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H_∞ Control for Aircraft Take off in Windshear

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ABSTRACT

Results are presented from an application of the most recently developed H_∞ control design methodology to a stabilizing controller for an aircraft during take off in a windshear. The emphasis is on the formulating of H_∞ optimal control synthesis problem in the state space. Simulation tests are performed in a six degree-of-freedom flight simulator for different windshear histories and different take off conditions, these results demonstrate that the controlled aircraft could take off in windshear safely, and the designed controller provides stability robustness to both external windshear disturbance as well as the model parameters variation with changes in flight conditions.

Keywords : Simulation research, Flight control, Robust control, H_∞ control, Take off, Windshear

概　　要

本報告は、ウインドシア中を離陸する航空機に対する安定化制御則の設計に、最近開発された H_∞ 制御則設計法を応用した結果について述べる。本報告では、特に状態空間における H_∞ 最適制御則構築の定式化を行い、6自由度を持つ飛行シミュレータを用いて異なるウインドシア時歴及び異なる離陸条件に対するシミュレーション試験を行った。その結果、制御対象の航空機はウインドシア中を安全に離陸できることが分かった。さらに、設計された制御則は、ウインドシアによる外乱及び飛行条件の変化によるモデルパラメータの変動の双方に対してロバストな安定性を持つことが示された。

1. Introduction

Control of an aircraft encountering windshear has gained considerable importance in the recent past since the variable wind has been found to be major contributing factors in many accidents involving large aircraft. Studies have been carried out on different aspects such as modeling and identifying windshear as well as the design of controllers to enhance the chance for survival of aircraft while encountering windshear. As we know, once the aircraft becomes airborne during take off, the pilot has no choice but to fly through the windshear, so the study of control problem is important since it suggests the piloting strategies for crew training and assists in the development of autopilots.

Primary among these studies, the so-called simplified gamma guidance scheme and acceleration guidance scheme are developed in References [1]－[3] based on attaining near optimal trajectories in the presence of a given windshear structure, another approach to solving the problem has been via deterministic control of uncertain system [4]-[7], the control of climb rate by means of the deviation of angle of attack from its nominal value is presented in Reference [4], in References [5] [6] all the state variables and only the relative path inclination are stabilized respectively by the control of angle of attack, Reference [7] considers the stabilization of climb rate about a desired value utilizing an adaptive strategy where only the climb rate information is used. But the emphasis of all these papers are on the feasibility of proposed control concepts, about how these control schemes can be realized by actual aircraft control and how to design the practical autopilots are not considered.

In the opinion of author, the control of aircraft during take off in the presence of windshear is a problem of stabilizing the

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flight of aircraft while to make the absolute path angle as high as possible, the latter can be realized by the power setting held at a value which gives the maximum thrust, and the former problem can be solved by the effective robustness control design theory using only the elevator control. In this paper the new developed H_∞ robust control technique is applied to the guidance of aircraft take off in the presence of windshear. Reference [8] presents the robust integrated flight/propulsion control design for a STOVL aircraft using H_∞ control design method in frequency domain, here the solution of this problem will be focused in state space, and no a prior information or assumptions about the bounds of the uncertain windshear is needed in deriving the controller. Having obtained a control design, two windshear models and different flight conditions are considered. For all these test cases the H_∞ control is found to be strong robust control strategy against the windshear encountered.

2. Formulation of aircraft during take off

The longitudinal dynamics of aircraft take off in variable winds are modeled using perturbation equations written in body axes. They are linearized about a reference equilibrium condition of constant flight speed, these equations may be written in the matrix form

$$\Delta\dot{x}(t) = A\Delta x(t) + B\Delta u(t) + E\Delta w(t) \tag{1}$$

where Δx is the perturbation state vector, Δu is the perturbation control vector, and Δw is the perturbation longitudinal wind velocity vector, which is expressed in inertial coordinates and assumed to be uniform over the length and span of aircraft.

$$\Delta x = (\Delta u_e, \Delta w_e, \Delta q, \Delta\theta, \Delta\delta_T)^T \tag{2a}$$
$$\Delta u = (\Delta\delta_e, \Delta\delta_{Tc})^T \tag{2b}$$
$$\Delta w = (\Delta w_x, \Delta w_z)^T \tag{2c}$$

Where Δu_e, Δw_e are the components of inertial speed in body axes, Δw_x and Δw_z are the horizontal and vertical wind components respectively. In this definition, $\Delta w_x > 0$ is tailwind, $\Delta w_z > 0$ is downdraft.

