disturbance equations is based on the Fourier decomposition in the spanwise direction. Each mode is characterized by a dimensional wavenumber which remains constant in the streamwise direction, in agreement with the experimental observations. Two methods have been used in the analysis of the streamwise evolution of a particular mode. The method based on local approximation [47, 51] treats variable coefficients in the disturbance equations as being independent of x ; this leads to disturbances in the form.

$$\begin{aligned} u'(x, y, z) &= u(y) \cos(\alpha z) \exp(\beta x), \\ v'(x, y, z) &= v(y) \cos(\alpha z) \exp(\beta x), \\ w'(x, y, z) &= w(y) \sin(\alpha z) \exp(\beta x), \\ p'(x, y, z) &= p(y) \cos(\alpha z) \exp(\beta x), \end{aligned}$$

an exponential approximation of the streamwise growth, reduction of the disturbance equations into ordinary differential equations and formation of a one-dimensional eigenvalue problem for α, β and G . Here, α denotes the wavenumber made dimensionless with respect to the boundary layer thickness δ and β stands for the spatial amplification rate made dimensionless with respect to the boundary layer evolution scale $Re\delta$. Since the above method does not provide a solution of the disturbance equations but only an approximation, the question arises as how good this approximation is. One may note that since the disturbance

equations (1)-(4) represent a truncated form of the Navier-Stokes equations, there is no need to solve them exactly and as long as the error of approximation is comparable to the truncation error, the error bounds for the whole analysis are not affected. Also, in the stability analysis one is interested in estimating the upper bounds on the possible disturbance growth rather than in determination of the evolution of a particular disturbance. In the second approach [67] the full parabolic problem for the streamwise evolution of the vortices is solved numerically using a marching procedure. Since an explicit specification of the initial conditions is required, one has difficulties in answering the stability question where the issue of effects of all possible initial conditions has to be investigated. Not surprisingly the initial stages of the vortex development show strong dependence on the assumed form of the initial conditions. Given a sufficiently large number of numerical experiments, one may nevertheless attempt to answer the stability question. Comparison of both approaches [37, 92, 93] shows that given physically realistic initial conditions for the second one both methods produce equivalent results. The following discussion will be based on results obtained using the method of local approximations with references being made to the results obtained using the marching procedure where appropriate. The principle of exchange of stabilities (see [39] for definition) is assumed throughout, but its existence has been proven only under very restrictive assumptions [81]. The existing experimental evidence (see Fig. 7) tends to support this assumption.

Figure 11 displays stability diagrams for the first (most unstable) mode for the Blasius boundary layer. Higher modes have been shown not to be relevant [55] and will not be discussed here. If the geometry of the wall (R), the type of the fluid (ν) and the velocity of the oncoming flow (U_∞) are specified, the Görtler number in these diagrams can be interpreted as a measure of

distance from the leading edge. A straight line with slope $3/2$ corresponds to a constant dimensional wavenumber in the (G, α) plane (dimensionless wavenumber α changes with x due to variation of the length scale δ). Alternatively, one can use the wavelength parameter $\Lambda = (U_\infty \lambda / \nu)(\lambda / R)^{1/2}$ which is constant in the streamwise direction. In the above, λ denotes the dimensional wavelength of the disturbance. Stability diagrams in the (G, α) and (G, Λ) planes are displayed in Figs. 11A and 11B, respectively. These diagrams show that typically as a fluid element travels downstream, it moves from the stable to unstable part of the flow field, with the instability intensifying with the distance travelled. Forms of the neutral curve for large α that have been obtained using asymptotic [65] and local approximation [47, 51] methods are identical. While the location of this segment of the curve does not agree with the available experimental data, as shown in Fig. 10, it is very likely that the experimental results were affected by nonlinear effects, as discussed in Section 3.1. The form of the neutral curve for $\alpha \rightarrow 0$ is uncertain. This part of the curve corresponds to large vortices that penetrate far outside the boundary layer. One reason for the uncertainty is that calculations carried out so far [47, 51, 67] represented mean flow outside the boundary layer as being identical to that at the edge of the boundary layer. Since the disturbance equations contain terms arising due to the third-order boundary layer (see [160] for explanation of the higher-order boundary layer theory), reasons should be given why such a simplistic representation of the external flow is acceptable. It has been demonstrated that, for example, flow due to displacement thickness has a strong effect on the disturbances for small wavenumbers [132]. Another reason for the uncertainty is that disturbance equations (1)-(4) may be incomplete in the limit $\alpha \rightarrow 0$. Large vortices are very weak and they may be affected by terms considered negligibly small and omitted so far from the model equations. Also, large vortices have never been

observed in experiments. Bippes [22] attempted to introduce such vortices artificially into the boundary layer and observed that they had always separated into smaller ones. Finally, one would not expect on physical grounds the neutral curve to level off as shown in Fig. 11. The small value of the Görtler number at which this levelling off occurs describes a location in the flow field that would be very close to the leading edge in most experimental facilities. The flow there may not be well approximated by the boundary-layer equations. An attempt to augment disturbance equations in the limit $\alpha \rightarrow 0$ was inconclusive [49]. In view of the above uncertainties it is premature to draw any conclusions regarding the character of the neutral curve for $\alpha \rightarrow 0$ on the basis of the presently available numerical results. The question of the character of the critical stability conditions requires further investigation. It will be assumed for the purposes of further discussion that the neutral curve has the form as shown in Fig. 11. The reader, however, should keep in mind the uncertainties discussed above.

Since the critical wavenumber is not defined, theoretical predictions regarding the character of the disturbance motion emerging from the instability can only be based on comparisons of relative growth of different modes [53]. Results shown in Fig. 11 demonstrate that given the same initial amplitude for all possible disturbances, the most amplified ones are observed in experiments. This has been independently confirmed by experiments described in [33, 34, 93]. Theoretical comparison of growth of different disturbances requires calculation of the total (integrated) growth, i.e., ratio of disturbance amplitude (or energy) at a given location and at a location corresponding to the neutral curve. It has been shown using the marching solution procedures that the computed total growth of the vortices is independent of the particular form of the neutral curve for $\alpha \rightarrow 0$ [93]. This is so because amplification of large vortices around the neutral curve is very small and, as a result, it brings negligible

contribution to the total growth. Predictions regarding the total growth that are based either on the local approximation method or on the direct solution of the disturbance equations are essentially identical and show that disturbances observed experimentally do correspond to the most amplified ones (see Figs. 13 and 11A) [53]. The existence of the wavelength selection mechanism based on the total growth of disturbances could, therefore, be considered as well documented. The above results also show that it is possible to use the semi-empirical transition prediction $\exp(n)$ method without settling the issue of the neutral curve, because the total growth depends very little on the form of this curve for $\alpha \rightarrow 0$.

The wavelength selection mechanism is very weak and it can be easily affected by results of the receptivity process, as discussed in Section 3.1. It has been shown [50, 53] that when the oncoming flow contains streamwise vorticity, growth of the disturbances becomes double exponential (i.e. spatial amplification β increases exponentially with x). Modelling of the receptivity problem through direct solutions of the disturbance equations with different initial conditions [37, 76] resulted in various suitably defined neutral curves. This points out the importance of the form of the oncoming flow, which is directly related to the receptivity problem. No convincing analysis of the receptivity process by itself is available.

Disturbances at low Görtler numbers, i.e., below the neutral curve, are damped and, perhaps, because of that their behaviour has not attracted much attention. It has been shown [51] that these disturbances might have a form of several rather than a single layer of vortices, as commonly assumed (this fact can be deduced by considering the asymptotic form of the solution of the disturbance equations in the limit $y \rightarrow \infty$; when β assumes sufficiently large negative values the eigenvalues take complex form which suggests existence of several layers of vortices; conditions under which this occurs are marked with a shaded line in Fig. 11). The character of these disturbances

Fig. 13 Total (integrated) growth of the vortices. Amplification factor defines increase of the amplitude of the vortices taking place between the neutral curve and the location defined in terms of the specified Görtler number (from [53]).

might hold the key to the solution of the receptivity problem.

The neutral curve has the form of an arc that begins at small wave and Görtler numbers, extends towards high wavenumbers at small Görtler numbers and then bends back towards small wavenumbers at high Görtler numbers (Fig. 11A). This bending can be better seen in the (G, Λ) plane (Fig. 11B). Because of such a shape of the neutral curve, high wavenumber disturbances corresponding to $\Lambda < 44.29$ will never be amplified [51] while those corresponding to wavenumbers somewhat smaller will eventually be damped. This self stabilization property might be of interest in applications.

Suction has been investigated as the primary method for control of the instability. Results obtained on the basis of Blasius boundary layer with self similar suction show that a significant reduction in the total growth of the vortices can be obtained, however, the level of suction that has to be applied is substantially higher than the level

of suction sufficient to eliminate the Tollmien-Schlichting waves [52]. Wall blowing has been shown to have an opposite effect [54].

Analysis of the stability of velocity profiles other than the Blasius boundary layer shows interesting possibilities. Floryan [56] used wall jets to represent flows with a nonmonotonic velocity distribution and Blasius boundary layer to represent flows with a monotonic velocity distribution. Neutral curves shown in Fig. 14 demonstrate (i) that the instability occurs in the case of convex as well as concave walls (see discussion in Section 2), (ii) the character of the disturbance motion for the same mean flow in the convex and concave wall cases could be significantly different, and (iii) the critical Görtler number in the concave wall case could be increased by making velocity distribution nonmonotonic. Detailed calculations carried out for the wall jet over a concave wall [57] show that (i) the first mode of the instability does not need to correspond to a single layer of vortices, (ii) disturbances split sequentially into

Fig. 14 Neutral stability curves for the Blasius boundary layer over a concave wall and wall jet over concave and convex walls (from [56]).

an ever larger number of vortex layers during streamwise evolution, and (iii) there exists a strong self stabilization mechanism eliminating small vortices (see Fig. 15).

Analyses of the effects of changes of the geometry of the wall have been carried out on the basis of disturbance equations (1)-(4). Since these equations have been derived in the context of a very particular flow configuration, it remains to be proven whether they apply to other geometries as well. Two issues related to the geometry that attracted most of the attention are effects of variations of the curvature and effects of streamwise pressure gradients. Kalburgi et al. [93] simulated both of them using marching solutions

of the disturbance equations. Variable curvature has been modelled by making the curvature appearing in the definition of the Görtler number a function of x . It is very likely that there might be other terms of a similar order of magnitude that should have been included in the disturbance equations for consistency. Effects of pressure variations entered only through introduction of different mean flows. It has been shown [70, 132] that the treatment of the disturbance equations outside the boundary layer has to be changed in such situations and that stability characteristics are very sensitive to the details of modelling of the external flow. The above uncertainties affect most strongly stability properties of large vortices

Fig. 15 Linear stability diagram for the wall jet over a concave wall (from [57]).

around the neutral curve. Since this part of the stability diagram has little or no effect on the total growth, such simplified analysis can still produce meaningful results. And indeed, good agreement with experiment has been reported in [93]. These results demonstrate stabilization of the flow in the presence of favourable and destabilization in the case of unfavourable pressure gradients.

