Dynamics of Satellites with Radial Wire Antennas

By

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Summary: Analysis is given for the dynamics of a spinning satellite with four radial wire antennas. In this analysis, generalized model is used, in which, mass properties of wire antennas are not equal and antennas' attachment points on the satellite have some amounts of offsets from the satellite CG (center of gravity) plane.

Each antenna is modeled as a physical pendulum. They have two degrees of freedom: in-spin plane and out-of-plane motion. Lagrangian with 14 degrees of freedom is obtained, and then equations of motion are derived. A set of equations can be divided into three groups. From a modal analysis for each group, the effects of wire antennas asymmetry and their offsets on the satellite attitude motion is revealed.

Numerical simulation results of GEOTALL satellite are also given.

1. INTRODUCTION

GEOTALL satellite is to be launched into a 8–200 earth radius double lunar swing-by orbit in the second quarter of 1992. This satellite is spin stabilized and has four long (50 meters) radial wire antennas to measure the electric field of geomagnetic tail region. Thruster operations after the deployment of the wire antennas are scheduled to adjust its orbit for lunar swing-by and to maintain its spin-axis perpendicular to the ecliptic plane.

The dynamics analysis for four equally spaced wire antennas with the same mass property in the CG plane is given in reference [*1*] and [*2*]. For GEOTALL satellite, antenna configuration is different from those treated in the references, i.e., four wire antennas are not the same in mass properties and have offsets from CG plane. In the analysis presented here, more generalized model is used to study the effect of asymmetric configuration of wire antennas and their CG-offsets.

2. EQUATION OF MOTION OF WIRES AND SATELLITE BODY

A satellite is modeled as a cylindrical rigid hub with four radial wire antennas. Antennas' configuration is assumed as follows.

1) Two opposing antennas are the same in mass properties, but between two pairs the mass properties are not the same.

2) Four antennas are located apart from satellite CG plane.

Fig. 2–1 gives the nomenclature used, where X_B, Y_B are nominal antenna

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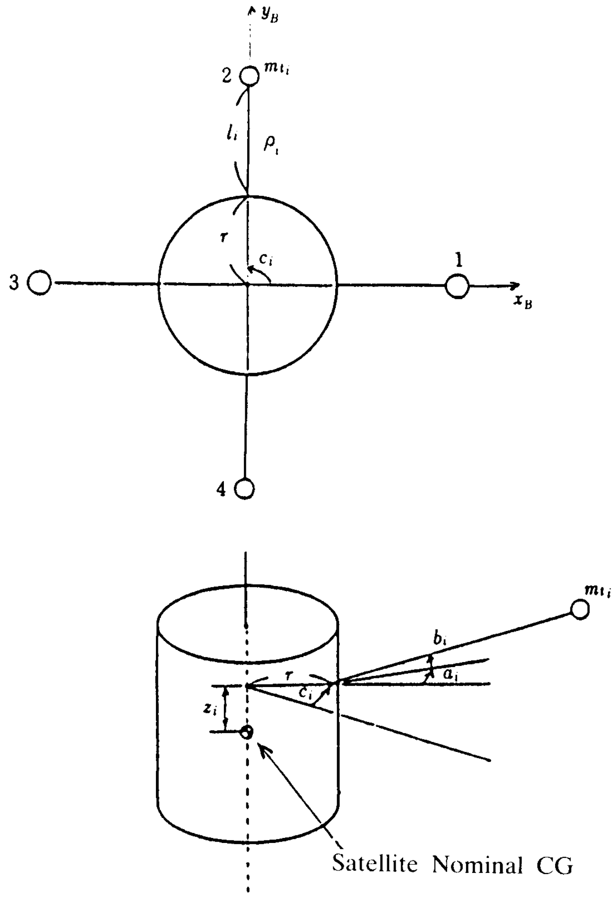

Fig. 2-1. Satellite model and symbols used.

directions, Z_B is satellite spin axis.

The Lagrangian T for this system is

$$T=\frac{1}{2}\int \rho\left(\frac{dR}{dt}\right)_I^2 dv \tag{1}$$

$$R=R_B+r, \tag{2}$$

where R_B is the vector from the origin of an inertial frame to satellite CG, r is the vector from satellite CG to a satellite element dv, and subscript I indicates the differentiation is performed in inertial frame.

