Institute of Space and Aeronautical Science, University of Tokyo
Report No. 479, May 1972

Effects of Transport Velocity of Wake Vortex on Aerofoil Oscillations

By

Yoshimichi TANIDA

Summary: The effects of transport velocity of wake vortex on an oscillating aerofoil is discussed theoretically. The calculated results show that, when an aerofoil performs translatory oscillations, the slow trnasportation of wake vortex relative to the aerofoil deteriorates the aerodynamic damping effect, especially for the faster oscillation.
Agreement with an experiment is obtained.

1. INTRODUCTION

When an aerofoil oscillates in real viscous flow, the unsteady aerodynamic force and moment are more or less different from those predicted by the potential flow theory, so that there will naturally result some influence on the flutter boundary of aerofoil. For example, an aerofoil oscillating in real fluid is not always free from flutter even in the range of mass ratio smaller than the critical values given by the theory, where it is expected to be flutter-free [*1*].

In order to compare the theoretical unsteady aerodynamic derivatives with the actual ones, experimental studies have been made by Halfmann et al. [*2*], Spurk [*3*], Tanida et al. [*4*], and others. According to these experiments, the unsteady lift force on an aerofoil oscillating in translatory mode is less than the theoretical one and ahead of the latter in phase, when the oscillating frequency is low. As the frequency increases, however, the unsteady lift becomes larger than the theoretical one, but it gets behind in phase. For further increase of the frequency, the unsteady lift becomes smaller again than the theoretical one.

When the oscillation of the aerofoil is sufficiently slow, e.g. $\omega a/U<0.1$ (ω; circular frequency, a; amplitude of oscillation, U; flight velocity), the unsteady aerodynamic characteristics of the aerofoil can be estimated by the potential flow theory, using the aerodynamic derivatives obtained from the experiments. For faster oscillation, however, that is not the case, even if the effects of the aerofoil thickness and others are taken into account.

On the other hand, the effects of viscosity of fluid have been studied theoretically on an oscillating flat-plate aerofoil by solving the Oseen's equations of motion, remarking the propriety of the Kutta's hypothesis at the trailing edge [*5*], [*6*]. Those analyses show too small viscous corrections for the unsteady aerodynamic forces, that is at most the order of $R_e^{-1/2}$ for sufficiently large Reynolds number, so that such a theoretical analysis is unlikely to be able to make clear the discrepancies between theory and experiment.

In all of the theoretical treatments mentioned above, it is assumed that the vorticity shed from an oscillating aerofoil is transported downstream, relative to the aerofoil, at the main stream velocity. In the real flow, however, there being viscous wake behind the aerofoil, the shed vorticity may be washed downstream at a velocity somewhat smaller than that of the main flow. The nearer it is to the trailing edge, the slower the flow is, so that there would be the concentration of shed vorticity just downstream of the trailing edge. The wake flow behind an oscillating aerofoil was studied by Bratt [7] and Ohashi [8] by visualization method. Bratt showed that the shed vortex is transported at about 70% of main stream velocity, and Ohashi that the transport velocity of the wake vortex can be larger than the main stream one when the oscillation becomes sufficiently fast. However, there still remains unknown the effects of the concentration of wake vorticity on the aerodynamic characteristics of an oscillating aerofoil.

Then, the present paper gives theoretical considerations on the effects of wake velocity on aerofoil oscillations, at which the shed vortex is conveyed downstream.

2. Transportation of Wake Vortex

An preliminary experiment was carried out to observe the behaviour of wake vortex by a visualization method, using a water table, in which water is either still or circulated by a pump. A test aerofoil (profile; double circular arcs, chord; 50 mm) is hung downward vertically from the carriage, which can move the aerofoil at an arbitrary speed. To visualize the flow, a drop of India ink is dripped on the water surface far upstream of the test aerofoil, because another method such as the use of Al-powder was not enough for examining the wake behind a small, thin aerofoil. The photographs of the flow are taken successively using a motor-driven camera.