Görtler instability ceases to play an important role when the mean flow becomes three-dimensional. This fact can be deduced from the structure of the disturbance motion (see Eqs. 1-4), where the total transverse flow is $O(Re^{-1})$ when the mean flow is two-dimensional. Addition of a transverse component of the mean flow that is $O(Re^{-1})$ or

larger destroys this structure [48]. An analytical proof of this fact has been given by Hall [68] in the limit of $\alpha \rightarrow \infty$. Details of the extension of this proof to $\alpha \rightarrow 0$ (1) remain to be worked out but the results are predictable. Numerical experimentation shows that when the crossflow Reynolds number is increased, the Görtler vortex structure begins to transition into co-rotating vortices and this transition is complete when the Reynolds number is in excess of 45 [28]. It should be emphasized that the destabilizing centrifugal effects are active even in the presence of a large crossflow, however, their role is limited to modifying the flow structures occurring due to other flow phenomena rather than producing a distinct one

Fig. 16 Formation of the "horseshoe" structures (from [83-86]).
Experimental conditions as in Fig. 7.

on their own. The scope and importance of these modifications remain to be determined.

Disturbance equations (1)-(4) have been generalized to the compressible case by El-Hady and Verma [41-45] who provided estimates of the stability characteristics using the local approximation method. Jallade et al. [88] obtained slightly different results from similar calculations for reasons not known. Previous results based on the parallel flow approximation are reviewed in [96]. Goglia and Verma [61] have shown that higher modes are irrelevant, thus, the following discussion will concentrate on the first mode only. The character of a typical stability diagram is similar

to the case of an incompressible fluid. The shape of the neutral curve has been determined analytically in the limit $\alpha \rightarrow \infty$ by Tumin and Chernov [159]. Similar results have been reported by Hall and Malik [75]. Direct solutions of the disturbance equations using marching procedures are given by Spall and Malik [143, 144]. The available results are inconclusive regarding the critical stability conditions for the same reasons as in the case of incompressible flow. The effects of compressibility are stabilizing when measured in terms of the computed total growth. Wall suction can be used for stabilization purposes and its amount has to increase with increasing Mach

Fig. 17 Visualization of the instability in the spanwise plane (from [83-86]). Experimental conditions as in Fig. 7. (a) smoke introduced simultaneously at $x = 200$ mm and $x = 800$ mm, (b) smoke introduced only at $x = 800$ mm. See also Fig. 9.

number [42]. Wall cooling is reported in [42] to have a small stabilizing effect, while according to [143, 144] it has a small destabilizing effect. Effects of pressure gradient have been modelled by modifying mean flow similarly as it was done in the incompressible flow case. The available

results [144] show a strong destabilizing effect of the adverse pressure gradient.

The above summary shows that some of the general tendencies in the character of the instability in the compressible flow case have been identified, however, a complete understanding of

Fig. 18 Visualization of the instability in the spanwise plane (from [83-86]). Experimental conditions as in Fig. 7. Smoke introduced at different y locations.

the instability has not yet been achieved and not all possible flow regimes have been studied. It has been noted for example that the vortex structure concentrates at the edge of the boundary layer as the Mach number is increasing, however, when wall cooling is applied this structure expands back into the wall region [144]. In the hypersonic flow regime the evolution of the vortices crucially depends on the viscosity model [78], but seems not to be affected by other real gas effects [113].

Occurrence of the Görtler instability in the form of oscillatory (or meandering) vortices as a primary state has been considered by Hall [65] who showed, on the basis of unsteady version of Eqs. (1)-(4), that in the limit $\alpha \rightarrow \infty$ steady vortices are most unstable. The case of $\alpha = 0$ (1) which is of practical interest has not been examined. It is commonly assumed, based on the experimental evidence, that steady vortices are always dominant. While this seems to be a plausible assumption, it requires a formal proof.

4. Nonlinear Stages of the Instability

The observed acceleration of the laminar-turbulent transition process in the case of concave walls is generally attributed to the presence of Görtler vortices. It is of interest to determine the mechanism describing the transition process in such situations and to come up with more accurate transition prediction criteria.

4.1 Experimental Evidence

The available experimental evidence suggests that Görtler vortices do not transition themselves but set up a flow field that is unstable with respect to other instabilities with higher growth rates. The sequence of events depends on how much the vortices are allowed to distort the mean flow before other instabilities begin to play a dominant role or, in other words, it depends on the relative values of the Görtler and Reynolds numbers.

In the case of a small curvature, i.e., small Görtler and large Reynolds numbers, the boundary

layer becomes unstable first with respect to Tollmien-Schlichting waves and then with respect to Görtler vortices. Vortices are found to make the development of the unstable Tollmien-Schlichting waves three-dimensional and to accelerate their growth, most strikingly in the spanwise locations corresponding to the vortex induced upwash where the boundary layer attains its maximum thickness (section a in Fig. 3) [154]. The resulting breakdown process does not produce any "spikes", which are characteristic of the breakdown in a two-dimensional boundary layer on a flat plate [95], and is similar to the breakdown in a boundary layer on a flat plate but with large spanwise thickness variations [101].

The case of a strong curvature is of more interest because the low Reynolds number keeps the flow stable with respect to Tollmien-Schlichting waves. This allows study of the nonlinear development of the Görtler vortices and their involvement in the transition process that is isolated from other disturbances. It is found that vortices significantly distort the flow and set-up localized high shear layers which, in turn, become unstable with respect to the inflection point instability and eventually result in turbulence. A detailed description of this process will now be given.

The nonlinear evolution of the vortices has been well documented by Ito [83-86]. Appearance of the vortices leads to a low momentum fluid moving away and high momentum fluid moving towards the wall in the upwash and downwash sections, respectively. The resulting formation of the high and low speed regions in the boundary layer exhibits a regular spanwise periodicity as documented by the measurements of the u velocity component at 500 mm (Fig. 8b). The nonlinear development begins with a change in the lateral spacing of the vortices, i.e., vortices on both sides of the upwash section (section a in Figs. 3 and 8) approach each other, form a pair and lift up a little [9] (see Figs. 7, 9, 16-18). This brings a change in the spanwise distribution of $u(z)$, i.e., the

accelerated part becomes wide and flat while the decelerated part becomes narrow and sharp (600 mm in Fig. 8b) [1]. The reduction of the distance between the paired vortices increases the upwash and promotes a rapid growth of the boundary layer in the upwash section beginning at 700 mm (Fig. 8a). Velocity distribution $u(y)$ at this section acquires an inflection point at 700 mm and at 800 mm it becomes nonmonotonic with a low speed fluid riding atop the high speed fluid and with a high shear layer appearing at the edge of the boundary layer [3]. At the same time high shear layers form in the spanwise distribution $u(z)$ on both sides of the upwash section (see measurements at 800 mm) while the velocity distribution at the downwash section becomes fuller and more stable. It is the instabilities of the high shear layers formed around the upwash section that lead to a chaotic breakdown and transition to turbulence. The additional mixing brought in by the breakdown process equilibrates the spanwise distribution $u(z)$ and produces the normal-to-the-wall distribution of $u(y)$ typical for turbulent boundary layers, as evidenced by measurements at 1000 mm and 1100 mm (Fig. 8). It can be seen in Figs. 7 and 16 that each pair of vortices transitions separately from all others, which supports the view that the breakdown process relies on instabilities of the local shear layers that were set up by the vortices and that the vortices by themselves play no direct role. It is interesting to note that Liepmann in his old experiments [107, 108] correlated the occurrence of transition with $G = 16.6$, which is exactly when the formation of the high shear layers occurs (see Fig. 8).

There is an interesting adjustment of the pressure field taking place in parallel with the above evolution of the vortices [6, 9]. When the nonlinear effects set in, pressure extrema still fall in between the vortices, as in the linear regime. When the secondary instability appears, centres of the vortices correspond to the minimum pressure (as in the case of an isolated vortex), according to preliminary measurements [9]. Detailed

Fig. 19 Schematic diagram of formation of the "horseshoe"-type structures (from [5]).
(a) Initial pattern, (b) Deformed pattern, (c) Breakdown.

measurements are required in order to verify this behaviour.

Several modes of secondary instability associated with the nonlinear development of Görtler vortices have been reported so far. Aihara and Koyama [5] observed formation of the so-called "horseshoe" vortices. These vortices can be seen in Fig. 7 between 850 and 1000 mm. A close-up view of the same section of the flow field is shown in Fig. 16. Formation of these vortices can be explained with the help of a sketch shown in Fig. 19. The high shear layer that forms atop the upwash section of a vortex pair is subject to the Kelvin-Helmholtz instability which generates spanwise vortices. These vortices are then bent towards the wall due to the rotating action of the original streamwise vortices. The breakdown is reported to appear first near the shoulder of the "horseshoe" vortex. It is interesting to note that this "horseshoe" structure is carried past the

transition region into the turbulent flow where it eventually collapses [7]. There, the basic vortex layer seems to lift up and another very thin one seems to be forming underneath [9]. Bippes [22] and Bippes and Görtler [21] observed oscillatory (or meandering) mode, as illustrated in Fig. 20. As the low speed fluid is elevated due to the action of streamwise vortices, it blocks the way of the high speed fluid which therefore must change direction of its motion. The resulting lateral deflections of the longitudinal vortex axis grow in the streamwise direction until they become so sharp that they can no longer reverse themselves and result in a breakdown. The character of this breakdown is not known. Different experimenters reported different frequencies characterizing the meandering motion [6, 7, 147, 150, 163]. Peerhossaini and Wesfreid [130] identified three stages in the evolution of this mode as a function of the free-stream velocity, i.e., an increase of the

Fig. 20 Secondary instability in the form of meandering mode (from [22]). Flow from left to right.

amplitude of oscillation with an increase of the velocity, then a decrease of the amplitude with a further increase of the velocity and then an onset of a “jump and stay” mode, where the upwash interface between the paired vortices jumps sideways with a long residency period on each side.

The two modes described above most commonly occur simultaneously in a transition process [1, 3, 4, 6, 106, 147, 150, 163]. Figure 16 shows presence of the meandering motion before formation of the “horseshoe” vortices. Meandering is responsible for low frequencies in the power spectra while “horseshoe” vortices give rise to higher frequencies. It has been found by Aihara and his co-workers [1, 3, 4, 6] that the frequencies associated with the “horseshoe”-type structures grow more rapidly and eventually lead to transition, with the most rapid oscillations occurring around the shoulders of the “horseshoe” vortices. Swearingen and Blackwelder [147, 150] similarly concluded that oscillations occurring due to a secondary instability of the free shear layer formed atop the low speed region eventually cause break-

down. They correlated location of the maximum of the amplitude of the oscillations with locations of vertical shear layers on both sides of the low speed streaks, which is in qualitative agreement with the observations made by Aihara et al. discussed above.