The simulated aircraft is a research aircraft Dornier-228-200 of 5700 (kg) gross weight. The linearization reference equilibrium condition used for robust controller design is named condition 1, here $h_0 = 15.24$ m (50ft), $\delta_f = 5$ (deg), $\delta_{T0} = 100$ (deg), $\delta_{e0} = 0$ (deg), $v_0 = 61.2$ m/s (120 kt), $\gamma_0 = 9.16$ (deg).

To test the robustness of designed controller in the flight condition different from the nominal design case, another reference equilibrium condition named condition 2 is also considered, where $h_0 = 15.24$ m (50ft), $\delta_f = 5$ (deg), $\delta_{T0} = 100$ (deg), $\delta_{e0} = 0$ (deg), $v_0 = 66.3$ m/s (130 kt), $\gamma_0 = 8.19$ (deg). The resulting modal characteristics are summarized in Table 1.

3. H_∞ Control Design

3.1 General Theoretical Background

For a linear system described by the state equations

$$\dot{x}(t) = Ax(t) + Bu(t) + Ew(t) \tag{3a}$$
$$z_1(t) = D_1x(t) \tag{3b}$$
$$z_2(t) = D_2u(t) \tag{3c}$$
$$y(t) = Cx(t) \tag{3d}$$

Where $x(t) \in R^n$ is the state ; $u(t) \in R^m$ is the control ;

Table 1.Summary of natural mode characteristics of Do-228 during take off

Flight Conditions	1		2	
Mode	short-period	phugoid	short-period	phugoid
Eigenvalues	$-1.4143 \pm 2.1240i$	$-0.0063 \pm 0.1673i$	$-1.5161 \pm 1.9798i$	$-0.0112 \pm 0.1547i$
ξ	0.5542	0.0376	0.6080	0.0722
$\omega_n(rad/s)$	2.5518	0.1674	2.4936	0.1551
$\omega(rad/s)$	2.1240	0.1673	1.9798	0.1547
$T_{1/2}$, $T_2(s)$	0.4900	110.0	0.4817	61.8750
$T(s)$	2.9567	37.5374	3.1720	40.5947

$w(t) \in R^q$ is the exogenous disturbance ; $y(t) \in R^r$ is the measured output and $z_1(t) \in R^{q1}$, $z_2(t) \in R^{q2}$ are the controlled outputs.

The standard H_∞ optimal problem is concerned with constructing a dynamic feedback compensator that these design criteria are satisfied.

(1) The closed loop system is asymptotically stable.

(2) The H_∞ norm of closed loop transfer function from w to z_1, z_2 is bounded by a prescribed positive constant λ.

$$\|T_{zw}\|_\infty \leqq \lambda \tag{4}$$

Where the H_∞ norm $\|T_{zw}\|_\infty$ which is the popular performance measures in optimal control theory, defined in the frequency domain for a stable transfer matrix as

$$\|T_{zw}\|_\infty = \sup_\omega \sigma_{max} \left[T_{zw}(j\omega)\right] \tag{5}$$

and σ_{max} denotes its maximum singular value.

When $D_2^T D_2 = I$, if this admissible controller is existent, the necessary and sufficient conditions are as follows [9].

(1) The unique stabilizing solutions to two algebraic matrix Riccati equations are positive semi-definite.

$$A^T X_\infty + X_\infty A + D_1^T D_1 - X_\infty (BB^T - \lambda^{-2} EE^T) X_\infty = 0 \tag{6a}$$
$$AY_\infty + Y_\infty A^T + EE^T - Y_\infty (C^T C - \lambda^{-2} D_1^T D_1) Y_\infty = 0 \tag{6b}$$

(2) The spectral radius of the product of X_∞ and Y_∞ is less than λ^2.

$$\rho(X_\infty Y_\infty) \leqq \lambda^2 \tag{7}$$

When these conditions hold, the controller may be constructed by linear static feedback of the form

$$\dot{x}_c = A_c x_c + B_c y \tag{8}$$
$$u = C_c x_c \tag{9}$$

where

$$A_c = A + \lambda^{-2} EE^T X_\infty + BC_c - B_c C \tag{10a}$$
$$B_c = Z_\infty Y_\infty C^T \tag{10b}$$
$$C_c = -B^T X_\infty \tag{10c}$$

and

$$Z_\infty = (I - \lambda^{-2} Y_\infty X_\infty)^{-1} \tag{11}$$

The quantity x_c is the output of an observer-type system, it may be viewed as the estimated state. The block diagram of closed control system is illustrated in Fig 1.

