

DYNAMIC STABILITY OF PIPES SUBJECTED TO INTERNAL FLOW AND EXTERNAL ANNULAR AXIAL FLOW SIMULTANEOUSLY

Katsuhisa Fujita⁺¹ and Akinori Moriasa⁺²

^{+1,+2}Mechanical and Physical Engineering, Graduate School of Engineering, Osaka City University,
3-3-138 Sugimoto-cho, Sumiyoshi-ku, Osaka, 558-8585, Japan

When slender pipes are subjected to internal flow, the pipes lose stability by flutter and divergence in increasing the fluid velocity. In addition, they also lose stability when they are subjected to external annular axial flow. In the development of a piping system in the field of ocean mining, and in the field of fluid energy utilization, and so forth, the double walled pipe structure system subjected to an internal flow and an external flow simultaneously is thought to be one of the important pipe structures. In this paper, the pipe structures are assumed to be composed of the cantilevered beam structures. For the analysis of the internal flow, the conventional inviscid stability analysis method is applied. For the analysis of the external annular axial flow, two kinds of solutions are applied. The one is the viscous solution using the Navier-Stokes equation of motion and it is called as NS solution hereafter. The other is the ideal fluid solution which the viscous influence is added to later and it is called as R&P solution hereafter. Changing the flow direction and the fluid velocity of the internal flow and the external flow, and the specifications of modeling, the stability of the double walled pipes is investigated and discussed.

Keyword: flutter, flow induced vibration, coupled vibration, axial flow, double wall pipe

1. INTRODUCTION

When slender pipes are subjected to internal flow, the pipes lose stability by flutter and divergence in increasing the fluid velocity. In addition, they also lose stability when they are subjected to external annular axial flow. In this paper, the dynamic stability of flexible pipe is investigated when a flexible pipe is subjected to an internal flow and an external flow simultaneously. Concerning the double walled pipe system subjected to an internal flow and an external flow, the piping system for transporting sea bottom resources such as methane hydrate, and the cooling and heating piping system for gas energy utilization so on are enumerated.

When the flexible pipe subjected to an internal flow and an external annular axial flow simultaneously, it is considered that the direction of flow has much effect on the dynamic stability in the case of the cantilevered pipe. Especially, the case that the direction of an internal flow is the opposite of an external flow is predicted to have stronger interaction than the same direction. Therefore, in this paper, the case that the direction of an internal flow is the opposite direction of an external flow is investigated.

The dynamic stability analyses of a flexible pipe subjected to an internal flow have been already reported by Paidoussis and Luu¹⁾, Paidoussis²⁾. In these studies, it has been reported that the stability analysis of an aspirating pipe can be performed by replacing the minus velocity in stead of the plus velocity in the analysis of a discharged pipe. However, the jet from the free end of the discharged pipe can not become the inverse jet at the time of aspirating fluid. The reverse is found to be not truth. Giacobbi et al.³⁾ has reported the more accurate analytical method on the dynamics of a cantilevered pipe aspirating fluid. The stability analysis of an internal flow is based on these references in this paper. On the other hand, the dynamic stability analyses of an external annular axial flow have been reported by Fujita and Shintani⁴⁾, Fujita and Ohkuma^{5, 6)}. Combining the two kind of the dynamic stability analysis method, the dynamic stability of the flexible cantilevered pipe subjected to an internal and an external annular axial flow simultaneously is investigated. Besides, in the case of an external annular flow, the viscous solution is derived by using the Navier-Stokes

⁺¹fujita@mech.eng.osaka-cu.ac.jp

equation under the assumption that the gap at the annulus is small. The drop of accuracy is predicted when the gap becomes wider. On the other hand, Rinaldi and Paidoussis⁷⁾ have reported the dynamic stability analysis method of the flexible cantilevered pipe subjected to an external annular axial flow by adding the viscous effect on the theory of ideal fluid. Therefore, applying this method, the effect of the gap so forth on the accuracy of the dynamic stability of the flexible pipe subjected to an internal flow and an external annular flow simultaneously is also investigated.

2. MODELING

(1) Flexible pipe subjected to internal flow and external flow simultaneously

As one of the piping structure subjected to an internal flow and an external flow, Fig. 1 shows the modeling of a double walled pipe structure in which an inner flexible pipe is subjected to an internal flow and an external flow. The outer pipe is assumed to be rigid and the inner pipe is assumed to be flexible, for the simplicity of calculations. The fluid flows through the inside of the inner pipe as an internal flow, and the fluid flows through the annular gap between the rigid outer pipe and the flexible inner pipe as an external flow.

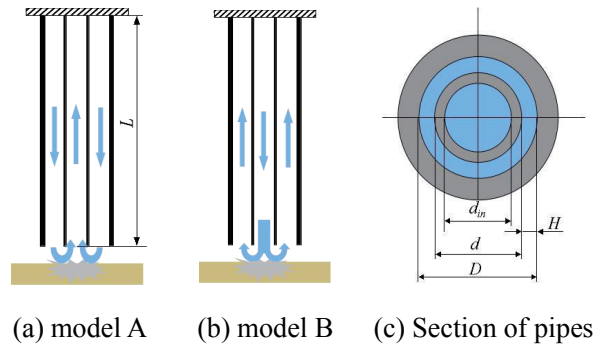


Figure 1: Structure of double walled pipe.