Fig. 1 shows one of the typical results of this experiment, when an aerofoil moves impulsively at a certain speed in the direction perpendicular to the flight. It is seen that the shed vortices, loci of which are indicated by full lines, are transported upstream (downstream relative to the aerofoil) at a velocity smaller than the flight speed. The transport velocity of the wake vortex decreases exponentially in course of time, being compared with the calculated locus of the fluid particle centered in the laminar wake (dotted line).

On the other hand, Fig. 2 illustrates that, when an aerofoil starts impulsively to move at a certain speed in still fluid, the starting vortex is likely to stay at the place where it is created. That may be attributed to the fact that the viscous wake is not yet formed completely just after the aerofoil started.

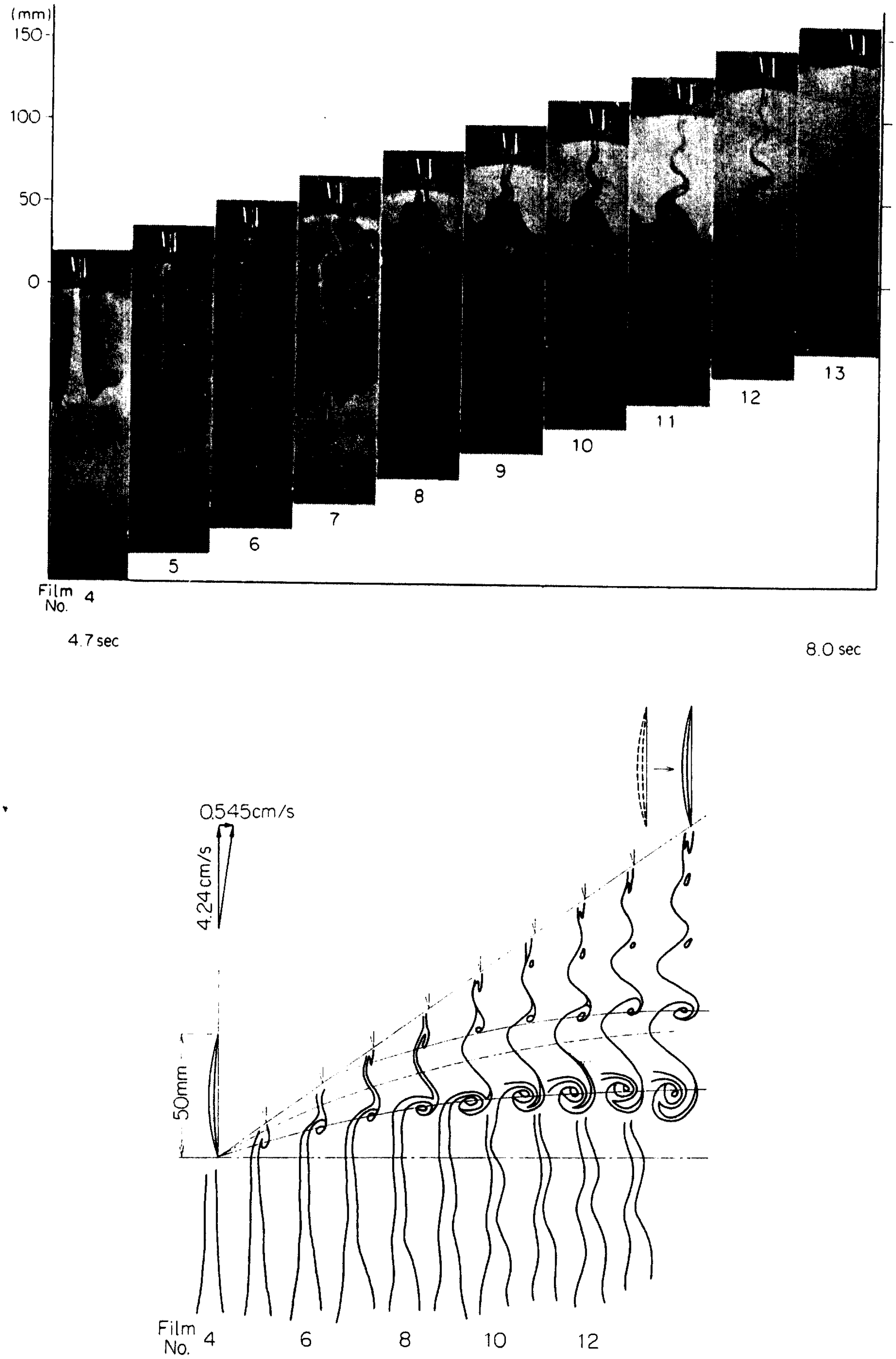

