Interplanetary Mission Designs by Multiple Earth-Venus-Jupiter Swingbys

By

Hiroki YOKOTA* and Toru TANABE*

(February 5, 1983)

Summary: This paper presents a possible usage of a multiple swingby technique which makes use of at most three gravity-assist planets (Earth, Venus, and Jupiter). Various combinations are considered (e.g., a multiple Venus swingby, a multiple Earth-Venus swingby, etc.), and each of the energy exchange mechanisms between a spacecraft and planet(s) is investigated as well as the angular momentum exchange mechanisms. Focusing on attainable orbital elements, the capability of each swingby technique is also investigated.

The major purpose of this technique is to reduce the launch velocity increment while maintaining a reasonable flight time. When only one planet is involved, a forward dynamic programming approach is successfully applied to determine the optimal swingby sequence. However, when more than two planets are used, computations become quite time consuming so that a simplified control strategy is proposed. This strategy is built up with several subsequences, in which a forward dynamic programming approach is partially applied.

Four specific interplanetary mission designs (M1 through M4) are presented:

M1: escaping from the solar system;
M2: falling into the sun;
M3: out-of-the-ecliptic mission; and
M4: retrograde mission;

by using a multiple Earth-Venus swingby and a multiple Earth-Venus-Jupiter swingby with as small as 3.0 km/sec launch velocity increment. Results are compared with a conventional direct launch technique in terms of the Earth launch velocity increment, flight time, and available launch window.

Computations are based on a patched conics method, and orbits of Earth, Venus, and Jupiter are assumed to be circular and coplanar unless otherwise specified.

1. Introduction

Beginning as early as in 1956 with Crocco, various studies have been carried out in this field. Some missions such as Pioneer, Voyager, etc., have already been achieved successfully and others such as Galileo, and the International Solar Polar Mission (ISPM) are now being proposed. In short, a swingby is a process which takes advantage of planetary gravitational perturbations to generate a desirable trajectory. It is shown that this process is very effective in saving launch energy as well as in reducing the flight time, although planetary ephemeris may limit the available launch window. In this section, taking a brief survey on previous studies, the scope of this paper is clarified.

Table 1 lists four basic modes of swingby technique. When a certain encountered planet is given, there may be two choices as to whether the swingby takes place

* Department of Aeronautics, University of Tokyo

a single time (mode I), or multiple times (mode II). Also, when several planets are selected, the swingby with one of the planets may occur just once (mode III) or more than once (mode IV). In each mode, active thrusting maneuvers may be combined. A spacecraft may use midcourse impulsive kick(s) before and/or after a swingby event. In a special case, the velocity impulse may be applied during the swingby phase (powered swingby). Also, continuous thrusting such as a solar electric propulsion system (SEPS) may be used, and combinations of both can be considered.

Swingby techniques which involve encounters with Earth, Venus, and Jupiter should be analyzed. There may be 26 possible techniques depending on the planet(s) to be encountered and on the number encountered (0, 1, and more than 2). Ten of these basic techniques are listed in Table 2.

First of all, neither SES nor MES makes any sense, because an Earth swingby simply changes direction of the relative velocity between the spacecraft and Earth so that a postswingby state can already be attained at the Earth launch moment. Next, since the mass of Venus is about a 400th of that of Jupiter, SVS does not

Table 1. Swingby techniques previously studied

mode	without active maneuvers	with midcourse impulse(s)	with continuous thrusting (SEPS)
I	Mariner 6,7 ISPM comet flyby	outer planets (ΔV-EGA)	comet flyby asteroid rendezvous (SEEGA)
II	OPEN solar probe		
III	Pioneer 10,11 Voyager 1,2 (VEGA)	solar probe (ΔV-EJGA)	solar probe Uranus mission (SEEJGA)
IV	Mariner 10 periodic orbit Galileo	out-of-ecliptic	

Table 2. Swingby techniques with Earth/Venus/Jupiter encounter(s)

mode	Encountered Planet* Earth	Venus	Jupiter
I	Single Earth Swingby (SES)	Single Venus Swingby (SVS)	Single Jupiter Swingby (SJS)
II	Multiple Earth Swingby (MES)	Multiple Venus Swingby (MVS)	Multiple Jupiter Swingby (MJS)
III, IV	Multiple Earth-Venus Swingby (MEVS) Multiple Venus-Jupiter Swingby (MVJS) Multiple Earth-Jupiter Swingby (MEJS)		
III, IV	Multiple Earth-Venus-Jupiter Swingby (MEVJS)		

* The spacecraft first encounters the underlined planet.

work effectively enough to achieve a favorable trajectory. Therefore, MVS is used to realize a successive energy exchange. However, even so, there is a limited region of accessible trajectories, and a considerable Earth launch velocity increment is required to increase this capability.