In the case of model A, the external flow outside of the flexible pipe flows from the clamped edge to the free edge as a discharged flow, and the internal flow of the flexible pipe flows from the free edge to the clamped edge as an aspirated flow. On the other hand, model B is totally reverse to model A. In model B, the external flow flows from the free edge to the clamped edge as an aspirated flow, and the internal flow flows from the clamped edge to the free edge as a discharged flow. Even if we change the specifications of model, the external flow rate is assumed to be always equal with the internal flow rate for each flow velocity. Besides, the specifications of the section of a double walled pipe structure are shown in Fig. 1(c). Where, d is the outer diameter of an inner flexible pipe, d_{in} is the inner diameter of the inner flexible pipe, L is the length of the inner flexible pipe, D is the inner diameter of an outer rigid pipe, and H is the gap between the inner pipe and the outer pipe, which means the width of flow passage.

(2) Stability of flexible pipe subjected to only internal flow

a) Equation of motion of flexible pipe subjected to only internal flow

The equation of motion of the cantilevered flexible pipe which discharges fluid from the free end as an internal flow is expressed based on the ideal fluid theory by Paidoussis²⁾ as shown in Eq. (1). And, also the nondimensional equation of motion is also shown in the following as Eqs. (2) and (3).

$$EI \frac{\partial^4 w}{\partial x^4} + \rho_f A_m V^2 \frac{\partial^2 w}{\partial x^2} + 2\rho_f A_m V \frac{\partial^2 w}{\partial x \partial t} + (\rho_s A_s + \rho_f A_m) \frac{\partial^2 w}{\partial t^2} = 0, \quad (1)$$

$$\xi = \frac{x}{L}, \quad \eta = \frac{w}{L}, \quad \tau = \left(\frac{EI}{\rho_f A_s} \right)^{1/2} \frac{t}{L^2}, \quad v = \left(\frac{\rho_f A_s}{EI} \right)^{1/2} LV, \quad m = \frac{\rho_s}{\rho_f}, \quad \sigma = \frac{d}{d_{in}}, \quad (2)$$

$$\frac{\partial^4 \eta}{\partial \xi^4} + \frac{1}{\sigma^2 - 1} \left(\frac{\partial}{\partial \tau} + v \frac{\partial}{\partial \xi} \right)^2 \eta + m \frac{\partial^2 \eta}{\partial \tau^2} = 0. \quad (3)$$

The equation of motion of the flexible cantilevered pipe which aspirates fluid from the free end as an internal flow is expressed using Giacobbi et al.³⁾ as follows because it is thought to be improved from the equations reported by Paidoussis and Luu¹⁾.

$$EI \frac{\partial^4 w}{\partial x^4} + \left[1 - (1 + \bar{\gamma}) \left(\frac{3}{2} - \alpha \right) \right] \rho_f A_{in} V^2 \frac{\partial^2 w}{\partial x^2} - 2 \rho_f A_{in} V \frac{\partial^2 w}{\partial t \partial x} + (\rho_s A_s + \rho_f A_{in}) \frac{\partial^2 w}{\partial t^2} + \left[\rho_f A_{in} V \frac{\partial w}{\partial t} - \alpha \rho_f A_{in} V^2 (1 - \psi) \frac{\partial w}{\partial x} \right] \delta(x - L) = 0. \quad (4)$$

And, nondimensional equation of motion is also shown in the following.

$$\frac{\partial^4 \eta}{\partial \xi^4} + \left[\left\{ 1 - (1 + \bar{\gamma}) \left(\frac{3}{2} - \alpha \right) \right\} \frac{1}{\sigma^2 - 1} v^2 \right] \frac{\partial^2 \eta}{\partial \xi^2} - 2 \frac{1}{\sigma^2 - 1} v \frac{\partial^2 \eta}{\partial \tau \partial \xi} + \left(m + \frac{1}{\sigma^2 - 1} \right) \frac{\partial^2 \eta}{\partial \tau^2} + \left[\frac{1}{\sigma^2 - 1} v \frac{\partial \eta}{\partial \tau} - \frac{1}{\sigma^2 - 1} \alpha v^2 (1 - \psi) \frac{\partial \eta}{\partial \xi} \right] \delta(\xi - 1) = 0, \quad (5)$$

where, EI is the bending rigidity of inner flexible pipe, ρ_f is the density of fluid, A_{in} is the sectional area of the passage, A_s is the sectional area of the flexible pipe, V is fluid velocity, ρ_s is the density of flexible pipe, w is the deflection of flexible pipe, and x is the axial coordinate of flexible pipe. And, ξ is the nondimensional axial coordinate, η is the nondimensional deflection of flexible pipe, σ is the ratio of outer diameter and inner diameter of flexible pipe, v is the nondimensional fluid velocity, $\delta(\cdot)$ is the delta function, and m is the ratio of the density of flexible pipe for the density of fluid. The numerical parameters in aspirating fluid are $\alpha = 0.86$, $\bar{\gamma} = 0.35$, $\psi = 0.94$.

b) Comparison of dynamic stability of cantilevered pipe subjected to only internal flow between aspirating and discharging flow

Figure 2 shows the root loci, these are Argand diagrams, of cantilevered flexible pipe subjected to an internal flow in both cases of aspirating and discharging flow at the free edge.

