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Navier-Stokes Simulation of Shock-Tube Flows over a Wedge-Type Step by Finite Volume Method

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Summary: Shock-tube flows over a wedge-type step are simulated numerically by the finite volume method with quadrilateral cells. The two-dimensional, time-dependent Navier-Stokes equations are solved. Time development of flow fields and pressure histories on walls are obtained and shown to be in good qualitative agreement with experiments.

1. INTRODUCTION

Numerical simulations of shock-tube flows over a step have been performed by using the Euler equations in most cases *[1]*. However, Navier-Stokes simulations are necessary when the viscous effects are important. To study shock-tube flows over a forward-facing step, Yamamoto, Hatakeyama and Oguchi *[2]* solved the Navier-Stokes equations by the finite volume method with rectangular cells. The time-dependent flow features calculated were compared with experiments and shown to be in good qualitative agreement. As the finite volume method allows us to deal with complicated flow geometries in physical coordinates, extensions of the work done by Yamamoto *et. al.* to flows over steps with more generalized geometries may be straightforward. As an example, in this paper we consider two-dimensional, unsteady, shock-tube flows over a wedge-type step. The Navier-Stokes equations are solved by the finite volume method with arbitrary quadrilateral cells, and results are compared with experiments.

2. BASIC EQUATIONS AND METHOD OF SOLUTION

2.1 *Navier-Stokes equations*

According to Yamamoto *et. al., [2]* the change of the conserved quantities for a region $\partial\Omega$ in a time interval δt is expressed as

$$\delta\int_{\Omega} Q\, d\Omega = -\int_{t}^{t+\delta t}\left[\int_{\partial\Omega} \mathbf{g}\cdot\mathbf{n}\, d\partial\Omega\right] dt \tag{1}$$

where Q is the conserved quantities, **g** the flux vector, and **n** a unit vector normal to the boundary. In the Cartesian coordinates (x, y), **g** is divided into two components:

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$$\mathbf{g} = \mathbf{E}\mathbf{i} + \mathbf{F}\mathbf{j}$$

where **i** and **j** are unit vectors in the x- and y-directions, respectively. The two-dimensional, unsteady Navier-Stokes equations can be written in integral form as

$$\frac{\partial}{\partial t}\iint_{\Omega} Q\ dxdy+\int_{\partial\Omega}(Edy-Fdx)=0 \tag{2}$$

and

$$Q=\begin{pmatrix}\rho\\ \rho u\\ \rho v\\ e\end{pmatrix} \tag{3.1}$$

$$E=\begin{pmatrix}\rho u\\ \rho u^2+p-(2\beta_1+\beta_2)u_x-\beta_2 v_y\\ \rho uv-\beta_1 v_x-\beta_1 u_y\\ (e+p)u-(2\beta_1+\beta_2)uu_x-\beta_1 vv_x-\beta_3 T_x-\beta_1 vu_y-\beta_2 uv_y\end{pmatrix} \tag{3.2}$$

$$F=\begin{pmatrix}\rho v\\ \rho uv-\beta_1 u_y-\beta_1 v_x\\ \rho v^2+p-(2\beta_1+\beta_2)v_y-\beta_2 u_x\\ (e+p)v-(2\beta_1+\beta_2)vv_y-\beta_1 uu_y-\beta_3 T_y-\beta_1 uv_x-\beta_2 vu_x\end{pmatrix} \tag{3.3}$$

In the above expressions, p, ρ, u, v, T and e are the pressure, density, velocity components, temperature and total energy. For a perfect gas, $e=p/(\gamma-1)+\rho(u^2+v^2)/2$. We divide the physical domain into a number of quadrilateral cells. If s and n are defined as coordinates along and normal to a boundary of each cell, the following relation between the coordinates (x, y) and (s, n) may hold (Fig. 1). For example,

$$\frac{\partial u}{\partial s}\Delta s=\left[\frac{\partial u}{\partial x}\frac{\partial x}{\partial s}+\frac{\partial u}{\partial y}\frac{\partial y}{\partial s}\right]\Delta s=\frac{\partial u}{\partial x}\Delta x+\frac{\partial u}{\partial y}\Delta y \tag{4.1}$$