Meanwhile, SJS turns out to be a powerful technique; however, at least 8.793 km/sec is needed to reach Jupiter. The capability of MJS is comparable with SJS, because Jupiter has such a large mass that a single swingby is in most cases sufficient to generate a favorable trajectory.

On the other hand, MEVS is cartainly capable of generating various interesting trajectories with as small as 2.495 km/sec launch velocity increment. For the case of MVJS, 3.893 km/sec Earth launch velocity increment is necessary, and for MEJS 8.793 km/sec is required as in SJS or MJS.

As for MEVJS, it literally contains all possible trajectories which can be achieved by using all other techniques. In this paper, this technique is considered as a sum of MEVS plus one of SJS, MVJS, or MEJS.

The encounter conditions may be expressed by using two state variables: a relative velocity ratio between the spacecraft and a planet, and the direction of the relative velocity ratio (an encounter angle). The semimajor axis and eccentricity become functions of those two variables. When the spacecraft has a rendezvous with just one planet (SVS, SJS, MVS, and MJS), the relative velocity ratio is kept constant throughout the whole flight. In such a case, the favorable swingby sequence can be denoted solely by a series of encounter angles. It is clear that these angles take discrete values based on flight time restrictions.

On the other hand, the relative velocity ratio becomes an active variable with MEVS, MVJS, and MEJS. Two sets of encounter conditions should be defined with respect to the orbits of both planets. For instance, α and θ, and β and ψ are computed for MEVS, and by using a Venus swingby, θ changes into a new value keeping α constant, and β and ψ change accordingly. It should be noted that the relative positions of Earth and Venus play an important role. In other words, even when β and ψ are uniquely determined in the Earth orbit, the post-swingby sequence may differ depending on the location of Venus.

The MVS and MEVS computations must now be compared. Assume that the initial state of the spacecraft is given in the Venus orbit. Then by using MVS, the spacecraft necessarily encounters Venus in the next stage; whereas by using MEVS, it has two possibilities: whether to encounter Earth or Venus. Furthermore, for both cases, there may be several ways of encountering the corresponding planet. Assume that it achieves n successive swingbys, and each time m possible encounters are considered either with Earth or Venus. Then, the total number of possible swingby sequences becomes m^n for MVS, and $(2m)^n$ for MEVS. In the case where $m=5$, and $n=7$ (an example for escaping from the solar system), the number of possible sequences are 78,125 and 10,000,000, respectively.

To cope with these computational difficulties, we apply a forward dynamic programming method (FDP). The reason that 'forward' rather than 'backward' is chosen is that the Earth launch condition is likely to be predetermined in the first stage. Results with MVS are quite successful. In the example above, the number of possible sequences is reduced from 78,125 to only 155. This surprising reduction is due to the fact that the subsequent swingby sequence does not depend on the prior history.

However, MEVS computation requires checking all 10,000,000 possible combinations. This is because the subsequent swingby sequence depends not only on the previous set of encounter conditions, but on the location of the planet encountered next. It may happen that an initial sequence with a longer flight time may result in a reduction of the total flight time. Presently, we propose a simplified control strategy, although this may yield a suboptimal sequence. This strategy is also applied in MEVJS which is considered a combination of MEVS, MEJS, etc.

2. Multiple Swingby Process

Assuming circular planetary orbits, the relative velocity ratios, α and β, and encounter angles, θ, and ψ are defined. Our concern is primarily focused on the Earth-Venus system; however, the general formulation is perfectly maintained solely by setting the proper radius ratio γ for the relevant two-planet system.

Relative Velocity Ratio and Encounter Angle

Suppose that a trajectory of a spacecraft crosses both the Earth and the Venus orbits, and let α, β, and γ be defined as

$$\alpha = v_{rv}/v_v \tag{1}$$

$$\beta = v_{re}/v_e \tag{2}$$

$$\gamma = r_v/r_e \tag{3}$$

where r_v: the Venus orbital radius; r_e: the Earth orbital radius; v_v: the Venus orbital velocity; v_e: the Earth orbital velocity; v_{rv}: the relative velocity of the spacecraft with respect to Venus; and v_{re}: the relative velocity with respect to Earth. Then, the relative velocity ratios α and β can be expressed as

