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On the open boundary conditions for incompressive unbounded flows

by
Shigeki HATAYAMA
National Aerospace Laboratory, 7-44-1 Jindaijihigashi-machi Chofu-shi Tokyo, Japan 182

Abstract

Four open boundary conditions for incompressible unbounded flows are evaluated in the framework of the Leith type third-order upwind scheme(QUICKEST scheme), and each effectiveness is compared by two means of difference of flows among open boundary conditions and between short and long open boundaries. Three test problems used for the open boundary condition evaluation are the backward-facing step flow, the blunt based body flow and the rectangular cylinder obstacle flow in a channel. The investigated open boundary conditions are four. The author proposes to take the uniform inlet velocity as the phase speed in the Sommerfeld radiation condition. As the conclusion, we show that this is the most excellent open boundary condition within four open boundary conditions.

1. Introducing remarks

In many computational problems, we are faced with infinite domains, which for computational reasons must be made finite. One possibility is to introduce an artificial boundary in order to reduce the infinite computational domain to a finite one. Then, the introduction of the artificial boundary makes it necessary to formulate appropriate artificial boundary conditions. However, mathematics does not tell us how to select artificial boundary conditions.

2. Some qualities that candidates as open boundary conditions should display

They should permit the flow to exit the domain gracefully and passively, and not have any effect on the behaviour of the solution in the domain near the open boundary, especially far from it. They should be transparent, and lead to the same solution inside the common domain no matter where truncation occurred.

3. Boundary conditions

Figure 1 shows geometry definition of three test problems. In (A), B1, B2, B3 and B5 are the no-slipe solid walls, B4 the inlet and B6 an open boundary. Coordinates of points 1 and 2 are (2JH,JH) and (IN,2JH), respectly. We take IN=14JH as the short open boundary and IN=20JH as the long open boundary. In (B), B1, B2, B3, B4 and B6 are the no-slipe solid walls, B5 the inlet and B7 an open boundary. Coordinates of points 1, 2 and 3 are (0,JH), (2JH,2JH) and (IN,3JH), respectly. We take IN=14JH as the short open boundary and IN=20JH as the long open boundary. In (C), B1, B2, B3, B4, B6 and B7 are the no-slipe solid walls, B5 the inlet and B8 an open boundary. Coordinates of points 1, 2 and 3 are (8JH,2JH), (9JH,3JH) and (IN,5JH), respectly. We take IN=30JH as the short open boundary and IN=35JH as the long open boundary.

In all the numerical computations, grid size is decided based on JH=40. At the inlet, a uniform inlet u-velocity profile

$$u(y) = 1 \tag{1}$$

is chosen. We note that truncation occures at x=IN.

4. Four candidates of open boundary conditions(OBC)

OBC:no.1

The following open boundary condition was firstly used by Thoman and Szewczyk(1966):

$$\frac{\partial v}{\partial x}|_{OB} = -\frac{\partial^2 \psi}{\partial x^2}|_{OB} = 0, \ \frac{\partial \zeta}{\partial x}|_{OB} = 0. \tag{2}$$

OBC:no.2

The following open boundary condition was proposed by Mehta and Lavan(1975):

$$\frac{\partial \zeta}{\partial t} = -\frac{\partial(u\zeta)}{\partial x} - \frac{\partial(v\zeta)}{\partial y}, \ \frac{\partial \psi}{\partial x} = -v, \frac{\partial v}{\partial t} = -u\frac{\partial v}{\partial x} - v\frac{\partial v}{\partial y}$$
$$= -\left(u\zeta + \frac{1}{2}\frac{\partial(u^2+v^2)}{\partial y}\right) \ at \ OB, \tag{3}$$

in case that at the open boundary the inertia terms are dominant.

