STUDY ON IDENTIFICATION OF BRIDGE DECK FLUTTER DERIVATIVES BY GUST RESPONSE

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This study presents the method to approach flutter derivatives (FDs) of bridge section from simulation method. The more challenging is the application the system identification method to extract FDs from the stochastic vibration technique via simulated buffeting responses for section model. The flow analysis includes the investigation on: the fluctuated wind velocities were simulated from target power spectrum, the buffeting responses of a bridge section model obtained from numerical dynamic solution at different mean wind speeds. Next, the gust responses data has been analysis by the system identification technique in extracting FDs and investigate the difficulties involved in this method are discussed. The time domain analysis of gust response, stochastic system identification is proposed to estimate two degree of freedom systems. Finally, some adverse effects of gust response data on the accuracy of these methods to obtain FDs was discussed and concluded. The result of the study can clarify the effects of turbulence on FDs and further apply to estimate FDs from response of full-scale under buffeting load.

Keyword: Flutter[1], Flutter derivative[2], System identification[3], Gust response[4], Turbulence[5]

1. INTRODUCTION

For the slender and flexible structures, such as long-span cable supported bridges are very sensitive to wind excitation. Base on aerodynamic the flutter and buffeting is vital important problem of great concern. The predictions of flutter instability and buffeting response related to flutter derivatives (FDs). The wind-tunnel test is the best choice to identify FDs. There are two type techniques for identification FDs of bridge section model, such as force vibration technique, free vibration technique [1]. Force vibration approach is a reliable one but it requires sophisticated driving equipment, somewhat expensive and time consuming. Free vibration approach seems to be more tractable and widely adopted technique but the free vibration response need to use the system identification techniques to extract modal parameters. Various system identifications to extract FDs from wind-tunnel experiment were developed by many authors [2-5], and in these systems the buffeting force and their response consider as external noise so this cause more difficulties at high wind velocity and particularly appears turbulence.

The regarding the turbulent effect on FDs was investigated by some authors. G. Bartoli and M. Righi [5] used CSIM is based on Sarkar MITD [3] to extract simultaneously all FDs from 2DOF (degree of freedom) section model. The conclusion is that identification of flutter derivatives in turbulent flow resulted satisfactory in spite of the difficulties encountered due to the process caused by the locally induced noise owing to signature of turbulence. The main reason is that, the CSIM is the deterministic system identification and the effects of turbulence are regarded as a more noisy-input signal to the system makes more problems in the identification process. N. Nikitas, J.H.G. Macdonal and J.B. Jakobsen [7] are employed to extract FDs from ambient vibration data from full-scale monitoring has been using more elaborate stochastic identification technique (CBHM) [6] and the study also illustrated the viability of system identification techniques for extracting valuable result from full-scale data. V. Boonyapinyo and T. Janesupasaeree [1] applied data-driven stochastic subspace identification technique (SSI-DATA in short) to extract the flutter derivatives of bridge deck from wind tunnel test under smooth and turbulent flow. The conclusion of this paper saying that the SSI-DATA can be used to estimate FDs from buffeting responses with reliable results and an advantage of stochastic system is that its considers the buffeting force and response like input instead of noise. So, the ratio

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of signal to noise is not effect by wind speed and the flutter derivatives at high wind speeds are readily available. From these consideration it was bring to the idea for applying the stochastic system identification (SSI in short) to estimate the FDs from simulated gust responses of bridge deck.

This study concentrates on the buffeting responses to obtain from the numerical method of bridge deck under fluctuated wind excitation. The turbulent wind speed was simulated from target given power spectrum. Afterward, the output only system identification SSI [8] has been applied to extract flutter derivatives.

2. THEORITICAL BACKGOUND

(1) Wind load on a line-like structure

Wind flow consist of a mean time-invariant component in the along wind direction and a fluctuating (turbulence) component in each of the flow direction. The components of flow are defined as: along-wind involve mean time-invariant and horizontal fluctuation U(t)=U+u(t), and vertical fluctuating w(t). Diana (1986) [10] extended the quasi-steady theory by using the relative wind velocity (V_{rel} in short) in the calculation of the forces. The aerodynamic forces acting on the structure including three parts: drag, lift and aerodynamic moment, see fig. 1.

