ISSN 0389-4010 UDC 533. 662. 6: 629. 735. 45. 035. 6:

## TECHNICAL REPORT OF NATIONAL AEROSPACE LABORATORY

#### TR-321T

# An Aerodynamic Design and the Overall Stage Performance of an Air-cooled Axial-flow Turbine

Atsumasa YAMAMOTO, Kitao TAKAHARA, Hiroyuki NOUSE, Shigeo INOUE, Hiroshi USUI and Fujio MIMURA

January 1981

#### NATIONAL AEROSPACE LABORATORY

CHŌFU, TOKYO, JAPAN

#### TABLE OF CONTENTS

	Page
Symbols	1
. Introduction	3
. Design	4
2.1. Calculation of Turbine Performance	4
2.2. Calculation of Velocity Triangles	6
2.3. Determining the Profile and the Blade	
Arrangement of Air-Cooled Blades	8
. Experiments	12
3.1. Experimental Equipment and Measuring Equipment	12
3.2. Experimental Methods	14
3.3. Methods of Analyzing the Experiments	16
3.4. Experimental Results	17
3.5. Studies and Discussion of the Experimental Results	22
. Conclusion	24
5. Postscript	25
REFERENCES	25
APPENDIX A. DETAILS OF THE DESIGN	27
APPENDIX B. EQUIVALENT CONDITIONS OF	
TURBINE OPERATING STATES	39

# AN AERODYNAMIC DESIGN AND THE OVERALL STAGE PERFORMANCE OF AN AIR-COOLED AXIAL-FLOW TURBINE\*

By Atsumasa Yamamoto\*\*, Kitao Takahara\*\*, Hiroyuki Nouse\*\*, Shigeo Inoue\*\*, Hiroshi Usui\*\* and Fujio Mimura

#### ABSTRACT

In order to investigate air-cooled turbines for application to high-temperature engines, a single stage turbine with a 0.56-m (22-inch) tip diameter was designed. The aerodynamical design procedures of the turbine are presented herein. The stator and rotor blades are characterized by low blade solidity, thick blade section, blunt leading edge and trailing edge, and low blade aspect ratio.

A cold air test without supply of cooling air was conducted to determine the turbine aerodynamic performance. The highest efficiency obtained over the range of conditions investigated was 0.865.

The turbine satisfied the equivalent design value of specific work output at the design condition (i.e. at equivalent design speed and equivalent design pressure ratio) with an efficiency of 0.856.

Detailed surveys of rotor-outlet gas flow were made with Pitot tubes and temperature sensors and the results are also presented.

From these results, it can be concluded that the present airfoil shapes designed for an air-cooled turbine by application of conventional technical design procedures worked well from an aerodynamical point of view and that the compromise, which was necessary in designing the aerofoil shapes because of cooling considerations, did not noticeably impair the turbine's overall performance.

#### 概 要

高温タービン用の空冷タービン翼列は、空気冷却という条件から非冷却タービン 翼列にくらべ、より低いソリディティをもち、翼の肉厚および翼前縁、後縁部も厚く、 かつ高圧、高負荷のためアスペクト比も低い、本報では、そうした特徴をもつタービ ン翼の設計方法の詳細を述べ、またその翼列の性能を確かめるための実機大回転試験 タービンによる空力性能実験の結果を述べた、その結果、最大断熱効率 86.5% を得、 種々な制約を受ける空冷タービン翼列も、従来から非冷却翼の設計に用いられて来た 方法の延長で設計しても、性能を大きく損なうことはないことが判った。

$\boldsymbol{A}$	Area
$A_a$	Annular area
$\boldsymbol{a}$	Sound velocity
$R_r R_r$	Leading and trailing edge angles

**SYMBOLS** 

$oldsymbol{C}$	Chord length
$C_p$ , $C_{pi}$	Pressure coefficients of compres-
	sible and incompressible fluids
$c_p$	Specific heat at constant pressure
$D_p, D_s, D_t$	Blade diffusion parameters
d	Diameter
$d_{\scriptscriptstyle L}, d_{\scriptscriptstyle T}$	Leading and trailing edge dia-
	meters of blade
$oldsymbol{d} heta$	Angle

of blade

<sup>\*</sup> Received 23, October 1980.

<sup>\*\*</sup> Aeroengine division.

0	Mushing mainh days and		dynamometer output and turbine
G	Turbine weight flow rate		expansion ratio
g H	Gravity acceleration	<b>1</b> 0	Adiabatic temperature efficiency
$\Delta H_T$	Enthalpy Turbine thermal head	$\eta_{1-3}$	based on total temperature ratio
h	Blade height		and total pressure ratio
i	Blade incidence, also indicates	heta	Angle
ι	i-th circle	$\theta, \theta_{cr}$	Velocity ratio (Correction Co-
J	Work equivalent of heat;	, o <sub>c7</sub>	efficient)
O	426.9 kg·m/kcal	κ	Specific heat ratio
L	Specific blade loss; $(1-\eta_t)/\sigma_M$	λ	Air fuel ratio
$\overline{l}$	Arm length of dynamometer;	$\mu$	Viscosity
•	1.4606 m	<b>ξ</b>	Angle of stagger
M	Mach number	$\pi_T$	Turbine expansion ratio
m	$\sin^2 \alpha_2$	$\rho$	Density
N	Turbine revolution speed	$\rho_R$	Degree of reaction
n	Number of blades	σ	Solidity; C/S
0	Blade throat width	τ	Turbine torque
$\boldsymbol{P}$	Pressure	$\phi,\phi_{cr}$	Correction coefficients for turbine
R	Gas constant	7, 70,	expansion ratio
$ar{R}_e$	Mean Reynolds number inside	$\varphi_N$	Velocity coefficient of stator blade
-	turbine	$\varphi_{N'}$	Total pressure ratio before and
$R_L, R_T$	Leading and trailing edge radii	,	after stator blade
•	of blade	$\phi$	Entropy function
RF	Temperature recovery coefficient	ω	Angular velocity
r	Blade radius in height direction		-
S	Entropy	(Subscr	ipts)
S	Blade pitch	a	Axial direction
T	Temperature	ad	Adiabatic
$oldsymbol{U}$	Peripheral velocity	air	Air
V	Absolute velocity	cr	Value at $M=1.0$
W	Relative velocity	$\mathbf{des}$	Design
$W_T$	Dynamometer load	i	Incompressible, or i-th
X, Y	Coordinates	M	Mean (mean diameter)
$Y_t$	Total pressure loss coefficient of	max	Maximum
	blade	N	Stator blade
$\boldsymbol{x}$	Length (mainly in axial direction)	R	Rotor blade
у	Length (mainly in blade height	r	Relative
	direction)	$\mathbf{rad}$	Radial direction
$\boldsymbol{z}$	Length	8	Static
/G 1 1		st	Standard state
(Greek l		T	Turbine
α	Absolute outflow angle	t	Total
β	Relative outflow angle	th	Theoretical
γ	Speciac weight	u	Peripheral velocity or peripheral
$\delta$	Ratio of turbine inlet total pres-		direction
	sure to standard pressure	1	Before stator blade
$\varepsilon$ , $\varepsilon_{c\tau}$	Correction coefficients for flow	2	After stator blade, before rotor
			blade
	rate		-
η	Efficiency	3	After rotor blade
$\eta \ \eta_T$	Efficiency Adiabatic efficiency based on en-	3 ∞	After rotor blade Infinite
•	Efficiency		After rotor blade

#### 1. INTRODUCTION

In recent aircraft there has been a demand for the development of engines with a decreased fuel flow rate by increasing the maximum cycle temperature, improving the thermal efficiency, and using fan engines with an improved propulsive efficiency. It is necessary to cool the turbine blades so that they can withstand this high gas temperature at the turbine inlet. In the aircraft turbines all over the world today, the air coming from the compressor is used to cool the blades, and the turbine inlet temperature has been raised rapidly year by year thanks to progress and improvement in the cooling methods.