OBC:no.3

The following open boundary condition is the Sommerfeld radiation condition firstly used by Orlanski(1976):

$$\frac{\partial \phi}{\partial t} + c\frac{\partial \phi}{\partial x} = 0 \ at \ OB, \tag{4}$$

where ϕ is any variable, and c is the phase velocity of the waves. Orlanski proposed the following method which numerically evaluates the phase speed at the closet interior points every time: Using a leapfrog finite-difference representation, we have

$$\frac{\phi_{OB}^{n+1} - \phi_{OB}^{n-1}}{2\Delta t} = -\frac{c}{2\Delta x}(\phi_{OB}^{n+1} + \phi_{OB}^{n-1} - 2\phi_{OB-1}^{n}). \tag{5}$$

Hence the phase speed is numerically evaluated at the closet interior points from the above equation as follows:

$$c = -\frac{\Delta x}{\Delta t}\frac{\phi^n_{OB-1} - \phi^{n-2}_{OB-1}}{\phi^n_{OB-1} + \phi^{n-2}_{OB-1} - 2\phi^{n-1}_{OB-2}}. \tag{6}$$

From the above two equations, we can also obtain the boundary conditions $\{\phi^{n+1}_{OB}\}$ as follows:

$$\phi^{n+1}_{OB} = \frac{1 - c\Delta t/\Delta x}{1 + c\Delta t/\Delta x}\phi^{n-1}_{OB} + \frac{2c\Delta t/\Delta x}{1 + c\Delta t/\Delta x}\phi^n_{OB-1}. \tag{7}$$

OBC:no.4

The following open boundary condition is the Sommerfeld radiation condition used by Bottaro(1990) and Kobayashi, Pereira and Sousa(1993):

$$\frac{\partial\phi}{\partial t} + c\frac{\partial\phi}{\partial x} = 0 \ at\ OB, \tag{8}$$

where ϕ is any variable, and c is the phase velocity of the waves. Bottaro took the average streamwise speed in the channel as c, and Kobayashi et al. the mean channel velocity as c. The author proposes to take the uniform inlet velocity as c. Therefore, c=1.

5. Results on the backward-facing step flows

In this problem, we show results for Re=1,000. Firstly we compare difference of flows among four OBCs in case of IN=20JH=800. From results, we can see that variation of flow does not yet arrive at the open boundary at t=30, find that variation of flow already arrives at the open boundary at t=35, and hence it is the same as well at t=40. As seen from these results,there is severe difference among four OBCs in flows in the domain near the open boundary. Hence we can not at all conclude which of four OBCs gives the most excellent solution. While it is surely true in the domain near the open boundary, we can also show complete coincidence in flows among four OBCs in the domain within x=14JH=560. That is, there is no difference of flows among four OBCs. This fact promotes us next step. (We here note that computation of flow by OBC:no.2 was breaked off due to occure overflow in computation of ζ at $t > 35.1$.)

Secondly we compare difference of flows between short and long open boundaries by each OBC. Clearly from results, there is severe difference of flows by OBC:no.1 in the domain near the open boundary of IN=560. Hence OBC:no.1 can not at all say as a good OBC. Next we examine the case of OBC:no.2. As seen from results, OBC:no.2 shows comparatively good coincedence of flows in the domain near the open boundary of IN=560. Regretably this OBC can not bear practically due to occure overflow at $t > 35.1$. Next we look into the case of OBC:no.3. Clearly from results, there is severe difference of flows by OBC:no.3, especially at the open boundary of IN=560. Hence OBC:no.3 is better than OBC:no.1, but we can not yet say that it is a good OBC.

Finally we examine the case of OBC:no.4. Figure 2 shows its difference of flows between short and long open boundaries. As seen from (C) which is drawed in piles the streamfunction-profile and the vorticity-profile of flow by each OBC on several vertical internal points, OBC:no.4 shows tolerable good coincedence of flows even at the open boundary of IN=560. Hence we can conclude that OBC:no.4 is the best OBC among four OBCs, and is the excellent OBC.

6. Results on the blunt based body flows

This problem is more complicated than the previous one, and hence OBC:no.2 could not bear practically for this problem due to this complexity. We discuss about OBCs for this problem similarly to the previous problem. We show here results for Re=1,000. Firstly we compare difference of flows among three OBCs in case of IN=20JH=800. From results, we can see that variation of flow does not yet arrive at the open boundary at t=30, find that variation of flow already arrives at the open boundary at t=35, and hence it is the same as well at t=40. As seen from results, there is severe difference among three OBCs in flows in the domain near the open boundary. Hence we can not at all conclude which of three OBCs gives the most excellent solution. While it is surely true in the domain near the open boundary, we can also show complete coincidence in flows among three OBCs in the domain within x=14JH=560. That is, there is no difference of flows among three OBCs. This fact promotes us next step.