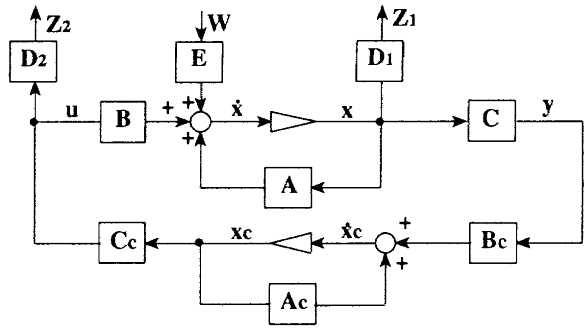

Fig.1 Block Diagram of H_∞ Control

3.2 Controller Design

During the flight of take off especially in the presence of windshear, usually the full throttle is used, so the available control input is only elevator angle $\Delta\delta_e$, throttle control variables $\Delta\delta_T$ and $\Delta\delta_{Tc}$ are dropped from the state vector and control vector respectively. The detailed numerical expressions of A, B, E are given in the Appendix. In the model used for control design, the measured output matrix

$$C = I \tag{12}$$

The design weights can be chosen by designer to reflect the performance specifications given for the problem, the state variables are penalized in the design so that good flight performance will occur, the control variables are weighted so that the maximum deflections and rates are indirectly included in the problem formulation, if the limits are exceeded during normal flight conditions, their weights in the design problem are increased until all the limitations are satisfied. The nominal performance weights are chosen as

$$D_1 = I \qquad D_2 = I \tag{13}$$

To construct the admissible controller, two Riccati matrix equations are needed to be solved iteratively. A large initial value of λ is selected so that the Riccati equations can be solved easily. As no technique is known to minimize the bound on H_∞ norm automatically so far, solutions should be found using recursive iteration manually with decreasing the value of λ, and the algorithm is executed repeatedly until a desired value of λ is reached or until one of the Riccati equations no long converge. The minimum λ is found to be 1.175 (1.401 dB) in this paper. The maximum singular value plot of the associated closed loop and open loop transfer functions are shown in Fig.2, where all singular values are given in decibels. It is clear the controlled aircraft has better response behavior than the open loop system when the frequency of windshear input between 0.03 and 40 (rad/s). The designed state space matrices of controller are given in the Appendix.

4. Simulations and Discussions

For the purposes of examining robustness of H_∞ controller,

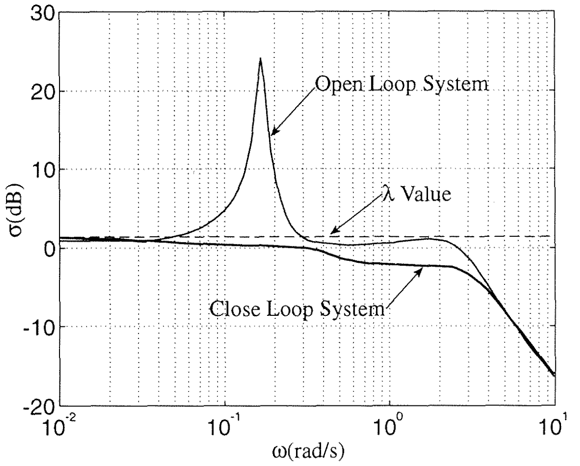

Fig.2 Maximum Singular Values of System

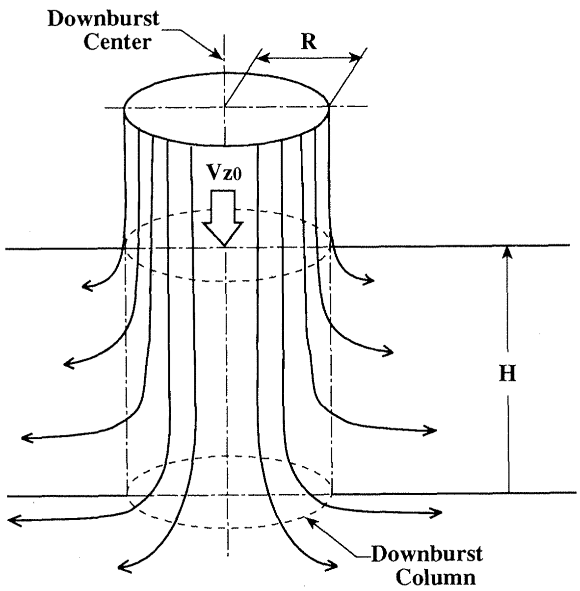

Fig.3 Structure of Windshear Model 1

two general downburst profiles are used in the simulation test.