In turbines using air-cooled blades, that is, in air-cooled turbines, higher turbine inlet temperatures and higher cycle pressure ratios are dealt with than is the case in non-cooled turbines. For this reason, the blades are required to have various special properties. The most important ones are the following. (1) Since it is necessary to introduce cooling air inside the blades, the leading and trailing edges of the blades will be thicker, and the blades as a whole will be thick. (2) At the higher cycle temperatures, the optimum thermodynamic expansion ratio will increase. Therefore, in order to eliminate as for as possible the necessity for cooling after the second stage, it is necessary to increase the work per stage. The deflection angles in the stator and rotor blades are increased in order for this work to be absorbed by the thick turbine blades. (3) In most cases, the cooling air used for cooling the turbine blades is extracted from the intermediate or final stages of the compressor. For this reason, the increased amount of cooling air will increase the total loss of the engine as a whole, and therefore the necessary amount of cooling air must be suppressed to the utmost minimum [1]. It is necessary to reduce the number of cooling blades for this purpose. (4) Together with the increase in the turbine inlet pressure, the high-pressure turbine blades will become smaller in size, and there will be a relative increase in the decline of the aerodynamic performance on account of the rotor blade tip clearance. (5) Not a very large twisting angle can be designed by the blades on account of problems connected with fabrication of the cooling holes, etc. (6) On account of injection and leakage of the cooling air from the blades or the inner and outer walls, there occurs aerodynamical interference between the cooling air and the main flow gas, and the aerodynamical performance of the turbine is affected.

Thus, air-cooled blades, which are required to fulfill conditions which are quite different from those of the noncooled blades, must incorporate more compromises in the design than was the case in the conventional non-cooled blades of the past, and there are also many difficult points in the production. There have been a number of studies overseas concerning turbines with such air-cooled blades [2-4], and their performance properties have been investigated. Nevertheless, the various design problems caused by the foregoing circumstances and the characteristics of the blades have not been clarified to any great degree, and the designing methods themselves have not been established.

In view of these circumstances, our Aeroengine Division is making studies of hightemperature air-cooled turbines as part of its aeronautical engine research. In this report, originally from ref. [30] (1973), we describe the designing methods and the results of experiments concerning the aerodynamic performance properties of the early air-cooled turbines. We first set up the fundamental policies which are necessary in air-cooled turbines and describe a method of designing under these policies in which the conventional designing methods of the past will be applied as far as possible. Next we describe the results of experimental studies to determine whether air-cooled turbines designed in this way display the anticipated aerodynamical overall performance properties or not. Furthermore, some of the problems in this designing method are pointed out on the basis of the results of measurements of the internal flow.

These experiments were conducted using unheated compressed air with a turbine

inlet temperature of about 100°C. The experiments were performed without supplying air for cooling the blades.

#### 2. DESIGN

At the early stage of research into high-temperature turbines in the Aeroengine Division of this Laboratory, the planning called for a turbine inlet gas temperature of 1,150°C. This turbine had a single stage, and the internal air-cooling system was adopted in both the stator and the rotor blades.

As for the sequence of the aerodynamical designing, we first carried out performance calculations one-dimensionally for several types of velocity triangles at the mean blade diameter; then the optimum one was selected from them. Next, we performed two-dimensional calculations under the assumption that the above performance will be realized by the mean blade diameter (MEAN) and determined the velocity triangle at the blade tip (TIP), the mean blade diameter (MEAN), and blade root (ROOT). Finally, the profile and the blade arrangement were determined on the basis of this velocity triangle. In Fig. 1 are shown the

inspection section radius and velocity triangle symbols for design calculations. In the designing at this time the flow of cooling air was not taken into consideration in calculation of the gas flow inside the turbine. In the following, an outline is given of this designing.

### 2.1. Calculation of Turbine Performance

Turbine performance calculations were performed for various turbine revolution speeds, stator blade outflow angles, and stator blade outflow Mach numbers, and those with great turbine work and adiabatic efficiency were selected.

The design hypotheses are as follows.

- (1) It is assumed that the state of the air flow in the inspection sections is uniform in each section, and the state of the air flow is calculated one-dimensionally as an adiabatic flow of a perfect gas at the mean blade diameter.
- (2) The stator blade velocity coefficient is assumed to be  $\varphi_N = 0.97$ .
- (3) The rotor blade total pressure loss coefficient is assumed to be  $Y_{tR} = 0.23$ .
- (4) The specific heat ratio is assumed to be constant inside the turbine. The

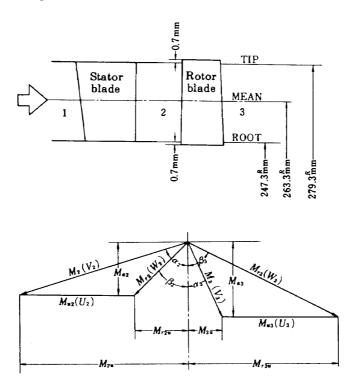
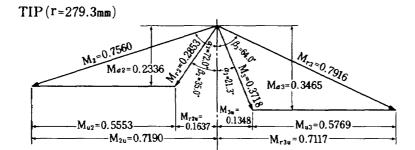
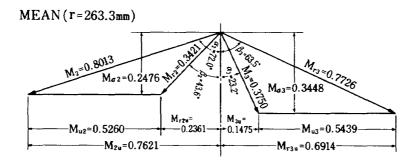


Fig. 1. Inspection sections for design calculations and velocity triangle symbols.

Item	Notation	Design value	Unit
Gas flow rate	G	3.95	kg/sec
Inlet total pressure	$P_{t1}$	25000	kg/m²
Inlet total temperature	$T_{t1}$	1423.15	K
Adiabatic efficiency	$\eta_T$	0.85	
Adiabatic thermal head	$\Delta H_{ad}$	63.25	kcal/kg
Revolutions per minute	N	13300	rpm
Peripheral velocity	$U_{M}$	366.7	m/sec
Expansion ratio	$\pi_T$	2.02	
Theoretical velocity	$V_{th}$	727.5	m/sec
Velocity ratio	$U_{M}/V_{th}$	0.504	
Degree of reaction	$\rho_{R,M}$	0.464	

Table 1. Major Design Specifications





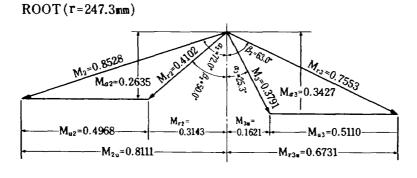


Fig. 2. Design velocity triangles.

Table 2 (a). Results of Calculations of Stator Blade Profile and Blade Arrangements (TIP; r=277.3 mm)