The equations of motion in Lagrangian notation are

$$\begin{gathered}
\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{x}_B}\right)-\frac{\partial T}{\partial x_B}=0 \quad \frac{d}{dt}\left(\frac{\partial T}{\partial \dot{y}_B}\right)-\frac{\partial T}{\partial y_B}=0 \quad \frac{d}{dt}\left(\frac{\partial T}{\partial \dot{z}_B}\right)-\frac{\partial T}{\partial z_B}=0 \\
\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{\theta}_1}\right)-\frac{\partial T}{\partial \theta_1}=0 \quad \frac{d}{dt}\left(\frac{\partial T}{\partial \dot{\theta}_2}\right)-\frac{\partial T}{\partial \theta_2}=0 \quad \frac{d}{dt}\left(\frac{\partial T}{\partial \dot{\theta}_3}\right)-\frac{\partial T}{\partial \theta_3}=0 \\
\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{a}_i}\right)-\frac{\partial T}{\partial a_i}=0 \quad \frac{d}{dt}\left(\frac{\partial T}{\partial \dot{b}_i}\right)-\frac{\partial T}{\partial b_i}=0 \quad (\text{for } i=1,\ 4)
\end{gathered} \tag{3}$$

When one introduces mode variables as below,

$$\begin{aligned} &\alpha_{\oplus 1}=\frac{a_1+a_3}{2} && \alpha_{\ominus 1}=\frac{a_1-a_3}{2} && \beta_{\oplus 1}=\frac{b_1+b_3}{2} && \beta_{\ominus 1}=\frac{b_1-b_3}{2} \\ &\alpha_{\oplus 2}=\frac{a_2+a_4}{2} && \alpha_{\ominus 2}=\frac{a_2-a_4}{2} && \beta_{\oplus 2}=\frac{b_2+b_4}{2} && \beta_{\ominus 2}=\frac{b_2-b_4}{2} \end{aligned} \tag{4}$$

(The physical meanings of these variables are shown in Fig. 2–2.)
select inertial frame so that momentum be $P_x=P_y=P_z=0$, and eliminate X_B, Y_B, z_B, the following equations of motion in three groups are obtained.

1) Equations about ω_x, ω_y, $\alpha_{\ominus i}$ and $\beta_{\ominus i}$

$$I_x\dot{\omega}_x+(I_z-I_y)\omega_0\omega_y=2m_1^{(2)}l_1z'_1(\ddot{\alpha}_{\ominus 1}-\omega_0^2\alpha_{\ominus 1})-4m_2^{(2)}l_2z'_2\dot{\alpha}_{\ominus 2}\omega_0-2m_2^{(2)}l_2r\ddot{\beta}_{\ominus 2} \tag{5}$$

$$I_y\dot{\omega}_y-(I_z-I_x)\omega_0\omega_x=2m_2^{(2)}l_2z'_2(\ddot{\alpha}_{\ominus 2}-\omega_0^2\alpha_{\ominus 2})+4m_1^{(2)}l_1z'_1\dot{\alpha}_{\ominus 1}\omega_0+2m_1^{(2)}l_1r\ddot{\beta}_{\ominus 1} \tag{6}$$

$$\begin{aligned} &\left(m_1^{(3)}-2\frac{m_1^{(2)2}}{M}\right)l_1\ddot{\alpha}_{\ominus 1}+4\frac{m_1^{(2)}m_2^{(2)}}{M}l_2\omega_0\dot{\alpha}_{\ominus 2} \\ &\quad+m_1^{(2)}\left(r+\frac{2m_1^{(2)}}{M}l_1\right)\omega_0^2\alpha_{\ominus 1}=m_1^{(2)}z'_1(\dot{\omega}_x-\omega_0\omega_y) \end{aligned} \tag{7}$$

$$\begin{aligned} &\left(m_2^{(3)}-2\frac{m_2^{(2)2}}{M}\right)l_2\ddot{\alpha}_{\ominus 2}-4\frac{m_1^{(2)}m_2^{(2)}}{M}l_1\omega_0\dot{\alpha}_{\ominus 1} \\ &\quad+m_2^{(2)}\left(r+\frac{2m_2^{(2)}}{M}l_2\right)\omega_0^2\alpha_{\ominus 2}=m_2^{(2)}z'_2(\dot{\omega}_y+\omega_0\omega_x) \end{aligned} \tag{8}$$