Wortmann [167] observed a second mode in the form of steady twisted interfaces between the longitudinal vortices and interpreted this to be a result of a subharmonic interaction. This mode has not been reproduced by other experimentors. Since the interfaces twisted to the left in the right side of the channel and to the right in the left side, it is possible that twisting resulted from an interaction with the side walls. This view is supported by the existence of a strong interaction between the side wall’s boundary layers and the instability, that has been recently reported [126].

4.2 Nonlinear Theory

Two issues of interest are (i) nonlinear evolution of Görtler vortices and (ii) interaction between Görtler vortices and Tollmien-Schlichting waves.

The following discussion begins with the nonlinear evolution of the vortices.

4.2.1 Nonlinear evolution of Görtler vortices

The available results show that the large wavenumber ($\alpha \rightarrow \infty$) vortices evolve differently from vortices of $\alpha = 0$ (1). Most of the analytical results deal with the former regime, however, such vortices have never been observed experimentally. The latter regime is of practical interest, but the results are rather limited and heavily dependent on numerical experimentation. It is convenient to begin discussion with the large wavenumber vortices.

A. Large Wavenumber Vortices ($\alpha \rightarrow \infty$)

Since the linear theory predicts a bifurcation point only, one may inquire at first how the vortices evolve in a small neighbourhood of the neutral curve (weakly nonlinear regime). In most hydrodynamic stability problems such a solution can be approximated in terms of disturbance amplitude expansion, where an interaction between the fundamental and its harmonics produces mean flow correction, e.g., Stuart-Watson approach [146]. Hall [66] has shown, on the basis of nonlinear version of Eqs. (1)-(4), that such structure of solution does not exist for large wavenumber vortices. Nonlinear effects resemble the mean flow interaction of Meksyn and Stuart [121], i.e., they are dominated by interactions between the fundamental and the mean flow and both of them have to be determined simultaneously rather than sequentially. Higher harmonics play no role at the leading order of approximation. The truncated governing equations retain a parabolic structure in the streamwise direction. The character of the possible solutions has been investigated through numerical experimentation with suitably defined initial conditions. The presented results do not explain the differences between the experimental and theoretical neutral curves shown in Fig. 10. The range of wavenumbers there is such that the large wavenumber theory might have been

relevant. The reader may recall that it has been postulated in Section 3.1 that because of large amplitude of disturbances used in the experiments, these results were very likely to be affected by nonlinear effects. Further investigation of the nonlinear effects is required in order to explain these differences.

Hall and Lakin [73] extended the analysis given in [66] into the strongly nonlinear regime. Results show that vortices evolve downstream into an ultimate state that is determined by interaction between the fundamental and the mean flow. Equations for the fundamental component generate equations for the basic state which suggests that vortices are dominant and effectively drive the boundary layer.

The idea of existence of an ultimate state in the evolution of any disturbance has been postulated by Malkus [114] who argued that in a turbulent flow the mean state adjusts itself so that all modes present become neutrally stable. Hall and Lakin [73] argue that a similar state exists in the Görtler instability where the flow locally adjusts itself to make the small wavenumber vortices neutral. This is an attractive concept, however its practicality remains to be proven. Since, according to the assumptions given in [73], the curvature of the wall has to increase at a sufficiently high rate in order to support such vortices, modelling of the overall geometry of the flow system must be checked for consistency. The character of variations of the curvature makes comparisons with the existing experiments virtually impossible. The postulated vortices have not yet been observed. Evidence discussed in Section 4.1 shows that vortices become subject to secondary instabilities before entering the large wavenumber regime. Navier-Stokes simulations of the same process [109, 135, 136] show emergence of other instabilities rather than the postulated asymptotic form of the vortex motion, in agreement with the experimental observations. The velocity field induced by such a vortex [73] does not show an inflection type velocity distribution

in the upwash section that has been consistently observed in many experiments and has been reproduced in the Navier-Stokes simulations.

B. Vortices with Wavenumber $\alpha = 0$ (1)

Weakly nonlinear theories for $\alpha = 0$ (1) and smaller have not been formulated, mainly because of uncertainties associated with the neutral curve (Section 3.2). The character of disturbance evolution in this regime can be deduced from the available numerical solutions. Hall [74] analyzed the problem on the basis of numerical solutions of the full nonlinear version of Eqs. (1)-(4), where the initial conditions for the fundamental and all harmonics had to be suitably defined. The reader may note that such arbitrariness in the initial state poses difficulties in formulating general conclusions. The results show a typical growth of the higher harmonics, which is in contrast to the high wavenumber case discussed previously. This rise of harmonics can be better seen in the results of the direct Navier-Stokes simulations carried out by Malik [111] and Malik and Hussaini [112], and is responsible, according to Aihara [2, 3] and Aihara and Koyama [5], for the observed pairing of the vortices and change in the spanwise distribution of u , i.e., development of a sharp peak in the upwash section and flattening of the downwash section, as shown in Fig. 8b. Comparison of the experimental and theoretical spanwise distributions of u could be used for validation of the accuracy of modelling, however, the existing theoretical papers do not provide sufficient information for such a comparison to be made.

Predictions regarding further downstream development of the vortices into the strongly nonlinear regime are somewhat uncertain. Disturbance equations used by Hall [74] describe spatial evolution of the vortices and should, in principle, provide correct predictions until unsteady disturbances appear. The solution procedure that has been applied involved representation of the spanwise variations in terms of truncated Fourier

series with four modes retained, and marching procedure adapted from [67] to solve for the spatial evolution. Results reported in [74] deal with the curvature of the wall increasing in the streamwise direction, which permits only qualitative comparisons with experiments. These results show development of an inflection type streamwise velocity profile in the upwash section, in qualitative agreement with the experiments. Further downstream disturbance energy begins to concentrate in the fundamental component and the mean flow correction, with higher harmonics playing a relatively minor role [74, 76]. This is expected. Because the thickness of the boundary layer grows in the streamwise direction, all vortices eventually enter the large wavenumber regime (they follow line with slope 3/2 in Fig. 11A), where the fundamental component and the mean flow correction should be dominant, according to the large wavenumber theory discussed in the previous subsection. The transition from the initial evolution that involves higher harmonics to the ultimate state that does not involve higher harmonics is not well documented. One would expect that if the integrations were carried out sufficiently far downstream, the disturbance structure predicted asymptotically by Hall and Lakin [73] would be recovered. A test like this might have provided some relevance to the large wavenumber theories but it has not been made. It should also be noted that the numerical prediction [74] gives an inflection-type velocity profile while the prediction based on the asymptotic solution [73] does not. Results given in [76] for the case that matches experimental conditions of Swearingen and Blackwelder [150] and, therefore, affords an opportunity for quantitative comparisons, produced only qualitative agreement. Moreover, calculations predicted occurrence of backflow at the upwash plane [76], which is very unexpected. Such backflow is unlikely to occur in a real flow, it has not been reported by Swearingen and Blackwelder [150] and is not present in the results of measurements

displayed in Fig. 8. All of it makes numerical results uncertain. The possible sources of uncertainties are the implicit weak nonlinear assumption in the spanwise plane, i.e., insufficient number of modes, and/or unrealistic initial conditions [136]. A careful re-evaluation of the numerical scheme is required specifically taking into account the resolution requirements for simulation of strongly nonlinear stages of transition [168].

The same problem has been simulated using numerical solutions of the full Navier-Stokes equations with finite-difference [135, 136] and spectral [109] discretizations. The results produced by both methods under conditions corresponding to the experiment of Swearingen and Blackwelder [150] produced essentially identical results, which also agreed surprisingly well with the experiment. Calculations revealed development of the characteristic inflection-type streamwise velocity profile in the upwash section and formation of high shear layers above it and on both of its sides. It has been pointed out in Section 4.1 that transition is dominated by secondary instabilities of these layers. Different stages of this process have been well reproduced in [109]. Results show that the dominant mode of the secondary instability is concentrated around inflection points in the spanwise distribution of $u(z)$. This is in agreement with experiments but contrary to expectations, where the Rayleigh modes associated with the inflection point in the normal-to-the-wall distribution of $u(y)$ are expected to be the most dangerous. It should be noted that the inflection points in the spanwise distribution of $u(z)$ exist from the very beginning of the instability, but play no role until strong vertical shear layers form around them. The mechanism of breakdown is certainly more complicated than just the inflection point instability and deserves further attention.

Sabry and Liu [136] simulated the case of a curved wall followed by a flat wall, as found in experiments of Peerhossaini and Wesfreid [130]. The results show that once the vortices develop, they persist for surprisingly large distances from

the location where centrifugal forces ceased to act, in qualitative agreement with the experiments. It appears that disturbances are able to maintain themselves through the Reynolds stress energy conversion mechanism, which is only gradually overcome by viscous dissipation.

A note of caution regarding the Navier-Stokes simulation of the Görtler instability is, perhaps, appropriate at this point. Algorithmical limitations lead to the parallel flow approximation, simplistic treatment of geometry and imposition of periodic boundary conditions in the streamwise direction. Spatial evolution of the flow is modelled by letting it evolve in time and then converting the temporal evolution into a spatial one by using a conveniently selected convection velocity. This approach is able to capture the qualitative features of the transition process in a flat plate boundary layer that is dominated by the Tollmien-Schlichting waves and indeed some theoretical justification based on the Gaster transformation exists in this case. The available results [109, 135, 136] show that similar procedures are able to reproduce the transition process that is dominated by the Görtler vortices with a surprising degree of accuracy. This apparent success, however, should not hide the arbitrariness of many assumptions where, for example, the only criterion used in selecting convection velocity is that numerical results match their experimental counterparts. This leads to adoption of the convection velocity equal to $0.567 U_\infty$ in [109] and $0.644 U_\infty$ in [135, 136] when simulating exactly the same experimental conditions. Elimination of the effects of boundary layer growth is also troublesome and precludes simulation of the early stages of the instability where many unanswered questions still remain; it also raises questions regarding the correct form of the initial conditions to be used for simulations.

C. Secondary Instabilities

Conditions leading to the initiation of the secondary instabilities of Görtler vortices have not yet been identified. Hall and Seddougui [71]

considered stability of the large wavenumber asymptotic state of Görtler vortices described previously by Hall and Lakin [73]. Preliminary results showed possible emergence of an oscillatory three-dimensional vortex motion. Assumptions used in the analysis limit validity of the results to the part of the parameter space that is not likely to be of practical interest. Vortices with twisted interfaces, such as those observed by Wortmann [167], have been produced by Hall [66] through manipulation of the initial conditions. As before, assumptions used in the analysis preclude any direct comparisons with the experiments. See also questions regarding the accuracy of Wortmann's results, as discussed in Section 4.1. Aihara and Koyama [5] analyzed emergence of the "horse-shoe"-like structures and showed, in qualitative terms, its consistency with the Kelvin-Helmholtz instability. Sabry et al. [137] considered linear stability of the inflectional velocity profiles produced by the vortex motion. The mean flow was assumed to be two-dimensional and was obtained by first calculating the complete flow field using the Navier-Stokes equations [136] and then taking sections either parallel to the wall at different distances from the wall or normal to the wall and parallel to the stream at the upwash section. Preliminary results show existence of high frequency unstable modes, in qualitative agreement with the experiments [150].

