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Experimental Studies on Driving Mechanism of the High Frequency Combustion Oscillation in a Premixed Gas Rocket

By

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Summary: The experimental studies were conducted to make clear the initiating and driving mechanism of the longitudinal high frequency combustion oscillation in a rocket motor burning a premixed gas as the propellant. The effects of several variables upon the critical oscillation period, which has been found in the previous studies to govern the occurrence of the oscillation, were studied in the beginning. Then the distribution of the gas temperature in the reaction zone during the oscillation was measured in detail. Finally, the photographic study of the reaction zone during the oscillation as well as during the steady combustion was made by the high speed schlieren motion picture.

The results have made clear that the shock wave theory in which the driving mechanism depends on chemical kinetic factors is invalid for describing the occurrence of the oscillation. The interaction between the pressure wave and the combustion process which provides the driving force of the oscillation was found to be of a rather complicated fluid-mechanical nature. The shock wave plays no important role in this interaction. The characteristic time which governs the occurrence of the oscillation was identified with the ignition time delay of the unburnt propellant in the hot combustion gas stream. The latter was found to be of a chemical nature depending chiefly upon the propellant equivalence ratio, although some small effects of the combustion chamber pressure and the injection Mach number of the propellant were found in the experiments.

1. INTRODUCTION

The idea that a rocket motor burning premixed gases as the propellants may be used for the fundamental study of the high frequency combustion oscillation in a liquid propellant rocket motor, was first suggested by Zucrow and Osborn [*1*]. By employing such a relatively simple combustion system of premixed gases, the experimental work is simplified, and the effects of several of the variables which enter into the combustion process of a liquid propellant rocket, such as atomization, vaporization, mixing, etc. are eliminated. In the premixed gas rocket, it is not so difficult to change one parameter at a time, and the pure effect of the parameter can be studied.

In a series of the experimental studies using a cylindrical test rocket motor conducted at the Jet Propulsion Center, Purdue University [*2*]~[*5*], the effects of several variables upon the occurrence and the characteristics of longitudinal and transverse mode oscillations were examined. The observed combustion oscillations were characterized by the presence of pressure waves of finite amplitude propagating in the combustion chamber. The pressure wave was sinusoidal type wave of small amplitude, or in certain conditions became shock type wave of large amplitude. The results showed that the high frequency combustion oscillation which occurs in the premixed gas rocket has much the same character as that observed in a liquid propellant rocket motor. Therefore, it was suggested that the essential natures of the high frequency combustion oscillation can be studied in the simplified combustion system of the premixed gas rocket.

On the basis of the obtained experimental results, Zucrow and Osborn [*1*] suggested a possible mechanism of driving the oscillation. This mechanism was first postulated to explain the existence of the lower critical chamber length of the longitudinal mode oscillation, below which no oscillations will appear, but it is believed that the same mechanism also explains the initiation and the sustenance of the transverse mode oscillation. In substance, the mechanism states that in the combustion chamber a pressure wave is initiated by some pressure disturbance and the subsequent propagation of the pressure wave through the burning and unburned propellants adiabatically compresses these gases. The increased temperature and pressure of gases cause an acceleration in chemical reaction rate (or heat release rate) behind the pressure front, which drives and sustains the pressure wave.

Crocco and Cheng [*6*] studied theoretically the initiation of the high frequency combustion oscillation of longitudinal mode in a liquid propellant rocket. They have developed a linear theory, called sensitive time lag theory, on the basis of the experimental observations that a liquid propellant rocket exhibits almost invariably the so-called time lag behavior, in the sense that there exists the upper critical chamber length as well as the lower critical chamber length. Crocco [*7*] criticized the above driving mechanism postulated by Zucrow and Osborn since it cannot account for the possible existence of the upper critical chamber length. He insisted that the occurrence of the combustion oscillation in the premixed gas rocket, as well as that in a liquid propellant rocket, can be well explained by the sensitive time lag theory.

Zucrow and Osborn [*8*],[*9*] suspected the validity of the sensitive time lag theory for describing the high frequency combustion oscillation, since no upper critical chamber lengths have been observed in their experiments of the premixed gas rocket. In the mean time, similar experimental studies on the high frequency combustion oscillation in the premixed gas rocket was initiated at the Princeton University. The results [*10*],[*11*] showed, in agreement with those of Purdue, that under appropriate equivalence ratio the operation of this type of rocket was always unstable above a certain critical chamber length, and hence did not exhibit the time lag behavior.

Crocco [*12*] suggested that the possible cause of the oscillation in the premixed gas rocket might be different from that in a liquid propellant rocket. The marked apparent discrepancy of the behavior between the liquid and the premixed gas rocket has stimulated theoretical studies on the driving mechanism of the combustion oscillation in the premixed gas rocket. In these studies, it has been considered that the driving mechanism of the oscillation should not be related to the characteristic time of some rate process, but rather to some phenomenon which does not present a characteristic time. The attentions of several investigators [*13*]~[*15*] have been directed to chemical kinetics as the driving force of the oscillation, after the process of heat exchange with the chamber wall has been found to be not important.

Sirignano and Crocco [*15*] treated the oscillation as a nonlinear wave phenomenon of the shock wave propagation. The specific case of a one dimensional combustion zone where chemical kinetics provides the driving force of the oscillation was studied. It was shown that if the rate at which chemical energy is being liberated within the gas is represented by an Arrhenius type rate function depending only on temperature, the oscillation that may result does not exhibit the time lag behavior. The longitudinal oscillation limits are defined in terms of a critical value for a rate parameter E/RT (E=an over-all activation energy for the propellant combination, T=combustion temperature, and R=universal gas constant). The theory predicts that the oscillation would occur at any chamber length for favorable temperature, and no critical chamber lengths exist.

Independently of the studies in the Princeton and the Purdue Universities, an experimental investigation on the high frequency combustion oscillation of the longitudinal mode has also been conducted by the present author by using the premixed gas rocket. The objects of the investigation are to make clear the essential natures of the excited oscillation in the premixed gas rocket and to find out the initiating and driving mechanism of the oscillation. The initial phase of the investigation [*16*],[*17*] has made clear the phenomenological aspect of the oscillation. The obtained results differ from those of the Purdue and the Princeton groups in one important aspect. The upper critical chamber length, as well as the lower critical chamber length, has been observed in the experiments. Moreover, the careful examination of the obtained results on the oscillation regions has led to the conclusion that the occurrence of the oscillation is governed by a certain characteristic time. That is, the premixed gas rocket do exhibit the time lag behavior.

The apparent difference in the observations of these three groups of investigators on the occurrence of the oscillation has resulted in some uncertainty in the interpretation of the oscillation data and hence about the driving mechanism of the oscillation in the premixed gas rocket. Recently, an explanation on this difference has been hazarded by Crocco *et al.* [*19*] in terms of the aforementioned shock wave theory. They concluded that the theory is indeed the correct one which can explain the occurrence of the oscillation in the premixed gas rocket.

However, their description cannot give a satisfactory explanation for the time lag behavior of the premixed gas rocket observed by the present author. Moreover, more recent study of the present author [*18*] has revealed that the excited pressure oscillation in the chamber is essentially an acoustic standing wave oscillation of smooth sinusoidal waveform, although the shock type pressure waveform was observed at a fixed position in the chamber. A small pressure pulse was found to propagate in the chamber being superimposed on the standing wave oscillation, which makes the observed pressure waveform the so-called shock type. The propagation of the pulse in the chamber was isentropic and the nonlinearity of the oscillation was not so important as was expected from the observed pressure waveform.

The actual nonsteady combustion process in the chamber must be so complicated that the one dimensional chemical kinetics model of their theory seems too unrealistic even in the relatively simple combustion system of the premixed gas rocket. The interaction between the pressure wave and the combustion process which provides the driving force of the oscillation may be of a more complicated nature. The details of this interaction can only be known by the direct observation of the combustion reaction zone during the oscillation and this should be done before any correct theoretical model is postulated for describing the oscillation.

The object of the present studies is to find out the initiating and driving mechanism of the oscillation through the experimental works. It was found in the previous studies [*16*],[*17*] that the occurrence of the oscillation was governed by the critical oscillation period. The longitudinal oscillation of a given mode occurred only when the oscillation period of that mode was larger than this critical period. In the present studies the effects of several variables on the critical period were examined in the beginning, so as to elucidate the essential natures of the critical period. The effect of propellant equivalence ratio has already been known, and hence the effects of other three controllable parameters, that is the injection section length, the mean combustion chamber pressure, and the injection Mach number of the propellant, were studied. Then the visual and photographic observation of the reaction zone during the steady combustion was made to know how the combustion proceeds in the reaction zone. The distribution of gas temperature in the reaction zone was measured also in detail in order to determine the effect of temperature on the occurrence of the oscillation. Finally, the instantaneous behavior of flames in the reaction zone during the oscillation was observed by the high speed schlieren motion picture. From these experiments, a definite conclusion was obtained about the initiating and driving mechanism of the oscillation.

2. Experimental Apparatus

The experimental apparatus used in the present studies were almost the same as those used in the previous studies [*16*]~[*18*].. Figure 1 presents a cross sectional drawing of the research rocket motor used in the present studies. The motor was composed of a mixing chamber, an injector, water cooled combustion chamber sections of 5 cm×8 cm internal cross section, and an exhaust nozzle. The propellant used was the premixed city gas and air. The air and the city gas were continuously supplied by compressors and their flow rates were metered separately by the calibrated orifices installed in their respective feed lines. The average composition (approximate) of the city gas was 9.0% CO_2, 2.8% O_2, 6.2% CO, 37.0% H_2, 24.0% CH_4, 4.0% C_2H_4, 4.0% C_3H_6, and 13.0% N_2. The stoichiometric gas air ratio of the city gas was found to be 1:4.33, and the equivalence ratio of the mixture was calculated on this basis.

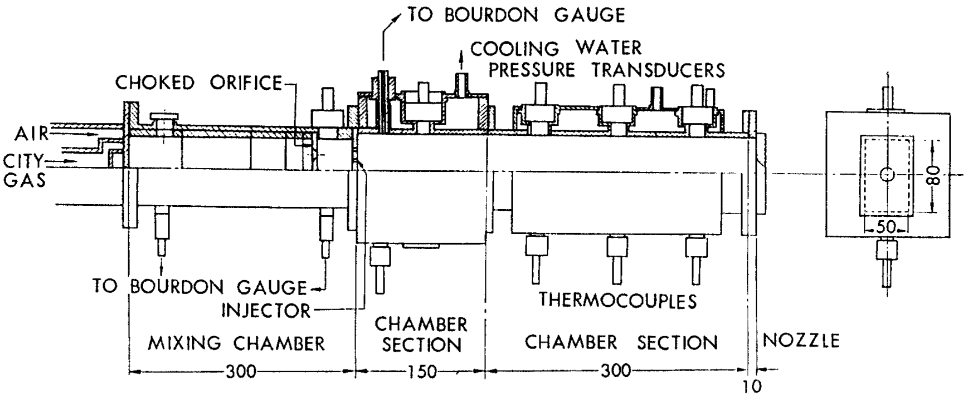

FIG. 1. Research rocket motor.

The city gas and the air were introduced into the mixing chamber impinging on each other, and then were mixed throughly. The premixed propellant entered into the injection section of the mixing chamber through a choked orifice, which was installed so as to suppress any low frequency combustion oscillation which might occur, and also to prevent the upstream propagating flame in the case of flashback. The mixing chamber was so designed as to make it possible to change the length of injection section and to measure the oscillating pressure and the mean pressure in the injection section. However, no important effect of the injection section length on the oscillation was found in the first experiment, and hence it was kept constant at the shortest length of 4.0 cm through the present studies.