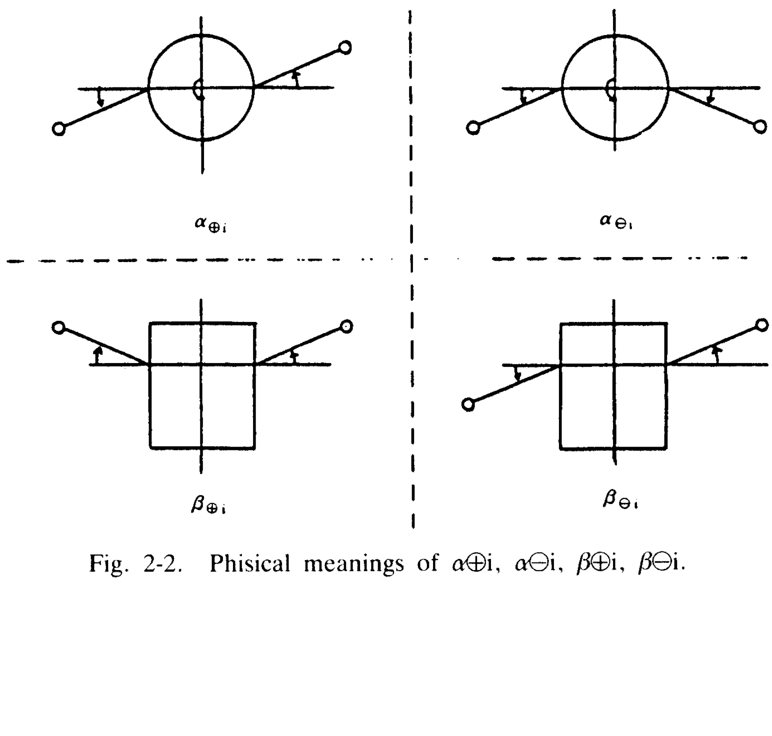

Fig. 2-2. Phisical meanings of $\alpha\oplus$i, $\alpha\ominus$i, $\beta\oplus$i, $\beta\ominus$i.

$$m_1^{(3)}l_1\ddot{\beta}_{\ominus 1}+(m_1^{(3)}l_1+m_1^{(2)}r)\omega_0^2\beta_{\ominus 1}=(m_1^{(3)}l_1+m_1^{(2)}r)(\dot{\omega}_y-\omega_0\omega_x) \tag{9}$$

$$m_2^{(3)}l_2\ddot{\beta}_{\ominus 2}+(m_2^{(3)}l_2+m_2^{(2)}r)\omega_0^2\beta_{\ominus 2}=-(m_2^{(3)}l_2+m_2^{(2)}r)(\dot{\omega}_x-\omega_0\omega_y) \tag{10}$$

where, ω_0 is an unperturbed spin rate of the satellite, and

$$M=m_B+\sum_i^4 (m_{ti}+\rho_i l_i) \tag{11}$$

$$m_i=m_{ti}+\rho_i l_i \qquad m_i^{(2)}=m_{ti}+\frac{1}{2}\rho_i l_i \qquad m_i^{(3)}=m_{ti}+\frac{1}{3}\rho_i l_i \tag{12}$$

$$I_x=I_{B1}+2\sum_i^2 m_i z_i^2-\left(2\sum_i^2 m_i z_i\right)^2/M+2(m_2^{(2)}l_2+m_2 r)r \tag{13}$$

$$I_y=I_{B1}+2\sum_i^2 m_i z_i^2-\left(2\sum_i^2 m_i z_i\right)^2/M+2(m_1^{(2)}l_2+m_1 r)r \tag{14}$$

$$I_z=I_{B3}+2\sum_i^2 (m_i^{(2)}l_i+m_i r)r \tag{15}$$

$$Z_i'=z_i-2\sum_i^2 m_i z_i/M \tag{16}$$

2) Equations about $\delta\theta_3$ and $\alpha_{\oplus i}$

$$I_3\delta\ddot{\theta}_3+2\sum_i^2 (m_i^{(3)}l_i+m_i^{(2)}r)l_i\ddot{\alpha}_{\oplus i}=0 \tag{17}$$

$$\left(m_1^{(3)}-\frac{2(m_1^{(3)}l_1+m_1^{(2)}r)^2}{I_3}\right)l_1\ddot{\alpha}_{\oplus 1} -\frac{2(m_1^{(3)}l_1+m_1^{(2)}r)(m_2^{(3)}l_2+m_2^{(2)}r)}{I_3}l_2\ddot{\alpha}_{\oplus 2}+m_1^{(2)}\mathrm{r}\omega_0^2\alpha_{\oplus 1}=0 \tag{18}$$

$$\left(m_2^{(3)}-\frac{2(m_2^{(3)}l_2+m_2^{(2)}r)^2}{I_3}\right)l_2\ddot{\alpha}_{\oplus 2} -\frac{2(m_1^{(3)}l_1+m_1^{(2)}r)(m_2^{(3)}l_2+m_2^{(2)}r)}{I_3}l_1\ddot{\alpha}_{\oplus 1}+m_2^{(2)}\mathrm{r}\omega_0^2\alpha_{\oplus 2}=0 \tag{19}$$