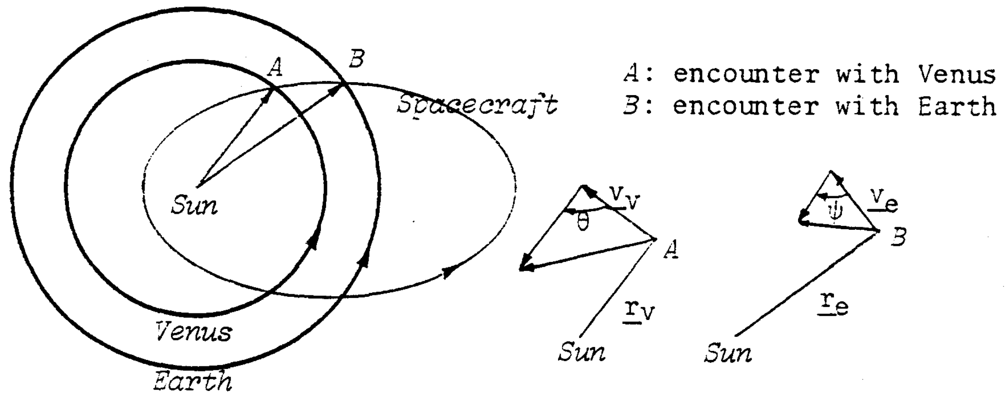

Fig. 1. Geometry of Earth & Venus encounters.

$$\alpha^2 = \gamma\beta^2 + 2\beta(1/\sqrt{\gamma} - \gamma)\cos\psi + 3 - 2/\sqrt{\gamma} - \gamma \tag{4}$$

$$\beta^2 = \alpha^2/\gamma + 2\alpha(\sqrt{\gamma} - 1/\gamma)\cos\theta + 3 - 2\sqrt{\gamma} - 1/\gamma \tag{5}$$

in which the encounter angles θ and ψ are defined by (see Fig. 1)

$$\cos\theta = \{1 - (1 - \beta\cos\psi)/\sqrt{\gamma}\}/\alpha \tag{6}$$

$$\cos\psi = \{1 - (1 - \alpha\cos\theta)\cdot\sqrt{\gamma}\}/\beta. \tag{7}$$

Note that the encounter angles are positive in a counter clockwise direction, and take values between $-\pi$ and π.

Orbital Elements

The orbital elements of the spacecraft (a: semimajor axis; and e: eccentricity) are defined in terms of β and ψ:

$$a = r_e/(1 + 2\beta\cos\psi - \alpha^2) \tag{8}$$

$$e = \beta\sqrt{1 - 2\beta\cos\psi + (\beta^2 + 3)\cos^2\psi - 2\beta\cos^3\psi}. \tag{9}$$

It is noted that any states could be expressed solely by either α and θ, or β and ψ in a two dimensional model.