FIG. 1. Transportation of Wake Vortex

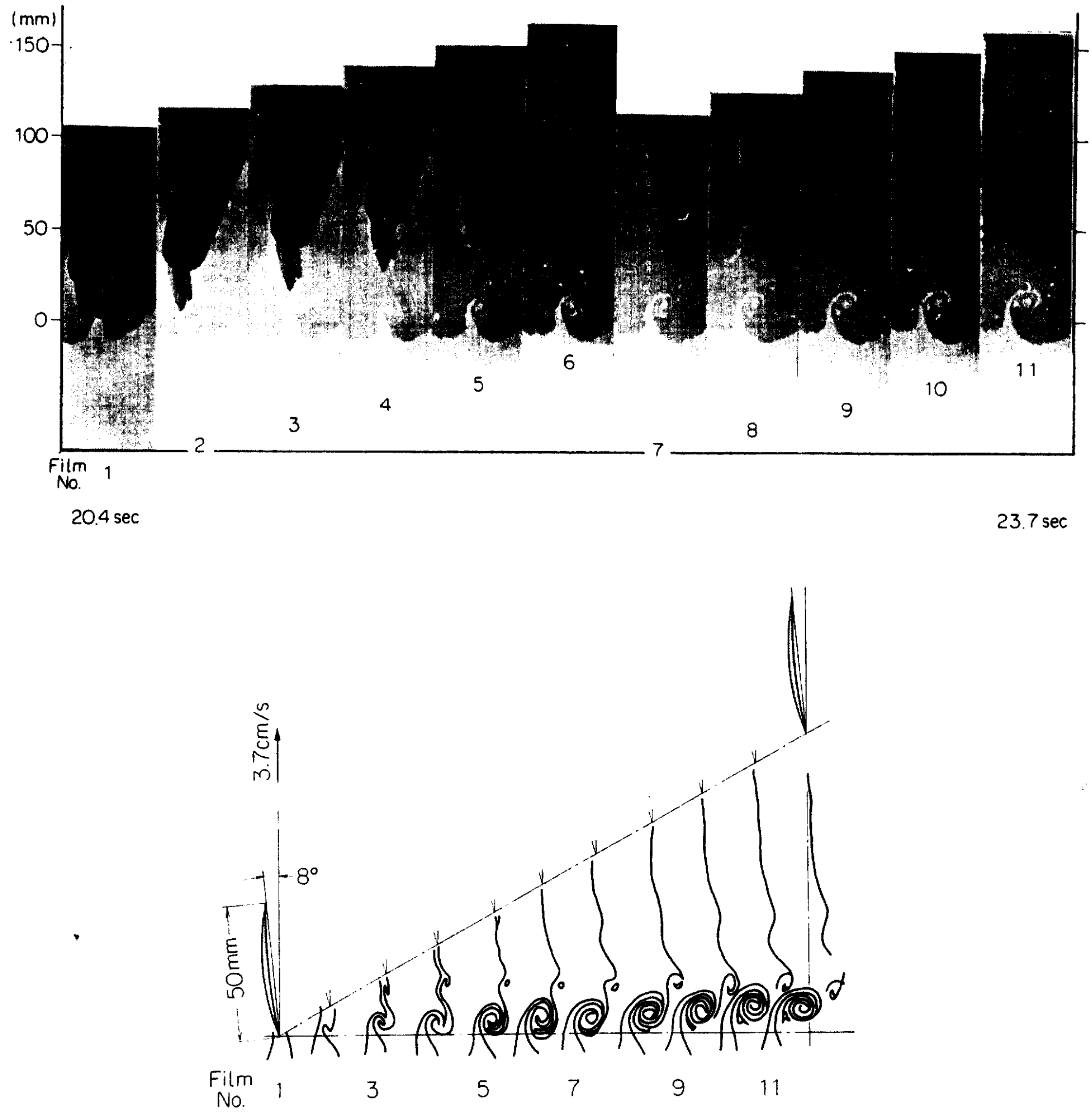

FIG. 2. Starting Vortex

3. THEORETICAL CONSIDERATIONS

The effects of the wake vortices will be considered for the simple case shown in Fig. 3, under the following assumptions:

(a) the flow is two-dimensional, incompressible and inviscid.

(b) the aerofoil is thin, and its vertical movement is small.

(c) the Kutta's hypothesis holds at the trailing edge of the aerofoil, and the shed vortices in the wake are transported on the x-axis at a velocity smaller than the flight speed.

Then, the usual linearized potential flow theory can be applied, by taking into account the velocity defect in the wake.

It will be assumed that the vorticity shed from the aerofoil is conveyed down-

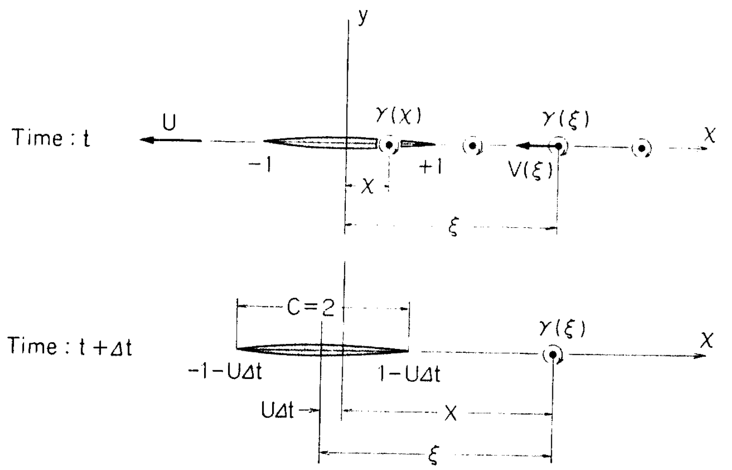

FIG. 3. System and Notation