Secondly we compare difference of flows between short and long open boundaries by each OBC. Clearly from resuluts, there is severe difference of flows by OBC:no.1 in the domain near the open boundary of IN=560. Hence OBC:no.1 can not at all say as a good OBC. Next we look into the case of OBC:no.3. Clearly from results, there is severe difference of flows by OBC:no.3, especially at the open boundary of IN=560. Hence OBC:no.3 is better than OBC:no.1, but we can not yet say that it is a good OBC.

Finally we examine the case of OBC:no.4. Figure 3 shows its difference of flows between short and long open boundaries. As seen from (C), OBC:no.4 shows tolerable coincedence of flows even at the open boundary of IN=560. Hence we can conclude that OBC:no.4 is the best OBC among three OBCs, and is the comparatively good OBC for this problem.

7. Results on the rectangular cylinder obstacle flows

This problem is the most complicated among three problem, and hence even OBC:no.1 could not bear practically for this problem due to this complexity. We discuss about OBCs for this problem similarly to the previous problem. We show here results for Re=1,000. Firstly we compare difference of flows between two OBCs in case of IN=35JH=1400. From results, we can see that variation of flow does not yet arrive at the open boundary at t=45, find that variation of flow already arrives at the open boundary at t=55, and hence it is the same as well at t=65. As seen from results, there is severe difference between two OBCs in flows in the domain near the open boundary. Hence we can not at all conclude which of two OBCs gives the most excellent solution. While it is surely true in the domain near the open boundary, we can also show comparatively good coincidence in flows between two OBCs in the domain within x=30JH=1200. That is, there is a little small difference of flows between two OBCs. This fact promotes us next step.

Secondly we compare difference of flows between short and long open boundaries by each OBC. Clearly from results, there is severe difference of flows by OBC:no.3, especially at the open boundary of IN=1200. Hence we can not yet say that OBC:no.3 is a good OBC.

Finally we examine the case of OBC:no.4. Figure 4 shows its difference of flows between short and long open boundaries. As seen from (C), OBC:no.4 shows considerably smaller difference of flows than OBC:no.3 at the open boundary of IN=1200. Hence we can conclude that OBC:no.4 is better than OBC:no.3, and bears more well practically for this problem.

8. Discussion

On the backward-facing step flows

(1) We also examined difference of flows between short and long open boundaries computed under the condition of Re=800 and OBC:no.1. We computed this flows in the framework of the first order upwind scheme. As seen from results, OBC:no.1 can bear well practically for such the problem as $Re \leq 800$.

(2) We examined the case of IN=400 where truncation occures. As seen from this, there is a little difference of flows, especially at the open boundary of IN=400. Hence we had better not shorten location of truncation to IN=400.

On the blunt based body flows

(1) As we compare results, we clearly see that the complete coincidence of flows between short and long open boundaries as Figure 2 can not obtain when the problem becomes more complicated.

(2) This fact suggests that OBC:no.4 no longer is the complete OBC for this problem, although it is the excellent OBC for the backward-facing step problem. Hence we must be studying to search for a better OBC.

On the rectangular cylinder obstacle flows

(1) Firstly we note that numerical solution of flows for $IN \leq 1000$ can not give the right solutions even by OBC:no.4. Because reflection occures at the open boundary, its effect is changed the behaviour of the solution in the domain far from the open boundary, and at last its accumulation leads the wrong solution.

(2)In this paper, the author proposes to take c=1 as the phase speed of the Sommerfeld radiation condition. When we compare each results of c=0.7, 1 and 1.3, c=1.3 seems to be the best among three phase speeds.

On the open boundary conditions

(1) Such the OBCs as OBC:no.1 and 2 force to prescribe any condition at the open boundary. Hence they seem to oppose some qualities that a ideal OBC would display. As its poofs, flows by these OBCs are necessarily influenced heavily whenever variation of flow arrives at the open boundary, as seen in Figure 4, 5 and 10.