3) Equations about $\beta_{\oplus i}$

$$\left(m_1^{(3)}-\frac{2m_1^{(2)2}}{M}\right)l_1\ddot{\beta}_{\oplus 1}-2\frac{m_1^{(2)}m_2^{(2)}}{M}l_2\ddot{\beta}_{\oplus 2}+(m_1^{(3)}l_1+m_1^{(2)}r)\omega_0^2\beta_{\oplus 1}=0 \tag{20}$$

$$\left(m_2^{(3)}-\frac{2m_2^{(2)2}}{M}\right)l_2\ddot{\beta}_{\oplus 2}-2\frac{m_1^{(2)}m_2^{(2)}}{M}l_1\ddot{\beta}_{\oplus 1}+(m_2^{(3)}l_2+m_2^{(2)}r)\omega_0^2\beta_{\oplus 2}=0 \tag{21}$$

The first group (5)–(10) relates the hub coning motion and the coupled in-plane and out-of-plane wire antenna motions. $\alpha_{\ominus i}$ mode variables and the hub coning motion are coupled through the antenna CG-offsets Z_{is}'. The second group (17)–(19) relates the hub spin ripple motions and in-plane wire antenna motions. The third group (20), (21) describes the out-of-plane wire antenna motions which are independent of the hub attitude coning and spin ripple motions.

If mass properties of four wire antennas are equal to each other and their locations are in the satellite CG plane, the equations of motion listed above are transformed to those given in reference [*1*].

3. Analysis of Satellite Motion

To examine the satellite motion, model analyses are tried for these three equation groups.

1) ω_x, ω_y, $\alpha_{\ominus i}$ and $\beta_{\ominus i}$ (center hub coning motion and wire antenna in-plane and out-of-plane motion)

In most satellites, antenna mass is much smaller than total satellite mass, i.e.,

$$m_i^{(2)}/M \ll 1 \tag{22}$$

If ignore above terms in equations (5)–(10), and assume ω_x, ω_y, $\alpha_{\ominus i}$, $\beta_{\ominus i}$ to oscillate as exp [iΩt], following four relations between the amplitude of the mode variables can be derived.

$$\alpha_{\ominus 1}=\frac{-\zeta_1}{x-x_1}(i\sqrt{x}\,\omega_x-\omega_y)/\omega_0 \tag{23}$$

$$\alpha_{\ominus 2}=\frac{-\zeta_2}{x-x_2}(i\sqrt{x}\,\omega_y+\omega_x)/\omega_0 \tag{24}$$

$$\beta_{\ominus 1}=\frac{-x_3}{x-x_3}(i\sqrt{x}\,\omega_y-\omega_x)/\omega_0 \tag{25}$$

$$\beta_{\ominus 2}=\frac{x_4}{x-x_4}(i\sqrt{x}\,\omega_x+\omega_y)/\omega_0 \tag{26}$$

where

$$x=(\Omega/\omega_0)^2,\ x_1=m_1^{(2)}r/m_1^{(3)}l_1,\ x_2=m_2^{(2)}r/m_2^{(3)}l_2,\ x_3=1+x_1,\ x_4=1+x_2$$
$$\zeta_1=m_1^{(2)}z_1'/m_1^{(3)}l_1,\ \zeta_2=m_2^{(2)}z_2'/m_2^{(3)}l_2 \tag{27}$$

Substituting them into equations (5), (6) of the center hub coning motion, following characteristic equation can be derived.

$$\begin{aligned}f(x)=&\left(I_z-I_y+\varepsilon_1\frac{x+1}{x-x_1}+2\varepsilon_2\frac{x}{x-x_2}-\delta_2\frac{x}{x-x_4}\right)\\&\times\left(I_z-I_x+\varepsilon_2\frac{x+1}{x-x_2}+2\varepsilon_1\frac{x}{x-x_1}-\delta_1\frac{x}{x-x_3}\right)\\&-x\left(I_x-\varepsilon_1\frac{x+1}{x-x_1}-2\varepsilon_2\frac{1}{x-x_2}-\delta_2\frac{x}{x-x_4}\right)\\&\times\left(I_y-\varepsilon_2\frac{x+1}{x-x_2}-2\varepsilon_1\frac{1}{x-x_1}-\delta_1\frac{x}{x-x_3}\right)=0\end{aligned} \tag{28}$$

where

$$\varepsilon_1=2m_1^{(3)}l_1^2\zeta_1^2,\ \ \varepsilon_2=2m_2^{(3)}l_2^2\zeta_2^2,\ \ \delta_1=2m_1^{(2)}l_1rx_3,\ \ \delta_2=2m_2^{(2)}l_2rx_4 \tag{29}$$