Model M 1.This downburst wind field model is referred to Ref. 10, this model simulates three wind components in the low-altitude wind field that have special variations in wind velocity similar to those measured in the atmosphere during severe convective disturbances. As shown in Fig.3, v_{z0} is 10 (m/s), neither the downdraft exists outside the shear column of 600 (m) radius, nor the horizontal flow higher than 300 (m).

Model M 2.This is the model in which Δw_x and Δw_z are given as functions of the time rather than position, the horizontal wind is given by

$$\Delta w_x = -\Delta w_{x0} \sin(2\pi t/T_0) \tag{14a}$$

and the vertical wind is given by

$$\Delta w_z = \Delta w_{z0}\ [1-\cos(2\pi t/T_0)]\ /2 \tag{14b}$$

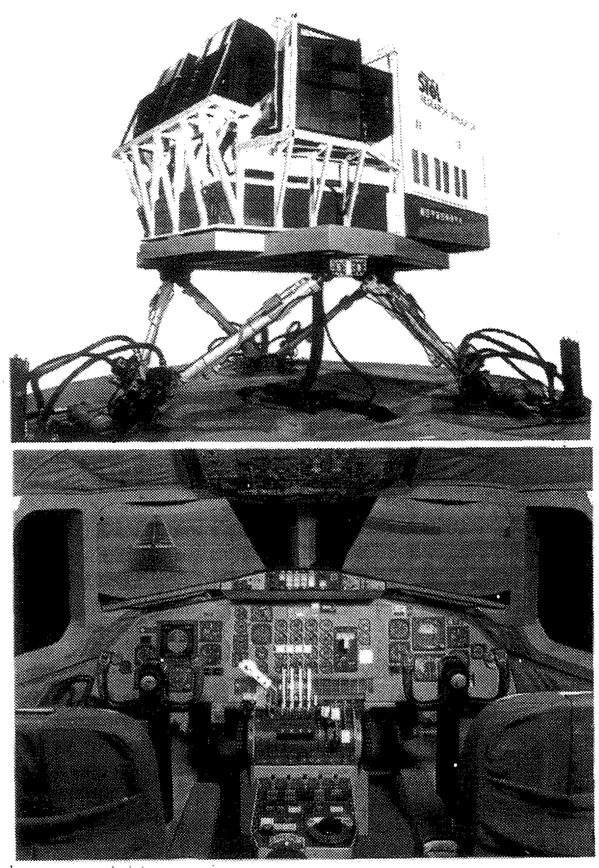

Fig.4 Six Degree-of-Freedom Flight Simulator

where Δw_{x0} and Δw_{z0} are given constants reflecting the windshear intensity, here $\Delta w_{x0}/\Delta w_{z0} = 12/8$ (m/s) is considered. T_0 is the total flight time through the downburst, usually it can be taken as 60 (s). The windshear models M2 is also considered in Refs. 4, 6, and 7.

In the relative harsh climate conditions, there are also some rough random turbulence existing besides the windshear. The severe turbulence model from MIL-F-8785 B (30 m) is considered in the simulation test, the intensity of turbulence $\sigma_{ug} =$ 1.43 (m/s), and the probability of exceedance is 10 %.

The performance of $\boldsymbol{H}_\infty$ guidance scheme for against windshear is examined by a six degree-of-freedom ground simulator, it is shown in Fig.4. For the application of dynamic $\boldsymbol{H}_\infty$ controller, sample time is very important, in this simulation test it is taken as 10 (ms), and the aircraft is assumed to be in an equilibrium state before it encounters any winds, the longitudinal motion is controlled by the designed autopilot and the lateral-directional motions are eliminated by the pilot, the test results are presented in Fig.5 to Fig.8.