The propellant was injected into the combustion chamber through the injector. Several combustion chamber sections of different types were used. These sections were combined appropriately in accordance with the aim of each experiment to form a combustion chamber. The injector used was also different for each experiment. The converging exhaust nozzle was a mere flat plate with a circular throat of 9.5 mm diameter in the center, so as to avoid the complex reflection of pressure waves at the entrance section of the nozzle.

For the studies determining the effects of the three parameters on the critical period, the four usual combustion chamber sections used in the previous study [*18*] were used. Two injectors of the same type but different injection area were used in order to change the injection Mach number of the propellant into the combustion chamber. The injectors had the same distribution of injection holes shown in Fig. 2, with different injection hole diameters of 2.0 mm and 2.4 mm. The experiments were conducted with six different combustion chamber lengths of 16.0 cm, 20.5 cm, 31.0 cm, 46.0 cm, 61.0 cm, and 76.0 cm, by combining the four sections.

Three special combustion chamber sections were made anew so as to examine how the combustion proceeds in the reaction zone. The cross sectional drawings of these sections with those of injectors are shown in Figs. 3~5. The combustion chamber section shown in Fig. 3 was used for the visual observation of the reaction zone when the combustion proceeds steadily at the atmospheric pressure. The injector used had the same distribution of injection holes as that shown in Fig. 2. The experiment was conducted with two usual chamber sections used so far attached

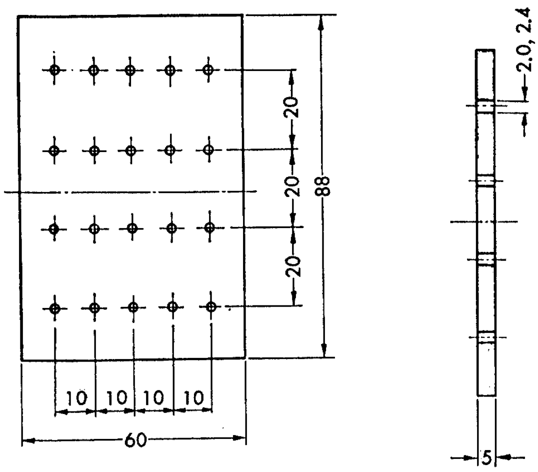

FIG. 2. Specifications for injector plates.

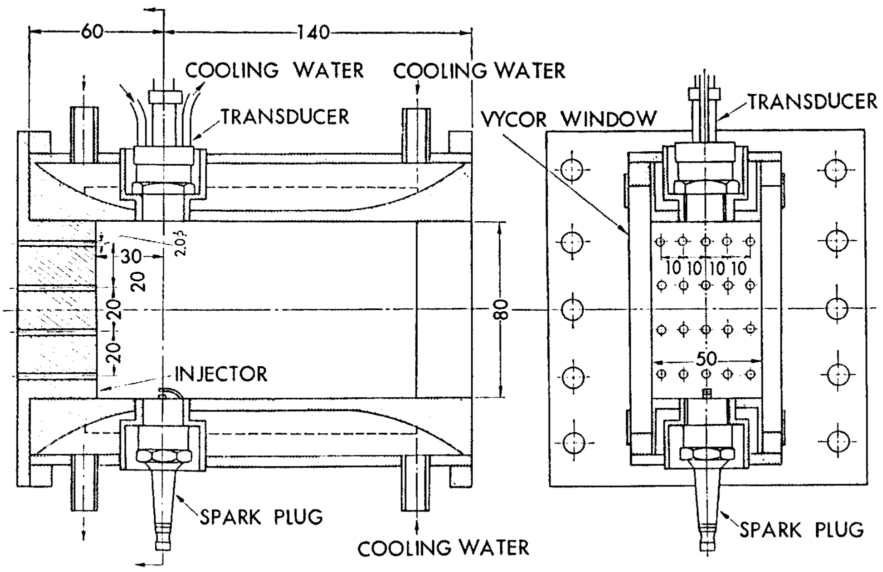

FIG. 3. Combusiton chamber section for photographic study of reaction zone at atmospheric chamber pressure.

behind this section. The total length of the combustion chamber was 78.5 cm.

The distribution of the gas temperature in the reaction zone was measured with the combustion chamber section shown in Fig. 4. In this section 10 thermocouples of *Pt/Pt-Rh* were inserted at intervals of 1 cm along flow direction starting at 2.0 cm from the injector face. The measurements were made at three different jutting heights of 20 mm, 15 mm, and 10 mm from the lower wall of the combustion chamber. The injectors used were the same as those shown in Fig. 2. The experiment was conducted with one usual combustion chamber section attached behind this section resulting in the total combustion chamber length of 46.0 cm.

For the photographic study of the reaction zone during the oscillation the combustion chamber section shown in Fig. 5 was used. This section had circular Vycor

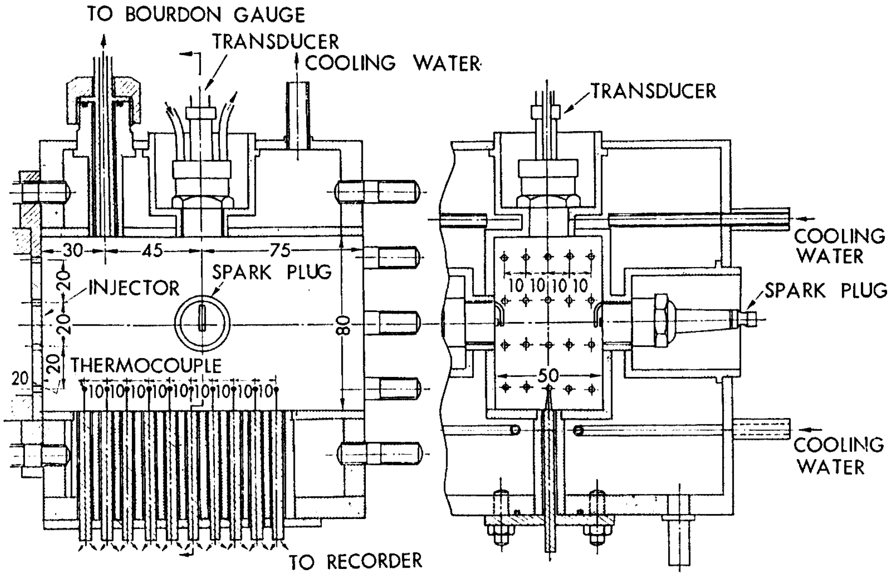

FIG. 4. Combustion chamber section for measurement of temperature distribution in reaction zone.

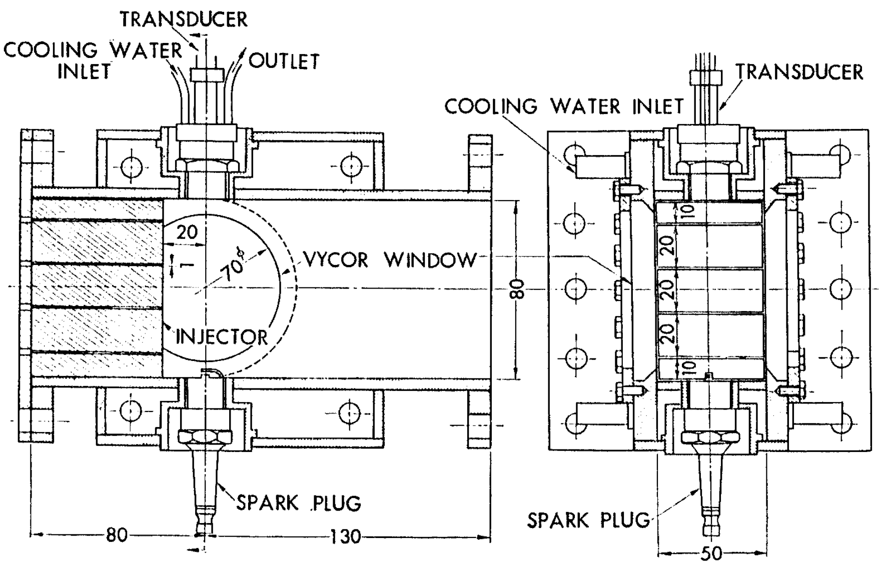

FIG. 5. Combustion chamber section for photographic study of reaction zone during oscillative combustion.

windows near the injector face to permit taking the schlieren picture. The special two dimensional injector of four slits of 1 mm width was used with this section so as to obtain two dimensional flames. Two pressure transducers were mounted flush with the upper wall at 2.0 cm from the injector face and 6.0 cm from the nozzle end of the chamber, respectively, to observe the pressure variation in the reaction zone and near the nozzle end of the chamber during the oscillation. In the experiment, two usual combustion chamber scetions were attached behind the section shown in Fig. 5 and the total length of the combustion chamber was 61.0 cm.

The measuring apparatus of the mean and the oscillating pressures and the gas temperature, as well as their recording system, were exactly the same as those used in the previous studies. The schlieren system used in the last experiment is described in reference [20]. The super high pressure mercury lamp (WAKO BHO-1000, DC 1 kw) was used as the light source of the system. The schlieren motion picture was taken with the high speed camera (HITACHI HIMAC 16HB) at about 9,000 frames per second. With this camera, the pressure variation in the reaction zone as well as the instantaneous behavior of flames was recorded simultaneously on the film.

3. Experimental Results

3.1. *Critical Oscillation Period*

(A) *Effect of Injection Section Length*

There is a possibility of high frequency combustion oscillation being excited when the propagation time of pressure wave in the propellant feeding system coincides with that in the combustion chamber. In the present investigation the longitudinal dimension of combustion chamber is several times or more as large as that of the injection section, but the sound velocity is several times as fast in the combustion chamber and hence the magnitude of the wave propagation time is the same order in the combustion chamber and in the injection section. Therefore, there is a possibility that the driving mechanism of the oscillation may be associated with the coupling of two pressure oscillations in the combustion chamber and in the injection section.

If the driving mechanism of the oscillation was connected with the coupling of two pressure oscillations in the combustion chamber and in the injection section, the critical oscillation period which was found to govern the occurrence of the oscillation should depend upon the length of the injection section. In the previous studies [16],[17], the mixing chamber with 2.0 cm injection section length was used. The experiment was conducted in the present studies with the same injector as before but with the longer injection section length of 4.0 cm, so as to determine the effect of this length on the critical period.

The oscillation boundaries of each mode oscillation in φ (propellant equivalence ratio)—P_c (mean combustion chamber pressure) plane were determined exactly in the same was as before for the six different chamber lengths L_c. Figures 6~8 present the obtained oscillation regions for the three representative chamber lengths. The oscillation region of each mode oscillation progressed steadily toward lower chamber pressure with the increase of the chamber length, extending its boundary both in the rich and in the lean sides. From these oscillation regions and the oscillation periods measured at the boundaries, the critical oscillation period τ_{cr} was obtained as the function of the equivalence ratio in the same way as before. The result for the case of $P_c=3.0$ kg/cm^2 abs. is shown in Fig. 9. The circles in the figure present the data obtained in this experiment while the solid curve represents the previous data [16],[17] obtained for the shorter injection section length of 2.0 cm. The positive integer n indicates the mode number of the oscillation. Although there are some scattering of data, it can be said that these two data are

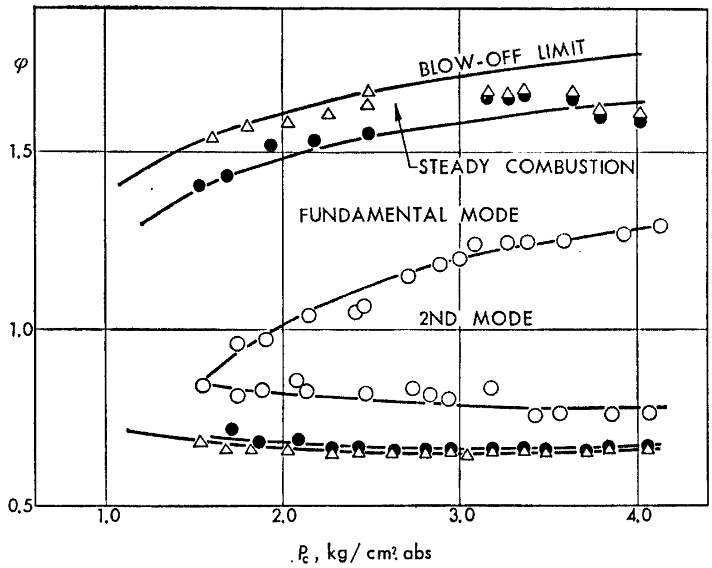

FIG. 6. Oscillation regions; L_c=46.0 cm, M_i=0.29.