4.2.2 Görtler Vortex–Tollmien-Schlichting (TS) wave interactions

Because of weakness of the Görtler instability, it is not only the evolution of the vortices by themselves but also their effect on evolution of the Tollmien-Schlichting waves that is of interest. The main concern is that the vortex–TS wave interactions could significantly accelerate growth of the otherwise benign TS waves and invalidate transition criteria based on the integrated linear growth. The interactions are crucially dependent on the amplitude of the primary mode and may take place when (i) Görtler vortices are finite

amplitude and TS waves are infinitesimally small, (ii) TS waves are finite amplitude and Görtler vortices are weak, and (iii) both Görtler vortices and TS waves are finite amplitude. There is a multitude of possible scenarios and only very few of them have been studied so far. The obvious objective here is to identify those that enhance the amplification process.

Nayfeh [127] suggested existence of a parametric subharmonic resonance where a finite amplitude Görtler vortex interacts with two oblique TS waves of the spanwise wavelength twice that of the Görtler vortex. This mechanism did not produce the expected significant increase of the growth rates of the TS waves according to the corrected calculations given in [111, 128]. The interaction did however destabilized the otherwise stable TS waves, but the growth rates were no more than the primary Görtler instability [112, 128]. It is still possible that resonance could be realized but with Görtler-vortex amplitude of at least 10% rather than the 1% studied so far [112].

Srivastava and Dallmann [145] analyzed the same problem and predicted combination resonance to occur when Görtler vortex and TS waves attain similar amplitudes. The numerical results show this effect to be very strong. Another look at this problem is required since some of these results have been shown to be inconsistent [112].

Malik and Hussaini [112] investigated possible excitation of a two dimensional TS wave in a flow modulated by very strong Görtler vortices. Since the mean flow exhibits an inflectional velocity profile in the normal-to-the-wall direction, a strong amplification of the TS waves might take place. The results of the Navier-Stokes simulations show an increase of the growth rate of the TS waves above the linear theory value to a level comparable to the growth rates of the oblique waves in the parametric resonance case discussed previously. Excitation of an oblique wave with the spanwise wavenumber equal to that of the Görtler vortex was also observed. This wave,

which does not appear in the absence of the primary two-dimensional wave may, under certain conditions, attain higher growth rates than the original two-dimensional wave. The computed growth rates of these excited modes remained small when compared to the growth of the Görtler vortex itself, at least in the time period covered by the calculation.

The above discussion shows that so far it has not been possible to pinpoint any major interaction mechanism. No final conclusions can be reached at present because of the limited scope of the available results. This problem certainly requires further investigation. Perhaps concepts developed in the analysis of similar problems in the channel flow case [14, 19, 20, 35, 36, 72] as well as in the flat plate boundary layer case [77, 140] can be adapted to the problem at hand.

5. Görtler Instability of Turbulent Boundary Layers

Centrifugal effects can be categorized into 3 types, i.e., (i) changes in the turbulence structure induced directly by the wall curvature, (ii) generation of the secondary flow (longitudinal vortices), and (iii) effects of the secondary flow on the turbulence structure. This review is concerned with the second effect, i.e., Görtler instability. There is experimental evidence [82] suggesting that the first effect may be dominant at least when the curvature is not too strong. A review by Bradshaw [24] offers a good introduction into this subject.

The occurrence of secondary flow due to the centrifugal effects has been reported for the first time by Pai [129] in 1943 for the flow between concentric rotating cylinders, and by Tani [152] in 1962 for boundary layers. Since the turbulent flow is more stable than the laminar one, much stronger curvature is required for generation of such a secondary flow. The conditions under which this flow does occur are of interest because of additional drag and additional mixing induced

by the vortices. Very little information about the quantitative character of these two effects is available. It is interesting to note that the presence of the secondary flow changes the large scale eddy structure, i.e., it has an effect qualitatively similar to that of the LEBU devices. It should be realized that Görtler instability is one of the very few mechanisms available for a nonintrusive manipulation of the large scale turbulent structures in boundary layers.

5.1 Experimental Evidence

Vortices in turbulent flow over a concave wall are bound to wander from side to side because of turbulence in the boundary layer and large scale turbulence or unsteadiness in the free stream. Experiments show [82] that vortices wander about preferred lateral positions, which are determined by steady nonuniformities in the oncoming flow. The spanwise wavelength is usually about twice the boundary-layer thickness, but the spacing and amplitude are influenced by the initial disturbances. As a matter of fact, in all experiments in which naturally occurring vortices have been observed [30, 122, 131, 141, 142, 152], their triggering could be traced to the upstream flow nonuniformities. A steadier and more regular vortex pattern is observed when artificial disturbances are introduced into the oncoming stream by, for example, placing small vortex generators in the settling chamber [11, 82]. If the level of unsteady disturbances is large compared to that of the steady disturbances, no vortices with the preferred spanwise locations can be observed [89, 99, 133]. The "vortices" then tend to wander about, merge, separate, appear and disappear in a seemingly random manner causing no periodicity in the spanwise direction for the mean flow [89, 99].

The above summary of the experimental observations allows one to formulate the following model for triggering of the vortices [11]. When the turbulent boundary layer is uniform and two-dimensional in the mean, the vortex cells are

triggered by an interaction between the centrifugal instability and the finite level of fluctuations created by the largest eddies present in the boundary layer. Eddies that involve motion consistent with the instability mechanism will be amplified and the most energetic ones will develop into vortex cells as they convect downstream. Because of random distribution of such eddies at the onset of curvature, no steady vortex pattern will emerge. When the upstream disturbances, from damping screens for example, cause spanwise nonuniformities in the boundary layer, the statistics of the large-scale eddies near the onset of curvature will not be uniform across the span and some vortex locations would become more probable than others. This will lead to spanwise variations in time-averaged measurements which signals presence of the vortices. The degree of spanwise variation will depend on the strength of the upstream disturbances and its effect on the probability distribution of the vortex position, as explicitly illustrated by the experiments with artificial disturbances.

The critical conditions, i.e., minimum curvature necessary for the appearance of the vortices, have not been determined. Experimental results provide a somewhat inconsistent picture, but in general suggest existence of vortices for the curvature parameter $\delta/R \geq 0.01$ in the case of boundary layers [82] and $b/R \geq 0.17$ in the case of wall jets [99]. Here δ is defined as the boundary layer thickness for which $U = 0.995$ and b stands for the half width of the wall jet.

The process of streamwise evolution of the vortices is not well understood. Hoffmann et al. [82] observed that response of the boundary layer to concave curvature is fairly slow. They argued that this implies reorganization of the eddy structure. While such a reorganization does take place, their observations also show that the growth process is noticeably slow. This indirectly shows the instability mechanism to be weak, in agreement with the conclusions reached in the laminar boundary layer case (Section 3.1).

Evolution of vortices far downstream poses another interesting question. So and Mellor [142] and Prabhu and Rao [131] observed that vortices eventually begin to decay. This suggests that vortices form an unstable system that is subject to secondary, tertiary, etc., instabilities which eventually lead to a breakdown of the “organized” motion and formation of a “new turbulent” boundary layer. This conclusion is contradicted by Hoffmann et al. [82], who have not observed any decay of the vortices. Smits et al. [141] worked with a curved wall followed by a straight wall and observed that vortices generated in the curved section persisted “indefinitely” downstream in the straight section. The last two experiments had curved sections much shorter than the previous two so it is possible that the vortices have not reached the state where decay begins. The issue of what happens with the vortices far downstream requires further clarification.

The effects of three-dimensionality of the mean flow on the centrifugal instability have been investigated by Baskaran and Bradshaw [12, 13] in the context of boundary layer flow over an infinite swept wing. The results show existence of a critical sweep angle (between 15° and 35°) above which Görtler vortices cease to exist. Occurrence of the cross-flow (co-rotating) vortices for larger sweep has not been observed. These results show that centrifugal instability can survive much stronger three-dimensional effects in the turbulent than in the laminar flow case. The mechanism that eliminates vortices is not known, but appears not to be related to the competition with the cross-flow instability that takes place in the laminar flow case.

Effects of convex curvature have been investigated only in the cases that are stable according to the inviscid stability criterion (Section 2). The results show the response of the flow to be qualitatively different from that in the concave wall case, i.e., it occurs almost instantaneously after introduction of the curvature and it involves mainly attenuation of the pre-existing turbulence

without producing large changes in statistical average eddy shape. Since this issue is not directly related to the topic of this review, the reader is referred for further information to [125] and references therein.

Steady vortices re-arrange mean flow and increase skin friction by 20% [122] to 40% [11] above the flat plate values. While the existing evidence shows that steady vortices occur only in the presence of pre-existing spanwise nonuniformities of the flow, such nonuniformities are present in all practical problems, where they occur due to, for example, nonuniformities in the laminar-turbulent transition process or structural irregularities, roughness or impact damage on aircraft wings and turbomachine blades. The increase of drag due to the Görtler instability should be therefore accounted for in any design process.

5.2 Theoretical Modelling

Concepts developed during modelling of the instability of laminar boundary layers apply directly to turbulent boundary layers and lead to essentially identical disturbance equations with additional terms arising due to the Reynolds shear stress. The main difficulty involves determination of the mean flow and the Reynolds shear stress distributions that can be then inserted into the disturbance equations. This problem ties in directly to the modelling of turbulence in the curved wall case, for which no satisfactory solution exists. Also, since the mean flow has to be explicitly determined for each value of the Reynolds number, the wall curvature rather than the Görtler number becomes the appropriate stability parameter.

A short digression to the issue of turbulence modelling is appropriate at this point. Older results have been reviewed by Bradshaw [24]. Recent experimental results can be found in [82, 125]. Moser and Moin [123, 124] carried out computer simulations based on the full Navier-Stokes equations of a fully developed turbulent flow in a curved channel and were able to evaluate all

terms in the Reynolds-stress transport equation. Their results show significant changes mainly in the Reynolds shear stress that are only partially attributable to Görtler vortices. It appears therefore that stability analysis of such a flow will produce incorrect results unless accompanied by accurate turbulence modelling.

The first stability analysis for a turbulent boundary layer is due to Tani [152] who adapted Görtler's stability results obtained for the laminar flow case [63] by replacing kinematic viscosity with a constant eddy viscosity. Sandmayr [139] improved this analysis by using mean flow and eddy viscosity distributions determined experimentally for the flat plate case. Kobayashi and Fujisawa [97] essentially repeated calculations of Sandmayr [139] using a much more accurate numerical procedure and produced a complete stability diagram. All these calculations predict existence of the instability, however, the results should be viewed only as qualitative.