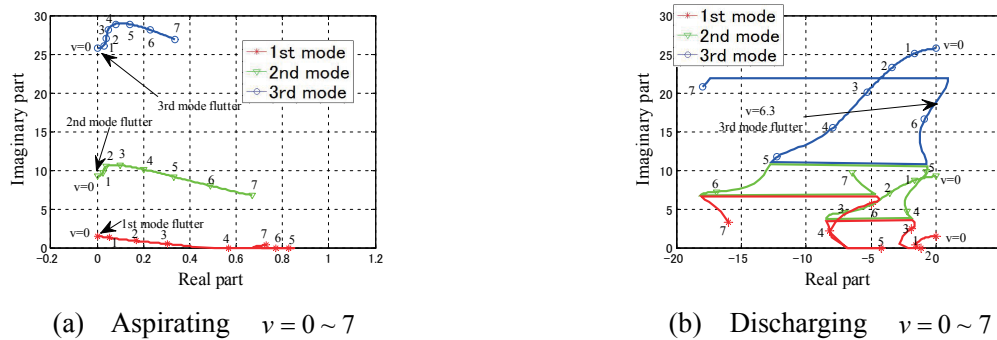


Figure 2: Root loci of cantilevered flexible pipe subjected to internal flow. ($\sigma = 1.1$)

This figure shows from the first mode to the higher modes. The first lowest frequency in complex eigenvalue analysis for each flow velocity is defined as the first mode, and the second lowest frequency is also

defined as the second mode, so forth. The stability analysis about the fluid discharged at the free edge of the cantilevered pipe is based on the solution by Paidoussis²⁾, and that about the fluid aspirated at the free edge is based on the solution by Giacobbi et al.³⁾. The ratio of the density of structure for the density of fluid m is 1, which means that the effect of fluid density is large.

From Fig. 2(a), it is found that the stability is lost from the first mode to the third mode when the fluid is aspirated from the free edge of the cantilevered flexible pipe as an internal flow. However, the real part which means the growth rate of vibration is smaller than that of the solution by Paidoussis and Luu¹⁾. The stability by Giacobbi³⁾ becomes better than that by Paidoussis and Luu¹⁾. On the other hand, in the case of the fluid discharged from the free edge of the cantilevered pipe, it is found that the third mode becomes flutter first at $\nu = 6.3$.

(3) Stability of flexible pipe subjected to only external annular axial flow

a) Equation of motion of cantilevered flexible pipe subjected to only external annular axial flow

The fluid structure coupled equations is derived by using the Euler-Bernoulli equation and the Navier-Stokes equation. These equations have been already reported by Fujita and Shintani⁴⁾, Fujita and Ohkuma^{5, 6)}. Besides, this solution is called as NS solution hereafter. X and Y are defined in the orthogonal coordinate system when the gap between the outer pipe and the inner pipe is changed from the cylindrical coordinate system to the orthogonal coordinate system. X denotes the circumferential direction, and Y the axial direction. When Q_X and Q_Y mean the circumferential and axial flow rate, respectively, P is the fluid pressure, ν is the kinematic viscosity, and H is the gap width as shown in Fig. 1(c), the equation of fluid is obtained as follows.

$$\frac{\partial Q_X}{\partial X} + \frac{\partial Q_Y}{\partial Y} + \frac{\partial H}{\partial t} = 0, \quad (6)$$

$$\frac{1}{\rho_f} \frac{\partial P}{\partial X} = -\frac{1}{H} \left\{ \frac{\partial Q_X}{\partial t} + \frac{\partial}{\partial X} \left(\frac{Q_X^2}{H} \right) + \frac{\partial}{\partial Y} \left(\frac{Q_X Q_Y}{H} \right) + \frac{12\nu Q_X}{H^2} \right\}, \quad (7)$$

$$\frac{1}{\rho_f} \frac{\partial P}{\partial Y} = -\frac{1}{H} \left\{ \frac{\partial Q_Y}{\partial t} + \frac{\partial}{\partial X} \left(\frac{Q_X Q_Y}{H} \right) + \frac{\partial}{\partial Y} \left(\frac{Q_Y^2}{H} \right) + \frac{12\nu Q_Y}{H^2} \right\}, \quad (8)$$

$$\begin{aligned} P(X, Y, t) &= \bar{P}(Y) + \Delta P(X, Y, t), \quad H(X, Y, t) = \bar{H} + \Delta H(X, Y, t), \\ Q_X(X, Y, t) &= \Delta Q_X(X, Y, t), \quad Q_Y(X, Y, t) = \bar{Q}_Y + \Delta Q_Y(X, Y, t), \end{aligned} \quad (9)$$

where, $\bar{(\quad)}$ denotes the steady component, and $\Delta(\quad)$ denotes the unsteady component. The fluid force acting on a flexible pipe can be obtained by Eqs. (6), (7) and (8), using the relationship of Eq. (9).