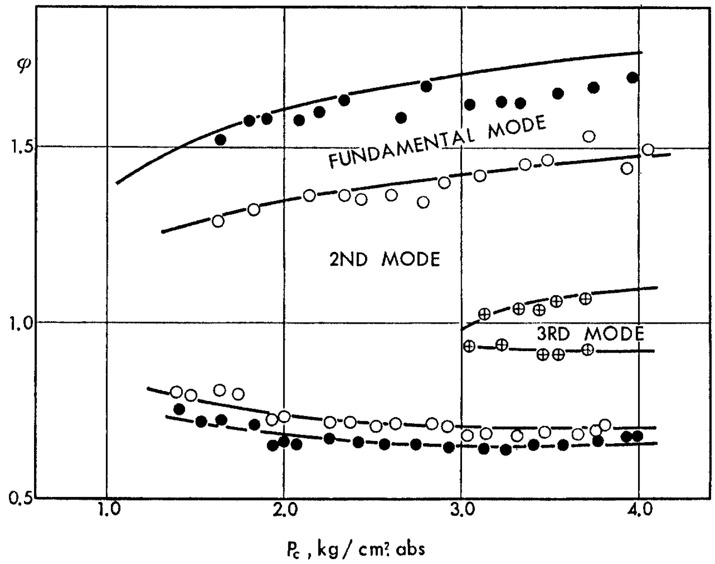

FIG. 7. Oscillation regions; L_c=61.0 cm, M_i=0.29.

identically the same.

In this experiment the pressure variation in the injection section, as well as in the combustion chamber, was observed simultaneously during the oscillation. Although the pressure in the injection section was found to vary rather periodically with the identical frequency of the oscillation in the combustion chamber, the amplitude of oscillation was very small (at most 1/5 of that in the combustion chamber), and no definite phase relations were observed between these two oscillations.

(B) *Effect of Combustion Chamber Pressure*

The critical oscillation period presented in Fig. 9 is that for the case of P_c= 3.0 kg/cm² abs. From the oscillation regions and the oscillation periods measured in the preceding experiment for the six chamber lengths, the critical oscillation

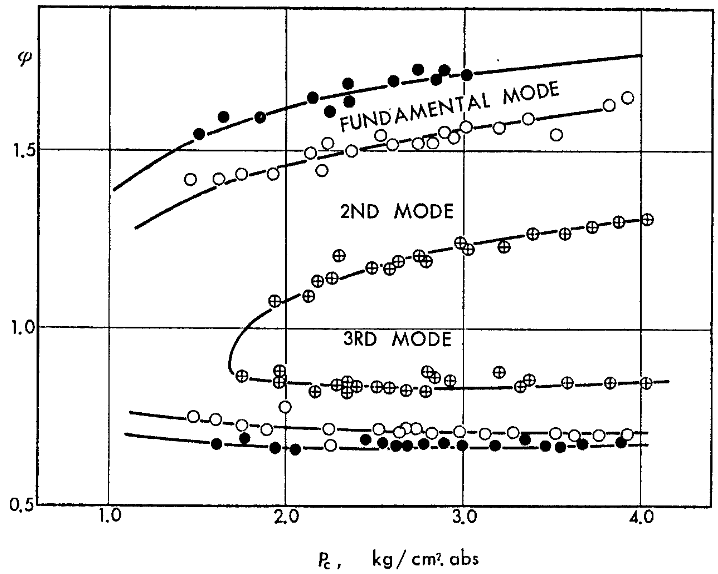

FIG. 8. Oscillation regions; L_c=76.0 cm, M_i=0.29.

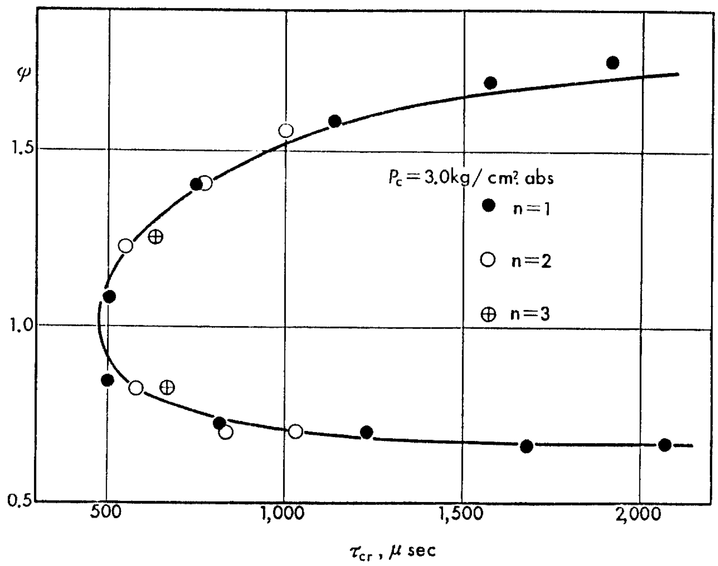

FIG. 9. Effect of injection section length on critical oscillation period.
solid curve; for 2.0 cm injection section length.
circles; for 4.0 cm injection section length.

period was obtained for other two chamber pressures, that is for P_c=2.0 kg/cm² abs. and P_c=4.0 kg/cm² abs. The result is shown in Fig. 10 together with that for P_c=3.0 kg/cm² abs. As is seen in the figure, the critical oscillation period decreases with the increase of the mean combustion chamber pressure.

(C) *Effect of Injection Mach Number*

The injection Mach number M_i of the unburnt propellant into the combustion chamber depends upon the ratio of two pressures in the injection section and in the combustion chamber. In the experiment this pressure ratio became independent of the combustion chamber pressure when the latter exceeded 2 kg/cm² abs., that is

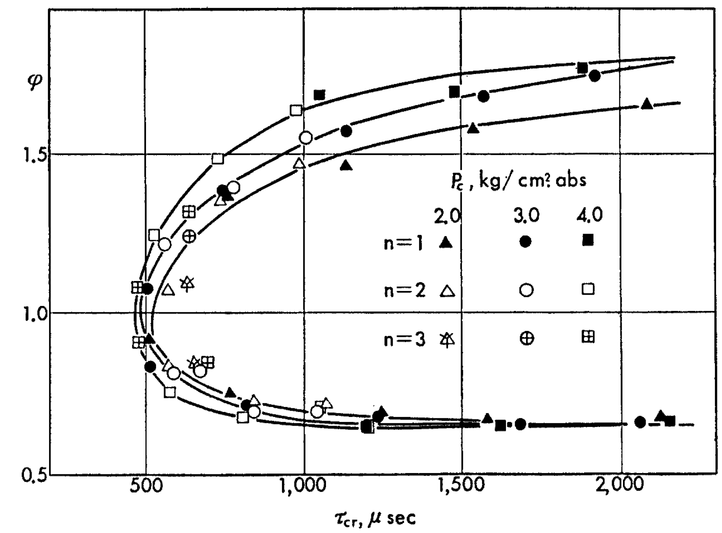

FIG. 10. Effect of combustion chamber pressure on critical oscillation period; $M_i=0.29$.

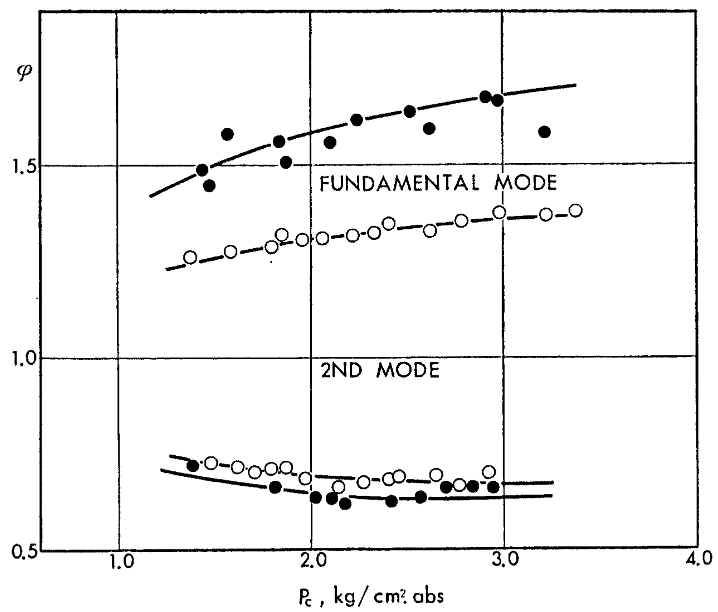

FIG. 11. Oscillation regions; $L_c=46.0$ cm, $M_i=0.37$.

when the flow at the exhaust nozzle became choked. Then the pressure ratio depended only upon the injection area of the injector for the constant flow area of the exhaust nozzle. In the present experiment, therefore, the injector with the smaller injection hole diameter was used so as to study the effect of the injection Mach number on the critical oscillation period.

The oscillation regions of each mode oscillation were determined in the same way as before for the six different chamber lengths. Figures 11~13 present the results for the three representative chamber lengths. These are compared with those shown in Figs. 6~8, which are the results for the lower injection Mach number. It can be seen that for the same chamber length the oscillation regions of each mode oscil-

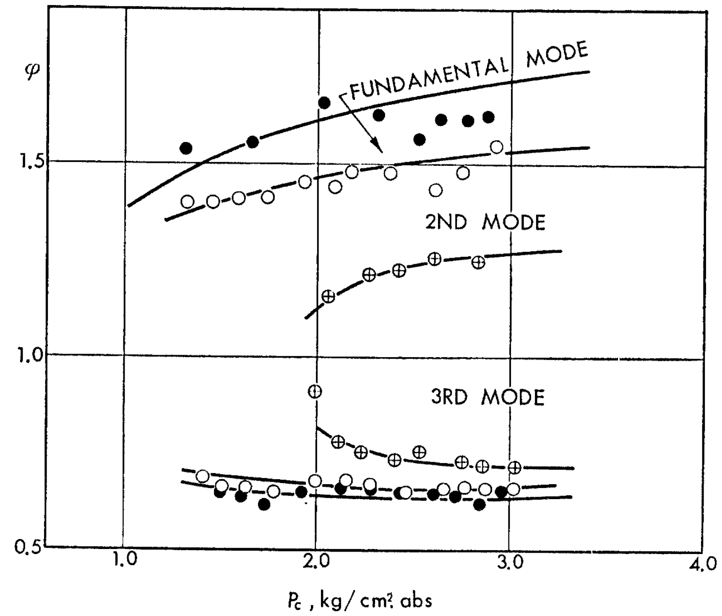

FIG. 12. Oscillation regions; $L_c=61.0$ cm, $M_i=0.37$.