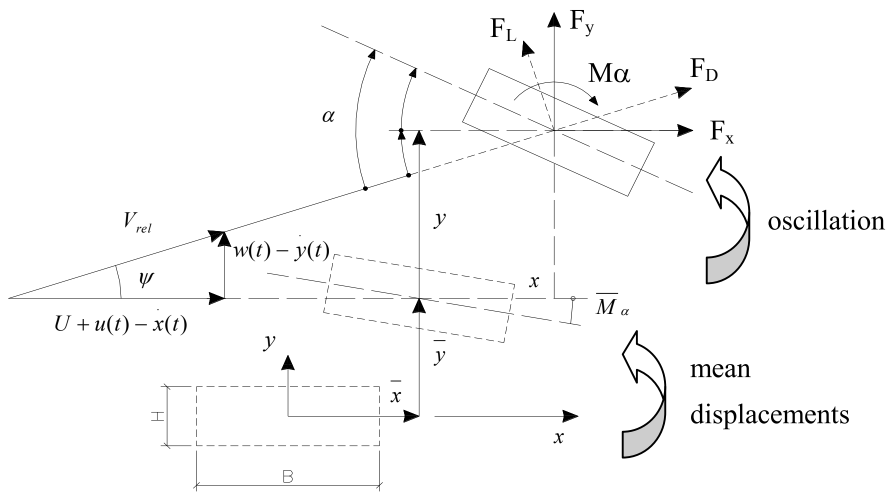

Figure 1: Instantaneous velocity and deck motion

From the quasi-steady theory and considering on the deck motion, the aerodynamic force per unit length can be express in the structural axis system as:

$$F_x = \frac{1}{2}\rho V_{rel}^2 H C_D(\alpha)\cos\psi - \frac{1}{2}\rho V_{rel}^2 B C_L(\alpha)\sin\psi \quad (1)$$

$$F_y = \frac{1}{2}\rho V_{rel}^2 H C_D(\alpha)\sin\psi + \frac{1}{2}\rho V_{rel}^2 B C_L(\alpha)\cos\psi \quad (2)$$

$$M_\alpha = \frac{1}{2}\rho V_{rel}^2 B^2 C_M(\alpha) \quad (3)$$

$$V_{rel}^2 = (U + u(t) - \dot{x}(t))^2 + (w(t) - \dot{y}(t))^2 \quad (4)$$

Where: α is the angle of attack, H is the deck height, B is the deck width, x is the horizontal displacement, y is the vertical displacement, $C_D(\alpha)$, $C_L(\alpha)$, $C_M(\alpha)$ is the draft, lift and moment force coefficient respectively.

Assume the fluctuating wind components and velocities of structure are small compared to mean velocity U, so the higher order terms will neglect. The resulting force contains a part of structural motion x(t)

dot and y(t) dot which is component of damping. These force due to self-excited force which is attributed to the aerodynamic damping.
The equation of motion of bridge deck with 2 DOF: bending mode h and torsion mode α can be written as follow:

$$m[\ddot{h}+2\xi_h\omega_h\dot{h}+\omega_h^2 h]=L_h \tag{5}$$

$$I[\ddot{\alpha}+2\xi_\alpha\omega_\alpha\dot{\alpha}+\omega_\alpha^2\alpha]=M_\alpha \tag{6}$$

where m and I is mass and mass moment of inertial per unit length; $\omega_h=2\pi f_h$ and $\omega_\alpha=2\pi f_\alpha$ is circular frequencies of heaving and pitching mode (in still air); ξ_h and ξ_α is critical damping ratio; L_h (or F_y) and M_α is the lift force and pitching moment per unit length; and the dot denotes derivative with time.
The lift and moment can be split into three parts: mean, buffeting and self-excited forces. By substituting the above equation Eq.2, Eq.3, Eq.4 into the Eq.5 and Eq.6, by moving the aerodynamic damping and stiffness terms to the left hand side Eq.5 and Eq.6 can be rewritten as follow:

$$[M]\{\ddot{q}(t)\}+[C^e]\{\dot{q}(t)\}+[K^e]\{q(t)\}=\{f(t)\}=B_2u(t) \tag{7}$$

where $\{q(t)\}=\{h(t) \quad \alpha(t)\}^T$ = generalized buffeting response; {f(t)} = generalized static and buffeting force; {f(t)} is factorized into matrix B_2 and input vector u(t); [M] = mass matrix; $[C^e]$ = gross damping matrix including the physical damping of structure and aerodynamic damping; $[K^e]$ = gross stiffness matrix.

Solution the Eq.7 by the constant acceleration method (Newmark-β) of numerical integration will obtain the buffeting response of bridge deck.

(2) Stochastic state-space models

The previous second-order of differential equation, Eq.7 is generalized n_2-DOF can be transformed into a first-order the state equation Eq.8.

$$\dot{x}(t)=\begin{Bmatrix}\dot{q}(t)\\ \ddot{q}(t)\end{Bmatrix}=\begin{bmatrix}0_{n_2xn_2} & I_{n_2xn_2}\\ -[M]^{-1}[K^e] & -[M]^{-1}[C^e]\end{bmatrix}_{nxn}\begin{Bmatrix}q(t)\\ \dot{q}(t)\end{Bmatrix}+\begin{bmatrix}0_{n_2xm}\\ [M]^{-1}B_2\end{bmatrix}u(t)$$
$$\dot{x}(t)=A_c x(t)+Bu(t) \tag{8}$$

The combination of the state equation and the observation equation fully describes the input and output behavior of the structural system and is as such named state-space system.

$$\begin{aligned}\dot{x}(t)&=A_c x(t)+Bu(t)\\ y(t)&=C_c x(t)+D(u(t))\end{aligned} \tag{9}$$

where A_c designated the state matrix is a n-by-n ($n=2n_2$); x(t) is the state vector; B is the input matrix; C_c is the output and D is the direct transmission matrix at continuous time.
In the modal analysis, sometime the input is unknown and measurements are mostly sampled at discrete-time. On the other hand, it is impossible to measure all DOFs and the last one, when measurements always have disturbance effects [8]. For all these reasons, the continuous deterministic system will be converted to suitable form: discrete-time stochastic state-space model as follow:

$$\begin{aligned}x_{k+1}&=Ax_k+w_k\\ y_k&=Cx_k+v_k\end{aligned} \tag{10}$$

where $x_k=x(k\Delta t)=\{q_k \ \dot{q}_k\}^T$ is the discrete-time state vector containing the discrete sample displacement q_k and velocity $\dot{q}_k$; w_k is the process noise due to disturbances and modelling inaccuracies; v_k is the measurement noise due to sensor inaccuracy. Following assumption w_k and v_k is zero mean and with covariance matrix:

$$E\left[\begin{pmatrix} w_p \\ v_p \end{pmatrix}\begin{pmatrix} w_q^T & v_q^T \end{pmatrix}\right] = \begin{pmatrix} Q & S \\ S^T & R \end{pmatrix}\delta_{pq} \tag{11}$$

where the index p and q are time-instants; E is the expected value; δ_{pq} is the Kronecker delta. As the correlation $E(w_p w_q^T)$ and $E(v_p v_q^T)$ are equal zero if different time-instant.