stream at a velocity of $U-V(\xi)$ relative to the aerofoil, where U is the velocity of flight and $V(\xi)$ the transport velocity of the wake vortex. Then, applying the Momentum theory [9] to the present case, the lift and the moment acting on the aerofoil per unit span are given as follows (see Appendix);

$$\left.\begin{aligned}
L=&-\rho\frac{d}{dt}\int_{-1}^{1}\gamma_0(x)xdx+\rho U\Gamma_0+\rho U\int_{1}^{\infty}\frac{\gamma(\xi)}{\sqrt{\xi^2-1}}d\xi\\
&+\rho\int_{1}^{\infty}V(\xi)\gamma(\xi)\frac{\xi}{\sqrt{\xi^2-1}}d\xi\\
M=&-\frac{1}{2}\rho\frac{d}{dt}\int_{-1}^{1}\gamma_0(x)\left(x^2-\frac{1}{2}\right)dx+\rho U\int_{-1}^{1}\gamma_0(x)xdx\\
&-\frac{1}{2}\rho U\int_{1}^{\infty}\frac{\gamma(\xi)}{\sqrt{\xi^2-1}}d\xi+\frac{1}{2}\rho\int_{1}^{\infty}V(\xi)\gamma(\xi)\frac{2\xi^2-1}{\sqrt{\xi^2-1}}d\xi
\end{aligned}\right\}\qquad(1)$$

where γ_0 is the so-called quasi-steady vorticity distribution and Γ_0 the quasi-steady circulation about the aerofoil, resulting from the former.