Function f(x) has the form shown in Fig. 3–1, Five roots of f(x), $\lambda_1 \ldots \lambda_5$, correspond to five natural frequency modes.

Two low frequency modes are the $\alpha_{\ominus 1}$ and $\alpha_{\ominus 2}$ predominating modes. These are excited by satellite radial translations force during radial ΔV thrusting. Intermediate frequency mode is the "nutation" mode. The center hub is nutating, but the wire antennas almost stand still. Two high frequency modes are $\beta_{\ominus 1}$ and $\beta_{\ominus 2}$ predominating modes. These modes are excited by the spin axis precession torque as well as the "nutation" mode during satellite attitude maneuver. These five mode shapes are shown in Fig. 3–2/

The static stability condition is derived from Fig. 3–1. For f(x) to have real roots, following inequality should be held, which limits the amounts of wire antennas offset from satellite CG plane.

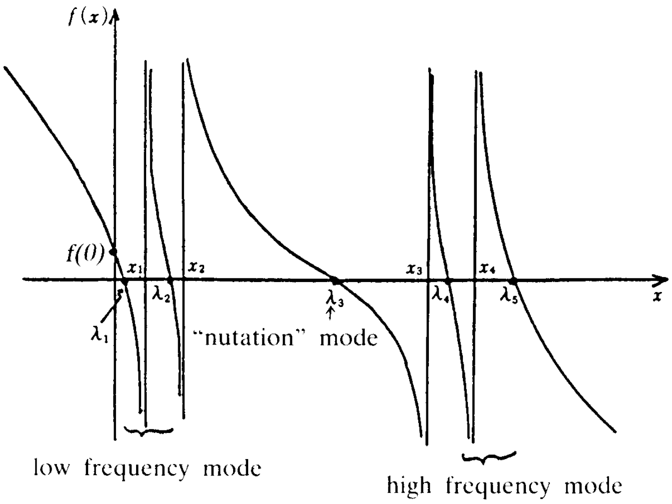

Fig. 3-1. Form of f(x) for ω_x, ω_y, $\alpha\Theta$, $\beta\Theta$.

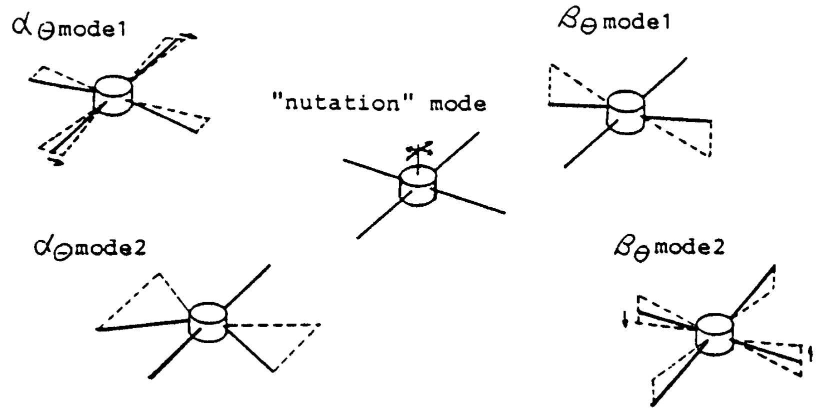

Fig. 3-2. Mode shapes of ω_x, ω_y, $\alpha\Theta$, $\beta\Theta$.

$$f(0)=\left\{(I_z-I_y)-\frac{\varepsilon_1}{x_1}\right\}\left\{(I_z-I_x)-\frac{\varepsilon_2}{x_2}\right\}>0 \tag{30}$$

2) $\delta\theta_3$and $\alpha_{\oplus i}$ (spin ripple and wire antenna in-plane motion)

From equations of motion (7)–(19), the following characteristic equation can be derived.