For the flexible pipe, the equation of motion based on the Euler-Bernoulli equation is applied as follows.

$$\rho_s A_s \frac{\partial^2 \Delta H}{\partial t^2} + EI \frac{\partial^4 \Delta H}{\partial Y^4} + W(L - Y) \frac{\partial^2 \Delta H}{\partial Y^2} = \Delta F, \quad (10)$$

where, ΔH is the unsteady displacement of a flexible pipe, W is the axial fluid force, and ΔF are the unsteady fluid forces which consist of the normal force, that is pressure, and the circumferential shear force. The fluid forces which are calculated using Eqs. (6), (7), (8) and (9) is coupled with the equation of motion of the flexible pipe by Eq. (10), and then the nondimensional equation of motion can be obtained using the following relationship.

$$\xi = \frac{Y}{L}, \quad \tau = \left(\frac{EI}{\rho_f A_s} \right)^{1/2} \frac{t}{L^2}, \quad u = \left(\frac{\rho_f A_s}{EI} \right)^{1/2} LU, \quad m = \frac{\rho_s}{\rho_f}, \quad \sigma = \frac{d}{d_m}, \quad \varepsilon = \frac{R}{L}, \quad r = \frac{R}{H}, \quad \beta = 12\nu \left(\frac{\rho_f A_s}{EI} \right)^{1/2}. \quad (11)$$

Applying the modal expansions using the Galerkin method, the coupled equation of motion between structure and fluid can be given as follows.

$$([M_s] + [M_a]) \frac{d^2}{d\tau^2} \{W_n(\tau)\} + ([C_s] + [C_a]) \frac{d}{d\tau} \{W_n(\tau)\} + ([K_s] + [K_a]) \{W_n(\tau)\} = \{0\}, \quad (12)$$

where $\{W_n(\tau)\}$ denotes the mode vector, $[M]$, $[C]$ and $[K]$ denote the mass, damping, and stiffness matrices, respectively. The suffix s shows the matrices of the flexible pipe, and the suffix a shows the added matrices by fluid.

b) Comparison of dynamic stability of cantilevered pipe subjected to only external annular flow between aspirating and discharging flow

Figure 3 shows the root loci of the cantilevered flexible pipe subjected to only an external flow in both cases of aspirating and discharging flow at the free edge. The ratio ε of the inner radius for the length of the flexible pipe is 0.02, and the ratio r of the inner radius for the gap between the outer pipe and the inner pipe is 2. And, the ratio m of the density of structure for the density of fluid is 1. The ratio σ of the outer diameter for the inner diameter of flexible pipe is 1.1. The nondimensional viscosity β is 5×10^{-6} .

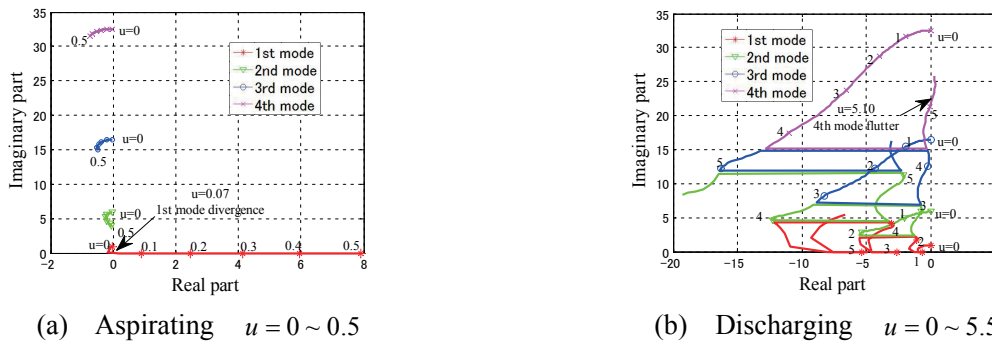


Figure 3: Root loci of cantilevered flexible pipe subjected to external flow by NS solution.
($\varepsilon = 0.02, r = 2$)

From Fig. 3(a), only the first mode is found to become unstable at very low velocity when the cantilevered flexible pipe is subjected to an external flow as an aspirated flow. Increasing the flow velocity a little, the real part which means the growth rate of vibration is found to become larger than the aspirating internal flow shown in Fig. 2(a). That is, this system is found to be more unstable. When the cantilevered pipe is subjected to an external flow as a discharged flow as shown in Fig. 3(b), each mode in the nondimensional root loci shows the same characteristics as that of the internal flow in Fig. 2(b). The flutter of fourth mode occurs first at $u = 5.10$.

3. NUMERICAL SIMULATIONS OF DOUBLE WALLED PIPE

(1) Comparison of root loci between model A and model B

The dynamic response of a double walled pipe is assumed to be calculated using principle of superposition. Figure 4 shows the comparison of the root loci between model A and model B which have the structure of double walled pipe as shown in Fig. 1. Figure 4 shows the case of $\varepsilon = 0.02, r = 2$ and $m = 1$, as

well as Fig. 3. When it is thought that an internal flow and an external flow circulate each other, the fluid velocity of an internal flow and that of an external flow are changed so that the external flow rate is always equal with the internal flow rate. Let's decide the critical velocity is displayed by the external velocity. Besides, as mentioned before, the parameters σ and β are fixed as $\sigma = 1.1$, $\beta = 5 \times 10^{-6}$.