(2) Such the OBCs as OBC:no.3 and 4(that is, the Sommerfeld radiation condition) do not force to prescribe any condition at the open boundary, but seem to aid to permit the the flow to exit the domain gracefully and passively. Such phenomenon is one of qualities that a ideal OBC would display

(3) The Sommerfled radiation condition is used by some researchers, but the method of deciding its phase speed is different by each researcher. However there is not a firm ground why the phase speed would be decided by their methods. Then the author proposes to take a constant as the phase speed, despite of being not able to state a firm ground.

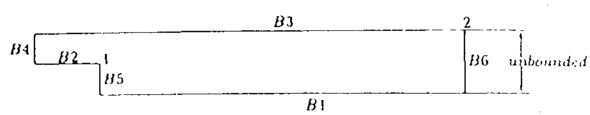
(4) Clearly from comparison of results, OBC:no.4 is more excellent than OBC:no.3 for all the problems. Moreover clearly from comparison of results, the case of c=1.3 seems to show the best result.

(5) Hence we will say that to take a constant as the phase speed is also better than to take a mean channel velocity every time as the phase speed. Because a mean value necessarily becomes to $c < 1$. From its reason, we can say that a larger value of c is profitable to premit the flow to exit the domain gracefully and passively.

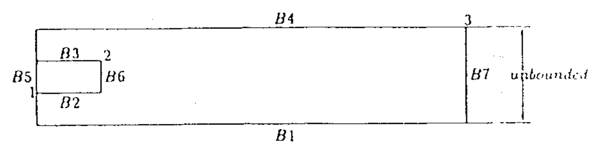
9. Concluding remarks

In this paper we studied about the open boundary conditions for incompressible unbounded flows, reported numerical solutions of flows by four OBCs for the backward-facing step problem, the blunt based body problem and the rectangular cylinder obstacle problem, and evaluated these results by means of difference of flows among four OBCs and between short and long open boundaries.

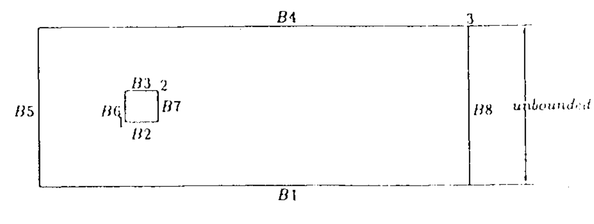
As the conclusion in all the cases, we showed that the OBC proposed by the author is the most excellent among the investigated OBCs. It is a very simple method which uses the Sommerfeld radiation condition as the OBC, and take a constant as its phase speed. This OBC showed to be the excellent OBC for the backward-facing step problem. However for the blunt based body problem and the rectangular cylinder obstacle problem, that is, as the problem becomes more complicated, this OBC no longer is the complete OBC. Hence we must be studying to search for a better OBC.