For the relative weak windshear M 1 the test pilot does not feel the effect of exogenous wind inputs when the autopilot is switched on, the controlled aircraft can climb continuously through the encountered wind field. For the relative severe

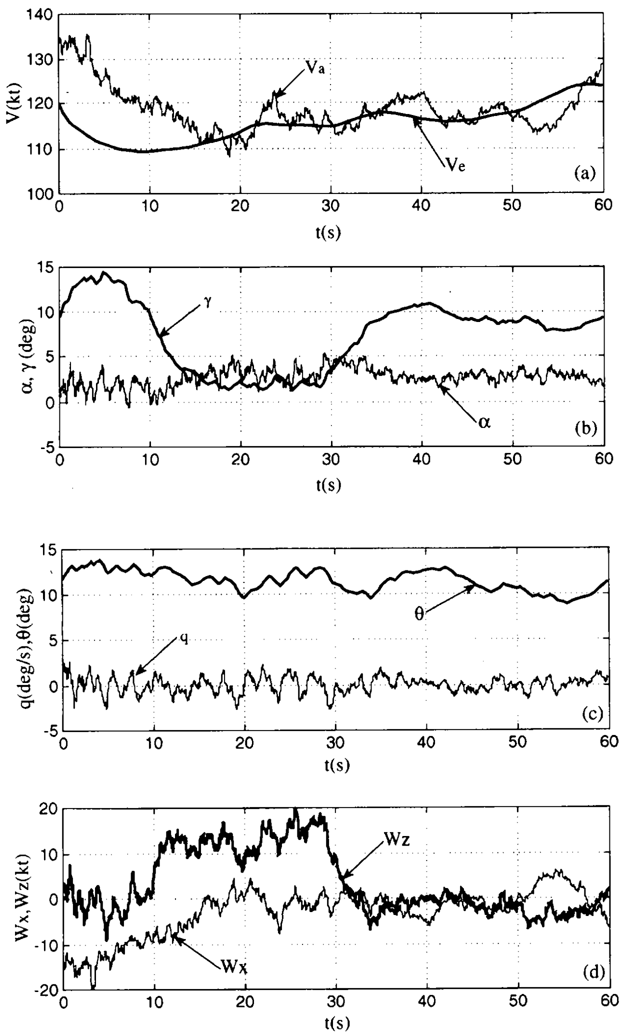

Fig.5 Simulation Result of Aircraft Take off in Windshear M 1

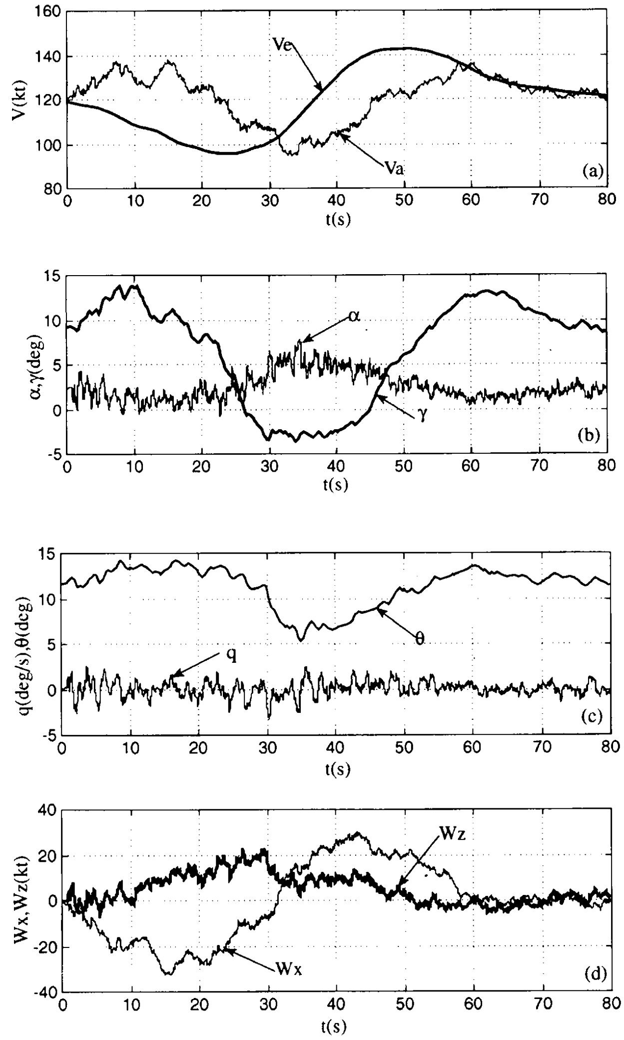

Fig.6 Simulation Result of Aircraft Take off in Windshear M 2

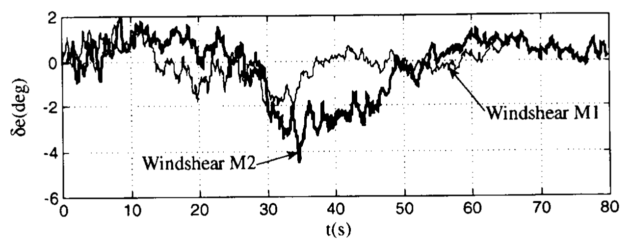

Fig.7 Elevator Control of Aircraft Take off in Windshear

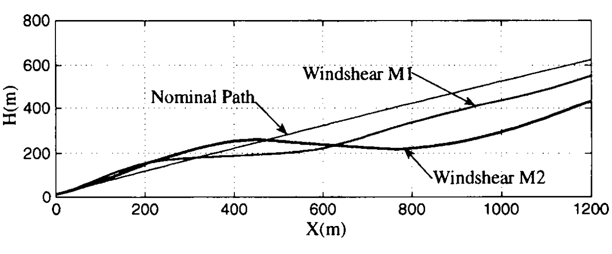

Fig.8 Flight Paths of Aircraft Take off in Windshear

windshear M 2 the aircraft flies deviating from the equilibrium flight condition as affected by the windshear, but H_∞ controller has enough robustness to suppress the deviations of state variables within certain ranges, as shown of Fig.6 (a), the minimum value of airspeed v_a is about 51 m/s (100 kt) which is higher than the limited safe flight speed 40.8 m/s (80 kt) greatly, and v_e is the ground speed. Comparing to the normal take off trajectory, there is some height increase in headwind and some height loss in tailwind and downdraft, the maximum height loss is about 45 (m) at the altitude more than 200 (m) for the relative severe windshear M 2, these results are presented in Fig.8. As the flight height is high enough, it is not so dangerous case.

The robustness of H_∞ controller applied in flight condition 2 is also tested, these results are almost the same as in condition 1, that means the designed controller has sufficient robustness to the windshear disturbance in different take off conditions.

5. Conclusions