C 33.60	ξ 00 -45.		$B_L$ 6.000	$B_T$ $-71.000$				
Blad	e Coordin	ates	*					
	Orig	ginal	Camb	er Line		Blade	Surface	V - 101/A
	(X*	, Y*)	CAM	(X, Y)	S.S	(X, Y)	P.S	(X, Y)
i	X	Y	X	Y	X	Y	X	Y
1	1.456	2.218	2.213	0.154	2.152	2.371	2.273	-2.063
2	2.463	2.860	3.628	0.144	3.754	3.002	3.501	-2.713
3	3.655	3.210	5.172	0.003	5.623	3.182	4.722	-3.175
4	4.878	3.356	6.620	0.279	7.438	2.976	5.801	-3.534
5	6.314	3.619	8.160	-0.769	9.469	2.605	6.851	-4.143
6	7.902	3.677	9.694	-1.481	11.450	1.750	7.937	-4.711
7	9.825	3.488	11.355	-2.540	<b>13.42</b> 0	0.270	9.290	-5.350
8	11.872	3.239	12.933	-3.857	<b>15.15</b> 0	-1.495	10.716	-6.219
9	14.056	3.020	14.444	-5.434	16.725	-3.455	12.162	-7.414
10	16.483	2.597	15.957	-7.354	18.064	-5.835	13.850	-8.873
11	18.882	2.276	17.314	-9.389	19.249	- 8.191	15.379	-10.588
12	21.510	1.999	18.673	-11.746	20.433	-10.798	16.913	-12.694
13	24.241	1.693	19.970	-14.312	21.499	-13.585	18.442	-15.039
14	27.113	1.343	21.228	-17.115	22.464	-16.590	19.992	-17.640
15	30.199	0.948	22.480	-20.228	23.366	-19.890	21.594	-20.566
16	33.147	0.525	23.594	-23.283	24.090	-23.110	23.098	-23.456
Ca	meer Line	$a_3 = -0$	0.00142 a	$_{2}=-0.01288$	$a_1 = 0.105$	10		
n	S	S/C		Throat	Position		Throat	Arccos
n	b	8/0	P.S	(X, Y)	S.S (	(X, Y)	Length O	O/S
10	24.540	0.730	23.098	-23.456	16. 459	-27.548	7.797	71.47
<b>7</b> 2	24.199	0.720	23.098	-23.456	16.593	-27.414	7.615	71.659
73	23.868	0.710	23.098	-23.456	16.715	-27.293	7.448	71.818
74	23.545	0.071	23.098	-23.456	16.835	-27.174	7.284	71.980
75	23.231	0.691	23.093	-23.456	16.965	-27.054	7.111	72.175

XYPLOT LABEL XYLBM 1

specific heat ratio corresponding to the mean temperature of the turbine inlet and outlet temperatures is taken and calculated as  $\kappa = 1.315$ .

(5) The effects of the cooling air on the main flow are ignored, and the amount of loss is taken into consideration later on when estimating the turbine adiabatic efficiency.

In Table 1 are shown the results of calculations obtained under the above hypotheses (for details refer to Appendix A.1.).

## 2.2. Calculation of the Velocity Triangles

On the basis of the several hypotheses given below, more detailed calculations were performed at the TIP, the MEAN, and the ROOT, based on the cycle values at MEAN determined in section 2.1, and the velocity triangles at each position were determined. In making these calculations, it was assumed that the stator blade outflow angle was constant in the radial direction. This is so because when such blades are adopted the stator blades will have almost a constant section in the blade

Table 2 (b). Results of Calculations of Stator Blade Profile and Blade Arrangement (MEAN; r=261.3 mm)

C 31.700	ξ -45.0	000	$B_L$ 8.000 -	$B_T$ $-71.000$				
	Coordina							
	Orig	inal	Cambo	er Line		Blade St	rface	
	(X*,	Y*)	CAM	(X, Y)	S.S (	(X, Y)	P.S	(X, Y)
i	X	Y	X	Y	X	Y	X	Y
1	1.326	2.092	2.082	0.206	1.976	2.295	2.188	-1.883
2	2.279	2.698	3.445	0.222	3.524	2.919	3.366	-2.474
3	3.405	3.028	4.919	0.104	5.324	3.104	4.513	-2.897
4	4.561	3.166	6.290	-0.159	7.062	2. 911	5.518	-3.230
5	5.917	3.413	7.743	-0.625	8.992	2.551	6.494	- 3.802
6	7.417	3.468	9.185	-1.305	10.863	1.730	7.506	-4.340
7	9.235	3.269	10.744	-2.316	12.713	0.319	8.775	-4.951
8	11.169	3.055	12.225	-3.571	14.331	-1.358	10.118	-5.784
9	13.233	2.849	13.643	-5.071	15.806	-3.217	11.479	-6.925
10	15.526	2.450	15.064	-6.894	17.058	-5.470	13.070	- 8.317
11	17.793	2.147	16.340	-8.824	18.169	-7.699	14.511	<b>- 9.948</b>
12	20.276	1.886	17.619	-11.056	19.281	-10.165	15.957	-11.946
13	22.857	1.597	18.840	-13.485	20.283	-12.801	17.397	-14.168
14	25.570	1.266	20.026	-16.135	21.192	-15.642	18.860	-16.629
15	28.487	0.895	21.208	-19.079	22.043	-18.760	20.372	-19.39'
16	31.272	0.495	22.259	-21.966	22.727	-21.802	21.792	-22.12
Car	nber Line	$a_3 = -$	-0.00152 a	$_{5} = -0.01681$	$a_1 = 0.140$	54		
	G	0.40		Throat	Position		Throat	Arcco
n	S	S/C	P.S	(X, Y)	S.S	(X, Y)	Length O	O/S
<u> </u>	23.124	0.729	21.792	-22.129	15.551	-25.922	7.303	71.58
72	22.803	0.719	21.792	-22.129	15.680	-25.795	7.127	71.78
73	22.450	0.709		-22.129	15.793	-25.685	6.974	71.93
74	22.186	0.700	21.792	-22.129	15.904	-25.577	6.823	72.08
75	21.891	0.691	21.792	-22.129	16.022	-25.470	6.667	72.26

XYPLOT LABEL XYLBM 21

height direction, and fabrication of the internal cooling holes will be easy. Consequently, such blades are suitable for air-cooled blades.

The design hypotheses are as follows. Hypotheses concerning the streamlines:

- (1) The flow inside the turbine is steady. It is also an adiabatic flow and is treated as a perfect gas.
- (2) The streamlines are parallel to the rotor blade axis, and the velocity components in the radial direction are ignored in each inspection section.

Hypotheses concerning the stator blades:

(3) The total temperature and total

pressure at the stator blade inlet are constant in the radial direction.

- (4) The total pressure loss of the stator blades is constant in the radial direction.
- (5) The absolute outflow angle of the stator blade is constant in the radial direction.
- (6) At the stator blade outlet, the flow satisfies the conditional equation for simple radial balance.

Hypotheses concerning the rotor blades:

- (7) The distribution of the rotor blade work (thermal head) is constant in the radial direction.
  - (8) The rotor blade adiabatic efficiency

Table 2 (c). Results of Calculations of Stator Blade Profile and Blade Arrangement (ROOT; r=245.3 mm)

C 29.8	$\frac{\xi}{-45}$ .	000	$B_L$ 8. 000	$B_T = -71.000$				-
Bla	de Coordina	ates						
	Orig	ginal	Cam	ber Line		Blade S	Surface	
	(X*,	Y*)	CAN	A(X, Y)	S.S	(X, Y)	P.S	(X, Y)
i	X	Y	X	Y	X	Y	X	Y
1	1.247	1.967	1.957	0.194	1.858	2.158	2.057	-1.771
2	2.142	2.536	3.239	0.209	3.314	2.744	3.164	-2.326
3	3.201	2.847	4.624	0.097	5.005	2.919	4.243	-2.724
4	4.287	2.976	5. 913	-0.150	6.635	2.737	5.188	-3.037
5	5.563	3.209	7.279	-0.588	8.453	2.399	6.104	-3.575
6	6.973	3.261	8.634	-1.227	10.212	1.627	7.056	- 4.081
7	8.682	3.093	10.100	-2.177	11.952	0.300	8.249	-4.655
8	10.500	2.873	11.492	-3.357	13.473	-1.277	9.511	-5.438
9	12.440	2.679	12.825	-4.768	14.859	- 3.024	10.791	-6.511
10	14. 596	2.304	14.161	-6.481	16.036	-5.142	12.286	-7.819
11	16.727	2.019	15.361	-8.295	17.081	-7.238	13.641	-9.352
12	19.061	1.773	16.563	-10.393	18.126	-9.556	15.000	-11.230
13	21.488	1.501	17.711	-12.677	19.069	-12.034	16.355	-13.319
14	24.038	1.191	18.826	-15.169	19.922	-14.704	17.730	-15.633
15	26.780	0.841	19.937	-17.936	20.723	-17.636	19.151	-18.235
16	29.398	0.466	20.925	-20.649	21.365	-20.496	20.486	-20.803
Ca	mber Line	$a_3 = -0$	0.00172 a	2 = -0.01789	$a_1 = 0.140$	54		
~	S	9.0		Throat	Position		Throat	Arccos
n	S	S/C	P.S	(X, Y)	S.S	(X, Y)	Length O	O/S
1	21.708	0.728	20.486	-20.803	14.628	-24.358	6.852	71.600
72	21.406	0.718	20.486	-20.803	14.738	-24.247	6.701	71.759
73	21.113	0.708	20.486	-20.803	14.851	-24.139	6.548	71.932
74	20.828	0.699	20.486	-20.803	14.974	-24.026	6.385	72.147
75	20.550	0.690	20.486	-20.803	15.070	-23.935	6.256	72.275

XYPLOT LABEL XYLBM 36

is constant in the radial direction.