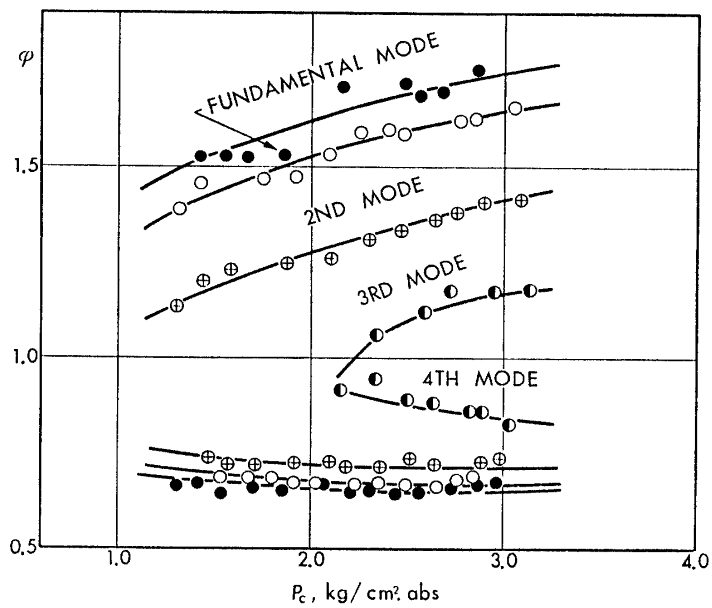

FIG. 13. Oscillation regions; $L_c=76.0$ cm, $M_i=0.37$.

lation are larger for the higher injection Mach number. The critical oscillation period was obtained as the function of φ exactly in the same way as before. Figure 14 presents the result together with that for the lower injection Mach number, for the case when $P_c=3.0$ kg/cm^2 abs. In the figure, the injection Mach number was calculated by the usual one dimensional theory with the assumption of isentropic expansion, using the observed value of the pressure ratio. As is seen the critical period decreases with the increase of the injection Mach number.

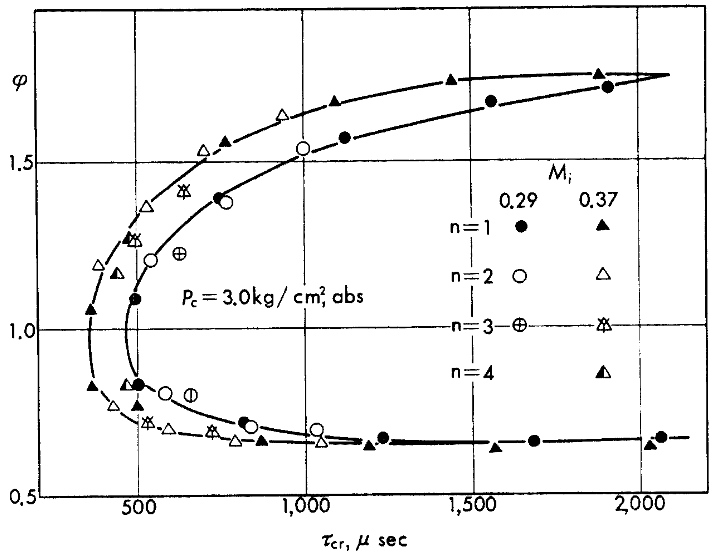

FIG. 14. Effect of injection Mach number on critical oscillation period; P_c=3.0 kg/cm² abs.

3-2. *Observation of Flames during Steady Combustion*

The visual and photographic observation was made in this experiment to study how the combustion proceeds in the reaction zone. The experiment was conducted without the exhaust nozzle, that is at the atmospheric combustion chamber pressure, so as to simplify the experimental works. The injector with the smaller injection holes was used and the flow rate of the propellant was so arranged as the injection Mach number of the propellant became equal to 0.37, which was the value when the motor was operated with the exhaust nozzle.

The visual observation of the reaction zone has made clear that the combustion proceeds through twenty turbulent flames formed near the injector face of the combustion chamber, corresponding to the twenty jets of the unburnt propellant. Each of these flames was stabilized by the recirculation zones of hot combustion gas which surrounded the flame. Figure 15 presents the direct photographs of the flames at six different equivalence ratios. The configuration of flames changed considerably with the equivalence ratio. The average flame length was shortest near the stoichiometric and became longer with the increase or the decrease of equivalence ratio from the stoichiometric.

The photographs in Fig. 15 are of the flames during the steady combustion at the atmospheric combustion chamber pressure. However, it had been confirmed by the visual observation in the preliminary experiment of the present investigation that the over-all configuration of flames depended little on whether combustion was stable or oscillative. Therefore, it can be considered that the time mean configurations of the flames during the oscillative combustion are similar to those shown in the figure.

3-3. *Distribution of Mean Gas Temperature in Reaction Zone*

In the present experiment, the distribution of gas temperature in the reaction zone

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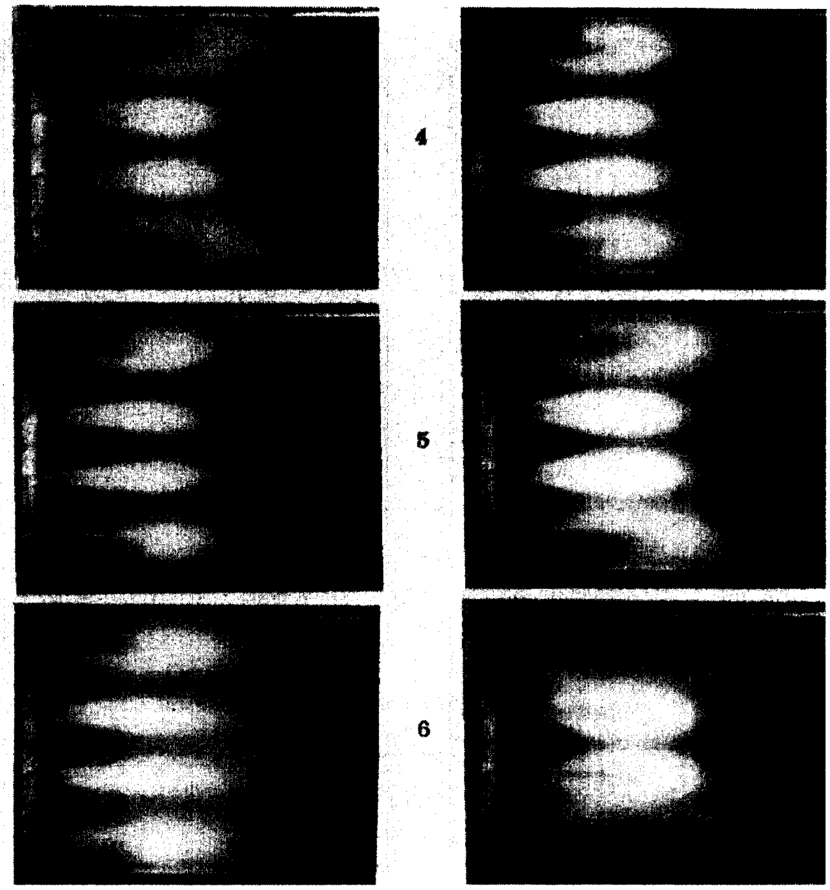


FIG. 15. Direct photographs of flames during steady combustion; $M_i=0.37$, $P_c=1.03$ kg/cm^2 abs.
1. $\varphi=0.60$ 4. $\varphi=1.13$
2. $\varphi=0.74$ 5. $\varphi=1.26$
3. $\varphi=0.98$ 6. $\varphi=1.43$

during the oscillative combustion was measured so as to elucidate further how the combustion proceeds in the combustion chamber. The measurement was made for the two injection Mach numbers of 0.29 and 0.37, and for the two combustion chamber pressures of 2.0 kg/cm^2 abs. and 3.0 kg/cm^2 abs. at the constant combustion chamber length of 46.0 cm. The mode of the oscillation excited in the combustion chamber depended upon the chamber length, but the combustion process itself concentrated near the injector face will have no direct connection with the chamber length. It may be considered, therefore, that the same temperature distribution in the reaction zone prevails for any other chamber length.

Figures 16~18 present the measured temperature distributions along flow direction in the reaction zone for various values of the equivalence ratio, for the case when $M_i=0.29$ and $P_c=3.0$ kg/cm^2 abs. Figure 16 presents the result for the case when the jutting height y of thermocouples from the lower wall was 10 mm. This position corresponded to the central axis of one of the twenty injecting jets of the propellant from the injector. The position of the maximum temperature may be considered to correspond to the time mean position of the turbulent flame tip.

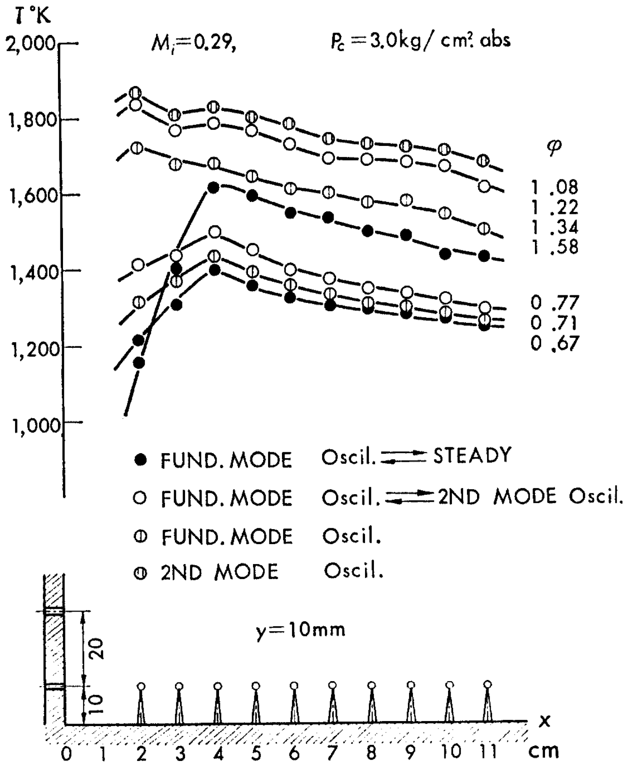
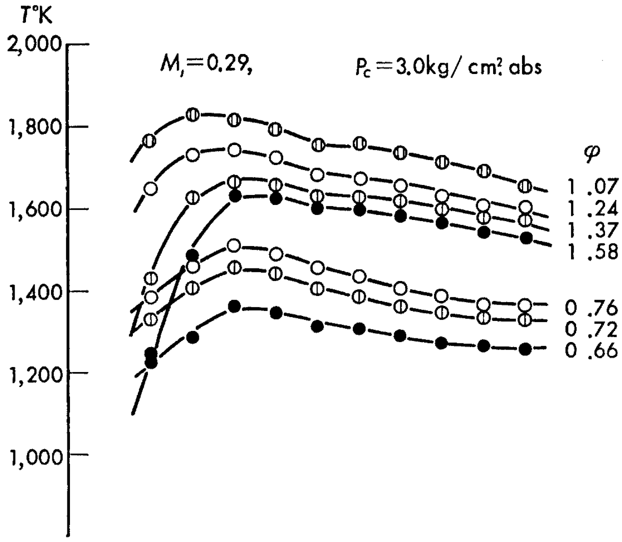


FIG. 16. Distribution of mean gas temperature in reaction zone; P_c=3.0 kg/cm² abs., M_i=0.29, y=10 mm.

In the figure, the temperature distribution at the lean and the rich oscillation boundaries which separate the steady combustion from the oscillative combustion of the fundamental mode are also shown. It can be seen that neither temperature distribution nor maximum temperature becomes equal at the two boundaries. This result conflicts sharply with the findings of Crocco *et al.* [*19*] who observed the same combustion temperature at the rich and the lean oscillation boundaries. The temperature distribution as well as the maximum temperature is also different for the rich and the lean oscillation boundaries which separate the fundamental mode oscillation from the second mode oscillation.