Further the stochastic model is assumed that $\{x_k\}$, $\{w_k\}$ and $\{v_k\}$ are mutual dependent: $E(x_k w_k^T)=0$ and $E(x_k v_k^T)=0$. According to B. Peeters and G.D. Roeck [8] proven that the output covariance $R=E[y_{k+i}\, y_k^T]$ for any arbitrary time-lags $i\Delta t$ can be considered as impulse response (Eq.12) of the deterministic linear time-invariance system A, C, G; where $G= E[x_{k+1}\, y_k^T]$ is the next state-output covariance matrix.

$$R_i = CA^{i-1}G \tag{12}$$

Therefore, the theoretical application of stochastic system can go back to eigen-system realization algorithm (ERA) method in [9]. The classification of stochastic system identification based on the key step of these methods, by following [8] they are covariance-driven stochastic subspace identification (COV-SSI) and data-driven stochastic subspace identification (DATA-SSI).

a) COV-SSI

The heart of COV-SSI method is the ERA developed by Juang and Pappa [8]. It is the famous technique for modal parameter from free vibration response or impulse response. The key step in the COV-SSI system is the computation output covariance can be expressed assuming ergodicity process as:

$$R_i = E[y_{k+1}\; y_k^T] = \lim_{N\to\infty}\frac{1}{N}\sum_{k=0}^{N-1} y_{k+i}y_k^T \approx \frac{1}{N}\sum_{k=0}^{N-1} y_{k+i}y_k^T \tag{13}$$

where i time lag; N finite number of data

All output covariance R_i in Eq.12 is stored in block Toeplitz matrix as:

$$T_{1|i} = \begin{bmatrix} R_i & R_{i-1}\cdots R_1 \\ R_{i+1} & R_i \;\cdots\; R_2 \\ \cdots & \cdots \quad \cdots \\ R_{2i-1} & R_{2i-2}\cdots R_i \end{bmatrix}_{ixi} \tag{14}$$

Combination Eq.12, and Eq.14, then block Toeplitz matrix can be decomposed as following:

$$T_{1|i} = \begin{bmatrix} C \\ CA \\ \cdots \\ CA^{i-1} \end{bmatrix}[A^{i-1}G \quad A^{i-2}G \cdots AG \quad G] = O_i C_i \tag{15}$$

On the other hand, the observability matrix O_i and controllability matrix C_i can be obtained from singular value decomposition (SVD) of the Toeplitz matrix:

$$T_{1|i} = [U_1 \quad U_2]\begin{bmatrix} S_1 & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} V_1^T \\ V_2^T \end{bmatrix} = U_1 S_1 V_1^T \tag{16}$$

S is a diagonal matrix containing singular value. The numbers of non-zero give the rank of the decomposed matrix and coincide with the size $n=2n_2$ of the state-space matrix A. Comparison the Eq.15 and Eq.16, we can rewrite that:

$$
\begin{aligned}
O_i &= U_1 S_1^{1/2} \\
C_i &= S_1^{1/2} V_1^T
\end{aligned}
\tag{17}
$$

Now the realization of all system matrixes A and C are achieved. The state matrix A can be obtained by decomposing a shift block Toeplitz matrix:

$$
T_{2|i+1} = O_i A C_i \tag{18}
$$

Combining Eq.17 and Eq.18 gives

$$
A = O_i^{+} T_{2|i+1} C_i = S_i^{-1/2} U_1^T T_{2|i+1} V_1 S_1^{-1/2} \tag{19}
$$

where $(.)^{+}$ is the pseudo-inverse of a matrix. The output matrix C equals the first l rows of O_i, where l number of outputs (in this study l=2).

b) Identification of flutter derivatives

The modal parameters of system can be obtained by solving the eigenvalue problem state matrix A:

$$
A = \Psi \Lambda \Psi^{-1}; \qquad \Phi = C\Psi \tag{20}
$$

where Ψ the complex eigenvector; Λ the complex eigenvalue is the diagonal matrix; Φ the mode shape matrix. When the complex modal parameters known, the gross damping C^e and gross stiffness K^e in Eq.8 is determined by following:

$$
\begin{bmatrix} K^e & C^e \end{bmatrix} = -M \begin{bmatrix} \Phi\Lambda^2 & \Phi^*(\Lambda^*)^2 \end{bmatrix} \begin{bmatrix} \Phi & \Phi^* \\ \Phi\Lambda & \Phi^*\Lambda^* \end{bmatrix}^{-1} \tag{21}
$$

$$
\text{Let} \quad
\begin{aligned}
\overline{C}^e &= M^{-1} C^e; \quad \overline{K}^e = M^{-1} K^e \\
\overline{C} &= M^{-1} C^0; \quad \overline{K} = M^{-1} K^0
\end{aligned}
$$

where C^0 and K^0 the mechanical damping and stiffness matrix of system under no-wind condition.