(9) At the rotor blade outlet, the flow satisfies the conditional equation for simple radial balance.

In Fig. 2 are shown the results of calculations obtained under the above hypotheses (for details refer to Appendix A.2).

## 2.3. Determining the Profile and the Blade Arrangement of Air-Cooled Blades

Since the blades are air-cooled blades, in determining the profiles for both the stator blades and the rotor blades, the following items were taken as the fundamental policies (for details refer to Appendix A.3).

- (1) The aforementioned velocity triangle shall be satisfied.
- (2) The number of blades shall be decreased as far as possible so that it will be possible to do with the minimum necessary amount of cooling air.
- (3) Since cooling air is passed through the interior of the blades, the leading and trailing edges of the blades will be made thicker than in ordinary non-cooled blades in view of the fabrication of the cooling holes.

- (4) The leading and trailing edges of the blades at each section shall be passed through in a straight line with respect to the blade span direction on account of the fabrication of the cooling holes.
- (5) The cooled blades shall have a construction in which there will be a low external heat transfer rate and a high internal heat transfer rate.

In Table 2(a)-(c) are shown the results of calculations of the stator blade profiles and blade arrangements obtained under the above fundamental policies. These results for the profiles and blade arrangements were plotted with an X-Y plotter, and an example of these (MEAN) is shown in Fig. 3. The chief dimensions of the

blades are organized in Table 3. Here, since during manufacturing of the blades the leading edge diameter of the blades  $(d_T)$  and the trailing edge thickness  $(d_L)$  were finally set to be constant in the blade span direction and were given values of 4 mm and 2 mm, respectively, they differ slightly from the computer results shown in Table 5.

The coordinates of the rotor blade profiles and blade arrangements are shown in Table 4, and the chief dimensions of the blades are shown in Table 5.

The arrangements of the stator and rotor blades at the mean blade diameters are shown in Fig. 4.

n=71	blades	TIP (r=277.3 mm)	MEAN (r=261.3 mm)	ROOT (r=245.3 mm)	Note 1
$d_L \phi$	(mm)	4.00	4.00	4.00	Note 2
$d_L(X,$	Y) (mm)	(2.21, 0.15)	(2.08, 0.21)	(1.96, 0.19)	
$d_T\phi$	(mm)	1.00	1.00	1.00	Note 3
$d_T(X, Y)$	(mm)	(23.59,-23.28)	(22.26,-21.97)	(20.93,-20.65)	
$d_{ ext{max}}\phi$	(mm)	7.36	6.94	6.52	
$d_{ exttt{max}}$ (X	, Y) (mm)	(9.69,-1.48)	(9.19,-1.31)	(8.63,-1.23)	
$\overline{C}$	(mm)	33.60	31.70	29.80	
S	(mm)	24. 540	23.124	21.708	
0	(mm)	7.798	7.303	6.852	
$B_L$	(deg.)	6.00	8.00	8.00	-
$B_T$	(deg.)	71.00	71.00	71.00	
ξ	(deg.)	45.00	45.00	45.00	

Table 3. Chief Dimensions of Stator Blades

- Note 1. r is 2 mm less than when the velocity triangles were calculated
  - 2. Value during manufacturing (for the values during calculation refer to i=1 in Table 2)
  - 3. Value during manufacturing (for the values during calculation refer to i=16 in Table 2).

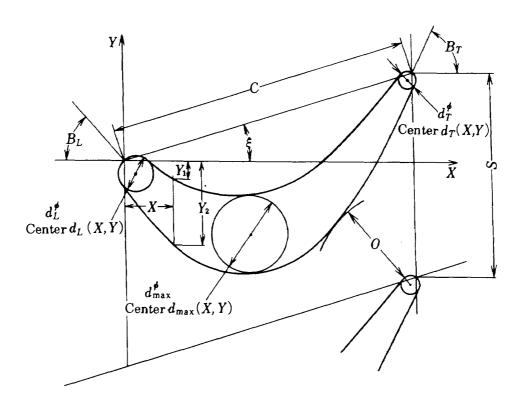


Table 4. Coordinates of Rotor Blade Profile and Blade Arrangements

		IP 3.3 mm)		EAN B.3 mm)		OT '.3 mm)
X (mm)	Y <sub>1</sub> (mm)	Y <sub>2</sub> (mm)	$Y_1$ (mm)	Y <sub>2</sub> (mm)	Y <sub>1</sub> (mm)	Y <sub>2</sub> (mm)
0.00	0.00	-1.20	0.00	-0.55	0.00	-1.90
1.00	0.33	-2.48	0.20	-3.00	0.08	-3.57
2.00	-0.20	-3.60	-0.49	-4.25	-0.85	-5.00
3.00	-0.67	-4.45	-1.13	-5.25	-1.68	-6.18
4.00	-1.00	-5.07	-1.60	-6.00	-2.30	-6.98
5.00	-1.17	-5.45	-1.95	-6.45	-2.75	-7.60
6.00	-1.23	-5.60	-2.12	-6.75	-3.05	-7.98
7.00	-1.14	-5.53	-2.12	-6.80	-3.22	-8.18
8.00	-0.83	-5.29	-1.98	-6.70	-3.20	-8.15
9.00	-0.35	-4.80	-1.68	-6.40	-3.03	-7.95
10.00	0.29	-4.14	-1.20	-5.93	-2.72	-7.58
11.00	1.05	-3.26	-0.65	-5.20	-2.27	-7.00
12.00	2.00	-2.15	0.13	-4.45	-1.60	-6.15
13.00	3.10	-1.00	1.05	-3.20	-0.75	-5.15
14.00	4.17	0.55	2.13	-1.80	0.22	-3.90
15.00	5.31	2.30	3.25	-0.10	1.25	-2.30
16.00	6.50	4.15	4.45	1.75	2.40	-0.50
17.00		6.30		3.70		1.30

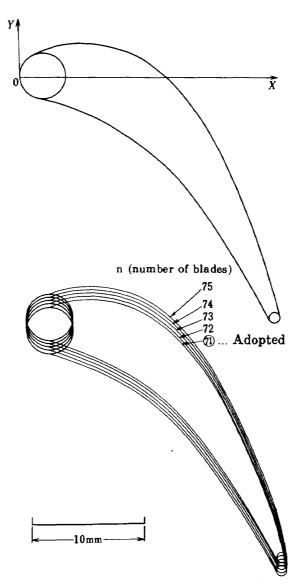
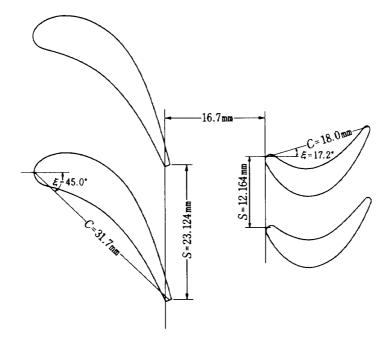


Fig. 3. Stator blade arrangements by X-Y plotter (MEAN: r=261.3 mm)