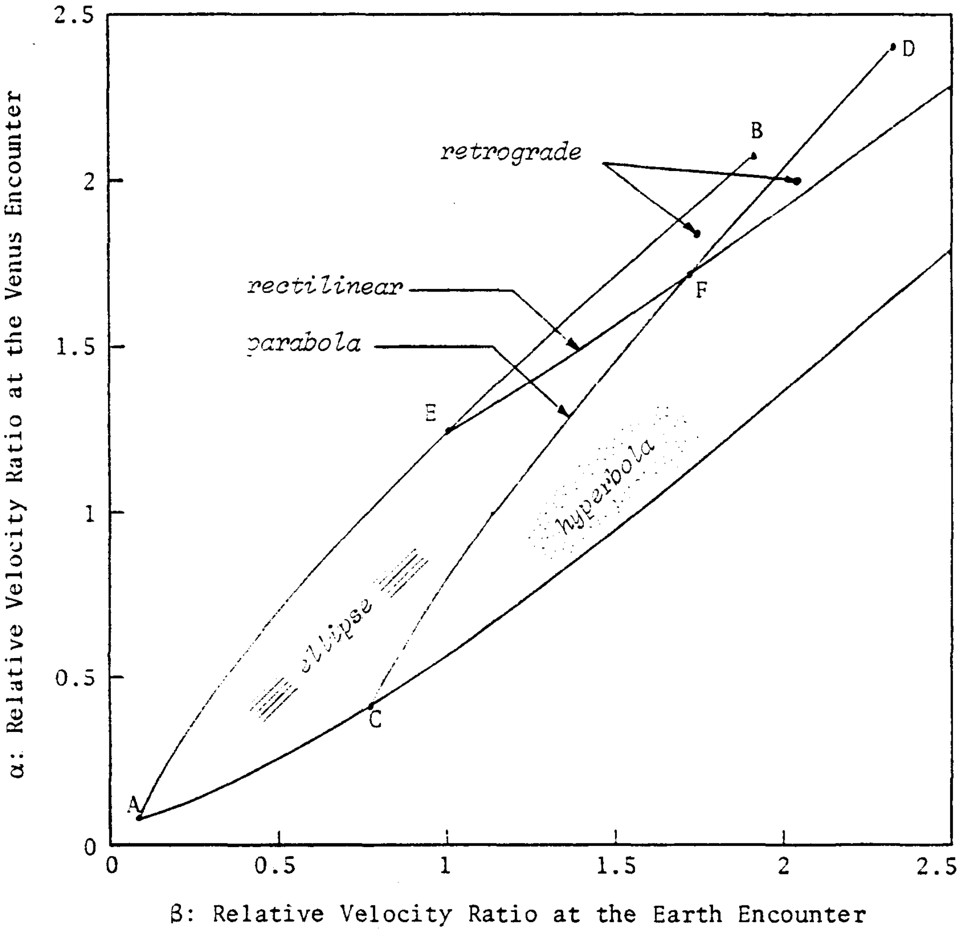

Fig. 2. Relation between β and α in the Earth-Venus system.

Figure 2 shows the relation between β and α in the Earth-Venus system. The point 'A' in Fig. 2 is expressed as

$$(\beta_0, \alpha_0)=(1-\sqrt{2\gamma/(1+\gamma)},\ \sqrt{2/(1+\gamma)}-1). \tag{10}$$

If the initial β at the launch from the Earth exceeds β_0 (2.485 km/sec), it is shown that the spacecraft can take any state within the area like a nose cone by choosing suitable encounter conditions. To this end, the following factors are taken into account.

1. The flight time between encounters with either Earth or Venus has to be carefully chosen in order to assure that the next swingby is properly aligned.
2. Special attention should be paid when the eccentricity of the spacecraft becomes greater than 1 because in such a case the spacecraft may eventually leave the solar system.
3. The Venus orbit has an inclination of 3.394° with respect to the ecliptic plane.
4. The sun is not a point mass, but has a radius of 696,000 km.

The discussion above may also be valid for a case of MEVJS by choosing a proper set of planets instead of Earth-Venus. By defining δ as a relative velocity ratio between Jupiter and the spacecraft, the relation between α and δ, or between β and δ can be drawn similar to Fig. 2.

Capability of MEVS & MEVJS

Focusing on achievable orbital elements, comparison is made here between MEVS and MEVJS capabilities with a launch velocity increment obtainable by existing chemical launchers.

It is shown that either MEVS or MEVJS gives the same minimum and maximum values except for perihelion distance, although the associated flight time differs depending on the swingby sequence. (In general, MEVS takes longer than MEVJS.) Referring to Table 3, (1) semimajor axis varies from $-\infty$ to ∞ both for MEVS and for MEVJS. However, a magnitude smaller than half a Venusian radius cannot be achieved. In other words, there is an energy interruption, and the energy with respect to the sun does not tend to infinity. (2) Eccentricity has a nonzero minimum value (in any case noncircular) which is met by obtaining $\beta=\beta_0$ (0.0838), and $\psi=\cos^{-1}(\beta_0/3)$ in both cases. (3) Any inclination is achievable if a proper relative velocity ratio is aligned to the designated direction through successive swingbys. (4) Aphelion distance changes no less than 0.723332 AU

Table 3. Capability of MEVS & MEVJS

orbital element	minimum	maximum
semimajor axis	$-\infty$ AU	∞ AU
eccentricity	0.083683	> 1
inclination	0°	180°
aphelion distance	0.723332 AU	∞ AU
perihelion distance (MEVS)	0.000816 AU	1 AU
(MEVJS)	0 AU	5.202833 AU

(the orbital radius of Venus). (5) As for perihelion distance, MEVJS can exactly hit the center of the sun in a rectilinear orbit (provided that the spacecraft survives enormous amounts of heat input), whereas MEVS cannot because of the limited deflection of relative velocity by an Earth swingby or by a Venus swingby. The minimum value of 0.000816 AU corresponds to 0.1753 radius of the sun with $\beta=1$, and $\psi=13.73°$. (The spacecraft collides with the sun.) Also, the maximum value differs between MEVS and MEVJS due to the differing radii of the encountered planets.

3. Sequence Determination

Swingby Types

Referring to Fig. 3, swingby types of elliptic transfers are first classified into four modes depending on the planet(s) to be encountered: modes 11, 12, 21, 22. (For instance, in mode 12 the spacecraft first encounters Earth and then Venus.) Next, each mode consists of four types (numbered from 1 to 4), which show the geometric relation of two successive swingbys. During a free-fall flight between swingbys, the spacecraft makes N complete revolutions around the sun, while the planet to be encountered does M complete revolutions.