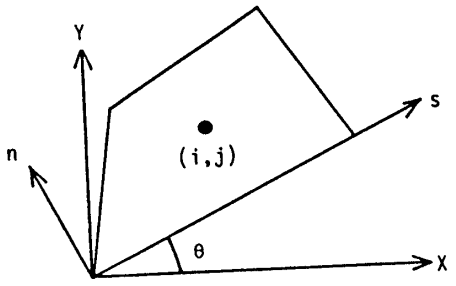

Fig. 1. Transformation of coordinates from (x, y) to (s, n).

$$\frac{\partial u}{\partial n}\Delta s=\left[\frac{\partial u}{\partial x}\frac{\partial x}{\partial n}+\frac{\partial u}{\partial y}\frac{\partial y}{\partial n}\right]\Delta s=-\frac{\partial u}{\partial x}\Delta y+\frac{\partial u}{\partial y}\Delta x \tag{4.2}$$

From Eq. (4), we obtain

$$\frac{\partial u}{\partial x}=-\sin\theta\frac{\partial u}{\partial n}+\cos\theta\frac{\partial u}{\partial s},\quad \frac{\partial u}{\partial y}=\cos\theta\frac{\partial u}{\partial n}+\sin\theta\frac{\partial u}{\partial s} \tag{5}$$

where θ is the angle between the cell boundary and the x-axis. The similar relation holds for the other flow quantities. Therefore, the Navier-Stokes equations can be rewritten in terms of s and n. With the finite volume concept, the flow quantities may be considered to be constant within each cell, and thus the gradients of the flow quantities along the cell boundary may be neglected (that is, $\partial/\partial s=0$). Therefore, the viscous terms are expressed only by the derivative normal to the cell boundary ($\partial/\partial n$).

Flow quantities are non-dimensionalized as follows.

$$t=t^*a_1/L,\ x=x^*/L,\ y=y^*/L,\ u=u^*/a_1,\ v=v^*/a_1,$$
$$\rho=\rho^*/\rho_1,\ p=p^*/\rho_1a_1^2,\ T=T^*/T_1,\ e=e^*/\rho_1a_1^2.$$

where a is the speed of sound. The asterisk denotes dimensional quantities, and the subscript 1 means the quantities upstream of the shock wave. The coefficients β_1, β_2 and β_3 are,

$$\beta_1=\mu/\mathrm{Re},\ \beta_2=\lambda/\mathrm{Re},\ \beta_3=\kappa/\mathrm{Pr}\cdot\mathrm{Re}\cdot(\gamma-1)$$

where γ is the ratio of the specific heats of the gas. The viscosity coefficients μ and λ are normalized by μ_1 and λ_1, respectively, and heat conductivity coefficient κ by κ_1. The Reynolds number Re and the Prandtl number Pr are defined as,

$$\mathrm{Re}=\rho_1a_1L/\mu_1,\ \mathrm{Pr}=\mu_1c_{p1}/\kappa_1$$

where c_p is the specific heat at constant pressure. For a monatomic gas, $2\beta_1+3\beta_2=0$.

2.2 *Numerical algorithm*

The flow quantities on a cell boundary were estimated by the same method as that proposed by Yamamoto, Hatakeyama & Oguchi *[2]*: for example,

$$\rho_{i-1/2,j}=w_{i-1/2,j}\rho_{i-1,j}+(1-w_{i-1/2,j})\rho_{i,j}$$

where $\rho_{i-1/2,j}$ is the density on the boundary between the cells (i−1, j) and (i, j). The weighting function $w_{i-1/2,j}$ is given by

$$w_{i-1/2,j}=\frac{c_{i-1,j}+|c_{i-1,j}|}{c_{i-1,j}+|c_{i-1,j}|+c_{i,j}+|c_{i,j}|}$$

where characteristic speeds normal to the boundary are expressed as,

$$c_{i-1,j}=\mathbf{u}_{i-1,j}\cdot\mathbf{n}_{i-1/2,j}+a_{i-1,j}$$
$$c_{i,j}=-\mathbf{u}_{i,j}\cdot\mathbf{n}_{i-1/2,j}+a_{i,j}$$