Table 5. Chief Dimensions of Rotor Blades

n=136 bla	des	rip $(r=279.3  mm)$	MEAN (r=263.3 mm)	$ \begin{array}{c} ROOT \\ (r=247.3 \text{ mm}) \end{array} $
$d_L\phi$	(mm)	2.00	2.00	2.00
$d_L(X, C)$	(mm)	(0.75, -0.65)	(0.65, -0.75)	(0.57, -0.85)
$d_T\phi$	(mm)	1.00	1.00	1.00
$d_T(X, Y)$	(mm)	(16.50, 6.33)	(16. 92, 5.75)	(17.35, 3.25)
$d_{ ext{max}}\phi$	(mm)	4.35	4.63	4.92
$d_{\max}\left(X, Y\right)$	(mm)	(6.45, -3.45)	(7.3, -4.45)	(7.6, -5.70)
$\overline{C}$	(mm)	18.05	18.00	18.00
S	(mm)	12.904	12.164	11.425
0	(mm)	5.75	5. 15	4.76
$B_L$	(deg.)	41.0	49.8	55.8
$B_T$	(deg.)	64.5	63.5	62.5
<b>ξ</b>	(deg.)	22.5	17.2	11.7



	Stator blade, $r=261.3 \text{ mm}$	Rotor blade, $r=263.3 \text{ mm}$
Leading edge diameter ratio $(d_L/C)$	0.126	0.111
Trailing edge thickness ratio $(d_T/C)$	0.032	0.056
Max. blade thickness ratio $(d_{\text{max}}/C)$	0.22	0.26
Solidity $(C/S)$	1.37	1.48
Aspect ratio $(h/C)$	1.03	1.86

Blade nondimensional values based on MEAN

Fig. 4. Blade arrangements (MEAN)

#### 3. EXPERIMENTS

## 3.1. Experimental Equipment and Measuring Equipment

An outline of the eexperimental equipment is shown in Fig. 5.

The measuring equipment is as follows. The turbine air flow rate was measured by means of the JIS standard disk type orifice. The turbine axial torque was measured by means of an automatic Ward-Leonard type 1,600 kW D.C. electric dynamometer. The turbine revolution speed was measured by means of the magnetic pulses from the reduction gear (reduction ratio 1:12.984) mounted on the dynamometer axis.

The gaseous states were measured before and after the turbine. The gas temperature

was measured by means of a CA thermocouple thermometer [5, 6], and the total pressure, static pressure, Mach number, and angles were measured by means of three-hole yaw meters [7]. The sensors for measuring gaseous states were fastened before the turbine at definite positions, and after the turbine they were arranged so that they would be able to traverse in the radial direction.

The types of these measuring instruments and the measuring positions are shown in Table 6 and Fig. 6. Photographs of the main parts of the measuring instruments and of the driving part of the motor-driven traversing device are shown in Fig. 7.

The measuring and data processing system used in the experiments is shown in Fig. 8[8]. These experiments were planned so that it would be possible to use com-

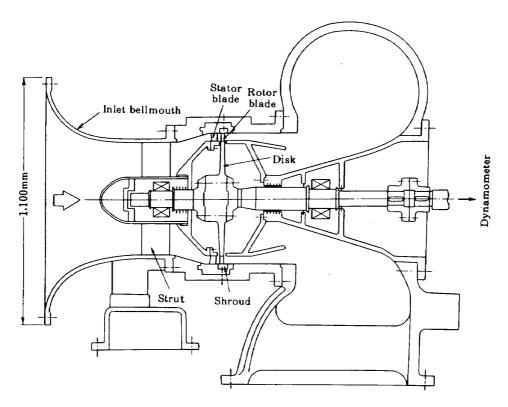


Fig. 5. Main unit of testing device for aerodynamic performance.

Table 6. Type of Measuring Devices

Appella- tions	Measuring positions in axial direction*	Measuring positions in circumferential direction*	Traversing direction	Type-form, etc.
$3P_{t1}$ -1	A	3	Radial direction	Three holes in a five-hole Pitot tube are used [7]
$3P_{t1}$ –2	A	5	"	
$P_{w_1}$ -1	В	1		Outer wall static pressure taps
$P_{w1}$ –2	В .	6		
T <sub>1</sub> -1	В	2	Radial direction	Stagnation type CA thermocouple $(RF=1,0)$ thermometer [5]
$T_1$ -2	В	4	"	(107 == 1.0) thermometer [0]
$3P_{t3}-1$	С	9	"	Three holes in a five-hole Pitot tube
$3P_{t3}$ – $2$	С	11	"	are used [7]
$P_{w^3}$ -1	D	7		Outer wall static pressure taps
$P_{w3}$ –2	D	12		
$T_{s}$ -1	D	8	Radial direction	Tip exposed type CA thermocouple $(RF = 0.7 \sim 0.82)$ thermometer
$T_3$ -2	D	10	"	(III = 0.11 0.02) thermometer
$7P_{t3}$ - $R$	E	13	Radial direction Radial, &	Circumferential direction comb type seven-hole Pitot tube, Three-hole
$3P_{t3}$ – $R$	F	14	circum- ferential directions	Pitot tube

<sup>\*</sup> Refer to Fig. 6.

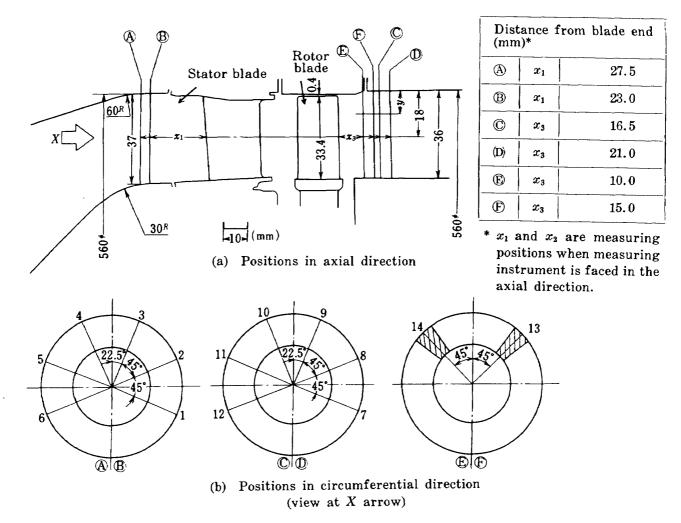


Fig. 6. Measuring positions of various measuring instruments (refer to Table 6)

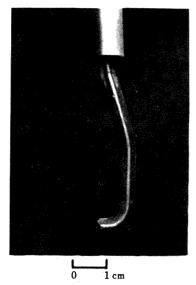
puters to make the measurements and record the results automatically within a short time.

#### 3.2. Experimental Methods

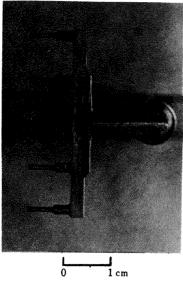
In these experiments, the turbine expansion ratio and revolution speeds were adjusted to conform with the target values. The expansion ratio of the experimental target was 1.4-2.1 with intervals of about 0.2, and the revolution speeds were 70-110% of the design value with intervals of about 10%. The measurements were performed by means of two thermometers and two pressure meters at the turbine outlet; traversing measurements were performed in the radial direction. Before these experiments it was ascertained that the temperatures and pressures at the turbine inlet are uniform in the radial direction and in the

circumferential direction.

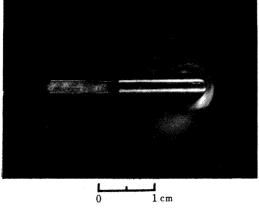
In cases when relatively low-temperature air is used, as in these experiments, drops of water are frequently attached to the sensors inserted after the turbines. order to prevent this, it was necessary to adjust the turbine expansion ratio and the revolution speed while paying careful attention to make sure that the turbine outlet temperature would not drop below the dew point temperature of that day even once while the experimental points were being set. In this way, traversing measurements were commenced in a state at which the turbine inlet temperature would be more or less stable at about 90-110°C. In cases when traversing measurements were performed at five points, the time required for one experimental point was approximately ten minutes.