A robust control strategy for an aircraft during take off in the presence of windshear has been studied. An autopilot for research aircraft Do-228 is designed by H_∞ control technique, no a prior information or assumptions about windshear structure or intensity is required. The simulation test results demonstrate that the designed controller has sufficient robustness to both external windshear which contains energy and is limited in scale within certain range, as well as the model parameters variation with change in flight conditions. The controlled aircraft is able to tolerate moderate to relatively severe windshears, the safety of aircraft encountering windshear during take off can be increased greatly.

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Appendix

The nominal design model of Do-228 :

$$A = \begin{bmatrix} -0.0401 & 0.1318 & 0.0578 & -9.6016 \\ -0.2350 & -1.2680 & 60.3206 & -1.9620 \\ 0.0016 & -0.0744 & -1.533 & 0.0 \\ 0.0 & 0.0 & 1.0 & 0.0 \end{bmatrix}$$

$$B = \begin{bmatrix} 0.2030 \\ -4.7400 \\ -4.9800 \\ 0.0 \end{bmatrix} \qquad E = \begin{bmatrix} 0.0129 & -0.1372 \\ 0.4841 & 1.1953 \\ 0.0133 & 0.0732 \\ 0.0 & 0.0 \end{bmatrix}$$

The designed H_∞ controller matrices :

$$A_c = \begin{bmatrix} -23.6412 & -1.8474 & 0.7451 & 1.9965 \\ 1.4981 & -3.7793 & 38.8444 & -229.6237 \\ 3.4586 & -1.5650 & -23.6737 & -206.8769 \\ 0.2888 & 0.0141 & 1.0030 & -0.0038 \end{bmatrix}$$

$$B_c = \begin{bmatrix} 23.4415 & 1.9875 & 0.2481 & -0.2888 \\ 1.9875 & 1.6715 & 0.0437 & -0.0141 \\ 0.2481 & 0.0437 & 0.0037 & -0.0030 \\ -0.2888 & -0.0141 & -0.0030 & 0.0038 \end{bmatrix}$$

$$C_c = \begin{bmatrix} -0.7422 & 0.2969 & 4.4410 & 41.1799 \end{bmatrix}$$

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