Following E. Simiu and R.H. Scanlan [13], the aerodynamic self-excited force and moment given by:

$$
\begin{aligned}
L_{se} &= \frac{1}{2}\rho U^2 B \left[K_h H_1^*(K_h)\frac{\dot{h}}{U} + K_\alpha H_2^*(K_\alpha)\frac{B\dot{\alpha}}{U} + K_\alpha^2 H_3^*(K_\alpha)\alpha + K_h^2 H_4^*(K_h)\frac{h}{B} \right] \\
M_{se} &= \frac{1}{2}\rho U^2 B^2 \left[K_h A_1^*(K_h)\frac{\dot{h}}{U} + K_\alpha A_2^*(K_\alpha)\frac{B\dot{\alpha}}{U} + K_\alpha^2 A_3^*(K_\alpha)\alpha + K_h^2 A_4^*(K_h)\frac{h}{B} \right]
\end{aligned}
\tag{22}
$$

where $K_i=\omega_i B/U$ the reduce frequency (i=h, α); H^*_i and A^*_i (i=1,2,3,4) are the flutter derivatives.

Substituting Eq.22 into Eq.7 and combining Eq.21, the flutter derivatives of two DOF can be defined as:

$$
\begin{aligned}
H_1^*(k_h) &= -\frac{2m}{\rho B^2 \omega_h}(\overline{C}_{11}^e - \overline{C}_{11}), & A_1^*(k_h) &= -\frac{2I}{\rho B^3 \omega_h}(\overline{C}_{21}^e - \overline{C}_{21}) \\
H_2^*(k_\alpha) &= -\frac{2m}{\rho B^3 \omega_\alpha}(\overline{C}_{12}^e - \overline{C}_{12}), & A_2^*(k_\alpha) &= -\frac{2I}{\rho B^4 \omega_\alpha}(\overline{C}_{22}^e - \overline{C}_{22}) \\
H_3^*(k_\alpha) &= -\frac{2m}{\rho B^3 \omega_\alpha^2}(\overline{K}_{12}^e - \overline{K}_{12}), & A_3^*(k_h) &= -\frac{2I}{\rho B^4 \omega_\alpha^2}(\overline{K}_{22}^e - \overline{K}_{22})
\end{aligned}
\tag{23}
$$

$$H_4^*(k_h) = -\frac{2m}{\rho B^3 \omega_h^2}(\overline{K}_{11}^e - \overline{K}_{11}), \qquad A_4^*(k_h) = -\frac{2I}{\rho B^4 \omega_h^2}(\overline{K}_{21}^e - \overline{K}_{21}) \tag{23}$$

3. TURBULENT WIND FIELD SIMULATION

Investigation on the aerodynamic bridge response in the time domain, turbulent wind field is obtained by simulation method first. The time histories of fluctuating wind velocity was generated from the target Kaimal's power spectral density (PSD) of horizontal and vertical fluctuating wind Eq.24

$$\frac{nS_u(n)}{\sigma_u^2} = \frac{105f}{(1+33f)^{5/3}}$$
$$\frac{nS_w(n)}{\sigma_w^2} = \frac{2f}{1+5.3f^{5/3}} \tag{24}$$

where $S_{u,w}(n)$ is the PSD of longitudinal and vertical velocity fluctuations u and w; n the frequency; σ_u the standard deviation of u; and f=nz/U: reduced frequency, and z: the reference height.

The generated wind velocity fluctuations are illustrated in Fig.2, fluctuated time series data and Fig.3, generated power spectrums and target spectrums.