(A) Backward-facing step problem

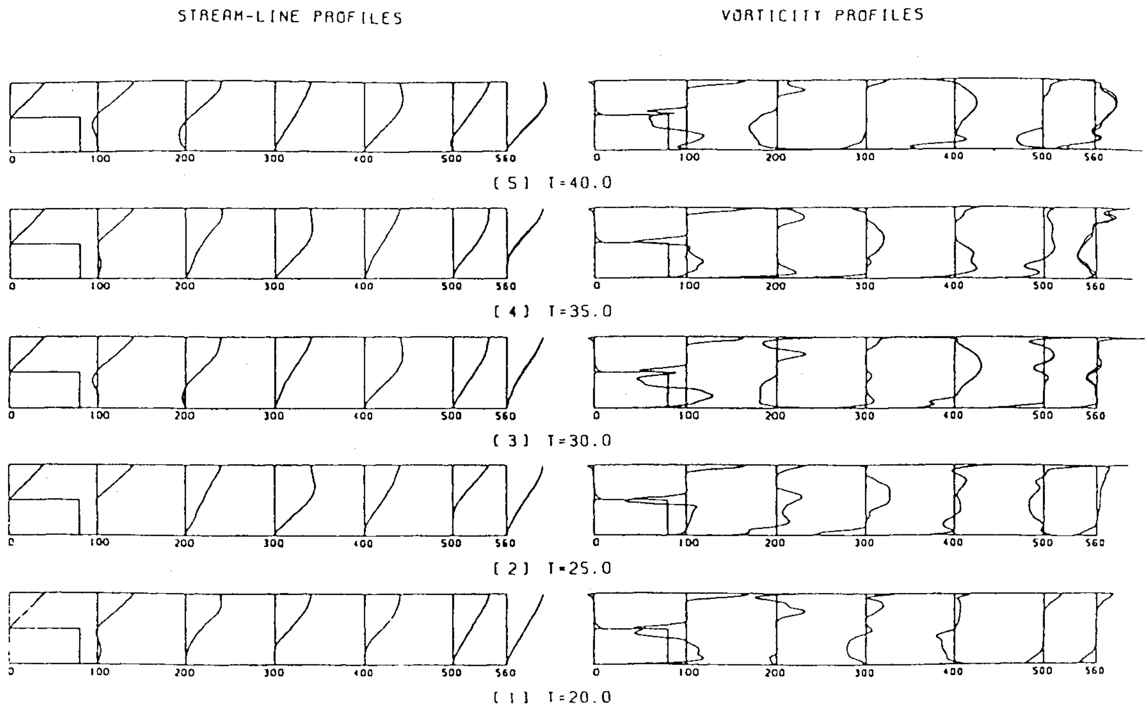


(B) Blunt based body problem

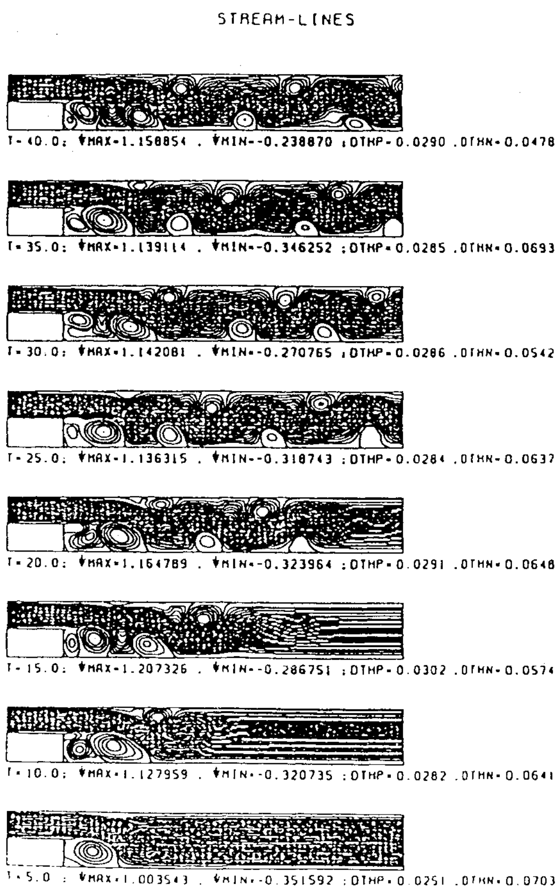


(C) Rectangular cylinder obstacle problem

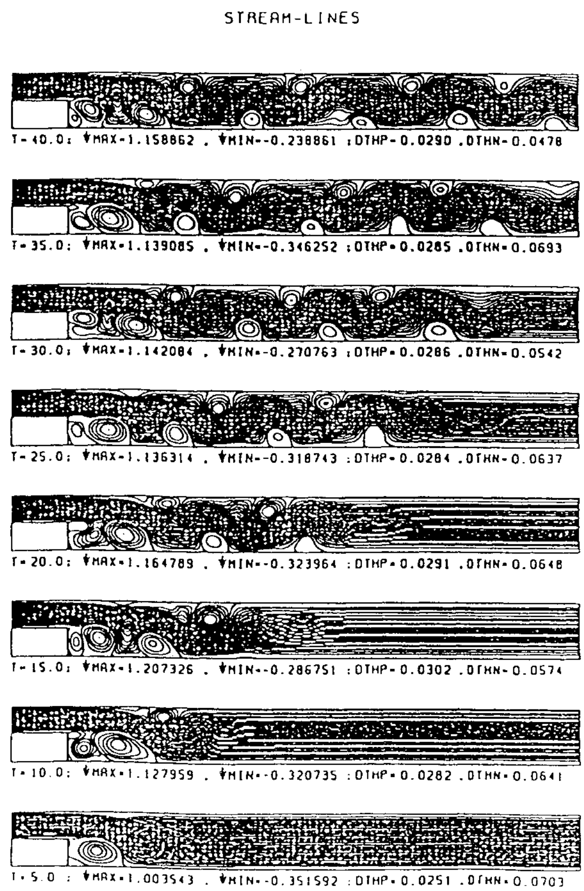
Fig. 1 Geometry definition of three test problems