Each term of RHS of Eq. (1) expresses the contribution of the apparent mass, the quasi-steady lift or moment, the contribution of the wake vorticity, and the effect of the velocity defect of the wake, in that order. The effect of the velocity concentration in the wake is of course included implicitly in the third term.

If the motion of the aerofoil is periodic, the resulting quasi-steady circulation about the aerofoil may be written

$$\Gamma_0=G_0e^{i\omega t}\qquad(2)$$

where G_0 is a constant. Similarly the strength of the vortex shed at the trailing edge can be expressed as

$$\gamma(1)=ge^{i\omega t}$$

where g is also a constant, and the shed vortex being washed downstream at a velocity $U-V(\xi)$ relative to the aerofoil, the vortex strength in the wake is

$$\gamma(\xi)=\frac{U-V(1)}{U-V(\xi)}ge^{i\omega t}\exp\left(-i\omega\int_1^{\xi}\frac{d\xi'}{U-V(\xi')}\right) \tag{4}$$

Then, according to the Kelvin's theorem that the total circulation of the whole system must be zero, the total circulation about the aerofoil is given by

$$\Gamma=e^{i\omega t}\left[G_0+g\int_1^{\infty}\left(\sqrt{\frac{\xi+1}{\xi-1}}-1\right)\frac{U-V(1)}{U-V(\xi)}\exp\left(-i\omega\int_1^{\xi}\frac{d\xi'}{U-V(\xi')}\right)d\xi\right] \tag{5}$$

The rate of change of circulation about the aerofoil must be equal and opposite to the rate of the vortex shedding or to the vortex strength at the trailing edge, so that

$$\frac{d\Gamma}{dt}=-[U-V(1)]\gamma(1)$$

Finally the following relation between G_0 and g is obtained;

$$\frac{G_0}{g}=-[U-V(1)]\left[\frac{1}{i\omega}+\int_1^{\infty}\left(\sqrt{\frac{\xi+1}{\xi-1}}-1\right)\frac{\exp\left(-i\omega\int_1^{\xi}\frac{d\xi'}{U-V(\xi')}\right)}{U-V(\xi)}d\xi\right] \tag{6}$$

Substituting $\gamma(\xi)$ obtained from Eqs. (4) and (6) into Eq. (1), the lift and the moment acting on an oscillating aerofoil are given for any case.

Some calculations are carried out for the case of an aerofoil performing the translatory oscillations normal to the flight direction, assuming that the transport velocity of the wake vortex is given by

$$\frac{V(\xi)}{U}=\alpha e^{-\beta(\xi-1)} \tag{7}$$

which is considered reasonable from the results of the aforementioned experiment.

Fig. 4 gives the calculated results of the unsteady lift, excluding the apparent mass contribution, as a function of the reduced frequency, $\omega c/2U$, and β, when α is assumed 0.5. In this figure, the length of the vector drawn from the origin to the point on the curve gives the amplitude of the lift (referred to that of the corresponding quasi-steady lift), and its angle with the horizontal axis gives the phase angle relative to the oscillating velocity of the aerofoil. The curve for $\beta\to\infty$ represents so-called Theodorsen function.

It is seen that the effect of the transportation of wake vortex becomes remarkable for higher reduced frequency and for β being smaller, acting to diminish the lift component in the same phase with the oscillating velocity or the aerodynamic damping force.