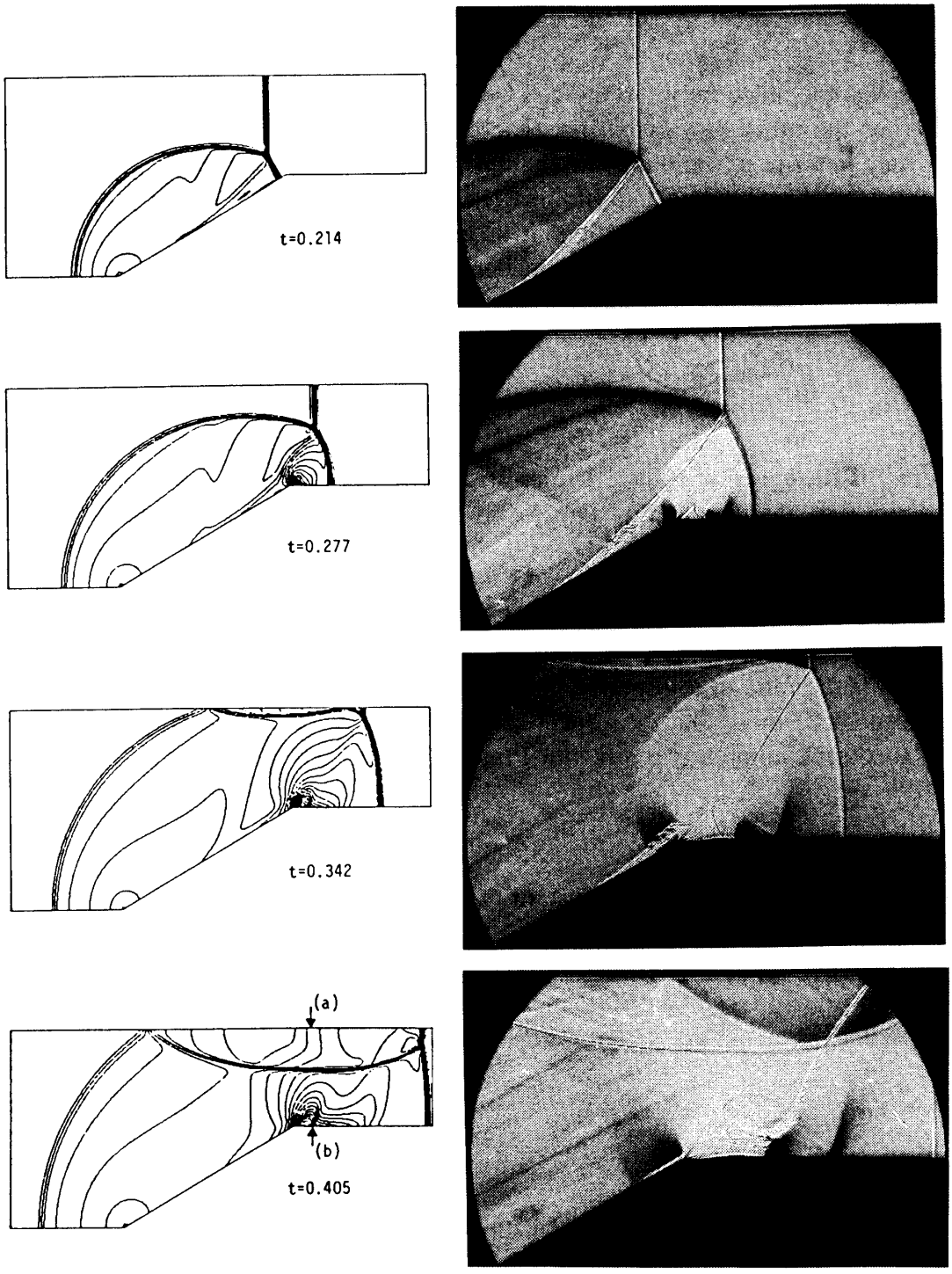

Fig. 2. Comparison of time-developing flow fields. $\theta_w=30°$. M=1.84.