(C) Difference of flows between IN=560 and IN=800 which is drawed in piles ψ-profile and ζ-profile on several vertical internal points each t

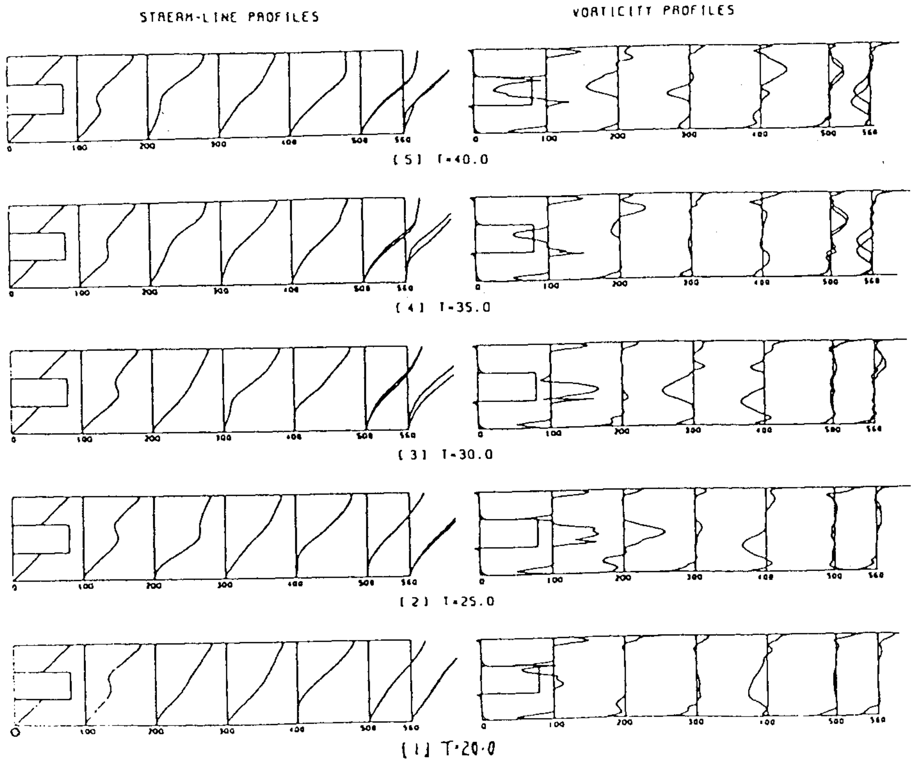


(A) Aspects of flow every t=5 (IN=560)

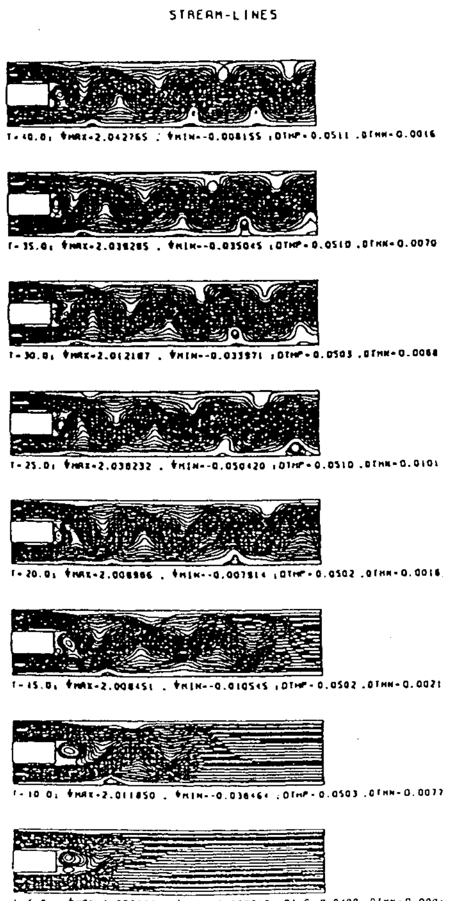


(B) Aspects of flow every t=5 (IN=800)