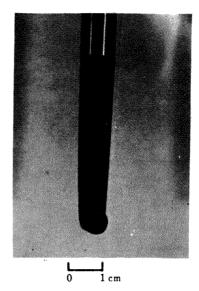
(a) Five-hole Pitot tube



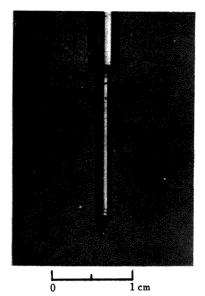
(b) Circumferential direction comb type seven-hole Pitot tube



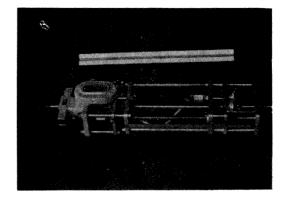
(c) Three-hole Pitot tube



(d) Stagnation type CA thermocouple thermometer



(e) Tip exposed type  ${\it CA}$  thermocouple thermometer



(f) Driving parts of motor-driven type traversing device

Fig. 7. Photographs of main parts of measuring instruments and of driving parts of motor-driven type traversing device.

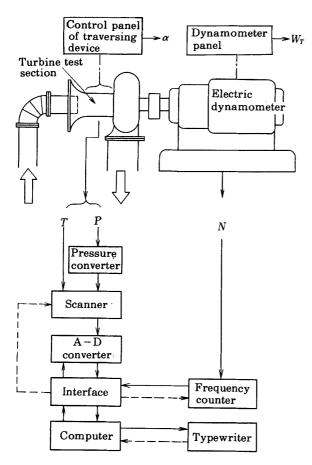


Fig. 8. Measuring-processing system

## 3.3. Methods of Analyzing the Experiments

The turbine flow rate G(kg/sec) was calculated by means of the orifice flow rate formula (refer to item 9 in bibliography).

The turbine torque  $(kg \cdot m)$  was sought by means of the following formula from the dynamometer load  $W_T(kg)$  and the arm length of the dy namometer l(m).

$$\tau = l \cdot W_T$$

The turbine specific output  $\Delta H_T$  (kcal/kg) was calculated following formula from the flow rate G(kg/sec), the torque  $\tau(kg \cdot m)$ , and the turbine revolution speed N(rpm).

$$\Delta H_T = \frac{2\pi}{60J} \cdot \frac{\tau \cdot N}{G}$$

The turbine expansion ratio  $\pi_T$  was sought by the following formula as the ratio of turbine inlet and outlet total pressures.

$$\pi_T = \frac{\overline{P}_{t1}}{\overline{P}_{t3}}$$

Here,  $\bar{P}_{t1}$  is the mean total pressure before the stator blade; we used the arithmetic mean value of the total pressures measured at the center of the flow channel by two Pitot tubes before the stator blade  $(3P_{t1}-1 \text{ and } 3P_{t1}-2)$ .  $\bar{P}_{t3}$  is the mean total pressure after the rotor blade; we use the arithmetic mean value of total ten data obtained at five traversing points in the radial direction by means of two of the Pitot tubes after the rotor blades.

The following adiatatic efficiency  $\eta_t$  was calculated as one of the adiabatic efficiencies of the turbine.

$$\eta_t \!=\! rac{ extit{\Delta} H_T}{c_p \!\cdot\! T_{t1} \left\{ 1 \!-\! \left(rac{1}{\pi_T}
ight)^{\epsilon-1/\epsilon} 
ight\}}$$

The following correction values taking into considerations the differences of specific heat ratios for design and for experiment conditions are used in order to compare the above experimental values and the design values as precisely as possible. (Refer to Appendix B.)

Flow rate  $G \rightarrow \text{Corrected flow rate}$   $\varepsilon_{cr} \cdot G \sqrt{\theta_{cr}} / \delta$ 

Revolution speed  $N{ o}{
m Corrected}$  revolution speed  $N/\sqrt{\theta_{cr}}$ 

Specific output  $\Delta H_T 
ightharpoonup$ Corrected specific output  $\Delta H_T/\theta_{cr}$ 

Torque  $\tau \rightarrow$  Corrected torque

 $\varepsilon_{cr} \cdot \tau / \delta$ 

Expansion ratio  $\pi_T \rightarrow \text{Corrected}$  expansion ratio  $\phi_{cr} \cdot \pi_T$ 

Here,  $\theta_{cr}$ ,  $\delta$ ,  $\varepsilon_{cr}$ , and  $\phi_{cr}$  are correction coefficients and are expressed by the following formulas.

$$egin{aligned} heta_{cr} = & \left(rac{\kappa}{\kappa+1}R \cdot T_{t1}
ight) \middle / \left(rac{\kappa_{st}}{\kappa_{st}+1}R_{st} \cdot T_{t,st}
ight) \ \delta = & P_{t1} \middle / P_{t,st} \end{aligned}$$

$$\varepsilon_{cr} = \left\{ \kappa_{st} \left( \frac{\kappa + 1}{2} \right)^{\kappa/\kappa - 1} \right\} / \left\{ \kappa \left( \frac{\kappa_{st} + 1}{2} \right)^{\kappa_{st}/\kappa_{st} - 1} \right\}$$

$$\phi_{cr} = \frac{1}{\pi_T} \left\{ 1 - \left( \frac{\kappa_{st} - 1}{\kappa_{st} + 1} \right) \left( \frac{\kappa + 1}{\kappa - 1} \right) \times (1 - \pi_T^{1 - \kappa/\epsilon}) \right\}^{\kappa_{st}/1 - \kappa_{st}}$$

The suffix at indicates the standard state. The following values were adopted here as the standard values:

> $T_{t,st} = 288.2^{\circ} \text{K}$   $P_{t,st} = 10332 \text{ kg/m}^2$   $R_{st} = 29.27 \text{ kg} \cdot \text{m/}^{\circ} \text{K} \cdot \text{kg}$  $K_{st} = 1.401$

To use these corrected values signifies that the design values and the experimental values will always be corrected to the values at the standard state and compared. In Table 7 are shown the various design values and their corrected values. In finding these corrected values, it was assumed that the mean specific heat ratio inside the turbine was 1.31 (refer to the value of No. 51 in Appendix Table 2) and that the gas constant was 29.27 kg·m/°K·kg.

#### 3.4. Experimental Results

#### (a) Flow Rate Characteristics (Fig. 9)

According to Fig. 9, the corrected flow rate is 3.98 kg/sec at the design corrected

revolution speed and the design corrected expansion ratio (2.09) and it is 7.3% greater than the design value. Differences in the flow rate curves caused by the revolution speed indicate that the flow rate is governed by the rotor blade. Further more, this turbine is choked by the rotor blade, and the expansion ratio at which it is believed that there is choking at all the revolution speeds in the experiment is about 2.3.

#### (b) Torque Characteristics (Fig. 10)

Concerning the flow rate, it is believed that choking will be caused by a corrected expansion ratio of about 2.3, as was mentioned above. As for the torque, however, it is displaced that a tendency towards increase continues even after this expansion ratio is exceeded. It always increases together with the expansion ratio at all the revolution speeds in the experiment. Thus, in this turbine, there is seen no limiting loading within the range of the experiments. Furthermore, the corrected torque in the experiments is 30.2 kg·m at the design corrected revolution speed and

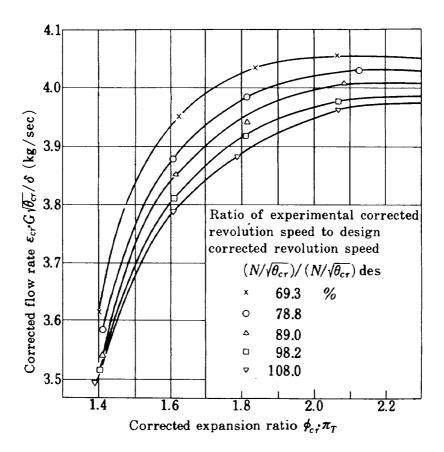


Fig. 9. Flow rate characteristics

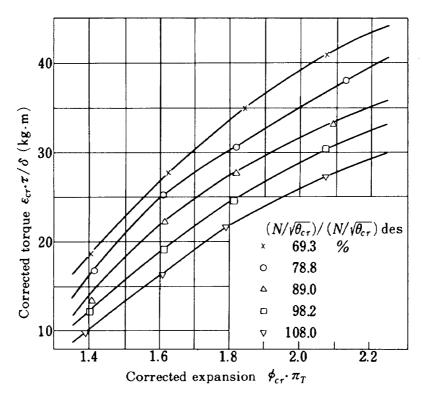


Fig. 10. Torque characteristics

the design corrected expansion ratio. This is a value approximately 8.2% greater than the design value.