Here $\mathbf{n}_{i-1/2,j}$ is the unit vector normal to the cell boundary.

Calculations are performed on a H-grid generated by solving the Poisson equation *[3]* and having 200 cells in the direction normal to the wall and 600 cells in the direction along the wall. The first-order Euler scheme was used for time integration.

3. Results and Discussion

The time-developing flow fields which are calculated for the case of $\gamma=5/3$ are compared with those obtained experimentally. The wedge angle θ_w was 30°, the shock Mach number 1.84. In the experiment argon was used for a test gas.

In Fig. 2, isopycnics calculated are compared with the Schliren photographs. Because of a small window in our shock-tube, the flow field covered in the Schliren photographs is about half of the computational field. We assumed t=0 when the shock-wave reached the wedge coming from the left hand side. As readily seen from Fig. 2, time-development of the flow field is simulated pretty well, though fine structures such as the roll-up of the slip flow at t=0.277 and the shock-wave of lambda type caused by the flow separation at t=0.277 and 0.342 are not captured by this simulation. The fine structures may be simulated with finer cells.

Pressure histories are also compared. Both in the experiment and in the computation, pressure histories are measured at two different points; one is located on the upper wall (marked by (a) in Fig. 2 at t=0.405) and another on the lower wall (marked by (b)). The results are shown in Fig. 3. On the upper wall both computational and experimental results show a good agreement (Fig. 3a). On the lower wall, quantitative agreement between the computation and the experiment is only fair (Fig. 3b). However, qualitative features of the pressure history which are observed experimentally are well simulated.

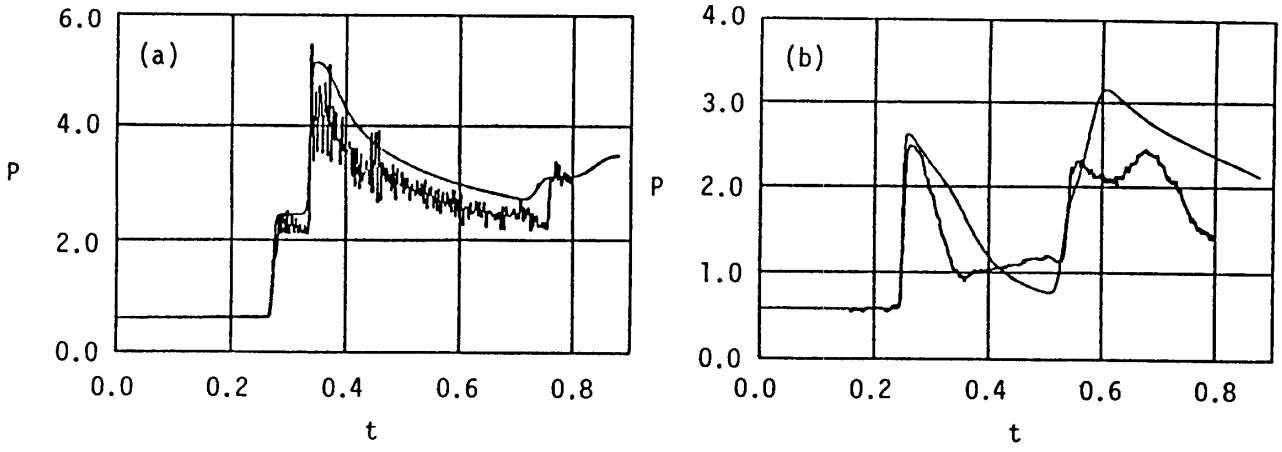

Fig. 3. Comparison of pressure histories. $\theta_w=30°$. M=1.84. — N. S. solution. ~ Experiment. a) upper wall, b) lower wall.

4. Concluding Remarks

The two-dimensional, Navier-Stokes equations are applied to the shock-tube flows over a wedge-type step, and solved by the finite volume method with arbitrary quadrilateral cells. Comparison of calculated results with experiments showed that qualitative features of the time-dependent flow field observed experimentally are well reproduced by the finite volume approach.

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