A series of papers on stability of turbulent wall jets illustrates the importance of turbulence modelling. Kobayashi and Fujisawa [97, 98] used mean velocity and Reynolds stress profiles from experiments on flat plates and obtained the critical value of curvature parameter $b/R = 0.31$ for the particular Reynolds number used in the calculations. Fujisawa et al. [58] repeated the same calculations with the mean flow and Reynolds stress distributions taken from experiments on concave walls and obtained critical conditions for $b/R = 0.08$. Calculations were based on eddy viscosity which was calculated from experimentally measured Reynolds stress using a standard relation that does not involve curvature effects. This relation is singular around the point of maximum velocity which necessitates additional approximations. Fujisawa and Shirai [59] eliminated the parallel flow approximation and treated eddy viscosity as a function of curvature. Its values were evaluated from the turbulent energy and dissipation in order to avoid problems around the point of maximum velocity. The required experimental

information came from the same experiments that formed the basis of calculations reported in [58]. Results predict instability for $b/R = 0.2$. The experimental results [99] predict instability in the range of $b/R = 0.17 - 0.27$. It appears therefore that the theoretical analysis can predict occurrence of the instability. The reader may note, however, that the above analyses contain a large empirical component whose structure is not completely understood and which has a strong effect on the stability characteristics. It is not likely that this empiricism can be eliminated from the analysis in the near future.

6. Heat Transfer Effects

There are two interrelated mechanisms that couple Görtler instability with heat transfer effects. In the first one, evolution of the vortices is affected by buoyancy forces, while in the second one, heat flow is affected by additional motion resulting from the action of vortices. The following discussion begins with the former mechanism.

Buoyancy forces may either reinforce or oppose centrifugal forces, depending on the direction of the gravity vector. DiPrima and Dunn [38] provided the first analysis of this problem on the basis of parallel flow approximation. Garg and DiPrima [60] extended Eqs. (1)-(4) to account for buoyancy effects, but provided a solution only in the large wavenumber limit. This and previous works done with the parallel flow approximation [90, 91, 94] show that Görtler instability could be either significantly delayed or significantly accelerated through judicious application of the wall heating/cooling.

The first explicit demonstration of the effects of Görtler vortices on heat transfer is due to McCormack et al. [120], who reported a 100%-150% increase in Nusselt number. This improvement in heat transfer has been subsequently confirmed by many other investigators [25, 39, 117-119]. Careful measurements of Crane and Sabzvari [32] showed that the heat transfer at the

downwash section of the vortex exceeds the heat transfer at the upwash section by as much as 2-3 times. The averaged heat transfer increases only after the vortex has reached a highly nonlinear stage in its development, which corresponds approximately to $x = 700$ mm in Fig. 8. The reader may note the appearance of the spanwise asymmetry between the upwash and downwash sections, with the downwash section where the local thinning of the boundary layer occurs occupying the greater part of the span. It is thought that this thinning of the boundary layer accounts for a substantial part of the heat transfer enhancement. Kottke [102] showed that this enhancement strongly depends on the wavelength and intensity of the disturbances present in the oncoming flow, with the biggest enhancement obtained for disturbances corresponding to the most amplified ones according to the linear theory. The available results are, however, insufficient for formulation of any quantitative correlations. Theoretical analysis of this problem done on the basis of linear theory [120] not surprisingly produced no net change in the heat transfer. Liu and Sabry [110] concluded on the basis of Navier-Stokes simulations that enhancement relies on additional advection generated by the vortices. Such advection is certainly a factor, however, rotational motion associated with the vortices is very weak as explicitly illustrated in Fig. 7. It seems more likely that vortices re-arrange the flow field which then acquires different heat transfer characteristics, rather than transport the heat directly. Further work is required in order to settle this question.

7. Concluding Remarks

Görtler instability is a classical subject by now, however, many questions even of a fundamental character remain to be settled. It should be stressed that this instability is a geometrically induced phenomenon and some of its characteristics might therefore depend on the overall geometry of the

flow system rather than on the wall curvature by itself. Effects of different geometrical parameters, such as those shown in Fig. 12, remain to be studied. Standardization of geometries subject to experimentation would be very helpful and would enable more meaningful comparisons to be made.

Significant progress in the understanding of this instability has been achieved in the past 15 years. The reader may note that a consistent formulation of the linear disturbance equations has been given only in 1979 [47]. Perhaps the most important aspect of this formulation is that it has demonstrated inapplicability of the concepts developed previously in the context of stability of parallel shear layers. A large body of literature that followed has been reviewed in detail in this paper. While the progress in flow visualization techniques offered qualitative insights into the different stages of the instability process including transition to turbulence, it has not yet been possible to accurately measure all three disturbance velocity components. The following summary describes issues that will probably draw attention in the near future. In the case of a simplest geometry, i.e., wall of constant curvature, the issue of critical conditions, i.e., whether the instability occurs at a zero or at a finite Görtler number, remains to be settled. Strong correlation between the form of the oncoming disturbances and the character of the unstable motion has been observed but the mechanics of this interaction is not understood. The experimentally determined large wavenumber branch of the neutral curve does not agree with the one calculated from the linear theory (Fig. 10); while the nonlinear effects might be responsible, the existing nonlinear theories fail to explain these differences. Conditions leading to the occurrence of secondary instabilities have not been determined. The occurrence of the instability in the case of convex walls remains to be experimentally demonstrated. It has been shown that different mean flows lead to the instability either in the form of a single layer or multiple layers of vortices, and that this topology could change with

distance along the wall; this issue has to be further quantified. The dynamics of vortices under stable distribution of centrifugal forces has received very little attention, however, it could hold the key to the receptivity problem. Practically no quantitative information dealing with turbulent flow is available. Effects of vortices on drag and heat transfer are acknowledged as being large, but practical correlations remain to be determined.

Görtler instability remains a very active research area as best illustrated by the recent Euromech 261 Colloquium; its Book of Abstracts [169] lists contributions from more than 50 research groups worldwide. One remains optimistic that many of the open issues listed above will be resolved in a not too distant future.

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References

- [1] Aihara, Y., "Transition in an Incompressible Boundary Layer along a Concave Wall", *Bul. Aeronautical Res. Inst., Tokyo Univ.*, 195-240, 1962.
- [2] Aihara, Y., "Nonlinear Analysis of Görtler Vortices", *Phys. Fluids*, v. 19, 1655-1660, 1976.
- [3] Aihara, Y., "Görtler Vortices in the Nonlinear Region", in "Theoretical and Experimental Fluid Mechanics", Muller, eds. Roesner and Schmidt, Springer-Verlag, 331-338, 1979.
- [4] Aihara, Y. and Sonoda, T., "Effects of Pressure Gradients on the Secondary Instability of Görtler Vortices", *AIAA Paper 81-0197*, AIAA 19th Aerospace Sciences Meeting,

- Reno, Jan. 12-15, 1981.
- [5] Aihara, Y. and Koyama, H., "Secondary Instability of Görtler Vortices: Formation of Periodic Three-Dimensional Coherent Structure", *Trans. Japan Soc. Aero. Space Sci.*, v. 24, 78-94, 1981.
 - [6] Aihara, Y. and Koyama, H., "Nonlinear Development and Secondary Instability of Görtler Vortices", IUTAM Symposium on "Stability in the Mechanics of Continua", Numbrecht, 1981, ed. Schroeder, F.H., (Springer-Verlag, 345-354, 1982).
 - [7] Aihara, Y. and Koyama, H., "Three-Dimensional Turbulent Boundary Layer along a Concave Wall", IUTAM Symposium on "Three-Dimensional Turbulent Boundary Layers", Berlin, 1982, eds. Fernholtz, H.H. and Krause, E., (Springer-Verlag, 210-220, 1982).
 - [8] Aihara, Y., Tomita, Y. and Ito, A., "Generation, Development and Distortion of Longitudinal Vortices in Boundary Layers along Concave and Flat Plates", IUTAM Symposium on "Laminar-Turbulent Transition", Novosibirsk, 1984, ed. Kozlov, V.V., (Springer-Verlag, 447-454, 1985).
 - [9] Aihara, Y., Private Communication, 1991.
 - [10] Babenko, V.V. and Iurchenko, N.F., "Experimental Investigation of Görtler Instability on Rigid and Elastic Flat Plates", *Gidromekhanika*, v. 41, 103-108, 1980.
 - [11] Barlow, R.S. and Johnston, J.P., "Roll-Cell Structure in a Concave Turbulent Boundary Layer", AIAA Paper 85-0297, AIAA 23rd Aerospace Sciences Meeting, Reno, Jan. 14-17, 1985.
 - [12] Baskaran, V. and Bradshaw, P., "Experimental Investigation of a Three-Dimensional Turbulent Shear Layer over an Infinite Swept Concave Surface". Proceedings of 6th Symposium on Turbulent Shear Flows, Toulouse, Sept. 7-9, 1987, (University Park, PA, Pennsylvania State University, 11-2-1 to 11-2-6, 1987).
 - [13] Baskaran, V. and Bradshaw, P., "Decay of Spanwise Wavy Inhomogeneities in a Three-Dimensional Turbulent Boundary Layer over an Infinite Swept Concave Wing", *Experiments in Fluids*, v. 6, 487-492, 1988.
 - [14] Bassom, A.P. and Hall, P., "On the Generation of Mean Flows by the Interaction of Görtler Vortices and Tollmien-Schlichting Waves in Curved Channel Flows", NASA CR-181690, 1988.
 - [15] Bayly, B.J., "Three-Dimensional Centrifugal Type Instabilities in Inviscid Two-Dimensional Flows", *Phys. Fluids*, v. 321, 56-64, 1987.
 - [16] Beckwith, I.E. and Holley, B.B., "Görtler Vortices and Transition in Wall Layers of Two Mach 5 Nozzles", NASA TP-1869, 1981.
 - [17] Beckwith, I.E., Creel, T.R., Chen, F.J. and Kendall, J.M., "Free Stream Noise and Transition Measurements in a Mach 3.5 Pilot Quiet Tunnel", AIAA Paper 83-0042, 1983, AIAA 21st Aerospace Sciences Meeting, Reno, Jan. 10-13, 1983.
 - [18] Beckwith, I.E., Malik, M.R., Chen, F.J. and Bushnell, D.M., "Effects of Nozzle Design Parameters on the Extent of Quiet Test Flow at Mach 3.5", IUTAM Symposium on "Laminar-Turbulent Transition", Novosibirsk, 1984, ed. Kozlov, V.V., (Springer-Verlag, 589-600, 1985).
 - [19] Bennett, J. and Hall, P., "On the Secondary Instability of Taylor-Görtler Vortices to Tollmien-Schlichting Waves in Fully Developed Flows", *J. Fluid Mech.*, v. 186, 445-469, 1988.
 - [20] Bennett, J., Hall, P. and Smith, F.T., "The Strong Nonlinear Interaction of Tollmien-Schlichting Waves and Taylor-Görtler Vortices in Curved Channel Flow", NASA CR-181697, 1988.
 - [21] Bippes, H. and Görtler, H., "Dreidimensionale Störungen in der Grenzschicht an einer konkaven Wand", *Acta Mechanica*, v. 14, 251-267, 1972.
 - [22] Bippes, H., "Experimentelle Untersuchung des