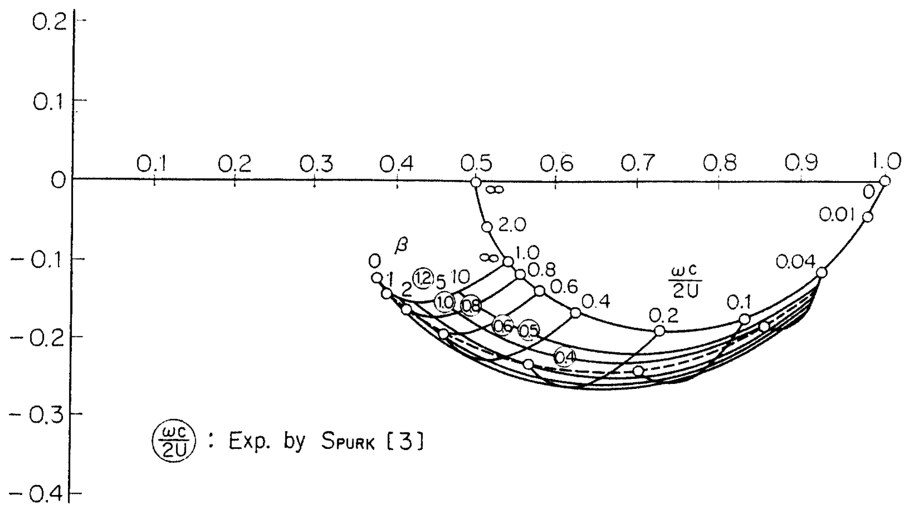

FIG. 4. Lift Diagram

In Fig. 4, there are also plotted the experimental results of the lift measured by Spurk [*3*]. It is noted that Spurk's results almost lie on the calculated curves of corresponding reduced frequency.

4. CONCLUDING REMARKS

Some theoretical considerations were made for the effects of the wake velocity at which the vorticity shed from an oscillating aerofoil is washed downstream.

The calculated results show that, when an aerofoil performs translatory oscillations, the slow transportation of the wake vortex relative to the aerofoil deteriorates the aerodynamic damping effect, especially for higher oscillating frequency.

The theoretical results presented in this paper agree in quality with some experimental ones. In order to give more positive proof of agreement, however, further considerations must be made for other effects, such as

(a) the finiteness of vortex trail, due to viscous dissipation and wake instability.

(b) the flow behaviour around the trailing edge, or relaxation of the Kutta condition.

and so forth.

Concerning the finite vortex trail, Jordan [*10*] showed in his theory that, on an aerofoil oscillating about its mid-chord point, the finiteness of vortex trail gives a remarkable effect to the damping moment, especially when the reduced frequency is small.

With regard to the relaxation of the Kutta condition, there seems to be no reliable explanation, both by theory and by experiment. However, it is likely that it could not bring remarkable effects on an oscillating aerofoil, except the case of sufficiently fast oscillation, say $\omega a/U \gg 0.1$.

The author is indebted to Mr. T. Obokata for his technical assistance.

Department of Jet Propulsion
Institute of Space and Aeronautical Science
University of Tokyo
March 10, 1972

References

[*1*] Woolston, D. S., & Castile, G. E.: Some Effects of Variations in Several Parameters Including Fluid Density on the Flutter Speed of Light, Uniform Cantilever Wings. N.A.C.A. T.N. 2558 (1951).

[*2*] Halfman, R. L., Johnson, H. C., & Haley, S. M.: Evaluation of High-angle-of-attack Aerodynamic-derivative Data and Stall-flutter Prediction Techniques. N.A.C.A. T.N. 2533 (1951).

[*3*] Spurk, J.: Messungen der Aerodynamischen Beiwerte Schwingender Flugelprofile im Windkanal und Vergleich mit der Theorie. Mitt. Max-Planck-Inst. für Strömungsforschung, Nr. 29 (1963).

[*4*] Tanida, Y., Hatta, K., & Asanuma, T.: Experimental Study on Flutter in Cascading Blades; Trans. Japan Soc. Mech. Engrs. Vol. 31, No. 227 (1965), p. 1081.