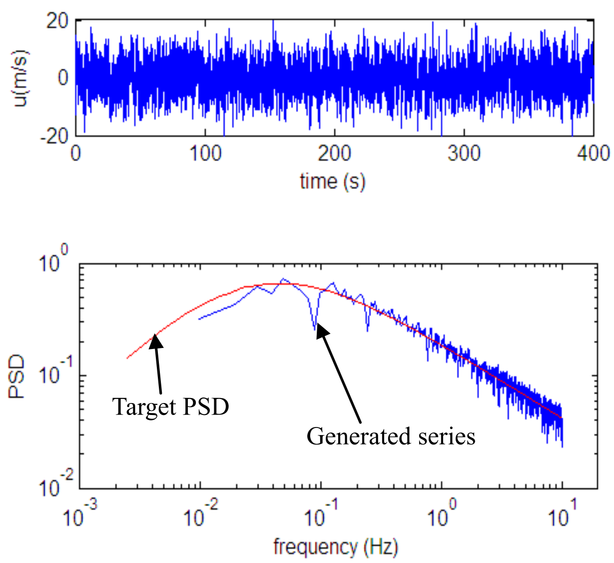

Figure 2: Longitudinal wind

Generated fluctuation wind velocity (upper)

Spectra of generated series and target (lower)

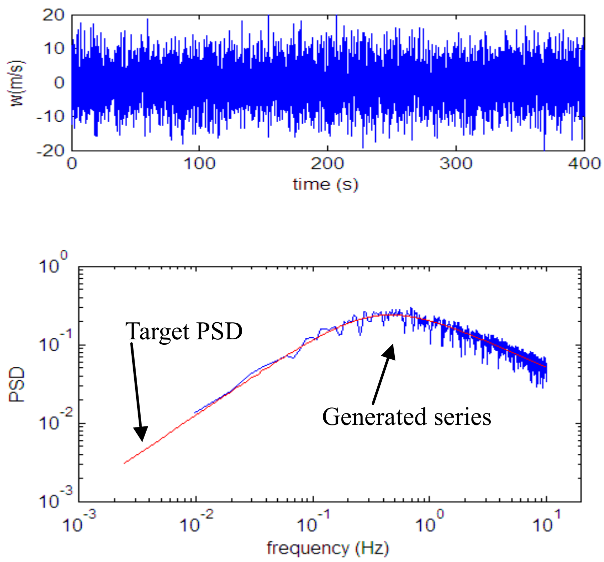

Figure 3: Vertical wind

Generated fluctuation wind velocity (upper)

Spectra of generated series and target (lower)

4. NUMERICAL SIMULATION RESULS

(1) White noise excitation

To verify the proposed method for identifying FDs, the response time-series of section model excited by lift and moment white-noise was computed by numerical integration method. The section model properties assumed following (Jakobsen and Hjorth-Hansen, 1995) [6]: f_h=1.947 Hz; f_α=5.76 Hz; logarithm decrements δ_h=0.035 and δ_a=0.033. The mean wind speed U=10.26 m/s, air density 1.181 kg/m^3. The effective stiffness and damping were pre-set at:

$$M_0 = \begin{bmatrix} 2.6526 & 0 \\ 0 & 0.0189 \end{bmatrix}; \quad C^e = \begin{bmatrix} 8.9308 & -0.0799 \\ 0.4345 & 0.0386 \end{bmatrix}; \quad K^e = \begin{bmatrix} 420.1002 & -59.1805 \\ 1.7552 & 19.6592 \end{bmatrix}$$

The SSI method applied to these response data and obtained the effective structural matrix and the deviation of identified matrices from the pre-set ones:

$$C_r^e = \begin{bmatrix} 8.9875 & -0.059 \\ 0.4468 & 0.0378 \end{bmatrix};\ K_r^e = \begin{bmatrix} 420.99 & -58.2082 \\ 2.0345 & 19.6516 \end{bmatrix};\ \Delta C\% = \begin{bmatrix} 0.63 & -26.15 \\ 2.83 & -2.07 \end{bmatrix};\ \Delta K\% = \begin{bmatrix} 0.21 & -1.64 \\ 15.91 & 0.07 \end{bmatrix}$$

The results are plausible to compare with the pre-set values. The maximum differences in the off-diagonal term C_{12} around 26%, this parameter related to H_2^*, but the magnitude value quite small so the effect is trivial.