- laminar-turbulenten Umschlags an einer parallel angeströmten konkaven Wand", Sitzungsberichte der Heidelberger Akademie der Wissenschaften Mathematisch-naturwissenschaftliche Klasse, 103-180, Jahrgang 1972, 3 Abhandlung, (also NASA-TM-72243, March 1978).
- [23] Bippes, H., Private Communication, 1988.
- [24] Bradshaw, P., "Effects of Streamline Curvature on Turbulent Flow", AGARDograph No. 169, 1973.
- [25] Brown, A. and Martin, B.W., "Flow Transition Phenomena and Heat Transfer over the Pressure Surfaces of Gas Turbine Blades", J. Engineering for Power, Transactions ASME, v. 104, 360-367, 1982.
- [26] Bötcher, J. and Wedemeyer, E., "The Flow Downstream of Screens and its Influence on the Flow in Stagnation Region of Cylindrical Bodies", J. Fluid Mech., v. 204, 501-522, 1989.
- [27] Clauser, M. and Clauser, F., "The Effects of Curvature on the Transition from Laminar to Turbulent Boundary Layer", NACA TN 613, 1937.
- [28] Collier, F.S. and Malik, M.R., "Stationary Disturbances in Three-Dimensional Boundary Layers over Concave Surfaces", AIAA Paper 87-1412, AIAA 19th Fluid Dynamics, Plasma Dynamics and Lasers Conference, Honolulu, June 8-10, 1987.
- [29] Crane, R.I. and Winoto, S.H., "Longitudinal Vortices in a Concave Surface Boundary Layer", AGARD-CD-271, Paper 9, Fluid Dynamics Panel Symposium, Netherlands, 1979.
- [30] Crane, R.I. and Sabzvari, J., "Laser-Doppler Measurements of Görtler Vortices in Laminar and Low-Reynolds Number Turbulent Boundary Layers", Proceedings Int. Symp. on Applications of Laser Doppler Anemometry to Fluid Mechanics, Lisbon, 1982; also in "Laser Anemometry in Fluid Mechanics", edited by Adrian, R.J. et al., LADOAN-Instituto Superior Técnico, Lisbon, 19-35, 1984.
- [31] Crane, R.I., Leoutsakos, G. and Sabzvari, J., "Transition in Pressure-Surface Boundary Layers", Paper No. 7, ASME, 31 International Gas Turbine Conference and Exhibit, Dusseldorf, Germany, June 8-12, 1986.
- [32] Crane, R.I. and Sabzvari, J., "Heat Transfer Visualization and Measurements in Unstable Concave-Wall Laminar Boundary Layers", ASME, Gas Turbine and Aeroengine Congress and Exposition, Amsterdam, Netherlands, June 6-9, 1988.
- [33] Dagenhart, J.R. and Mangalam, S.M., "Disturbance Functions of Görtler Instability on an Airfoil", AIAA Paper 86-1048, AIAA 4th Fluid Mechanics, Plasma Dynamics and Lasers Conference, Atlanta, May 12-14, 1986.
- [34] Dagenhart, J.R. and Mangalam, S.M., "Disturbance Functions of the Görtler Instability on an Airfoil", ICAS-86-1.8.1, 15th Congress of the International Council of the Aeronautical Sciences, London, U.K., September 1986.
- [35] Daudpota, Q.I., Zang, T.A. and Hall, P., "Interaction of Görtler Vortices and Tollmien-Schlichting Waves in Curved Channel Flow", AIAA Paper 87-1205, AIAA 19th Fluid Dynamics, Plasma Dynamics and Lasers Conference, Honolulu, June 8-10, 1987.
- [36] Daudpota, Q.I., Hall, P. and Zang, T.A., "On the Nonlinear Interaction of Görtler Vortices and Tollmien-Schlichting Waves in Curved Channel Flows at Finite Reynolds Numbers", J. Fluid Mech., v. 193, 569-595, 1988.
- [37] Day, H.P., Herbert, T. and Saric, W.S., "Comparing Local and Marching Analyses of Görtler Instability", AIAA J., v. 28, 1010-1015, 1990.
- [38] DiPrima, R.C. and Dunn, D.W., "The Effect of Heating and Cooling on the Stability of the Boundary Layer Flow of a Liquid over a Curved Surface", J. Aero. Sciences, v. 23, 913-916, 1956.
- [39] Drazin, P.G. and Reid, W.H., "Hydrodynamic Stability", Cambridge, 1981.

- [40] Durao, M.C., "Investigation of Heat Transfer in Straight and Curved Rectangular Ducts Using Liquid Crystals Thermography", M.Sc. Thesis, Naval Postgraduate School, Monterey, 1977.
- [41] El-Hady, N.M. and Verma, A.K., "Growth of Görtler Vortices in Compressible Boundary Layers along Curved Surfaces", AIAA Paper 81-1278, AIAA 14th Fluid and Plasma Dynamics Conference, Palo Alto, June 23-25, 1981.
- [42] El-Hady, N.M. and Verma, A.K., "Görtler Instability in Compressible Boundary Layers along Curved Surfaces with Suction and Cooling", NASA CR-3544, 1982.
- [43] El-Hady, N.M. and Verma, A.K., "Instability of Compressible Boundary Layers along Curved Walls with Suction or Cooling", AIAA Paper 82-1010, AIAA/ASME 3rd Joint Thermophysics, Fluids, Plasma and Heat Transfer Conference, St. Louis, June 7-11, 1982.
- [44] El-Hady N.M. and Verma, A.K., "Growth of Görtler Vortices in Compressible Boundary Layers along Curved Surfaces", J. Eng. Applied Sci., v. 2, 213-238, 1983.
- [45] El-Hady, N.M. and Verma, A.K., "Görtler Instability of Compressible Boundary Layers", AIAA J., v. 22, 1354-1355, 1984.
- [46] Euteneuer, G.A., "Einige Ergebnisse experimenteller Untersuchungen an instationären Görtler-Taylor-Wirbeln", ZAMM, v. 50, T177-T180, 1970.
- [47] Floryan, J.M. and Saric, W.S., "Stability of Görtler Vortices in Boundary Layers with Suction", AIAA Paper 79-1479, AIAA 12th Fluid and Plasma Dynamics Conference, Williamsburg, July 23-25, 1979.
- [48] Floryan, J.M. and Saric, W.S., "Stability of Görtler Vortices in Three-Dimensional Flows", Bul. Amer. Phys. Soc., v. 24, 1127, 1979.
- [49] Floryan, J.M., "Stability of Boundary Layer Flows over Curved Walls", Ph. D. Dissertation, Virginia Polytechnic Institute and State University, 1980.
- [50] Floryan, J.M. and Saric, W.S., "Wavelength Selection and Growth of Görtler Vortices", AIAA Paper 80-1376, AIAA 13th Fluid and Plasma Dynamics Conference, Snowmass, July 14-16, 1980.
- [51] Floryan, J.M. and Saric, W.S., "Stability of Görtler Vortices in Boundary Layers", AIAA J., v. 20, 316-324, 1982.
- [52] Floryan, J.M. and Saric, W.S., "Effects of Suction on the Görtler Instability of Boundary Layers", AIAA J., v. 21, 1635-1639, 1983.
- [53] Floryan, J.M. and Saric, W.S., "Wavelength Selection and Growth of Görtler Vortices", AIAA J., v. 22, 1529-1538, 1984.
- [54] Floryan, J.M., "Effects of Blowing on the Görtler Instability of Boundary Layers", AIAA J., v. 23, 1287-1288, 1985.
- [55] Floryan, J.M., "The Second Mode of the Görtler Instability of Boundary Layers", AIAA J., v. 23, 1828-1830, 1985.
- [56] Floryan, J.M., "Görtler Instability of Boundary Layers over Concave and Convex Walls", Phys. Fluids, v. 29, 2380-2387, 1986.
- [57] Floryan, J.M., "Görtler Instability of Wall Jets", AIAA J., v. 27, 112-114, 1989.
- [58] Fujisawa, N., Kobayashi, R. and Shirai, H., "Centrifugal Instability of Turbulent Wall Jets along Strongly Concave Surfaces", J. Applied Mechanics, Transactions ASME, v. 52, 492-493, 1985.
- [59] Fujisawa, N. and Shirai, H., "On the Stability of Turbulent Wall Jets along Concave Surfaces", Bulletin of JSME, v. 29, 3761-3766, 1986.
- [60] Garg, V.K. and DiPrima, R.C., "The Effects of Heating on the Stability of Flow over Concave Walls", Phys. Fluids, v. 27, 812-820, 1984.
- [61] Goglia, G.L. and Verma, A.K., "On Higher Eigenstates of Görtler Instability in Compressible Flow", NASA CR-169480, 1982.
- [62] Goglia, G.L. and Mangalam, J.M., "Experimental Studies on Taylor-Görtler Vortices",