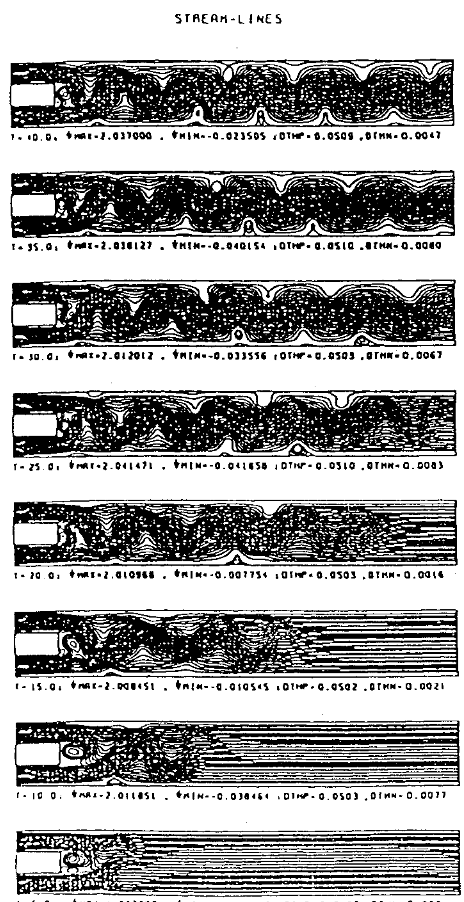
Fig.2 Difference of flows between short and long open boundaries (Re=1,000, OBC:no.4)



(C) Difference of flows between IN=560 and IN=800 which is drawed in piles ψ-profile and ζ-profile on several vertical internal points each t