(c) Map of Overall Performance (Fig. 11) In Fig. 11 is shown a map of the overall performance compiled from the flow rate characteristics in Fig. 9 and from the torque characteristics in Fig. 10. Here the corrected specific output  $\Delta H_r/\theta_{cr}$  is plotted on the ordinate, and the corrected flow rate revolution speed parameter (corrected flow rate x corrected revolution speed)  $\varepsilon_{cr}GN/\delta$ is plotted on the abscissa. The results are plotted in terms of equal corrected revolution speed lines, equal corrected expansion ratio lines, and equal adiabatic efficiency lines. According to this, the adiabatic efficiency  $\eta_t$  is 0.7-0.865 within the range of the experiments. The maximum adiabatic efficiency is obtained in the vicinity of a corrected revolution speed of 90% and of a corrected expansion ratio of 1.72. Its value is 0.865.

The D point, that is, the point of the design corrected revolution speed and the design corrected expansion ratio, was shown by the  $\circ$  mark. The corrected specific output obtained at this time is  $11.22 \, \text{kcal/kg}$ ,

and the adiabatic efficiency is 0.856.

(d) Absolute Outflow Angle of Rotor Blade (Fig. 12)

According to the results of measurements of the absolute outflow angles from the rotor-blade by means of the four Pitot tubes installed in the circumferential direction at the rotor blade outlet, there are almost no differences in the measuring angles of the four Pitot tubes at the center of the channel, and it is thought that one may regard the absolute outflow angles in the circumferential direction as being almost uniform at this position. For reason, it was decided to select the outflow angle at the center of the channel as the value representing the absolute outflow angles at the rotor blade outlets. It is clear from this figure that the rotor blade absolute outflow angle is 21.5° at the design corrected revolution speed and the design corrected expansion ratio. This is 1.8° smaller than the design value at the mean blade diameter. As will be mentioned below, this is the result of the fact that the deflection angle of the rotor blade has been smaller than that expected.

(e) Rotor Blade Absolute Outflow Mach

Table 7. Design Values and Their Corrected Values  $(\kappa_{des}=1.31)$ 

Design items	Design values	Corrected values
Inlet total təmperature $T_{t1}$ (°k)	1423.2	288.2
Inlet total pressure $P_{t1}$ (kg/m <sup>2</sup> )	25000	10332
Specific output $\Delta H_T$ (kcal/kg)	53.8	11.2
Flow rate G (kg/sec)	3.95	3.71
Revolution speed N (rpm)	13300	6071
Expansion ratio $\pi_T$	2.02	2.09

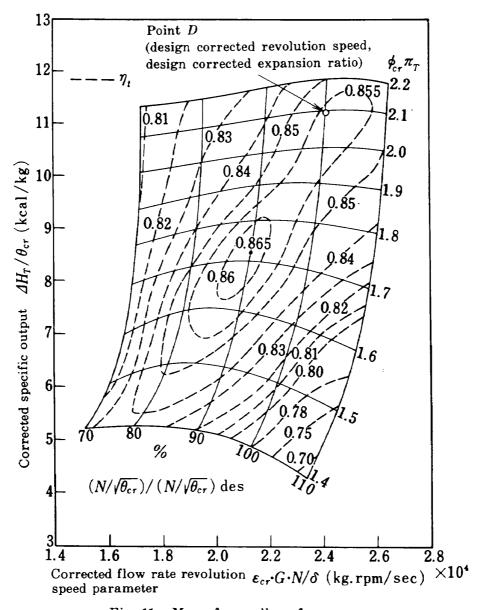


Fig. 11. Map of overall performance

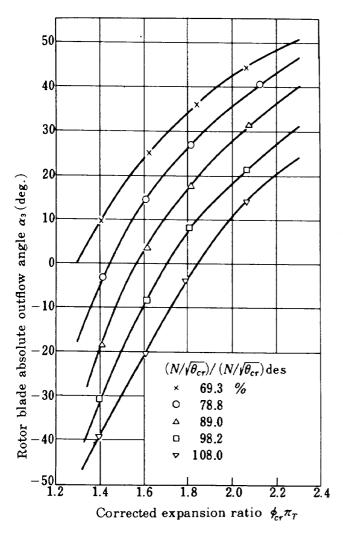


Fig. 12. Rotor blade absolute outflow angle (MEAN)

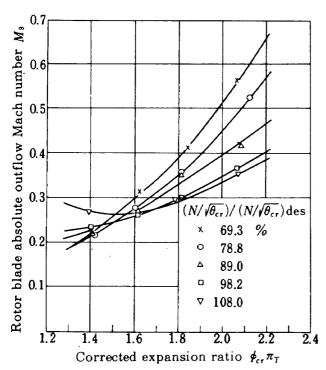


Fig. 13. Rotor blade absolute outflow Mach number (MEAN)

Number (Fig. 13)

According to the experiments, the absolute outflow Mach number at the rotor blade outlet had a distribution in the radial direction, but at the center of the channel it could be regarded as being almost uniform in the circumferential direction, just like the rotor blade absolute outflow angle. Here are shown the rotor blade absolute outflow Mach numbers at the center of the channel. It is clear from this figure that the rotor blade absolute outflow Mach number at the design corrected revolution speed and the design corrected expansion ratio is 0.375. This value agrees well with the design value.

(f) Velocity Triangle (Fig. 14)

Using the experimental results obtained as described above, we sought the velocity triangle at the point of the design corrected revolution speed and the design corrected expansion ratio. That is, we calculated the velocity triangles at the stator blade outlet and the rotor blade outlet using the experimental values of the flow rate and specific output at this point, of the rotor blade absolute outflow angle and of the rotor blade absolute outflow Mach number at the MEAN position, as well as the annuals area at the stator blade outlet and the total pressure loss coefficient of the stator blades. The only hypothesis in these calculations was that it was assumed that the total pressure loss coefficient of the stator blades  $(Y_{tN})$  was 0.0874. This value consists of the profile loss obtained by means of blade cascade experiments to which the secondary flow loss calculated in accordance with item [10] in the bibliography was added. Note that, depending on this hypothetical value, there are not very great differences in the velocity triangles which are obtained.

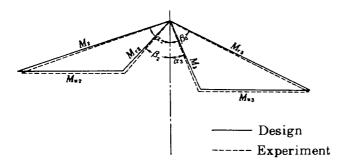
In Fig. 14, the experimental velocity triangles obtained in this way are shown in comparison with the design velocity triangles at the mean blade diameter. According to this figure the outflow angle of the stator blade is about 1.1° smaller than the design outflow angle, and this indicates that the flow has been turned to this extent in the axial direction. In this

manner, in the stator blades, the flow cannot be deflected as much as was anticipated (decrease of the deflection angle), while at the same time, there is also a decrease in the expansion ratio inside the stator blades (under-expanding), as is clear from the absolute outflow Mach number. On the other hand, it is clear that the deflection angle of the rotor blade has also decreased about 3.5° from the design value.