Besides elliptic transfers, two hyperbolic transfer types (types 5 and 6) are considered. In these cases, M is always equal to 0.

Swingby Sequence

Let a swingby sequence be denoted by a series of 7 digit numbers in the following way. The first and the second digits show the planet departed from, and arrived at, respectively. When Earth is the relevant planet, an integer 1 is given, 2 for Venus, and 3 for Jupiter. The next two digits represent the value of M, N is represented by the fifth and the sixth digits, and the last digit designates the swingby type.

Control Constraints

Two control constraints are assumed. First, the closest approach distance to an encountered planet is assumed to be greater than the radius of that planet. Secondly, the maximum number of M is set to a certain integer (4 in this paper)

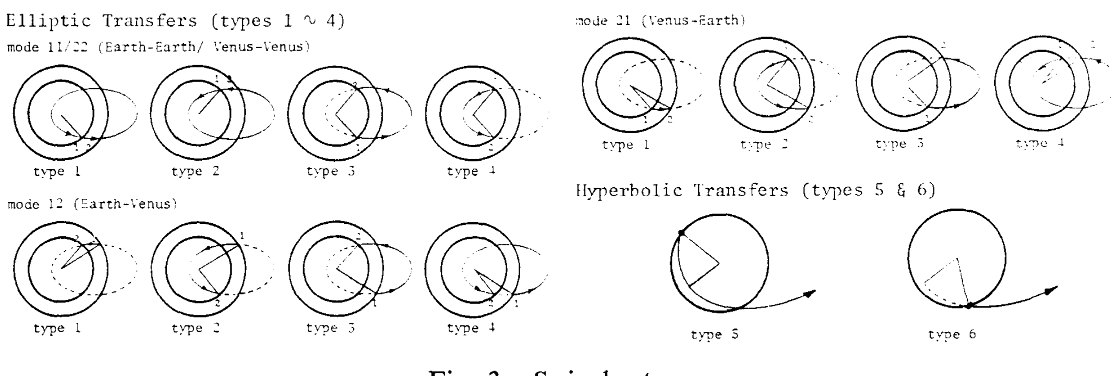

Fig. 3. Swingby types.

in order to exclude swingby sequences which require an excessively long flight time. However, the number of revolutions of the spacecraft is not constrained.

Control Strategy

Proposed here is a control strategy using MEVJS, which generates a favorable swingby sequence with a given launch velocity increment. Accordingly, a swingby sequence is divided into three subsequences, and each subsequence corresponds to one of the three following phases (C1, C2, and C3):

C1. encountering Venus with the smallest possible Δv (~ 3 km/sec),

C2. increasing a relative velocity ratio using MEVS in order to encounter Jupiter with δ greater that $\sqrt{2}-1$ for mission M1, and greater than 1 for other missions,

C3. completion of the purpose of a mission by using a SJS.

Note that in order to make $\delta \geqq \sqrt{2}-1$, α, and β ought to be greater than 0.32510, and 0.29521, and for $\delta \geqq 1$, they should be greater than 0.36509, and 0.35811, respectively. The advantage of the proposed strategy is in saving computational search time, while the disadvantage is that the sequence may be suboptimal in terms of minimization of flight time.

In the C1 phase, the spacecraft may make a few revolutions around the sun before reaching Venus so as to select favorable planetary locations. However, such time-adjusting revolutions are not used here, for our mission designs do not consider a planetary ephemeris.

The C2 phase is a process of MEVS, and FDP is applied in this phase. As shown in Fig. 4, the history of α and β may become like a multi-step function: during Venus swingbys, α remains constant, whereas β is constant during Earth swingbys. As long as the relative velocity ratio is kept constant, FDP can be directly applied to find a desirable subsequence. However, there is a problem in dealing with a switching point where the planet to be encountered changes. This is because our FDP does not take into account the location of the other planet which is not presently being used. To simplify the problem, in spite of the above mentioned circumstances, we make the following two assumptions (A1 & A2).

A1. A multi-step function in Fig. 4 continually moves up.

A2. A subsequence taken on one step is selected to minimize the associated flight time regardless of the other subsequences on other steps.

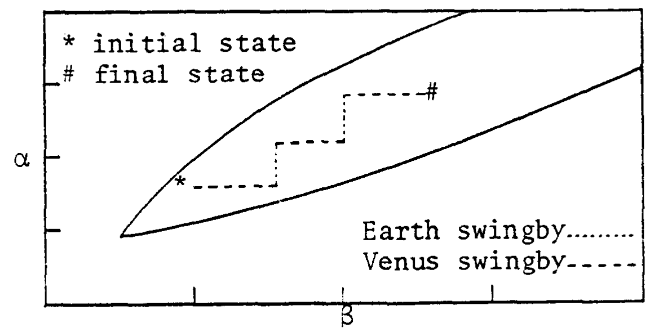

Fig. 4. History of α and β.