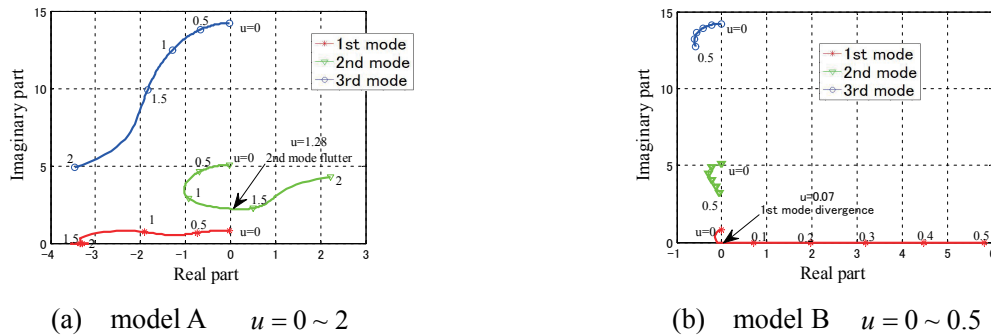


Figure 4: Root loci subjected to external flow by NS solution. ($\varepsilon = 0.02$, $r = 2$)

The root locus of model A in Fig. 4(a) is a combination of the case subjected to the aspirated internal flow shown in Fig. 2(a) and the case subjected to the discharged external flow shown in Fig. 3(b). The second mode is found to be destabilized due to the influence of the aspirated internal flow in Fig. 2(a). On the other hand, the root locus of model B in Fig. 4(b) is a combination of the case subjected to the discharged internal flow shown in Fig. 2(b) and the case subjected to the aspirated external flow shown in Fig. 3(a). The root locus of model B is similar with that of the aspirated external flow of Fig. 3(a), and the first mode is destabilized due to the influence of Fig. 3(a).

(2) Comparison of critical velocity between model A and model B

First, let's investigate the influence of the specifications of structure on the critical velocity in the case of $m = 1$ which means the smaller ratio of the density of structure for the density of fluid. After rearranging the root loci in Fig. 4, the influences on the critical velocities in model A and model B by the ratio ε and the ratio r are shown in Fig. 5. As mentioned above, ε is the ratio of the inner radius for the length and r is the ratio of the inner radius for the annular gap. The horizontal axis shows the nondimensional viscosity β , and the vertical axis shows the nondimensional critical velocity.

From Fig. 5, model A is found to be more stable than model B. The larger r means the narrower gap, and the smaller value of ε means the longer length in Fig. 5. As the flow velocities are calculated under the condition that the internal flow rate is always equal to the external flow rate, the external flow velocity becomes larger when the gap becomes narrower, that is, the influence of external flow becomes large. Therefore, the influence of the discharged external flow shown in Fig. 3(b) increases in model A when the gap becomes narrower. As a results, when the value of ratio r increases, it is found that the higher modes become critical in model A. In model B, when the gap becomes narrower, the influence of the aspirated external flow shown in Fig. 3(a) increases. Therefore, the first mode is always unstable even if the specifications are changed.