$$\begin{aligned} f(x)=&\left\{\left(m_1^{(3)}-\frac{2(m_1^{(3)}l_1+m_1^{(2)}r)^2}{I_3}\right)l_1x-m_1^{(2)}r\right\}\\ &\times\left\{\left(m_2^{(3)}-\frac{2(m_2^{(3)}l_1+m_2^{(2)}r)^2}{I_3}\right)l_2x-m_2^{(2)}r\right\}\\ &-\left\{\frac{2(m_1^{(3)}l_1+m_1^{(2)}r)(m_2^{(3)}l_2+m_2^{(2)}r)}{I_3}\right\}^2l_1l_2x^2=0 \end{aligned} \tag{31}$$

Two natural frequency modes exist in this group, whose mode shapes are shown in Fig. 3–3. In the low frequency mode, two wire antenna pairs oscillate in opposite direction ("counter phase mode"), whereas in the high frequency mode, they oscillate in the same direction ("in phase mode"). Both modes are excited by spin up or down maneuver and accompanied by the hub spin ripple motion. In the "counter phase mode", however, the satellite spin ripple motion which is due to the effect of the antenna asymmetry is, in general, very small. If four antennas have the same mass properties, the spin ripple motion vanishes in the "counter phase mode".

3) $\beta_{\oplus i}$ (wire antenna out-of-plane motion)

From equations of motion (20), (21), characteristic equation of this motion is derived as follows.

$$\begin{aligned} f(x)=&\left\{\left(m_1^{(3)}-\frac{2m_1^{(3)2}}{M}\right)l_1x-(m_1^{(3)}l_1+m_1^{2}r)\right\}\\ &\left\{\left(m_2^{(3)}-\frac{2m_2^{(3)2}}{M}\right)l_2x-(m_2^{(3)}l_2+m_2^{(2)}r)\right\}\\ &-\left\{\frac{2m_1^{(2)}m_2^{(2)}}{M}\right\}^2l_1l_2x^2=0 \end{aligned} \tag{32}$$

The mode shapes are shown in Fig. 3–4. As in the case of second equation group, there are two modes: "in phase mode" and "counter phase mode". Both modes are excited by a satellite axial translation force and are not coupled to the hub attitude

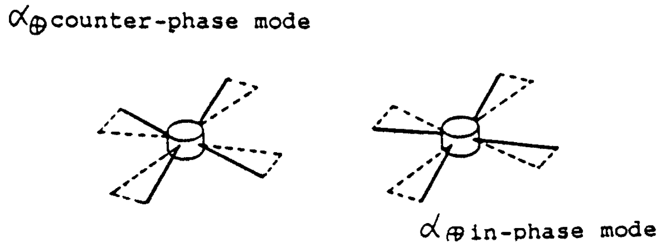

Fig. 3-3. Mode shapes of $\beta\oplus$.

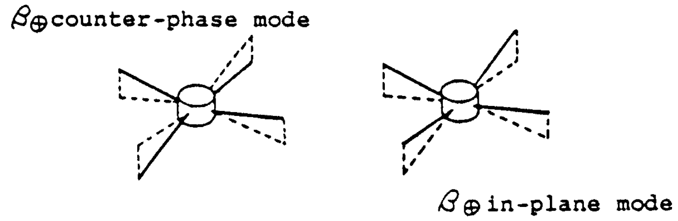

Fig. 3-4. Mode shapes of $\beta\oplus$.

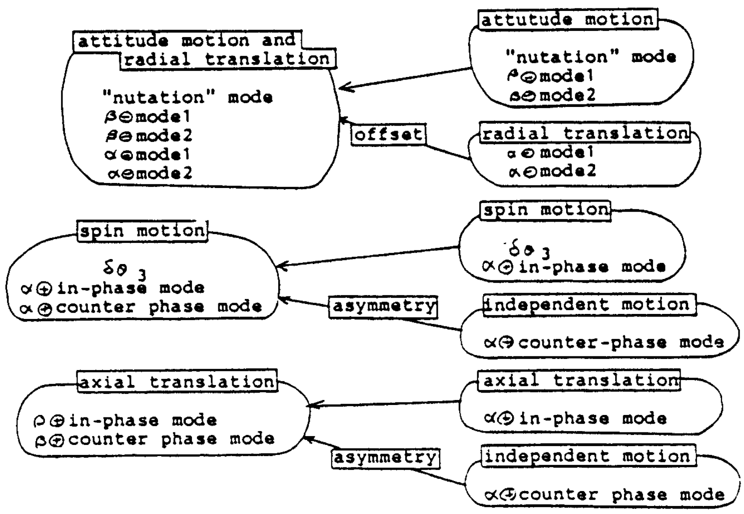

Fig. 3-5. Effect of antenna CG-offset and antenna asymmetry on natural frequency modes.

motions. As the excitation of "counter phase mode" during axial ΔV thrusting is brought by the effect of antenna asymmetry, its amplitude is very small. If four antennas have the same mass properties, the "counter phase mode" becomes independent of the axial translation motions of the center hub.