The temperature distribution as well as the maximum temperature in the reaction zone largely depended upon the propellant equivalence ratio. They did not show any appreciable change by passing over the oscillation boundaries. Any change, if it had been observed, might be attributed to the change in the equivalence ratio.

Figure 17 presents the observed temperature distributions at y=15 mm. The general character of the distribution is rather similar to that of Fig. 16. Figure 18 presents the observed results for y=20 mm. At this jutting height of thermocouples, all junctions were immersed in the hot combustion gas stream. The obtained distribution will show that of the burnt gas. Especially, the maximum temperature at the top will indicate the temperature of combustion gas in the recirculation zone. In this case too, the effect of the equivalence ratio is remarkable.



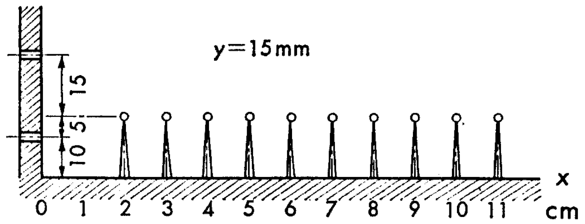


FIG. 17. Distribution of mean gas temperature in reaction zone; P_c=3.0 kg/cm² abs., M_i=0.29, y=15 mm.

The measurements were made exactly in the same way for the higher injection Mach number of 0.37 and for the lower combustion chamber pressure of 2.0 kg/cm² abs. However, no appreciable effects of the injection Mach number and the combustion chamber pressure were observed, and hence the results are not presented here. Instead, only the maximum temperatures observed in these distributions are shown as the function of the equivalence ratio in Fig. 19. The above results of temperature measurements have revealed that the temperature distribution in the reaction zone, as well as the flame tip temperature, depends chiefly upon the propellant equivalence ratio, being independent of whether combustion is stable or oscillative and of the mode of the oscillation. It is not the temperature which is crucial for the occurrence of the oscillation.

3-4. *Instantaneous Behavior of Flames during Oscillation*

In order to study the instantaneous behavior of flames during the oscillation, the high speed schlieren motion picture of the reaction zone was taken. Figure 20 presents the representative sequences of the picture taken at 9,000 frames/sec during the steady combustion. The dark strips in the photographs correspond to the two dimensional flames. In the scope of photographs only two of four flames can be seen clearly. The combustion proceeded by these turbulent flames formed near the injector face of the combustion chamber, just as was made clear by the direct

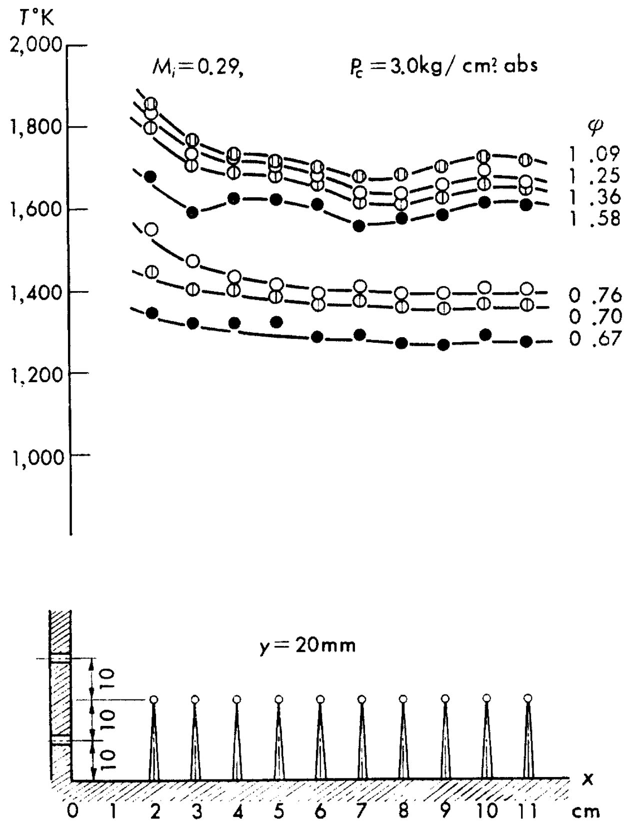


FIG. 18. Distribution of mean gas temperature in reaction zone; P_c=3.0 kg/cm² abs., M_i=0.29, y=20 mm.

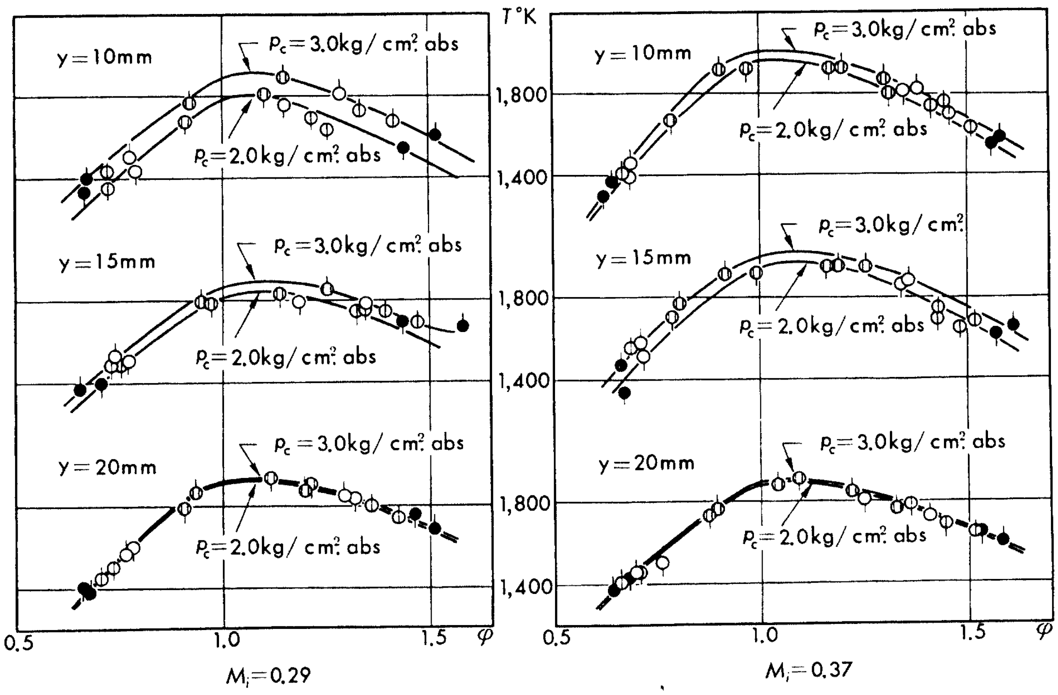


FIG. 19. Maximum temperatures in reaction zone.

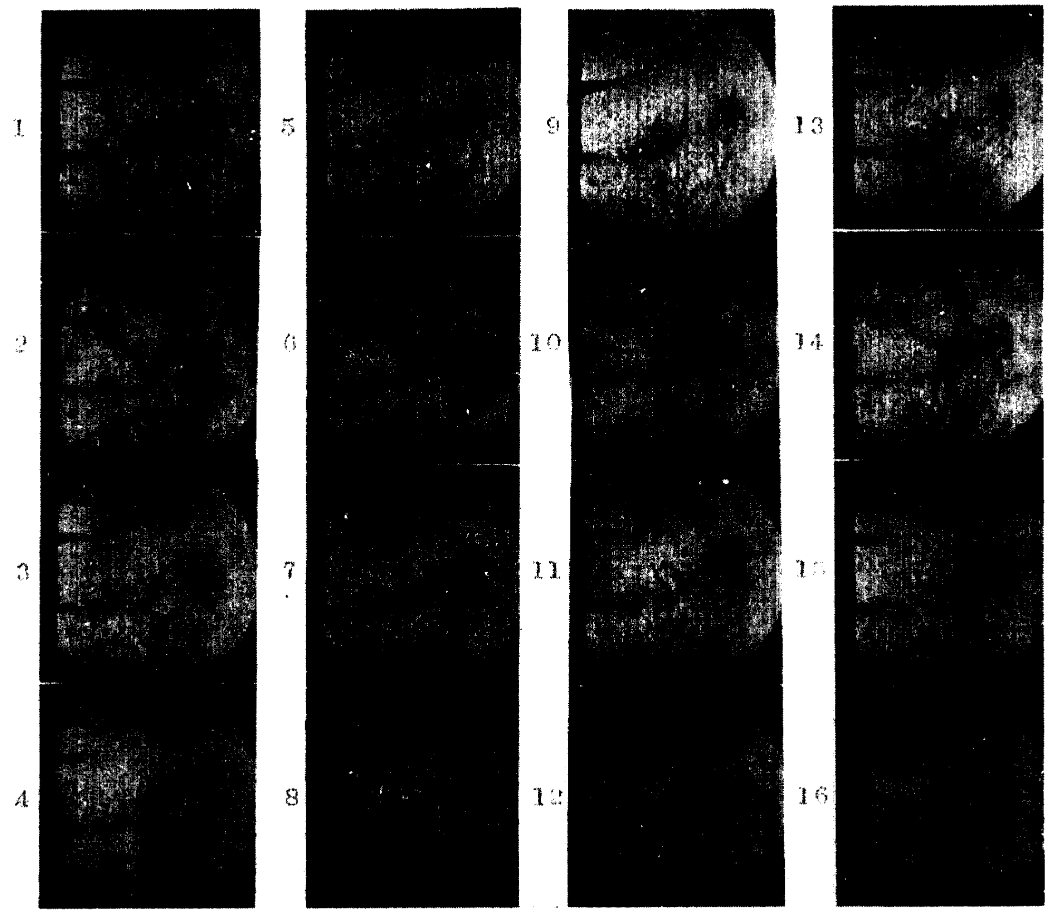


FIG. 20. Schlieren photographs of flames during steady combustion;
$P_c = 1.9\ \mathrm{kg/cm^2}$ abs., $\varphi = 0.65$, $M_i = 0.19$.
Approx. 9,000 frames/sec.

photographs in the previous experiment. Although the flames underwent continued random fluctuations, no organized behavior of flames with well defined periodicity was observed during the steady combustion.

Figures 21~24 present the representative sequences of the picture obtained during the oscillative combustion of the fundamental mode. In the figures, the consecutive sixty four sequences with the time interval of about 110 μsec are presented. As is seen in the figures, the flames oscillate regularly with a well defined periodicity. Each sixteen consecutive sequences in the figure constitute one cycle of the flame oscillation with the period of about 1,800 μsec. The signal of the pressure variation observed simultaneously in the reaction zone is superimposed on the photographs with the white trace. The observed pressure variation p' obtained from this trace with the over-all flame length l' is plotted against time under the photographs. Although many random fluctuations are superimposed on the pressure variation, it also has the well defined periodicity of the identical period with the flame oscillation.