- NASA CR-181254, 1985.
- [63] Görtler, H., "Instabilität Laminarer Grenzschichten an konkaven Wänden Gegenüber gewissen dreidimensionalen Störungen", ZAMM, v. 21, 250-252, 1941, (also NACA TM 1375, 1954).
 - [64] Gregory, N. and Walker, W.S., "The Effect on Transition of Isolated Surface Excrescences in the Boundary Layer", ARC Repl., R&M No. 2779, 1-10, 1956.
 - [65] Hall, P., "Taylor-Görtler Vortices in Fully Developed or Boundary-Layer Flows: Linear Theory", J. Fluid Mech., v. 124, 475-494, 1982.
 - [66] Hall, P., "On the Nonlinear Evolution of Görtler Vortices in Non-Parallel Boundary Layers", IMA J. Applied Math., v. 29, 173-196, 1982.
 - [67] Hall, P., "The Linear Development of Görtler Vortices in Growing Boundary Layers", J. Fluid Mech., v. 130, 41-58, 1983.
 - [68] Hall, P., "The Görtler Vortex Instability Mechanism in Three-Dimensional Boundary Layers", Proc. Roy. Soc. London, A399, 135-152, 1985.
 - [69] Hall, P., "Taylor-Görtler Instabilities of Tollmien-Schlichting Waves and Other Flows Governed by the Interactive Boundary Layer Equations", NASA CR-178010, 1985.
 - [70] Hall, P. and Bennett, J., "Taylor-Görtler Instabilities of Tollmien-Schlichting Waves and Other Flows Governed by the Interactive Boundary-Layer Equations", J. Fluid Mech., v. 171, 441-457, 1986.
 - [71] Hall, P. and Seddougui, S., "On the Onset of Three Dimensionality and Time Dependence in the Görtler Vortex Problem", J. Fluid Mech., v. 204, 405-420, 1989.
 - [72] Hall, P. and Smith, F.T., "The Nonlinear Interaction of Tollmien-Schlichting Waves and Taylor-Görtler Vortices in Curved Channel Flows", Proc. R. Soc. London, v. A417, 255-282, 1988.
 - [73] Hall, P. and Lakin, W.D., "The Fully Nonlinear Development of Görtler Vortices in Growing Boundary Layers", Proc. R. Soc. London, v. A415, 421-449, 1988.
 - [74] Hall, P., "The Nonlinear Development of Görtler Vortices in Growing Boundary Layers", J. Fluid Mech., v. 193, 243-266, 1988.
 - [75] Hall, P. and Malik, M.R., "The Growth of Görtler Vortices in Compressible Boundary Layers", J. Eng. Math., v. 23, 239-245, 1989.
 - [76] Hall, P., "Görtler Vortices in Growing Boundary Layers: The Leading Edge Receptivity Problem, Linear Growth and the Nonlinear Breakdown Stage", NASA CR-181962, 1989.
 - [77] Hall, P. and Smith, F.T., "Nonlinear Tollmien-Schlichting/Vortex Interaction in Boundary Layers", Eur. J. Mech., B/Fluids, v. 8, 179-205, 1989.
 - [78] Hall, P. and Fu, Y.B., "On the Görtler Vortex Instability Mechanism at Hypersonic Speeds", IUTAM Symposium on "Laminar-Turbulent Transition", Toulouse, 1989, eds. Arnal, D. and Michel, E., (Springer-Verlag, 453-463, 1990).
 - [79] Han, L.S. and Cox, W.R., "A Visual Study of Turbine Blade Pressure-Side Boundary Layers", J. Eng. for Power, Transactions of ASME, v. 105, 47-52, 1983.
 - [80] Herbert, T., "On the Stability of Boundary Layer along a Concave Wall", Archives of Mechanics, v. 28, 1039-1055, 1976.
 - [81] Herron, I.H., "Exchange of Stabilities for Görtler Flow", SIAM J. Appl. Math., v. 45, 775-779, 1985.
 - [82] Hoffmann, P.H., Muck, K.C. and Bradshaw, P., "The Effects of Concave Surface Curvature on Turbulent Boundary Layers", J. Fluid Mech., v. 164, 371-403, 1985.
 - [83] Ito, A., "The Generation and Breakdown of Longitudinal Vortices along a Concave Wall", J. Japan Soc. Aero. Space Sci., v. 28, 327-333, 1980.
 - [84] Ito, A., "Breakdown Structure of Longitudinal Vortices along a Concave Wall", J. Japan

- Soc. Aero. Space Sci., v. 33, 166-173, 1985.
- [85] Ito, A., "Visualization of Boundary Layer Transition along a Concave Wall", *Proceedings of the Fourth International Symposium on Flow Visualization*, Paris, 339-344, 1986, (Hemisphere, Washington, 1987).
- [86] Ito, A., "Breakdown Structure of Longitudinal Vortices along a Concave Wall; On the Relation of Horseshoe-Type Vortices and Fluctuating Flows", *J. Japan Soc. Aero. Space Sci.*, v. 36, 274-279, 1988.
- [87] Iurchenko, N.F., Babenko, V.V. and Kozlov, L.F., "Experimental Investigation of the Görtler Instability in Boundary Layers", in "Stratified and Turbulent Flows", Kiev, *KiD Naukova Dumka*, 50-59, 1979.
- [88] Jallade, S., Arnal, D. and Ha Minh, H., "Theoretical Study of Görtler Vortices: Linear Stability Approach", *IUTAM Symposium on "Laminar-Turbulent Transition"*, Toulouse, 1989, eds. Arnal, D. and Michel, E., (Springer-Verlag, 563-572, 1990).
- [89] Jeans, A.H. and Jonston, J.P., "The Effects of Concave Curvature on Turbulent Boundary Layer Structure", in *IUTAM Symposium on "Structure of Complex Turbulent Shear Flow"*, Marseille, 1982, eds. Dumas, R. and Fulachier, L., (Springer-Verlag, 89-99, 1983).
- [90] Kahawita, R.A. and Meroney, R.N., "Longitudinal Vortex Instabilities in Laminar Boundary Layers over Curved Heated Surfaces", *Phys. Fluids*, v. 17, 1661-1666, 1974.
- [91] Kahawita, R. and Meroney, R., "The Influence of Heating on the Stability of Laminar Boundary Layers along Concave Curved Walls", *J. Applied Mechanics*, *Transactions ASME*, v. 44, 11-17, 1977.
- [92] Kalburgi, V., Mangalam, S.M. and Dagenhart, J.R., "A Comparative Study of Theoretical Methods on Görtler Instability", *AIAA Paper 88-0407*, *AIAA 26th Aerospace Sciences Meeting*, Reno, Jan. 11-14, 1988.
- [93] Kalburgi, V., Mangalam, S.M. and Dagenhart, J.R., "Görtler Instability on an Airfoil: Comparison of Marching Solution with Experimental Observations", *AGARD CP-438*, Paper No. 8, 1989, (*Proceedings of AGARD Meeting*, Cesme, Turkey, 1988).
- [94] Kamotani, Y., Lin, J.K. and Ostrach, S., "Effects of Destabilizing Heating on Görtler Vortices", *J. Heat Transfer*, *Transactions ASME*, v. 107, 877-882, 1985.
- [95] Klebanoff, P.S., Tidstrom, K.D. and Sargent, L.M., "Three-Dimensional Nature of Boundary Layer Instability", *J. Fluid Mech.*, v. 12, 1-34, 1962.
- [96] Kobayashi, R. and Kohama, Y., "Taylor-Görtler Instability of Compressible Boundary Layers", *AIAA J.*, v. 15, 1723-1727, 1977.
- [97] Kobayashi, R. and Fujisawa, N., "Taylor-Görtler Instability of Turbulent Wall Jets along Concave Surfaces", Rept. No. 333 of the Institute of High Speed Mechanics, Tohoku University, Sendai, Japan, 1980.
- [98] Kobayashi, R. and Fujisawa, N., "Taylor-Görtler Instability of Turbulent Wall Jets along Concave Surfaces", *J. Applied Mechanics*, *Transactions ASME*, v. 47, 671-672, 1980.
- [99] Kobayashi, R. and Fujisawa, N., "Turbulence Measurements in Wall Jets along Strongly Concave Surfaces", *Acta Mechanica*, v. 47, 39-52, 1983.
- [100] Kohama, Y., "Three-Dimensional Boundary Layer Transition on a Concave-Convex Curved Wall", *IUTAM Symposium on "Turbulence Management and Relaminarization"*, India, 1987, eds. Liepmann, H.W. and Narasimha, R., (Springer-Verlag, 215-226, 1988).
- [101] Komoda, H., "Nonlinear Development of Disturbance in a Laminar Boundary Layer", *Phys. Fluids*, v. 10, Suppl. 87-94, 1967.
- [102] Kottke, V., "Taylor-Görtler Vortices and Their Effect on Heat and Mass Transfer", *Proceedings of the Eight International Heat Transfer Conference*, San Francisco, 1139-1144, 1986.
- [103] Kottke, V. and Mpourdis, B., "On the Existence of Taylor-Görtler Vortices on Concave Walls", *Proceedings of the Fourth International*

- Symposium on Flow Visualization, Paris, 475-480, 1986, (Hemisphere, Washington, 1987).
- [104] Kottke, V., "On the Instability of Laminar Boundary Layers along Concave Walls towards Görtler Vortices", in "Propagation in Nonequilibrium Systems", Springer-Verlag, Berlin, 1988.
- [105] Kozlov, L.F., Glushko, V.N. and Nikishova, O.D., "Experimental Study of the Generation of Görtler Vortices in an Unsteady Boundary Layer", *Fluid Mechanics—Soviet Research*, v. 10, 89-95, 1981.
- [106] Leoutsakos, G. and Crane, R.I., "Three-Dimensional Boundary Layer Transition on a Concave Surface", AGARD CP-438, 1989, Paper No. 9, (Proceedings of AGARD Meeting, Cesme, Turkey, 1988).
- [107] Liepmann, H.W., "Investigations on Laminar Boundary Layer Stability and Transition on Curved Boundaries", NACA Wartime Rept. W-107, 1943.
- [108] Liepmann, H.W., "Investigation of Boundary Layer Transition on Concave Walls", NACA Wartime Rept. W-87, 1945.
- [109] Liu, W. and Domaradzki, A., "Direct Numerical Simulation of Transition to Turbulence in Görtler Flows", AIAA Paper 90-0114, AIAA 28th Aerospace Sciences Meeting, Reno, Jan. 14-17, 1990.
- [110] Liu, J.C.T. and Sabry A.S., "Concentration and Heat Transfer in Nonlinear Görtler Vortex Flow and the Analogy with Longitudinal Momentum Transfer", *Proc. R. Soc. London*, v. A432, 1-12, 1991.
- [111] Malik, M.R., "Wave Interactions in Three-Dimensional Boundary Layers", AIAA Paper 86-1129, AIAA/ASME 4th Fluid Mechanics, Plasma Dynamics and Ladders Conference, Atlanta, May 12-14, 1986.
- [112] Malik, M.R. and Hussaini, M.Y., "Numerical Simulation of Interactions between Görtler Vortices and Tollmien-Schlichting Waves", *J. Fluid Mech.*, v. 210, 183-199, 1990.
- [113] Malik, M.R., no title, in IUTAM Symposium on "Laminar-Turbulent Transition", Toulouse, 1989, eds. Arnal, D. and Michel, E., (Springer-Verlag, 251-260, 1990).
- [114] Malkus, W., "Outline of a Theory of Turbulent Shear Flow", *J. Fluid Mech.*, v. 1, 521-539, 1956.
- [115] Mangalam, S.M., Dagenhart, J.R., Hepner, T.E. and Meyers, J.F., "The Görtler Instability on an Airfoil", AIAA Paper 85-0941, AIAA 23rd Aerospace Sciences Meeting, Reno, Jan. 14-17, 1985.
- [116] Mangalam, S.M., Dagenhart, J.R. and Kalburgi, V., "Influence of Suction and Curvature on the Growth of Görtler Vortices on an Airfoil", AIAA Paper 87-0481, AIAA 25th Aerospace Sciences Meeting, Reno, Jan. 12-15, 1987.
- [117] Martin, B.W. and Brown, A., "Factors Influencing Heat Transfer to the Pressure Surfaces of Gas Turbine Blades", *Int. J. Heat and Fluid Flow*, v. 1, 107-114, 1979.
- [118] Martin, B.W. and Brown, A., "Factors Influencing Heat Transfer to the Pressure Surfaces of Gas Turbine Blades", *Int. J. Turbo and Jet Engines*, v. 1, 247-258, 1984.
- [119] Mayle, R.E., Blair, M.F. and Kopper, F.C., "Turbulent Boundary Layer Heat Transfer on Curved Surfaces", *J. Heat Transfer, Transactions ASME*, v. 101, 521-525, 1979.
- [120] McCormack, P.D., Walker, H. and Kelleher, M., "Taylor-Görtler Vortices and Their Effect on Heat Transfer", *J. Heat Transfer, Transactions ASME*, v. 92, 101-112, 1970.
- [121] Meksyn, D. and Stuart, J.T., "Stability of Viscous Motion between Parallel Planes for Finite Disturbances", *Proc. R. Soc. London*, v. A208, 517-526, 1951.
- [122] Meroney, R.N. and Bradshaw, P., "Turbulent Boundary Layer Growth over Longitudinally Curved Surface", *AIAA J.*, v. 13, 1448-1453, 1975.
- [123] Moser, R.D. and Moin, P., "Effects of Wall Curvature on Turbulence Statistics", *Proceedings of the 5th Symposium on Turbulent Shear*