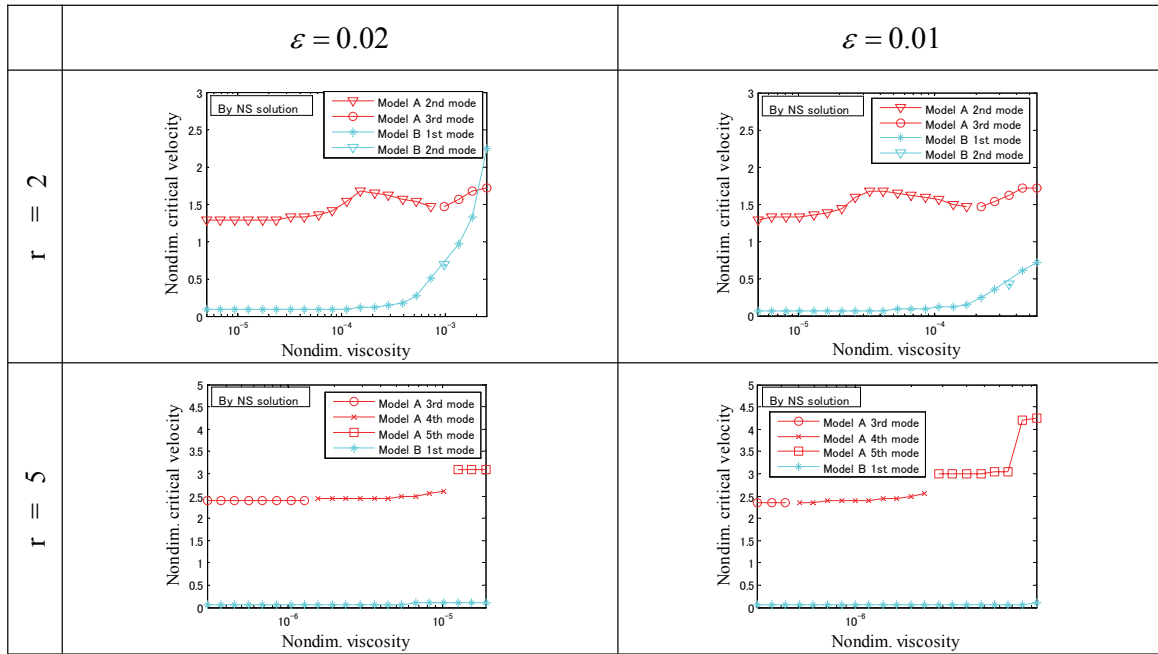


Figure 5: Comparison of critical velocity between model A and model B by using NS solution for the external flow. ($m = 1$)

Next, Fig. 6 shows the similar comparison as well as Fig. 5 in the case of $m = 1000$ which corresponds to the higher ratio of the density of structure for the density of fluid. From Fig. 6, model A is found to be more stable than model B in the case of $m = 1000$, as well as $m = 1$. When the nondimensional viscosity which is shown in Eq. (11)₈ increases, the critical velocity of model A approaches to that of model B.

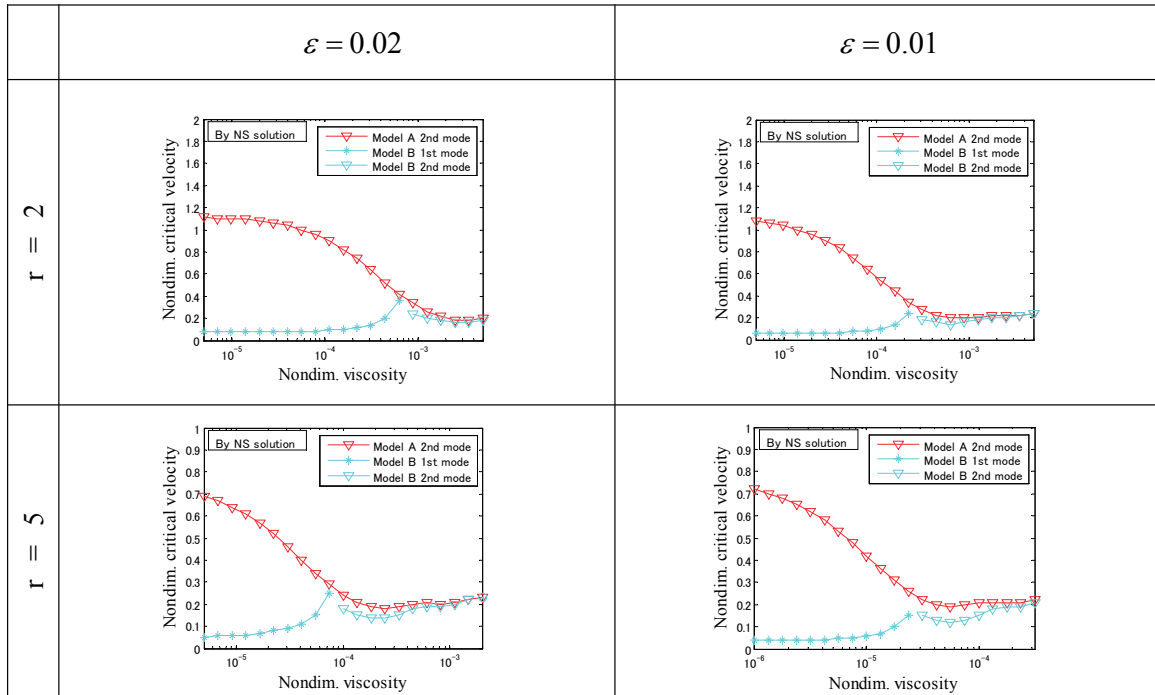


Figure 6: Comparison of critical velocity between model A and model B by using NS solution for the external flow. ($m = 1000$)

It is understood that the critical flow velocity of model A decreases when the nondimensional viscosity increases. In other words, the destabilization effect of the fluid viscosity is recognized. Especially, the effect becomes intense in the region of the large value of the ratio r which means the narrow gap between the inner pipe and the outer pipe decreases.

4. CONSIDRATIONS

(1) R&P solution on dynamic stability of cantilevered pipe subjected to only external flow

Rinaldi and Paidoussis⁷⁾ derived the dynamic stability analysis method of the flexible cantilevered pipe subjected to an external annular axial flow by adding the viscous effect on the ideal fluid acting on the pipe for a turbulent flow. In this paper, applying this solution to a laminar flow, the results are compared with NS solution which has already been reported in the above section. Besides, the solution which is obtained by applying the theory by Rinaldi and Paidoussis⁷⁾ to a laminar flow is called as R&P solution hereafter.

First, let's investigate the stability of a cantilevered flexible pipe subjected to only an external annular axial flow.