The pressure variation in the reaction zone was accompanied by so many small irregular fluctuations. These fluctuations may be attributed partly to the noise of

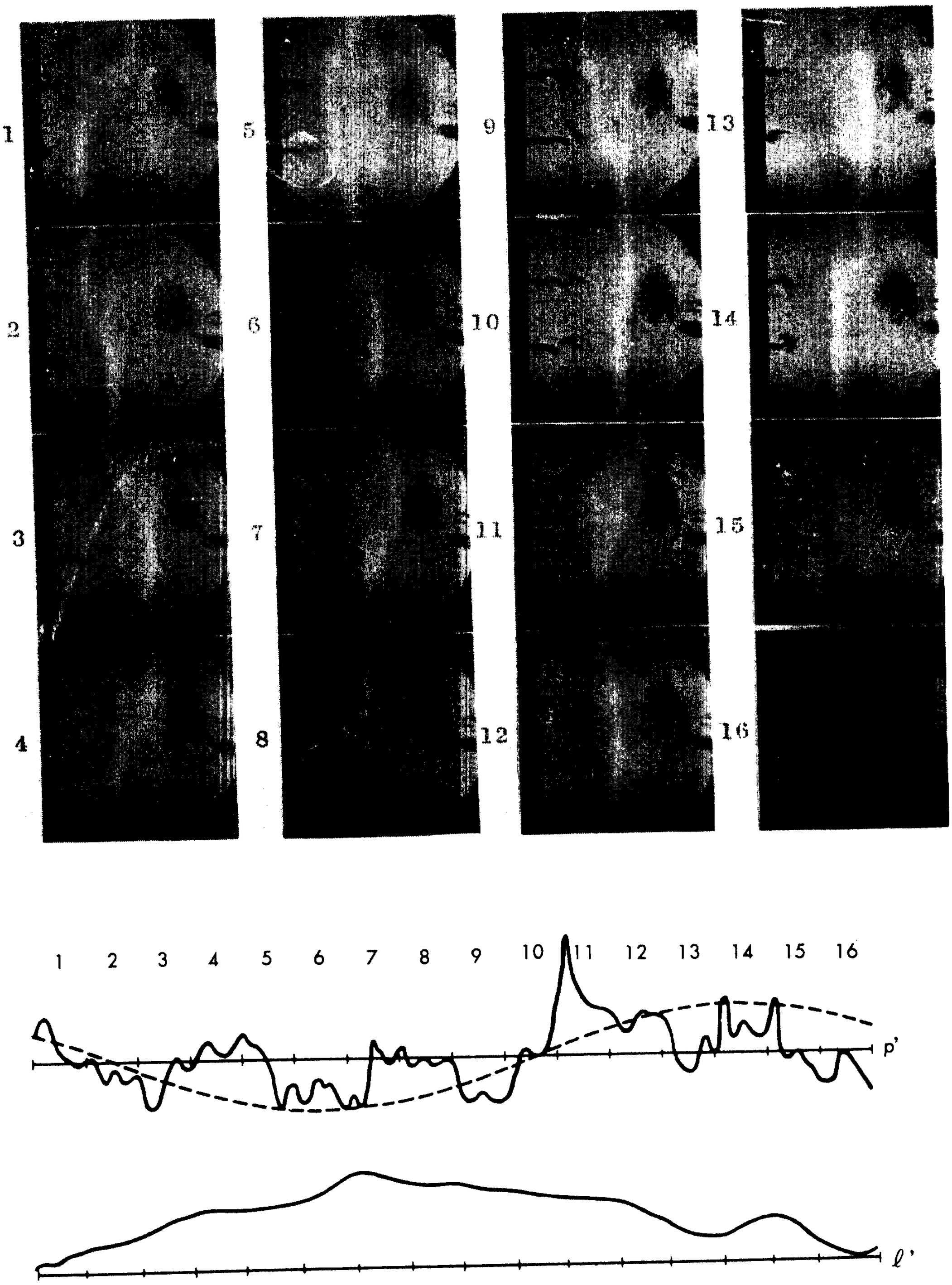


FIG. 21~FIG. 24. Schlieren photographs of flames and pressure variation in reaction zone during oscillative combustion;

P_c=1.8 kg/cm^2 abs., φ=0.81, M_i=0.27. Approx. 9,000 frames/sec.

—— p' Pressure variation in reaction zone observed simultaneously with schliren photographs. (The white trace in each sequence represents the varying pressure at the time about four sequences before it.)

- - - Standing pressure oscillation excited in chamber.

l' Variation of over-all flame length from injector face.

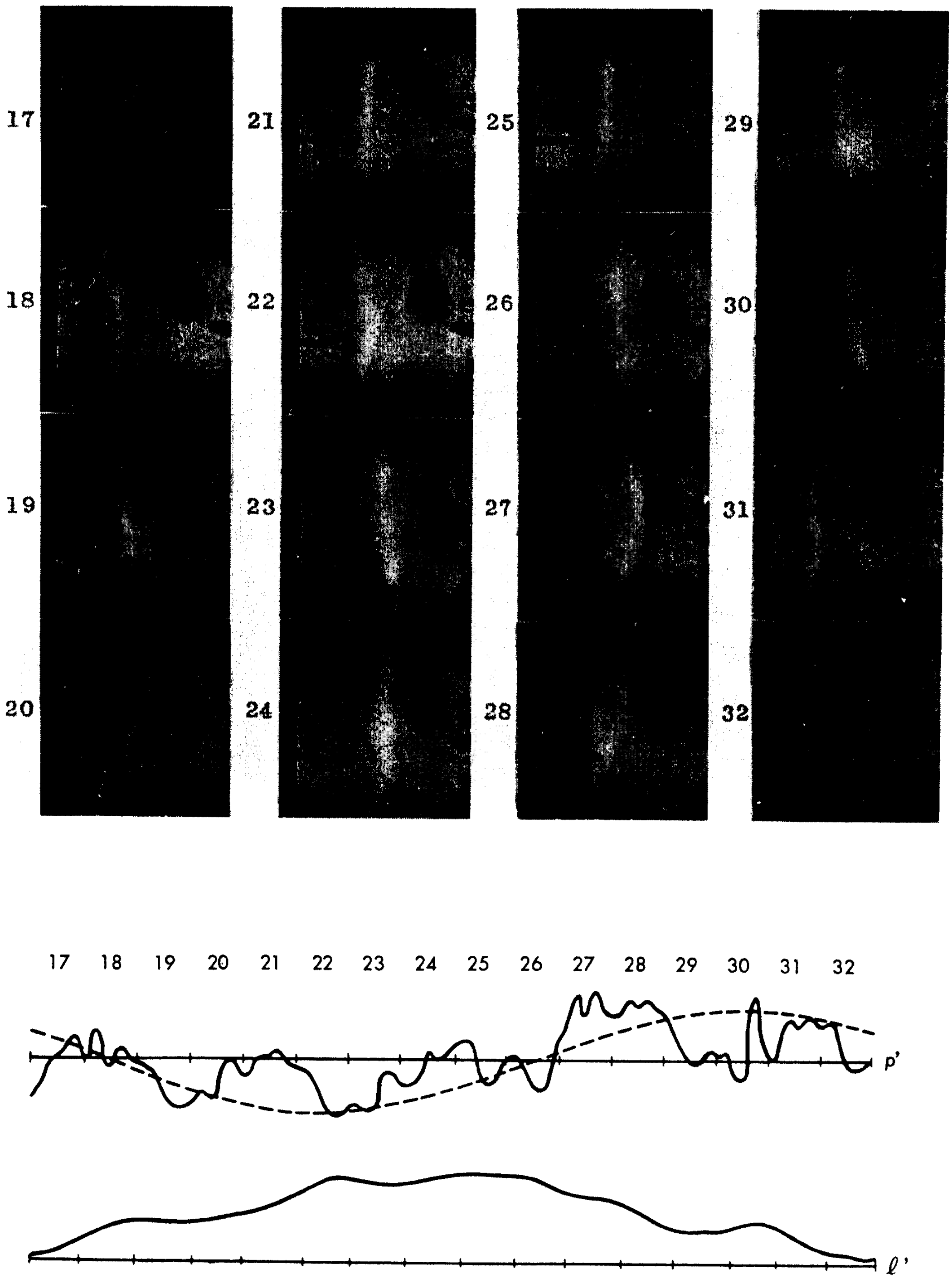

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FIG. 22.

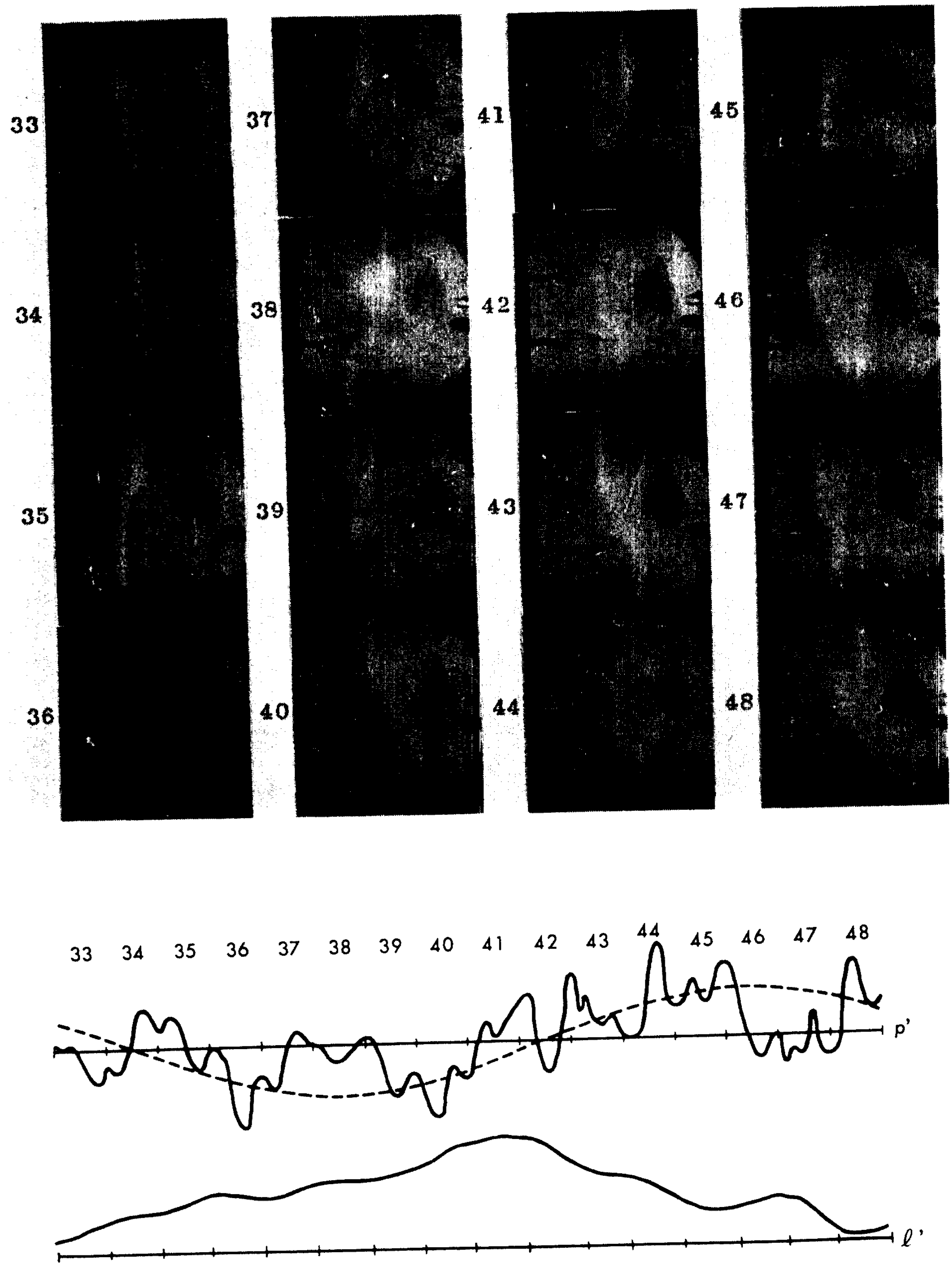
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Fig. 23.