4. Results

Four mission designs (M1 through M4) are presented here, in which the launch velocity increment is set to 10% of the Earth orbital velocity (2.978 km/sec). Note that indices plotted in Figs. 5 through 9 show locations where swingbys are achieved.

M1 with MEVJS

In this mission, the spacecraft is scheduled to escape from our solar system by using four successive swingbys. The swingby sequence goes 1200002–2100003–1102012–1300002–3000006 including a double Earth swingby, a single Jupiter swingby. Note that the initial position of Jupiter is arbitrarily chosen so as to obtain the ideal planetary geometry. The total flight time turns out to be 5.225 years until it encounters Jupiter, and the final orbital elements become: $a=-8.781$ AU, and $e=1.593$. The spacecraft is departing from the solar system with a hyperbolic excess velocity of 10.051 km/sec (2.119 AU/year).

M2 with MEVJS

Without any swingbys, reaching the center of our stellar system is quite difficult in terms of the required velocity increment. However, by using MEVJS, this velocity requirement is drastically reduced. The swingby sequence is 1200002–2100003–1201011–2101004–1103012–1300002–3000000, encountering Venus, Earth, Venus, Earth, Earth, and Jupiter in that order. Eventually, the rectilinear ellipse is achieved in the type of 3000000 with $e=1$, and $a=3.132$ AU, and the spacecraft will collide with the sun 8.994 years after the Earth launch. Note that by using MEVS, the spacecraft realizes 0.00335 AU perihelion distance (the radius of the sun is 0.00465 AU) with a 33.272 year flight time including 28 swingbys.

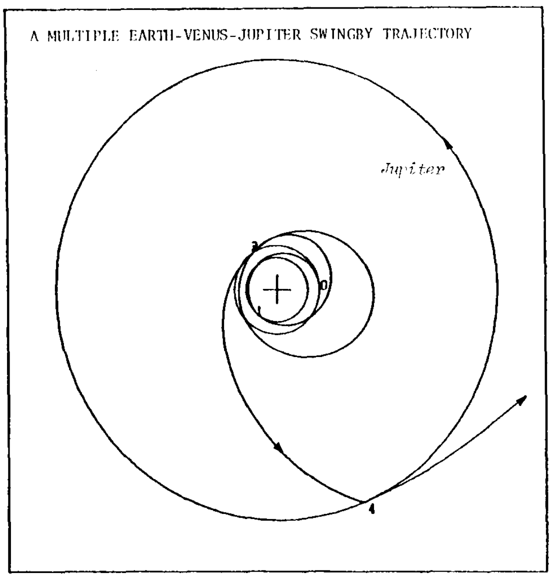

Fig. 5. M1 with MEVJS.

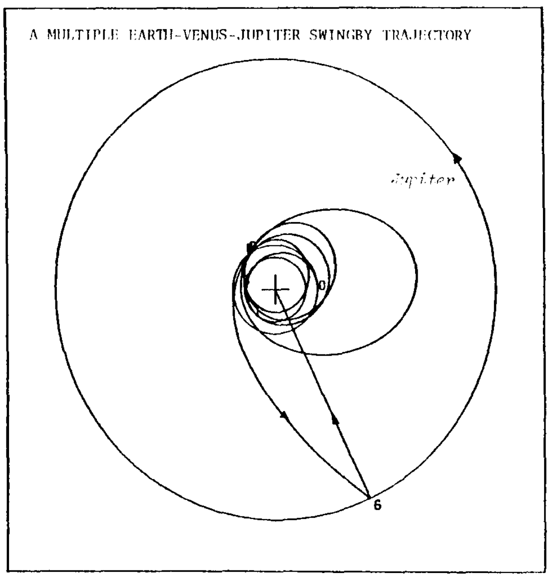

Fig. 6. M2 with MEVJS.

It turns out that by simply adding a single Jupiter swingby, the flight time becomes less than a third of that even with the same launch velocity increment. At the moment the spacecraft touches the surface of the sun, its velocity is estimated to be 617 km/sec (perhaps the greatest velocity any spacecraft has ever attained).