The effect of the antenna CG-offset and asymmetry on the satellite dynamics in these equations groups are summarized in Fig. 3–5. Owing to the antenna CG-offset, radial translation of the satellite and wire antenna $\alpha_{\ominus}$ motions are couple to the center *hub coning*. From the effect of wire antenna asymmetry, the "counter phase mode" of $\alpha_{\oplus i}$ is coupled to the spin ripple motion, and that of $\beta_{\oplus i}$ to the axial translation motion.

4. Example of The Numerical Simulation for Geotall Satellite

To confirm the analytical results, numerical simulations are performed. The parameters used here are those of GEOTAIL satellite, which are listed in Table. 4–1. The antenna configuration of GEOTAIL satellite is shown in Fig. 4–1.

The motions of center hub and wire antennas during precession maneuver with axial thrusters are shown in Fig. 4–2, where from 20 sec to 80 sec the maneuver is ON. Wire antenna $\beta_{\ominus i}$ mode variables oscillate at the rate close to the spin frequency, and

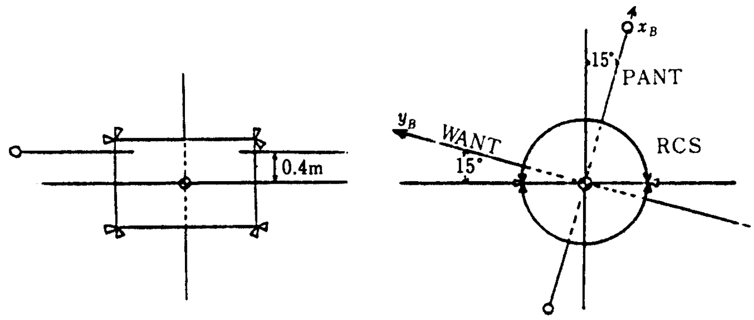

Fig. 4-1. Antenna configuration of GEOTAIL satellite.

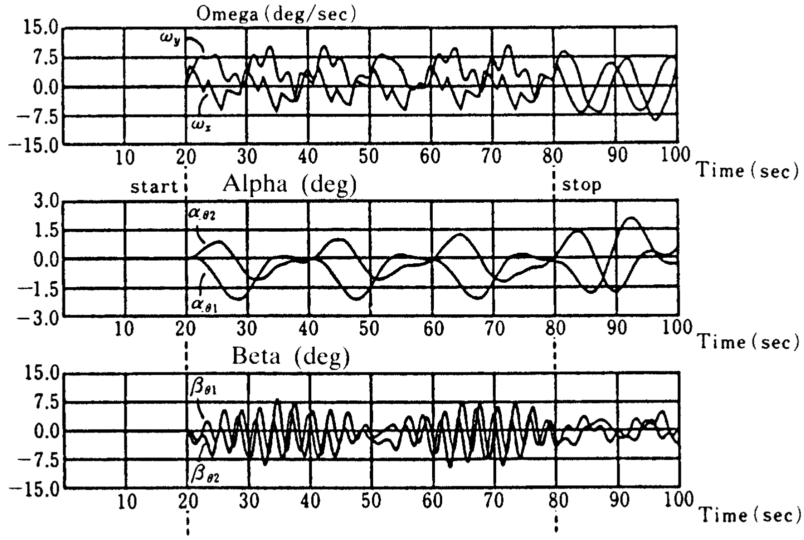

Fig. 4-2. Simulation result during precession maneuver.