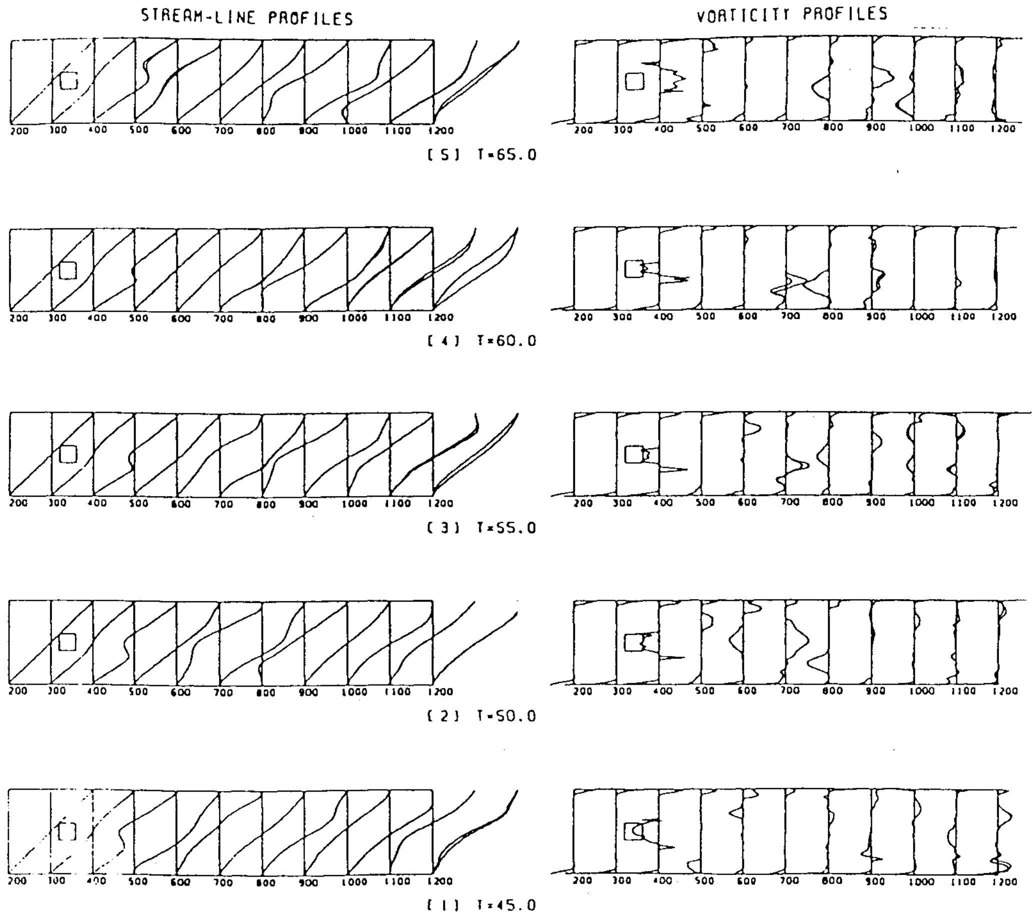


(A) Aspects of flow every t=5 (IN=560)

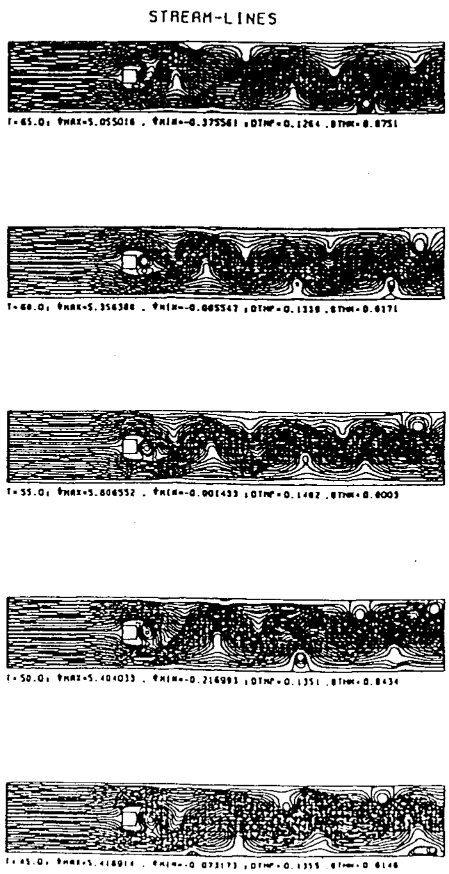


(B) Aspects of flow every t=5 (IN=800)

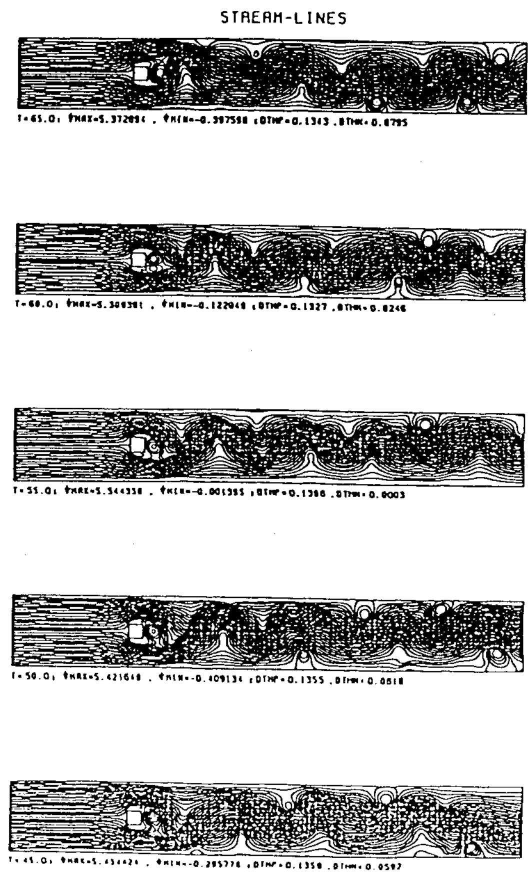
Fig.3 Difference of flows between short and long open boundaries (Re=1,000, OBC:no.4)



(C) Difference of flows between IN=1200 and IN=1400 which is drawed in piles ψ-profile and ζ-profile on several vertical internal points each t



(A) Aspects of flow every t=5 (IN=1200)



(B) Aspects of flow every t=5 (IN=1400)

Fig.4 Difference of flows between short and long open boundaries (Re=1,000, OBC:no.4)