M3 with MEVJS

In mission M3, the spacecraft moves out of the plane of the ecliptic. The sequence here is chosen to be identical to M2 with MEVJS except for the final encounter conditions with Jupiter. To attain the inclination of 90°, the last encounter angle is tilted by 30.10° to the ecliptic plane. The maximum heliographic latitude is achieved 10.092 years after launch at a distance of 1.762 AU from the sun, and the perihelion distance becomes 1.060 AU. Note that at the encounter with Jupiter, two alternatives are considered as to whether the spacecraft passes over the top of Jupiter or under the bottom. However, either passage results in an equivalent trajectory which is symmetric with respect to the ecliptic plane. In this paper, we neglect a very small velocity correction prior to the encounter which is needed to bring the spacecraft into the desired trajectory.

M4 with MEVJS

This is one of the most interesting missions. Although all planets go in the same direction, the final orbit goes in the reverse direction as shown in Fig. 8. In this case, the sequence becomes 1200001–2102022–1201012–2202012–2102012–1104003–1300002–3000008 which includes 3 encounters both with Venus and Earth, and one encounter with Jupiter. Taking 13.131 years until the last Jupiter swingby, the spacecraft realizes an inclination of 180°.

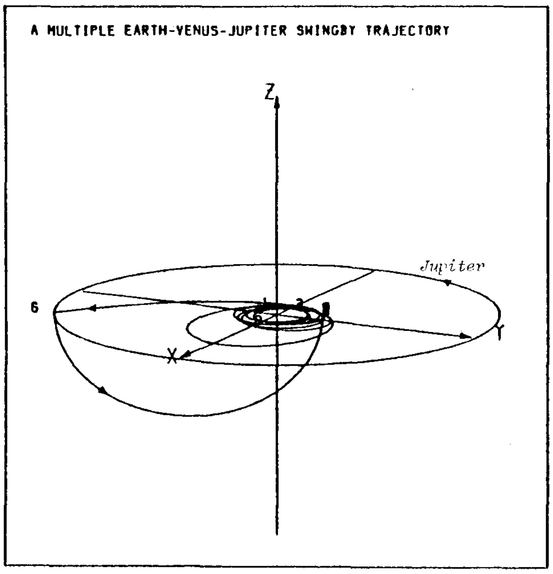

Fig. 7. M3 with MEVJS.

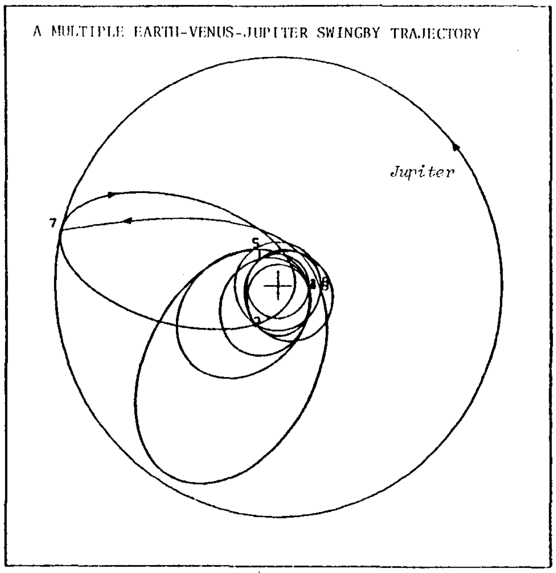

Fig. 8. M4 with MEVJS.

5. Discussion

Launch Velocity Increment

First, Table 4 lists Δv in km/sec with SJS, MEVS, MEVJS, and without any swingbys. Note that for the cases of MEVS and MEVJS, Δ_v of 2.50 km/sec is derived as the minimum requirement to encounter Venus regardless of mission specification. The Δv of mission M2 is met by obtaining the relative velocity ratio with one of three planets no less than 1. In mission M3, the final semilatus rectum is made greater than 1 AU with 90° inclination, and in mission M4, the closest approach to the sun is set to a distance no less than the Venus orbital radius. At the present time, it is virtually impossible to attain more than 20 km/sec launch velocity increment. So M2, M3, and M4 missions need some sort of gravity assist in cases without any active thrusting maneuvers. Although SJS reduces Δv, at least 8 km/sec is required to reach Jupiter for any mission. On the other hand, by using either MEVS or MEVJS all four missions here are achievable with as small as 2.50 km/sec.

Flight Time

Table 5 then records the total flight time in years for each mission design (M1 through M4). The termination of the flight time is set to crossing the Jupiter orbit for M1, the perihelion passage for M2 and M4, and the solar polar passage for M3. The flight time in this table corresponds to Δv in table 4 for cases of direct launch and SJS, but a Δv of 2.978 km/sec is chosen for MEVS and MEVJS. Note that data indicated by * are not available, but will take a much longer flight time compared with MEVJS.