[*5*] Chu, W. H.: An Aerodynamic Analysis for Flutter in Oseen-type Viscous Flow. J. Aero. Sci. Vol. 29 (1962), p. 781.

[*6*] Shen, S. F., & Crimi, P.: The Theory for an Oscillating Thin Airfoil as Derived from the Ossen Equations. J. Fluid Mech. Vol. 23, Pt. 3 (1965), p. 585.

[*7*] Bratt, J. B.: Flow Patterns in the Wake of an Oscillating Aerofoil. A.R.C. R.&M. 2773 (1950).

[*8*] Ohashi, H., & Ishikawa, N.: Visualizing Study on the Flow around the Trailing Edge of an Oscillating Aerofoil. J. Japan Soc. Mech. Engrs. Vol. 74, No. 634 (1971), p. 1500.

[*9*] v. Karman, Th., & Sears, W. R.: Airfoil Theory for Non-uniform Motion. J. Aeron. Sci. Vol. 5, No. 10 (1938), p. 379.

[*10*] Jordan, P. F.: The Harmonically Oscillating Wing with Finite Vortex Trail. A.R.C. R.&M. 3038 (1953).

Appendix Application of Momentum Theory [*9*] to the Present Case

Under the assumptions described in Chapter 3, the system will be observed from the stationary co-ordinates, as shown in Fig. 3. The total circulation of the whole system is invariably equal to zero in the present case. Hence

$$\int_{-1}^{1}\gamma(x)dx+\int_{1}^{\infty}\gamma(\xi)d\xi=0 \tag{A-1}$$

The lift and the moment acting on an oscillating aerofoil per unit span are given by the rate of change of the total momentum of all vortices located on the x-axis and that of the total moment of momentum, respectively.

$$\left.\begin{aligned} L&=-\rho\frac{d}{dt}\left\{\int_{-1}^{1}\gamma(x)xdx+\int_{1}^{\infty}\gamma(\xi)\xi d\xi\right\} \\ M&=-\frac{1}{2}\rho\frac{d}{dt}\left\{\int_{-1}^{1}\gamma(x)x^2dx+\int_{1}^{\infty}\gamma(\xi)\xi^2 d\xi\right\} \\ &\quad+\rho U\left\{\int_{-1}^{1}\gamma(x)xdx+\int_{1}^{\infty}\gamma(\xi)\xi d\xi\right\} \end{aligned}\right\} \tag{A–2}$$

The vorticity distribution over the aerofoil, $\gamma_1(x)$, which is induced by the distributed wake vorticity, $\gamma(\xi)$, is

$$\gamma_1(x)=\frac{1}{\pi}\int_1^{\infty}\sqrt{\frac{1-x}{1+x}}\sqrt{\frac{\xi+1}{\xi-1}}\frac{\gamma(\xi)}{\xi-x}d\xi \tag{A–3}$$

from which the circulation about the aerofoil results as

$$\Gamma_1=\int_{-1}^{1}\gamma_1(x)dx=\int_1^{\infty}\left(\sqrt{\frac{\xi+1}{\xi-1}}-1\right)d(\xi)d\xi \tag{A–4}$$

Total circulation about the aerofoil is then

$$\Gamma=\Gamma_0+\Gamma_1 \tag{A–5}$$

where Γ_0 is the circulation resulting from the quasi-steady vorticity distribution, $\gamma_0(x)$. Using Eq. (A–1), it is seen that the following relation holds between the quasi-steady circulation and the wake vorticity;

$$\Gamma_0=\int_{-1}^{1}\gamma_0(x)dx=-\int_1^{\infty}\sqrt{\frac{\xi+1}{\xi-1}}\gamma(\xi)d\xi \tag{A–6}$$