(2) Buffeting excitation

The buffeting response obtained by following the procedure in section 2.1 with different mean wind speed. Assuming the section prototype parameters per unit length of prototype bridge are following: mass is 4300 kg/m; mass moment of inertia 4.11×10^4 kg.m^2/m; the width of deck B is 30m; the height of deck is 3.2m; vertical mode frequency f_h=0.15Hz; torsional mode frequency f_α=0.35Hz; vertical damping ratio ξ_h=0.15; torsional damping ratio ξ_α=0.25.

The vertical and torsional responses were obtained at mean wind speeds varying from 10m/s to 90m/s and added turbulent of along and vertical wind. The buffeting responses were simulated at a sampling frequency of 20Hz. Fig.4 shows the simulated buffeting responses of two DOFs by numerical integration with constant acceleration method at mean wind speed U=10m/s.

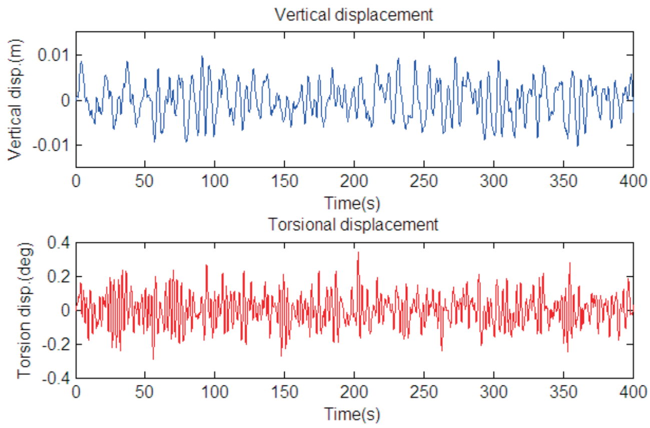

Figure 4: Buffeting response vertical displacement (upper), torsional displacement (lower))

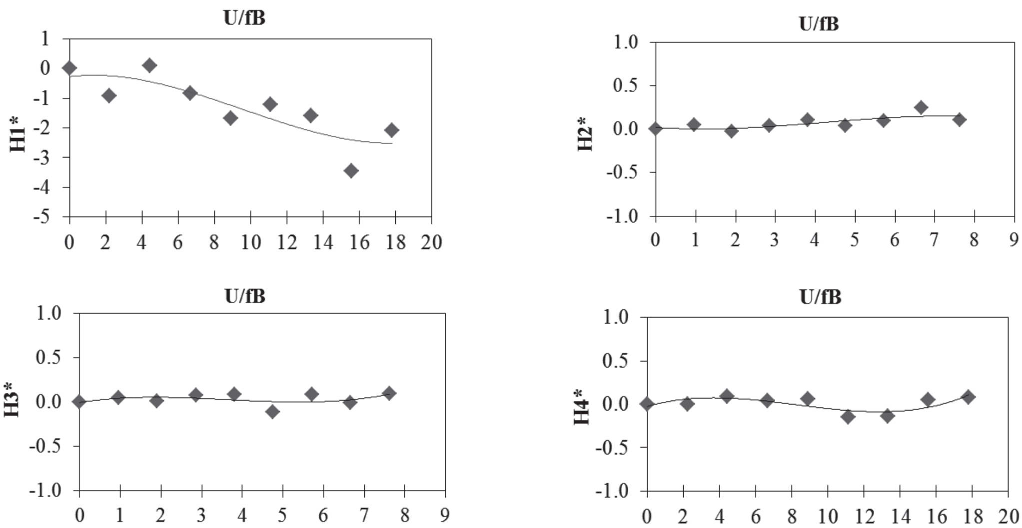

Figure 5: Vertical flutter derivatives and three order fitted polynomial, indicated by solid line