Available Launch Window

The synodic period between Earth and Jupiter is approximately 1.092 years (399 days), and the same period between Earth and Venus is nearly 1.599 years (584 days). Therefore, an available launch opportunity appears once during each of those periods for SJS and MEVS. For a case of MEVJS, the situation is more complicated. In a period of 23.173 years, a favorable launch occasion happens with 12.978° maximum misalignment with respect to the Jupiter's location. This misalignment angle is not very serious, for either a Venus-Jupiter leg or an

Table 4. Comparison of Δv with conventional techniques (km/sec).

mission	direct launch	SJS	MEVS MEVJS
M1	12.34	8.76	2.50
M2	29.78	10.67	2.50
M3	42.12	11.10	2.50
M4	57.57	13.34	2.50

Table 5. Comparison of flight time with conventional techniques (years).

mission	direct launch	SJS	MEVS	MEVJS
M1	1.108	2.731	10.593	5.225
M2	0.177	3.448	33.272	8.994
M3	0.250	3.713	*	10.092[1]
M4	0.400	3.565	*	26.771[2]

[1] with 1.762 AU semilatus rectum.
[2] with 0.692 AU perihelion distance.

Earth-Jupiter leg is able to cancel it. In order to shorten a long waiting time, one proposal here is to use time adjusting complete revolution(s) before first encountering Venus. Also, there are plenty of other flight sequences which satisfy the proposed mission, although they may require a slightly longer flight time.

Velocity Correction Maneuvers

Taking into account the Venus inclination, one trajectory, which is generated in a two dimensional planetary model, is recomputed in a three dimensional model. The flight sequence here is 1200002–2100003–1202021–2201014–2101011 with a 3.0 km/sec Earth launch velocity increment, and the initial location of the spacecraft is chosen to be 0°, 60°, and 120° with respect to the ascending node of the Venus orbit. Results show that the Earth launch velocity should be increased by 417 m/sec, 600 m/sec, and 30 m/sec in that order, and 2 out of 12 encounters require 105 m/sec and 175 m/sec velocity corrections.

6. Conclusions

Several single/multiple swingby techniques were investigated. Among them, MEVS, MEVJS, and SJS turned out to have a great potential for achieving trajectories which were not possible without swingbys. Especially, MEVS and MEVJS were considered as a sort of gravitational resonance process. By using either of them, the semimajor axis was controlled from $-\infty$ to ∞ except for the region of magnitude below half a Venus orbital radius. Eccentricity took any value above 0.083, inclination varied between 0° and 180°. In addition, SJS was also very useful; however, about 9.0 km/sec was at least required to reach Jupiter.

In general, it became difficult to find the optimal swingby sequence with an increasing number of gravity assist planets. When only one planet was used, FDP was successfully applied to obtain a favorable swingby sequence. However, when more than two planets were involved, computations became quite time consuming and then a simplified control strategy was proposed. This strategy consisted of several control phases, and in each phase FDP was partially applied. For a given launch velocity increment, a criterion function was defined as the total flight time.

Four interplanetary mission designs (M1 to M4) were presented. Note that the initial location of planets was arbitrarily chosen. Results show that missions with MEVS took considerably longer flight time (10.793 years, and 33.272 years for M1, and M2 respectively, and 1.108 years and 0.177 years for a case of direct launch), although the launch velocity increment was quite small (12.34 km/sec, and 29.78 km/sec for M1 and M2 without swingbys, whereas only 2.978 km/sec was required both for M1, and M2 with MEVS). On the other hand, missions using MEVJS with the same launch velocity increment shortened the flight time (5.225 years for M1, and 8.994 years for M2). Also, with the use of MEVJS, M3 with 1.762 AU semilatus rectum and M4 with 0.692 AU perihelion distance were achieved within 10.092 years and 26.771 years, respectively. Through the use of SJS, taking about $3 \sim 4$ year flight time, all four missions were achieved with $9 \sim 14$ km/sec. Eventually, the problem became a trade-off between launch velocity increment, flight time, and available launch window. In any case, by increasing launch velocity increment, flight time could be reduced.

As for further research, it is recommended to use a more rigorous planetary model including inclination, eccentricity, and the real calendar of the solar system. Preliminary error analyses revealed that due of the Venus inclination, at most a few 100's m/sec velocity correction was required at some encounters.

References

[1] Toru Tanabe, and Hiroki Yokota: 'Optimal Multiple Venus Swingby Sequences in Solar Probe Missions', The 31st IAF Congress, Paper No. 80-E-212, Tokyo, Japan, September 1980.

[2] David F. Bender: 'Out of Ecliptic Missions Using Venus or Earch Gravity Assists', AIAA Paper No. 76-189, January 1976.

[3] James E. Randolph: 'To Encounter a Star—The Solar Probe Mission', AAS Paper No. 79-118, 1979.