The total momentum of the system of continuously distributed vortices, which consist of the aerofoil and the wake, is

$$I=\rho\int_{-1}^{1}\gamma(x)xdx+\rho\int_1^{\infty}\gamma(\xi)\xi d\xi \tag{A–7}$$

Substituting $\gamma=\gamma_0+\gamma_1$ and using Eq. (A–3) for γ_1, Eq. (A–7) becomes

$$I=\rho\int_{-1}^{1}\gamma_0(x)xdx+\rho\int_1^{\infty}\gamma(\xi)\sqrt{\xi^2-1}\,d\xi \tag{A–7$'$}$$

Now, it is desired to differentiate Eq. (A–7′) with respect to time, to obtain the lift. Consider that X is the distance of an arbitrary point in the wake from a fixed origin at the instant of time t, when the mid-chord point of the aerofoil moving with $-U$ coincides with the origin. After a time interval Δt, the aerofoil has moved through a distance $U\Delta t$ leftwards, so that the ordinate of X relative to the aerofoil is $\xi=X+U\Delta t$, and if the wake vortices move at $-V(\xi)$, the vorticity at X becomes

$\gamma(X)+d/dX\{V(X)\gamma(X)\}\varDelta t$. Hence the differentiation of the second term of Eq. (A-7′) gives

$$\varDelta\int_1^\infty \gamma(\xi)f(\xi)d\xi=\int_{1-U\varDelta t}^\infty\left[\gamma(X)+\frac{d}{dX}\{V(X)\gamma(X)\}\varDelta t\right]f(X+U\varDelta t)dX-\int_1^\infty \gamma(X)f(X)dX$$

where $\sqrt{X^2-1}$ is replaced by a general form $f(X)$. If $\gamma(X)$ is finite and $f(1)=0$, and also if $V(X)$ is finite to be a small quantity of higher order than $1/f(X)$ for $X\to\infty$, then in the limit $\varDelta t\to 0$, and neglecting the terms of second order and higher,

$$\frac{d}{dt}\int_1^\infty \gamma(\xi)\sqrt{\xi^2-1}\,d\xi=\int_1^\infty [U-V(X)]\gamma(X)f'(X)dX$$

Then the differentiation of Eq. (A-7) is

$$\frac{dI}{dt}=\rho\frac{d}{dt}\int_{-1}^1 \gamma_0(x)xdx+\rho\int_1^\infty [U-V(\xi)]\gamma(\xi)\frac{\xi}{\sqrt{\xi^2-1}}d\xi \tag{A-8}$$

Finally from Eq. (A-2), (A-6) and (A-8), the lift is given in the form

$$L=-\frac{dI}{dt}=-\rho\frac{d}{dt}\int_{-1}^1 \gamma_0(x)xdx+\rho U\Gamma_0$$
$$+\rho U\int_1^\infty \frac{\gamma(\xi)}{\sqrt{\xi^2-1}}d\xi+\rho\int_1^\infty V(\xi)\gamma(\xi)\frac{\xi}{\sqrt{\xi^2-1}}d\xi \tag{A-9}$$

In a similar manner, the moment acting on the aerofoil, referred to its mid-point, is given by (a diving moment is considered positive)

$$M=-\frac{1}{2}\rho\int_{-1}^1 \gamma_0(x)\left(x^2-\frac{1}{2}\right)dx+\rho U\int_{-1}^1 \gamma_0(x)xdx$$
$$-\frac{1}{2}\rho U\int_1^\infty \frac{\gamma(\xi)}{\sqrt{\xi^2-1}}d\xi+\frac{1}{2}\rho\int_1^\infty V(\xi)\gamma(\xi)\frac{2\xi^2-1}{\sqrt{\xi^2-1}}d\xi \tag{A-10}$$

Each term of RHS of Eqs. (A-9) and (A-10) expresses the contribution of the apparent mass, the quasi-steady lift or moment, the contribution of the wake vorticity, and the effect of the velocity defect of the wake, respectively.