- Flows, 21.7-21.12, Cornell University, Aug. 7-9, 1985, (published by Pennsylvania State University, University Park, Pennsylvania, 1985).
- [124] Moser, R.D. and Moin, P., "The Effects of Curvature in Wall-Bounded Turbulent Flows", *J. Fluid Mech.*, v. 175, 479-510, 1987.
- [125] Muck, K.C., Hoffmann, P.H. and Bradshaw, P., "The Effect of Convex Surface Curvature on Turbulent Boundary Layers", *J. Fluid Mech.*, v. 161, 347-369, 1985.
- [126] Myose, R.Y. and Blackwelder, R.F., "Controlling the Spacing of Streamwise Vortices on Concave Walls", submitted to AIAA J., 1990.
- [127] Nayfeh, A.H., "Effects of Streamwise Vortices on Tollmien-Schlichting Waves", *J. Fluid Mech.*, v. 107, 441-453, 1981.
- [128] Nayfeh, A.H. and Al-Maaitah, A., "Influence of Streamwise Vortices on Tollmien-Schlichting Waves", IAAA Paper 87-1206, AIAA 19th Fluid Dynamics, Plasma Dynamics and Lasers Conference, Honolulu, June 8-10, 1987.
- [129] Pai Shin-I, "Turbulent Flow between Rotating Cylinders", NACA TN-892, 1943.
- [130] Peerhossaini, H. and Wesfreid, J.E., "On the Inner Structure of Streamwise Görtler Rolls", *Int. J. Heat and Fluid Flow*, v. 9, 12-18, 1988.
- [131] Prabhu, A. and Rao, B.N.S., "Effect of Concave Streamline Curvature on Turbulent Boundary Layers", AIAA Paper 81-1193, AIAA 14th Fluid and Plasma Dynamics Conference, Palo Alto, June 23-25, 1981.
- [132] Ragab, S.A. and Nayfeh, A.H., "Görtler Instability", *Phys. Fluids*, v. 24, 1405-1417, 1981.
- [133] Ramaprian, B.R. and Shivaprasad, B.G., "The Structure of Turbulent Boundary Layers along Mildly Curved Surfaces", *J. Fluid Mech.*, v. 85, 273-303, 1978.
- [134] Rayleigh, Lord, "On the Stability, or Instability, of Certain Fluid Motions", *Proc. London Math. Soc.*, v. 11, 57-70, 1880, (also "Scientific Papers", v. 1, 474-487, Cambridge University Press, 1899).
- [135] Sabry, A. and Liu, J.C.T., "Nonlinear Development of Görtler Vortices and the Generation of High Shear Layers in the Boundary Layer", *Proc. Symp. in Honour of C.C.Lin*, World Scientific Books, 175-183, 1987.
- [136] Sabry, A.S. and Liu, J.C.T., "Longitudinal Vorticity Elements in Boundary Layers: Their Nonlinear Development from Initial Görtler Vortices as a Prototype Problem", Division of Engineering Rept. FM89-11, Brown University, 1989.
- [137] Sabry, A.S., Yu, X. and Liu, J.C.T., "Secondary Instabilities of Three-Dimensional Inflectional Velocity Profiles Resulting from Longitudinal Vorticity Elements in Boundary Layers", IUTAM Symposium on "Laminar-Turbulent Transition", Toulouse, 1989, eds. Arnal, D. and Michel, E., (Springer-Verlag, 441-451, 1990).
- [138] Sabzvari, J. and Crane, R.I., "Effect of Görtler Vortices on Transitional Boundary Layers", in "Three-Dimensional Flow Phenomena in Fluid Machinery", Proceedings of Winter Annual Meeting of the American Society of Mechanical Engineers (FED, v. 32, 113-119, Florida, 1985).
- [139] Sandmayr, G., "On the Occurrence of Longitudinal Vortices in Turbulent Boundary Layers at Concave Walls", NACA TT F-14, 1972, (translation of DLR Report FB-66-41/June 1966).
- [140] Smith, F.T., "Nonlinear Interaction of Near-Planar Waves and Longitudinal Vortices in Boundary-Layer Transition", NASA CR-181751, 1988.
- [141] Smits, A.J., Young, S.T.B. and Bradshaw, P., "The Effects of Short Regions of High Surface Curvature on Turbulent Boundary Layers", *J. Fluid Mech.*, v. 94, 209-242, 1979.
- [142] So, R.M.C. and Mellor, G.L., "Experiment on Turbulent Boundary Layers on a Concave Wall", *Aeronautical Quarterly*, v. 26, 25-40,

- 1975.
- [143] Spall, R.E. and Malik, M.R., "Görtler Vortices in Supersonic Boundary Layers", Proceedings of the 1st National Fluid Dynamics Congress, 1103-1110, Cincinnati, July 25-28, 1988.
 - [144] Spall, R.E. and Malik, M.R., "Görtler Vortices in Supersonic and Hypersonic Boundary Layers", *Phys. Fluids*, v. A1, 1822-1835, 1989.
 - [145] Srivastava, K.M. and Dallmann, U., "Effects of Streamwise Vortices on Tollmien-Schlichting Waves in Growing Boundary Layers", *Phys. Fluids*, v. 30, 1005-1016, 1987.
 - [146] Stuart, J.T. and Watson, J., "On the Nonlinear Mechanics of Wave Disturbances in Stable and Unstable Parallel Flows. Part 1. The Basic Behaviour in Plane Poiseuille Flow. Part 2. The Development of Solution for Plane Poiseuille Flow and for Plane Couette Flow", *J. Fluid Mech.*, v. 9, 353-389, 1960.
 - [147] Swearingen, J.D. and Blackwelder, R.F., "Breakdown of Streamwise Vortices Near a Wall", IUTAM Symposium on "Structure of Complex Shear Flows", Marseille, 1982, eds. Dumas, R. and Fulachier, L., (Springer-Verlag, 10-19, 1983).
 - [148] Swearingen, J.D. and Blackwelder, R.F., "Parameters Controlling the Spacing of Streamwise Vortices on Concave Walls", AIAA 21st Aerospace Sciences Meeting, Reno, Jan. 10-13, 1983.
 - [149] Seearingen, J.D. and Blackwelder, R.F., "Spacing of Streamwise Vortices on Concave Walls", *AIAA J.*, V. 24, 1706-1709, 1986.
 - [150] Swearingen, J.D. and Blackwelder, R.F., "The Growth and Breakdown of Streamwise Vortices in the Presence of a Wall", *J. Fluid Mech.*, v. 182, 255-290, 1987.
 - [151] Tani, I., "Some Aspects of Boundary Layer Transition at Subsonic Speeds", *Advances in Aeronautical Sciences*, v. 3, 143-160, 1987.
 - [152] Tani, I., "Production of Longitudinal Vortices in a Boundary Layer along a Curved Wall", *J. Geophys. Res.*, v. 67, 3075-3080, 1962.
 - [153] Tani, I. and Sakagami, J., "Boundary Layer Instability at Subsonic Speeds", *Proc. Int. Council Aero. Sci., Third Congress, Stockholm, 1962*, 391-403, (Spartan, Washington, D.C., 1964).
 - [154] Tani, I. and Aihara, Y., "Görtler Vortices and Boundary Layer Transition", *ZAMP*, v. 20, 609-618, 1969.
 - [155] Taylor, G.I., "Stability of a Viscous Liquid Contained between Rotating Cylinders", *Philos. Trans. R. Soc. London, Ser. A*, v. 223, 289-343, 1923.
 - [156] Tillmann, W., "Development of Turbulence During Build-up of a Boundary Layer at a Concave Wall", *Phys. Fluids Suppl.*, v. 10, 108-111, 1967.
 - [157] Tomita, Y., "Influence of Polymer Additives on Neutral Stability of Görtler Vortices", in "The Influence of Polymer Additives on Velocity and Temperature Fields", IUTAM Symposium, Essen, 1984, ed. Gampert, B., (Springer-Verlag, 251-260, 1985).
 - [158] Tomita, Y., Funatsu, K., Tsuzuki, S. and Miyazaki, K., "Neutral Stability of Laminar Boundary Layers along the Concave Wall", *Bulletin of JSME*, v. 28, 2288-2293, 1985.
 - [159] Tumin, A.M. and Chernov, Yu. P., "Asymptotic Analysis of Flow Instability in a Compressible Boundary Layer on a Curved Surface", *Zhurnal Prikladnoi Mekhaniki i Technicheskoi Fiziki*, v. 3, 84-89, 1988.
 - [160] Van Dyke, M., "Perturbation Methods in Fluid Dynamics", Parabolic Press, Stanford, USA, 1975.
 - [161] Wilson, S.D.R. and Gladwell, I., "The Stability of a Two-Dimensional Stagnation Flow to Three-Dimensional Disturbances", *J. Fluid Mech.*, v. 84, 517-527, 1978.
 - [162] Winoto, S.H., Durao, D.F.G. and Crane, R.I., "Measurements within Görtler Vortices", *J. Fluid Engineering, Transactions ASME*, v. 100, 517-520, 1979.
 - [163] Winoto, S.H. and Crane, R.I., "Vortex Structure in Laminar Boundary Layers on a Concave Wall", *Int. J. Heat and Fluid Flow*,

- v. 2, 221-231, 1980.
- [164] Winoto, S.H., "Qualitative Investigation on Görtler Vortices", Proceedings of the Eight Australian Fluid Mechanics Conference, University of Newcastle, 7A1-7A.4, 1983.
- [165] Wortmann, F.X., "Experimentelle Untersuchungen Laminarer Grenzschichten bei instabiler Schichtung", Proc. XI Int. Cong. Appl. Mech., 815-825, 1964.
- [166] Wortmann, F.X., "Experimental Investigation of Vortex Occurrence at Transition in Unstable Laminar Boundary Layers", AFOSR Rept. 64-1280, AF 61 (052)-220, 1964.
- [167] Wortmann, F.X., "Visualization of Transition", J. Fluid Mech., v. 38, 473-480, 1969.
- [168] Zang, T.A., Krist, S.E. and Hussaini, M.Y., "Resolution Requirements for Numerical Simulation of Transition", J. Sci. Comput., v. 4, 197-217, 1989.
- [169] Euromech 261, "Colloquium on Görtler Vortex Flows, Synopsis of Contributions", University of Nantes, ISITEM, Nantes, France, June 11-14, 1990.

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