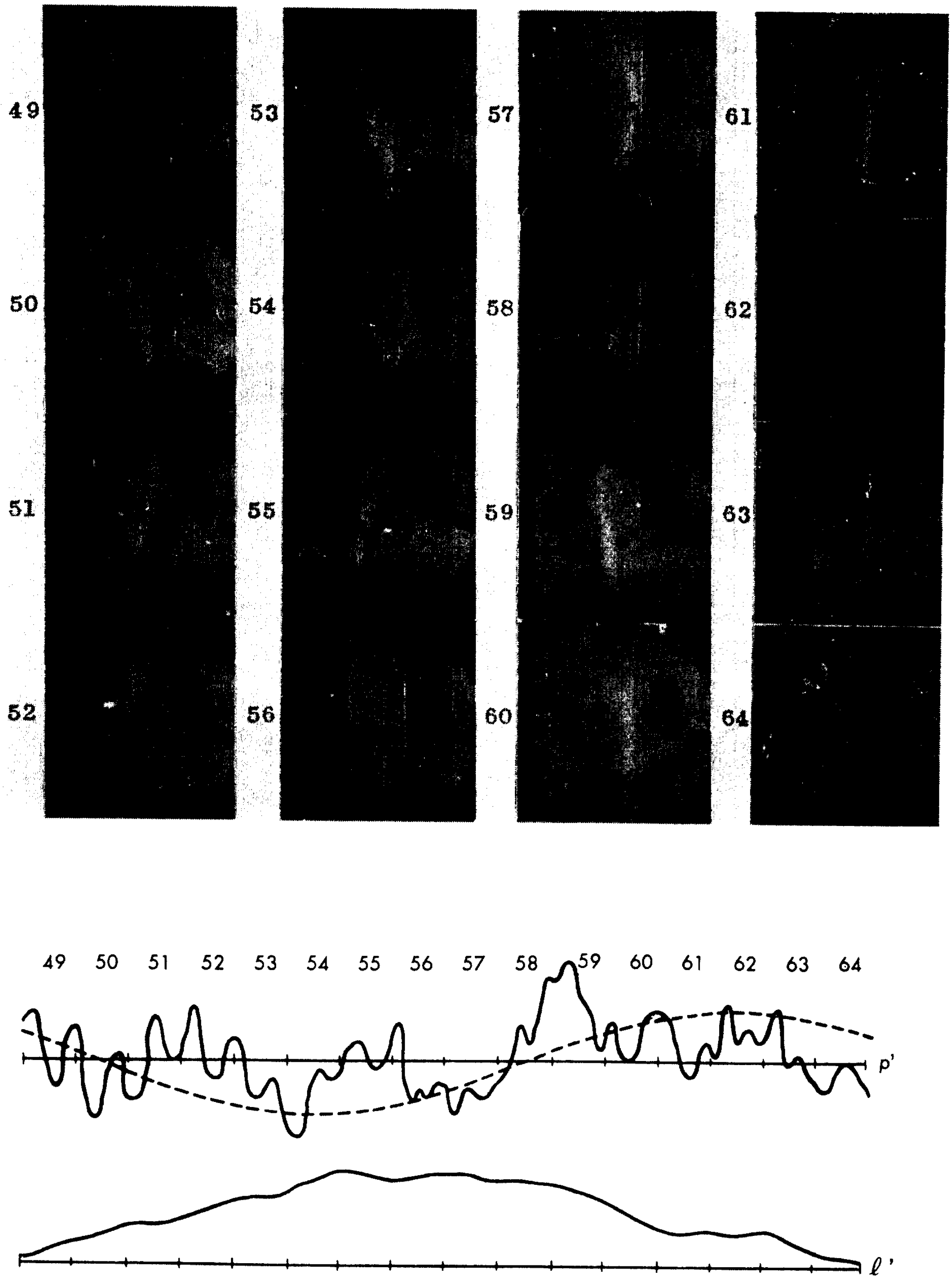

FIG. 24.

the transducer itself, but mostly to the complicated interaction between the pressure wave and the flames in the reaction zone. The pressure trace observed simultaneously near the nozzle end of the chamber exhibited more smooth sinusoidal waveform with a small pulse being superimposed on it. The excited pressure oscillation in the combustion chamber was essentially a standing wave oscillation of smooth sinusoidal waveform with a small propagating pressure pulse being superimposed on it, as was already made clear in the previous study [*18*].

The excited sinusoidal standing pressure oscillation in the chamber, which was obtained from the pressure trace near the nozzle end, is shown schematically under the photographs by the dotted lines, as compared with the pressure variation in the reaction zone represented by the solid curve. The rapid pressure increase which appeared in each cycle of the pressure oscillation in the reaction zone was confirmed to correspond to the small pulse propagating in the chamber being superimposed on the standing pressure oscillation.

There is no doubt that the flame oscillation is closely connected with the excitation of the standing pressure oscillation in the combustion chamber. Two possible causes may be considered for the flame oscillation. The first depends upon the variation of burning velocity of the flame in accordance with the pressure or temperature variation during one cycle of the oscillation. The second depends upon the variation of injection velocity of the propellant into the combustion chamber. In the case of the present studies, however, it is clear that the pressure variation of such a small amplitude (about a tenth of the mean combustion chamber pressure) will not produce any appreciable change in the chemical reaction rate of combustion. The burning velocity of the flame should remain almost constant during one cycle of the oscillation. On the other hand, the pressure oscillation in the combustion chamber will affect considerably the injection velocity of the propellant, since the magnitude of pressure drop through the injector was the same order with the amplitude of pressure oscillation.

During the standing pressure oscillation in the combustion chamber the pressure at the injector exit varies periodically, while the pressure in the injection section remains almost constant. Thus the pressure drop through the injector, and hence the injection velocity of the propellant into the combustion chamber, will vary periodically. The flame should oscillate in accordance with this oscillation of the injection velocity, since the burning velocity of the propellant remains almost constant. Therefore, it can be concluded that during the oscillative combustion the flame is oscillating with the identical period of the pressure oscillation on account of the variation of the injection velocity, which is, after all, caused by the pressure oscillation in the chamber.

4. Discussions

4-1. *Time Lag Behavior of the Gas Rocket*

The results of the experiments on the critical oscillation period have confirmed the conclusion of the previous studies that the gas rocket exhibits the time lag behavior. The occurrence of the oscillation is governed by the critical oscillation period. This means that the driving mechanism of the oscillation is related to some rate process which presents a characteristic time. This characteristic time should be closely related with the critical period, although the details of this relation is not clear at present.

The result of the first experiment has shown that the critical period, and hence the characteristic time, is independent of the injection section length. The driving mechanism of the oscillation is not associated with the coupling of two pressure oscillations in the combustion chamber and in the injection section. During the oscillation, some pressure waves may travel back into the injection section through the injector. These waves, however, will be damped quickly and no standing pressure oscillations are established in the injection section. The driving mechanism of the oscillation, therefore, should be sought in the interaction between the pressure wave and the combustion process in the reaction zone.

The interaction which provides the driving force of the oscillation must be accompanied with the characteristic time. The studies on the critical period have revealed that the characteristic time depends strongly on the propellant equivalence ratio. The effects of the chamber pressure and the propellant injection Mach number are not so important in the observed range of the experiments. It may be said that the characteristic time is rather of a chemical nature. Therefore it may be naturally expected that the interaction is also of a chemical nature. The fact is, however, that the interaction is essentially a fluidmechanical one against this expectation.

The results of the temperature measurement in the reaction zone have shown that the temperature alone does not govern the occurrence of the oscillation. The shock wave theory based on the one dimensional chemical kinetics model is not valid for describing the oscillation. In the reaction zone the combustion proceeds through the turbulent multiflames as is shown in the photographs of Fig. 15. These flames, stabilized by the recirculated hot combustion gas, are very sensitive to the disturbances in the flow field surrounding them. As is seen in the schlieren photographs of Fig. 20, they always undergo random fluctuations even in the steady combustion, on account of the turbulent condition in the reaction zone. In such a situation, the combustion process should be subject to overwhelming fluid dynamic influences and the chemical kinetics will not play an important role in the interaction between the pressure wave and the flames.

During the oscillation, the flames are oscillating with the identical period of the pressure oscillation on account of the variation of the injection velocity of the propellant, and not of the chemical reaction rate. The variation of the injection velocity is caused by the standing pressure oscillation in the combustion chamber. On the contrary, it is still unknown what effect has the flame oscillation on the standing pressure oscillation. When the effect has been known, the interaction between the pressure wave and the flame will be made clear. Then the physical nature of the characteristic time will also be revealed. The question will be discussed in the following.

4-2. *Heat Driven Oscillation by Spontaneous Ignition*

In order to sustain the standing pressure oscillation in the chamber, some driving force should be provided to overcome losses of the system. The standing pressure oscillation, in general, may be driven by the periodical variation of mass or heat addition rate to the gas in the chamber. In the case of the present studies, the flame oscillates periodically during the pressure oscillation in accordance with the variation of the injection velocity. The variation of the injection velocity should, of course, be accompanied by a variation of mass addition rate to the gas in the chamber. This variation of mass addition rate may, under favorable condition, drive the standing pressure oscillation in the chamber.

It is clear, however, that in the case of the present studies the variation of mass addition rate does not directly drive the oscillation. If it were so, the oscillation should occur even when there is no combustion in the chamber. The variation of mass addition rate supplies only the source for driving the oscillation. The increased mass addition will produce, after some time delay of combustion, an increased heat addition to the gas in the chamber. It is this increased heat addition which actually drives the oscillation. In other words, the observed oscillation in the chamber is a kind of heat driven oscillation and not of mass driven oscillation.

In the case of heat driven oscillation, there should exist some accelerating mechanism of combustion to provide the increased heat addition rate to the gas. In the studies of the heat driven oscillation by flame, it has often been suggested [*21*]~[*24*] that the increased heat addition rate to the gas is provided by the increase of flame front area. If the burning velocity is assumed to remain constant during one cycle of the oscillation, the instantaneous heat addition rate to the gas will be proportional to the flame front area at that instant. In the case of the present studies, the flame front area of the two dimensional flame will correspond approximately to the over-all flame length plotted against time under the schlieren photographs in Figs. 21~24. The periodical variation of the flame length may drive the oscillation if the Rayleigh's two criteria [*25*] are satisfied. The second time criterion requires that the variation of heat release rate is in phase with the pressure variation. As is seen in the figures, however, the variation of the flame length is rather out of phase with the standing pressure oscillation. Therefore, the variation of flame front area cannot drive such a severe oscillation as was observed in the present studies. Moreover, with this rather smooth variation of heat addition rate the appearance of the rapid pressure increase in each cycle of the oscillation cannot be explained.

The rapid pressure increase in the reaction zone, which produces the pressure pulse propagating in the chamber during the oscillation, can only be caused by some rapid heat addition to the gas in the reaction zone. In the schlieren photographs, however, no drastic increase of the flame length is observed at the instant of rapid pressure increase. At first glance, it seems that there are no sources of producing such a rapid pressure increase.

However, a detailed examination of the schlieren photographs will make one to notice an important aspect of the flame oscillation. With extending its length in the first several sequences of each cycle, the flames are forced to tear off at the tips throwing out the dark spots in the combustion gas stream. Although the still photographs reproduced in the figures are not so well contrasted, the motion picture of the original film shows more clearly the appearance and the subsequent behavior of these spots at the flame tips. In each cycle of the oscillation, the flames begin to tear off in the third or the fourth sequences throwing out the spots. These spots, in the subsequent sequences, become larger and thinner as they move towards downstream. They disappear in the eleventh or the twelfth sequences, accompanying the simultaneous rapid pressure increase in the reaction zone.

The above described behavior of the flame can be interpreted as follows. In the first several sequences, the decrease of pressure at the injector exit brings about the increase in the injection velocity of the propellant. The flames become longer in accordance with this increase, since the burning velocity remains almost constant. But the increase is so rapid that the chemical reaction at the flame tips cannot sustain all of this increase, and hence the flames are forced to tear off at the tips, some of the unburnt propellant volumes being thrown out in the hot combustion gas stream. These unburnt volumes contact directly with the hot combustion gas and become diluted and heated gradually on account of the turbulent mixing. They are ignited spontaneously and almost simultaneously after some time delay. A considerable amount of heat is added to the gas at the instant of this ignition, which causes the rapid pressure increase in the reaction zone.