(2) Equation of motion of cantilevered pipe subjected to only external annular flow by using R&P solution

The following equation of motion can be obtained by applying the theory of Rinaldi and Paidoussis⁷⁾ on the cantilevered flexible pipe subjected to an external annular axial flow when the flow is laminar.

$$EI \frac{\partial^4 w}{\partial x^4} + \chi \rho_f A U^2 \frac{\partial^2 w}{\partial x^2} + 2 \chi \rho_f A U \frac{\partial^2 w}{\partial t \partial x} + (\chi \rho_f A + \rho_s A) \frac{\partial^2 w}{\partial t^2} + \left[\frac{12 \pi \mu d}{D-d} U \left(1 + \frac{d}{D-d} \right) \right] \frac{\partial w}{\partial x} - \left(\left[\frac{12 \pi \mu d}{D-d} U \left(1 + \frac{d}{D-d} \right) \right] (L-x) \right) \frac{\partial^2 w}{\partial x^2} + \frac{12 \pi \mu d}{D-d} \frac{\partial w}{\partial t} = 0, \quad (13)$$

where, $A = A_s + A_{in}$. μ is the viscosity. χ is the confinement parameter. The nondimensional parameters and the nondimensional equation of motion are as follows.

$$\xi = \frac{x}{L}, \quad \eta = \frac{w}{L}, \quad \tau = \left(\frac{EI}{\rho_f A_s} \right)^{1/2} \frac{t}{L^2}, \quad u = \left(\frac{\rho_f A_s}{EI} \right)^{1/2} L U, \quad m = \frac{\rho_s}{\rho_f}, \quad \varepsilon = \frac{R}{L}, \quad \chi = \frac{1+2r+2r^2}{1+2r}, \quad (14)$$

$$r = \frac{R}{H}, \quad \beta = 12 \nu \left(\frac{\rho_f A_s}{EI} \right)^{1/2}, \quad \sigma = \frac{d}{d_m}, \quad \gamma = \frac{1}{\sigma^2 - 1},$$

$$\frac{\partial^4 \eta}{\partial \xi^4} + \chi(1+\gamma)u^2 \frac{\partial^2 \eta}{\partial \xi^2} + 2\chi(1+\gamma)u \frac{\partial^2 \eta}{\partial \tau \partial \xi} + [\chi(1+\gamma)+m] \frac{\partial^2 \eta}{\partial \tau^2} + \left[\beta(1+\gamma) \frac{ru}{\varepsilon^2} (1+r) \right] \frac{\partial \eta}{\partial \xi} - \left(\left[\beta(1+\gamma) \frac{ru}{\varepsilon^2} (1+r) \right] (1-\xi) \right) \frac{\partial^2 \eta}{\partial \xi^2} + \beta(1+\gamma) \frac{r}{\varepsilon^2} \frac{\partial \eta}{\partial \tau} = 0. \quad (15)$$

(3) Root loci of cantilevered pipe subjected to only external annular flow by using R&P solution

Figure 7 shows the root loci of the cantilevered pipe subjected to the aspirated and the discharged external flow by R&P solution. This figure shows the case of $\varepsilon = 0.02$, $r = 2$ as well as Fig. 3 and 4. Besides in this case, $m = 1$, $\sigma = 1.1$ and $\beta = 5 \times 10^{-6}$. As the root loci by NS solution have been shown in Fig. 3, the comparison between NS solution and R&P solution can be clarified when Fig. 7 is compared with Fig. 3.

In aspirating fluid, the first mode becomes unstable in NS solution, and all modes of the first to third modes become unstable in R&P solution. NS solution is found to be stable a little than R&P solution. However, as the real part shows the high growth rate of vibration in NS solution, NS solution cannot be necessarily thought to be stable. In discharging fluid, Fig. 3(b) by NS solution is similar to Fig. 7(b) by R&P

solution. The fourth mode of NS solution in Fig. 3(b) becomes flutter at $u = 5.10$. On the other hand, the fourth mode of R&P solution in Fig. 7(b) becomes flutter at $u = 4.62$.

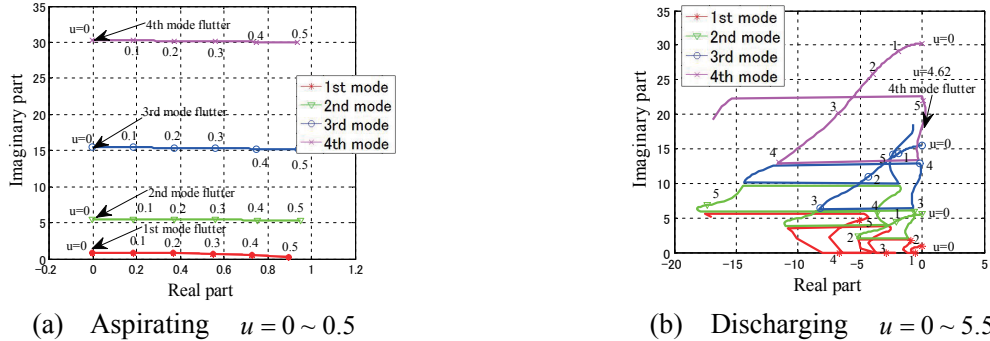


Figure 7: Root loci of cantilevered pipe subjected to only external flow by using R&P solution. ($\varepsilon = 0.02, r = 2$)

(4) Root loci of model A and model B subjected to internal and external annular flows by using R&P solution

Figure 8 shows the root loci of model A and model B in which the internal flow is based on the ideal fluid theory by Eq. (1)~(5), and the external flow is based on R&P solution in stead of NS solution. This figure shows the case of $\varepsilon = 0.02, r = 2$ as well as Fig. 3 and 4.