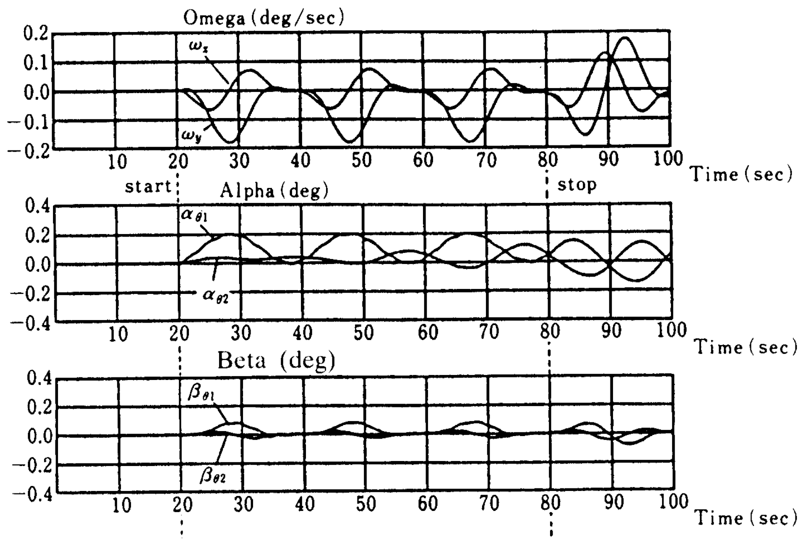

Fig. 4-3. Simulation result during radial ΔV.

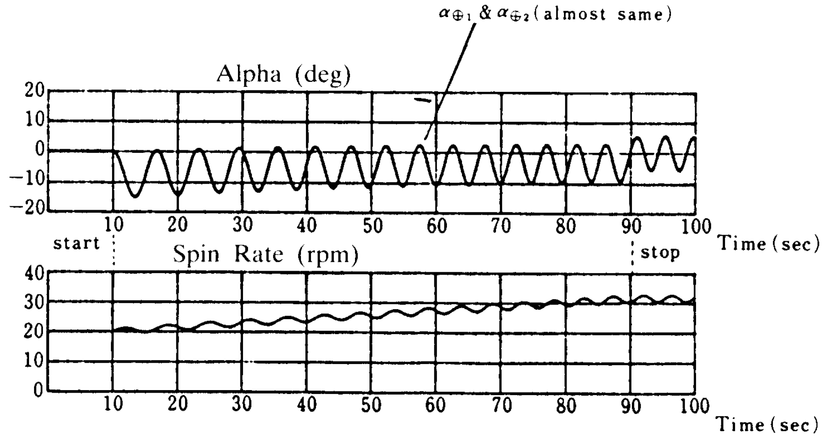

Fig. 4-4. Simulation result during spin-up.

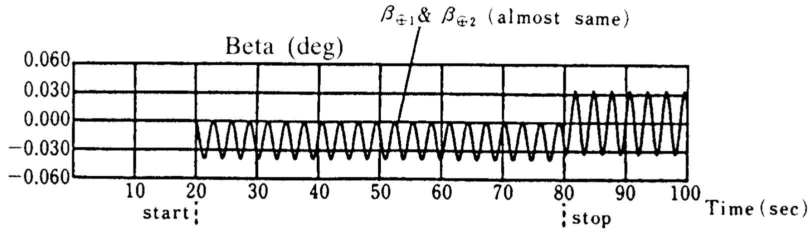

Fig. 4-5. Simulation result during axial ΔV.

$\alpha_{\ominus i}$ mode variables oscillate at lower frequencies.

A simulation result during radial ΔV thrusting is shown in Fig. 4–3. $\alpha_{\oplus i}$ mode variables are excited by the spin up torque and oscillates at its characteristic frequency. The center hub spin-rate also oscillates with the same frequency by the coupling with $\alpha_{\ominus i}$. The characteristic frequency is gradually increased due to the center hub spin-up.

A simulation result during axial ΔV thrusting is shown in fig. 4–5. Only the wire antenna mode is excited. The oscillations during the thrusting are biased by the force applied to the center hub. Natural frequency modes follows after the completion of thrusting, the amplitudes of which depend on the thruster-OFF timing in the foregoing ON-thrusting oscillation.

5. Conclusions

From the analysis mentioned above, several features of dynamical behavior of the satellite with four radial wire antennas are revealed.

The antenna CG-offsets introduce the coupling between the satellite translation motion and hub coning motion through the oscillations of antennas. Beside that, CG-offsets are closely related to the satellite stability condition, i.e., large offsets greater than the level determined from the center hub and wire antennas mass properties will destabilize the satellite attitude.

The antenna asymmetry couples the two antenna oscillation modes which are independent of the hub attitude motions for the symmetric antenna configuration, with the hub spin ripple and hub axial translation motions. However, the couplings thus introduced by the antenna asymmetry are very weak in the case where each antenna mass is much smaller than the total satellite mass as is the case for most of the satellites.

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[2] Fedor J. V., "Wire antenna motion damper for IMP-J spacecraft", 1985 X-732-73-293.