Such spontaneous ignition will sure drive the standing pressure oscillation if they occur at the proper instant of the pressure variation. The standing pressure oscillation, in turn, causes the periodical increase in the injection velocity, and hence the intermittent throwing of the unburnt propellant in the gas stream. The latter causes after some time delay the rapid heat addition to the gas. Thus closes the cycle of a self excited oscillation. The above postulated driving mechanism can explain the appearance of the small pressure pulse propagating in the chamber being superimposed on the sinusoidal standing pressure oscillation.

The postulated driving mechanism based on the spontaneous ignition is somewhat similar to that postulated by Rogers and Marble [26] for the occurrence of jet engine screech. In their mechanism, the unburnt volume is carried periodically into the hot recirculation zone behind the flameholder by the vortex formation in the flame front at the holder tip. In the present case, on the other hand, the unburnt volume of the propellant is thrown out in the hot combustion gas stream by the flame tip breaking. This sort of breaking has been recognized by Schelkin and Troshin [27] for a highly turbulent flame structure consisting of a main flame front trailed by islands of unburnt gas.

Schelkin and Troshin [27] introduced a criterion in which the breaking of the flame tip is expected when it is satisfied. Such a criterion is satisfied when, during the time of turbulent pulsation, the flame with normal burning velocity has not enough time to propagate by a distance comparable with the width of normal reaction zone. In the present case, a similar consideration leads to the following criterion:

$$t_f = \frac{l'}{v'} < \frac{\lambda}{u_n} = t_c$$

where l' and v' are the increase of the flame length and the injection velocity, respectively. u_n is the normal burning velocity of the propellant and λ is the width of the normal reaction zone. The criterion states that the flame tip breaking will occur when the time t_f required for the fluidmechanical movement is shorter than the time t_c required for the chemical reaction.

When the criterion is applied to the flame oscillation and the representative numerical values of $l'=2$ cm, $v'=100$ m/sec, $\lambda=0.01$ cm, $u_n=50$ cm/sec are substituted, both t_f and t_c become 200 μsec. Therefore, the magnitude of the fluid-mechanical time and the chemical time is the same order and there is a fair chance for the flame to be torn off at the tip as the result of the injection velocity increase. The above discussions back up the suggestion that the driving mechanism of the oscillation depends upon the spontaneous ignition of the unburnt volumes in the hot combustion gas stream.

It is clear that the driving mechanism of this sort is essentially of nonlinear character. However, it by no means signifies that the shock wave plays any important role in the driving of the oscillation. The main pressure oscillation excited in the chamber is the smooth sinusoidal standing pressure oscillation as was made clear in the previous study [*18*]. It is this standing pressure oscillation, and not the shock wave, which actually brings about the variation in the injection velocity and hence the variation in the heat release rate. It can be said that so long as the present investigation is concerned the shock wave plays no important role in the driving of the oscillation as well as in the characteristics of the oscillation.

The energy of the oscillation is supplied in the reaction zone from some of the chemical energy of propellant to overcome the losses in the rest of the system and to continue the oscillation. In the fully developed oscillation, the amplitude of the oscillation is determined by the balance between this input energy and the losses. The latter comprise the contributions from the viscous stress, the radiation at the exhaust nozzle, and from the mean flow which carries the energy of oscillation out of the chamber. The experimental study [*27*], as well as the theoretical analysis [*28*], has made clear that the viscous loss is negligible as compared with the nozzle losses. In the present studies, the flat plate nozzle has been used and therefore the radiation loss at the subsonic portion of the nozzle must be small as compared with that carried away by the mean flow. In the fully developed oscillation, therefore, it may be considered that the input energy in the reaction zone is balanced chiefly by the energy loss due to the mean flow.

4-3. *Characteristic Time of Oscillation*

A periodical variation of heat release rate does not always drive the standing pressure oscillation. There are famous two criteria given by Rayleigh [*25*] which must be satisfied in order that the oscillation may be driven. The first space criterion states that the oscillation of a given mode is most likely to occur when the variation in heat release rate occurs at the antinodal position of the pressure oscillation. In the case of the present investigation, this criterion is satisfied automatically, since the variation occurs in the reaction zone concentrated near the injector face, which is always the antinodal position for any mode of the longitudinal standing pressure oscillation in the chamber.

The second time criterion states that the standing pressure oscillation is driven or damped dynamically depending on whether the increase in heat release rate occurs during the time interval of the pressure increasement or decreasement. In the postulated driving mechanism, the phase relation between the pressure and the heat release rate variation is determined by the response time delay of the propellant. The latter may be regarded as the time elapsed from the instant of the minimum pressure at the injector exit until the instant of the maximum heat release rate, or the instant of the spontaneous ignition.

Figure 25 presents an explanatory diagram of the flame and the pressure oscillations. The represented one cycle corresponds to that shown in Fig. 21. In the diagram, the response time delay τ^* is taken as to start at sixth sequence when the oscillating pressure becomes minimum and ends at eleventh sequence when the ignition is occurred. At the instant of the ignition, the standing pressure is increasing and hence the ignition is actually driving the oscillation. The value of τ^* in this case is approximately 500 μsec.

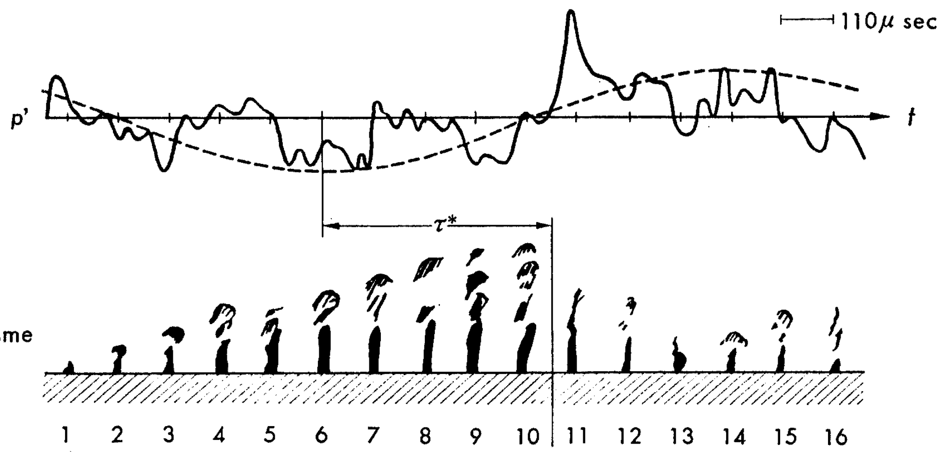

FIG. 25. Explanatory diagram of flame and pressure oscillations.

Of course, the above explanatory diagram is too much simplified. In actual situation, there are so many unburnt volumes of different sizes which are thrown out in the combustion gas stream. These volumes will not necessarily be ignited simultaneously. The spontaneous ignitions should distribute for a certain period of time. The response time delay, therefore, will be different for different volumes of the propellant and it will be spread in a definite range of time. However, the most powerful ignition should occur at the instant of rapid pressure increase, and at all events this ignition is produced as the result of the pressure decrease at the injector exit. Therefore, we can definite a representative value for these spread time delays as the time elapsed from the instant of minimum pressure till the instant of rapid pressure increase.

The postulated driving mechanism discussed so far is the one for the fully developed oscillation. It may be considered that the same mechanism can be applied to the incipient instability. As was made clear in the previous studies [16]~[18], when the operating condition of motor is near the oscillation boundary for the fundamental mode oscillation, the excited pressure oscillation in the chamber is a small sinusoidal one without the rapid pressure increase. This means that at the oscillation boundary the time criterion of driving is critical and hence the pressure oscillation in the chamber is almost neutral without developing into the large oscillation. The resulting variation in heat release rate also will be a small sinusoidal one. In such a case, the unburnt volumes thrown out in the combustion gas stream as the result of the flame tip breaking will become smaller in scale and larger in number, as compared with those in the case of fully developed oscillation. The small numerous spontaneous ignitions should spread over the time, resulting in the smooth sinusoidal variation of total heat release rate. The response time delay can be defined in this case in the same way as before, as the time elapsed from the instant of minimum pressure till the instant of maximum heat release rate.

The standing pressure oscillation is excited only when the time criterion is satisfied. The phase relation between the pressure and the heat release rate variations is determined by the response time delay. It is no doubt that this response time delay is the very characteristic time, which governs the occurrence of the oscillation. In order to initiate the oscillation, the ratio between the time delay and the period of the pressure oscillation should lie in a certain range. The period of the oscillation is determined by the geometrical dimension of the combustion chamber [16],[17], while the time delay will be independent of it. For a given mode of oscillation, therefore, there should exist the upper critical chamber length, as well as the lower critical chamber length, for the occurrence of the oscillation.

The experimental studies on the critical oscillation period have revealed that the characteristic time depends chiefly on the propellant equivalence ratio, although there are some small effects of the combustion chamber pressure and the injection Mach number of the propellant. These observed features of the characteristic time will be compared with the supposed features of the response time delay.

In Fig. 25, the substance of response time delay τ^* may be divided into two parts. The first is the time required for the injecting propellant jet to respond to the change of the pressure at the injector exit, to increase its injection velocity and to tear off the flame tip. The second is the time required for the unburnt propellant thrown out in the hot combustion gas stream to be diluted, heated and ignited spontaneously. The first is of a fluidmechanical nature while the second is chiefly of a chemical nature. The examination of Fig. 25 reveals that at the sixth sequence, when the response time delay starts, some of the unburnt volumes are already thrown out in the gas stream. In other words, during the fully developed oscillation the response time delay is for the most part the second chemical time, which corresponds to the so called ignition time delay.

The ignition time delay of the unburnt propellant in the hot turbulent gas stream will depend upon the chemical nature of the propellant and also upon the temperature of the gas stream. In the case of the present studies, the temperature of the gas stream as well as the chemical nature of the injecting unburnt propellant depends upon the propellant equivalence ratio. Therefore, the effect of the latter will be very important. The mean chamber pressure and the injection Mach number of the propellant, on the other hand, will also affect the ignition time delay, since they should have some effects on the turbulent condition at the flame tip and hence the heating rate of the propellant.

The above supposed features of the response time delay correlate very well with the observed features of the critical oscillation period and hence of the characteristic time which governs the occurrence of the oscillation. Although the details of the relation between the critical period and the response time delay is not clear at present, the experimental results confirm the postulated initiating and driving mechanism of the oscillation.

5. Conclusions

The experimental studies on the initiating and driving mechanism of the high frequency combustion oscillation of longitudinal mode in the rocket motor burning premixed gaseous propellant have brought about the following conclusions.

The high frequency combustion oscillation observed in the present studies is a kind of heat driven oscillation, in which the driving force is provided by the periodical spontaneous ignition of the unburnt propellant that is thrown out in the hot combustion gas stream from the flame tip. The throwing out of the propellant is caused by the rapid increase of the propellant injection velocity into the combustion chamber as the result of the standing pressure oscillation in the combustion chamber. Although the driving mechanism is substantially of a nonlinear nature, the shock wave does not play any important role in the driving of the oscillation, as well as in the characteristics of the excited oscillation.

The characteristic time which governs the occurrence of the oscillation is the response time delay of the propellant, the substance of which is the ignition time delay of the unburnt propellant in the hot combustion gas stream. This time delay depends chiefly upon the propellant equivalence ratio, and depends also upon the combustion chamber pressure and the injection Mach number of the propellant. The effects of the latter two are small as compared with that of the equivalence ratio.

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December 20, 1967

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