When NS solution in Fig. 4 is compared with R&P solution in Fig. 8, it is found that both root loci show the similar results in both model A and model B macroscopically. Concerning model A, the second mode becomes flutter at $u = 1.28$ in both solutions. In the case of model B, the first mode of NS solution becomes divergence at the low flow velocity, although all modes of the first to third mode of R&P solution become flutter at the low flow velocity.

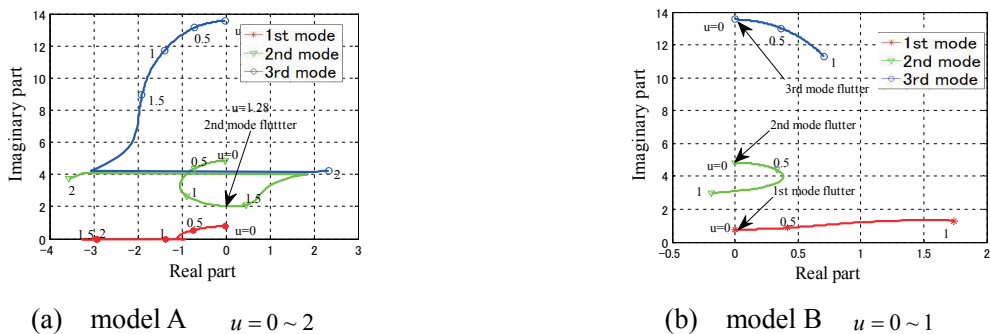


Figure 8: Root loci of double walled pipe subjected to internal flow, and to external flow by using R&P solution. ($\varepsilon = 0.02, r = 2$)

5. CONCLUSIONS

When the double walled pipe is subjected to the aspirated flow and the discharged flow simultaneously, the dynamic stability of pipe becomes complex. Therefore, when such a piping system is adopted in actual plant, it is necessary to pay enough consideration about the dynamic stability.

The piping system of the double walled pipe structure subjected to the discharged external flow and the aspirated internal flow simultaneously can be said to be one of the dynamically stable piping systems in the field of ocean mining and in the field of fluid energy utilization, and so forth. In other words, if the combination of the internal flow and the external flow is devised, it is found that the more dynamically stable system of the double walled pipe structure may be provided.

When the cantilevered flexible pipe discharges a fluid through the gap between the inner pipe and the outer pipe, NS solution and R&P solution are thought to show considerably a good agreement. However, when the cantilevered pipe aspirates a fluid, the both solutions show a little difference. Therefore, it can be understood that there are a little differences in the critical flow velocities of model A and model B, which are a double walled pipe structure, between NS solution and R&P solution.

ACKNOWLEDGMENT

On performing this study, we thank Professor Tadao Kawai of Osaka City University for his much support.

REFERENCES

- 1) Paidoussis, M. P. and Luu T. P. : Dynamics of a pipe aspirating fluid such as might be used in ocean mining, *Transactions of the ASME, Journal of Energy Resources Technology*, Vol. 107, pp. 250-255, 1985.
- 2) Paidoussis, M. P. : Fluid-structure interactions slender structures and axial flow, Vol. 1, Elsevier Academic Press, 1998.
- 3) Giacobbi, D. B., Rinaldi, S., Semler, C. and Paidoussis, M., P. : The dynamics of cantilevered pipe aspirating fluid studied by experimental, numerical and analytical methods, *Journal of Fluids and Structures*, Vol. 30, pp. 73-96, 2012.
- 4) Fujita, K. and Shintani, A. : Axial leakage flow-induced vibration of the elastic rod as the axisymmetric continuous flexible beam, *Transactions of the ASME, Journal of Pressure Vessel Technology*, Vol. 123, No.4, pp. 421-428, 2001.
- 5) Fujita, K. and Ohkuma, A. : Effect of structural dimensions on dynamic stability of elastic beam subjected to axial flow in confined narrow passage, *Transactions of the JSME, Journal of System Design and Dynamics*, Vol.4, No.6, pp. 809-821, 2012.
- 6) Fujita, K. and Ohkuma, A. : Influence of fluid viscosity and structural specifications on stability in the vicinity of critical velocity of an elastic beam subjected to confined axial flow, *Transactions of the JSME, Journal of System Design and Dynamics*, Vol.5, No.8, pp. 1593-1604, 2011.
- 7) Rinaldi, S. and Paidoussis, M., P. : Theory and experiments on the dynamics of a free-clamped cylinder in confined axial air-flow, *Journal of Fluids and Structures*, Vol. 28, pp. 167-179, 2012.