In this study, the actual model order is known and is equal to the order of state matrix A (=4) and also number of out-puts equal two (h and α). In order to extract the FDs, the SSI technique a computer program developed in Matlab's program following the procedure mention in section 2. From the eigenvalue and mode shapes, the effective damping and stiffness were determined from Eq.21 and physical matrices, the values of all FDs at a particular wind speed were obtained following Eq.23. Fig.5 and Fig.6 shows the FDs values at discretely located of each parameter respect to reduce wind speed and also the three order polynomials are depicted as continuous curves.

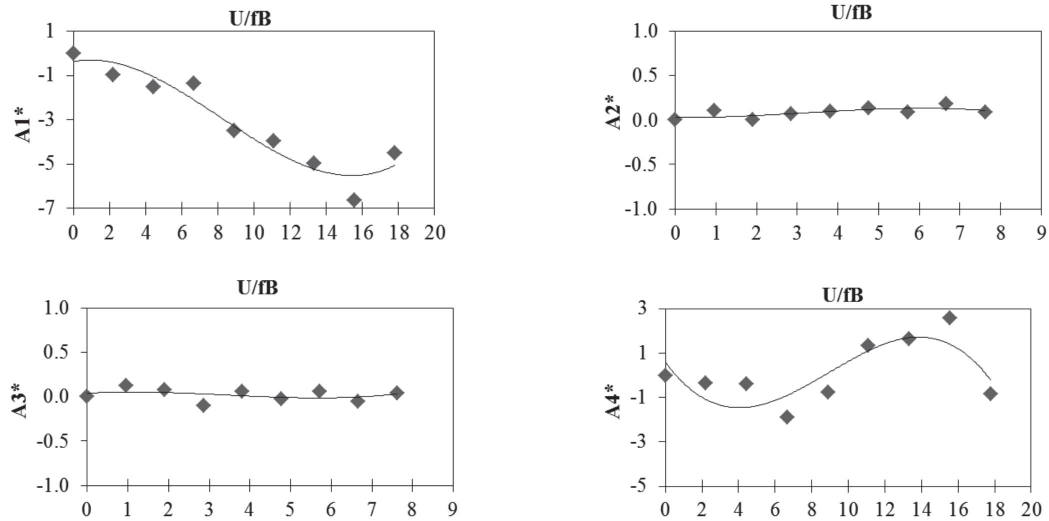

Figure 6: Torsional flutter derivatives and three order fitted polynomial, indicated by solid line

Identifying flutter derivatives of prototype section under turbulent wind has been hard work; the results $H1^*$ and $A1^*$ is plausible. As can be seen from the chart, the $H1^*$ derivative trend intended to larger negative with increase reduce wind speed, this indicated aerodynamic damping in vertical bending vibration is positive. $A4^*$ shows small scatter and changes trend at high reduce wind speed. The $H2^*$, $H3^*$ derivatives, which control the coupling from torsional to vertical have very small value. The $A2^*$, $A3^*$ derivatives, which related to aerodynamic torsional stiffness and torsional damping is also small.

5. CONCLUTIONS

These research attempts to apply the stochastic system identification method have been determined flutter derivatives of two DOFs system. The method uses only out-put from buffeting responses data. The flutter derivatives $H1^*$ and $A1^*$ are plausible, but the coupling term between the torsional and vertical mode $H2^*$, $H3^*$ derivatives are quite small. The aerodynamic effected on torsional mode $A2^*$ and $A3^*$ derivatives are also small. Extraction the FDs from buffeting of the prototype bridge is more challenging and the errors in the values may be attributed to the stochastic method itself. The authors will effort to minimize the errors by adopting better signal processing technique. The results show the potential to extract FDs from ambient vibration data on full-scale bridge.

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