#### (g) Internal Flow (Fig. 15)

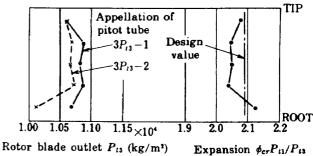
The internal flow at the rotor blade outlet was measured at approximately the design corrected revolution speed and the design corrected expansion ratio. The results of the measurements are shown in Fig. 15.

The two distributions of the rotor blade outlet total pressures  $P_{t3}$  shows exactly the total pressure distributions by the two Pitot

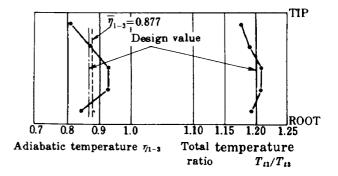


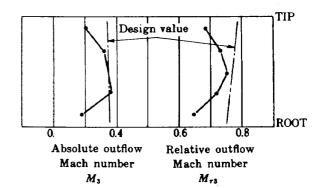
	Symbol	Design value	Experi- mental value
	$M_2$	0.8013	0.785
	$M_3$	0.3750	0.375
	$M_{r2}$	0.3421	0.338
	$M_{r3}$	0.7726	0.766
	$M_{a2}$	0.2476	0.257
Mach number	$M_{a3}$	0.3448	0.349
	$M_{u2}$	0.5260	0.521
	$M_{u3}$	0.5439	0.545
	$M_{2u}$	0.7621	0.742
	$M_{3u}$	0.1475	0.137
	$M_{r2u}$	0.2361	0.221
	$M_{r3u}$	0.6914	0.682
	$\alpha_2$	72.0	70.9
Angle (deg.)	$\alpha_3$	23.2	21.5
(8-/	$eta_2$	43.6	40.7
	β3	63.5	62.9

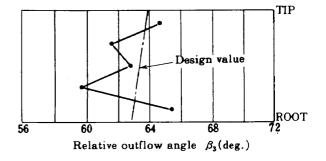
Fig. 14. Comparison of velocity triangle (comparison with design)



Expansion  $\phi_{cr}P_{t1}/P_{t3}$ total pressure ratio







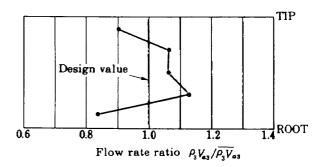


Fig. 15. Internal flow  $(\phi_{cr}\pi_T=2.07,(N/\sqrt{\theta}_{cr})/(N/\sqrt{\theta}_{cr})_{des}=0.98)$ 

tubes installed in the circumferential direction. As is shown in this example, the measured values are not uniform in the circumferential direction, and there is a certain amount of distribution. In the following let us express the distribution in the radial direction in terms of these local mean values (the arithmetic eman values of the measured values at the identical blade height). It is clear from a glance at Fig. 15 that the measured values all differ considerably in the radial direction.

As is seen in the distribution of the expansion ratio and the total temperature ratio, there is a greater drop in the total pressure near the walls than there is in the vicinity of the center of the channel, while, there is a small thermal head, as is seen in the distribution of the total temperature ratio. Consequently, the adiabatic temperature efficiency also is quite low near the walls. The mass flow rate mean value of this efficiency (as is shown in the flow rate ratio distribution, the flow rate differs considerably in the radial direction; therefore, the value was averaged by applying the weight of the flow local efficiency in the radial direction) is 0.877. This value in itself satisfies the design specifications, but one should note carefully that there are local values which differ considerably from the design value.

On the other hand, in the distribution of the blade outflow Mach numbers, it is indicated that the flow velocity is slow near the walls. Furthermore, the distribution of the relative outflow angles also has some complex shapes.

The distribution in the radial direction of the flow rate per unit area was also calculated using the above local values of the measured values. It was shown in terms of the flow rate ratio with respect to the mean flow rate per unit area. It is clear from this that there are big differences in the flow rate in the radial direction and that it is quite small in the vicinity of both walls. If this fact as well as the total temperature ratio distribution above are taken into consideration, it is clear that the amount of work also is quite small at the blade tip part and the blade root

part as compared with the vicinity of the mean blade diameter.

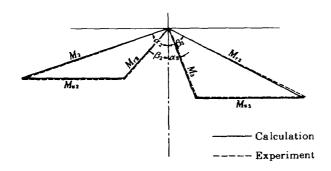
## 3.5. Studies and Discussion of the Experimental Results

In these experiments, a quite good turbine adiabatic efficiency was obtained throughout almost all regions of the experiments. In these experiments, the mean Reynolds number in the turbine at the design corrected revolution speed and the corrected expansion ratio (using the Reyolds number as defined in item [10] in the bibliography) was approximately  $4.3 \times 10^5$ . On the other hand, the number at the design conditions was approximately  $1.1 \times 10^5$ . Therefore, it is assumed that, because of these differences in the Reynolds numbers, at the design operating conditions there will be a decrease of about 2-3% from the adiabatic efficiency of these experiments.1 These design operating conditions were adjusted to the operating conditions of high-temperature tests using actual model turbines [11] which were performed separately at our Aeroengine Division. In this case, since the pressure and temperature at the turbine inlets are considerably less uniform than in the present experiments, it is believed that there is some further decrease in the efficiency [10]. In view of these facts, when operated at the design conditions, it is assumed that the adiabatic efficiency of this turbine will be approximately 82-83%. However, when this high-temperature turbine is actually adopted in an airplane, the turbine inlet total pressure will not be as low as that in these design conditions, but will have a

Oncerning the effects of the Reynolds number on the turbine efficiency, various formulas most of them empirical formulas, have been proposed. Although there are some differences between these formulas, when the Reynolds' number is less than  $2\times10^5$  its influence on the efficiency may generally be expressed in terms of the law of approximately -0.2 power of the Reynolds number. As the Reynolds number becomes greater, its influence becomes less, and when the value is approximately  $10^6-10^7$ , it is believed that its influence is entirely negligible.

value several times greater or even more. Therefore, the mean Reynolds number inside the turbine will also increase together with it, and it is believed that it will be approximately the same as, or even larger than, the Reynolds number in these experiments. Thus, even when one keeps in mind the lack of uniformity of the gaseous states at the turbine inlet, it is believed that, in the actual engines, the required efficiency of 85% will be satisfied.

On the other hand, the turbine inlet flow rate departs considerably from the design speciacations. As is pointed out also in [10] in the bibliography, this flow rate is extremely sensitive to the blade mounting angle, that is to the blade throat area. Especially in the upstream blades, devia-



	Symbol	Design value	Experi- mental value
	M <sub>2</sub>	0.778	0.785
	$M_3$	0.377	0.375
	$M_{r2}$	0.333	0.338
	$M_{r3}$	0.760	0.766
	$M_{a2}$	0.251	0.257
Mach number	$M_{a3}$	0.353	0.349
mach humber	$M_{u2}$	0.517	0.521
	$M_{u3}$	0.540	0.545
	$M_{2u}$	0.737	0.742
	$M_{3u}$	0.133	0.137
	$M_{r2u}$	0.220	0.221
	$M_{r3u}$	0.673	9.682
	α2	71.2	70.9
Angle (deg.)	$\alpha_3$	20.6	21.5
ringic (deg.)	$\beta_2$	41.2	40.7
	$\beta_3$	62.3	62.9

Fig. 16. Map of overall performance by calculations [10]

tions in this angle have a big effect on the flow rate. It is assumed from this that, since the blade mounting angle was mismounted a little toward the axial direction during the manufacture of this turbine, the throat area increased as a result, and there was a considerable flow rate as in the ex-In actual fact, it has been periments. ascertained that in this turbine the measured values of the throat area of the stator blades are 5.0% greater than the values which were planned during designing. Incidentally, when these measured areas are used to calculate the performance of the turbine, the results are as shown in Fig. 16. At point D, that is, the point of the design corrected revolution speed and the design corrected expansion ratio, the flow rate is about 5.4% greater than the design flow rate, and this excess flow rate is close to the amount obtained in the ex-Besides, a deviation in the periments.

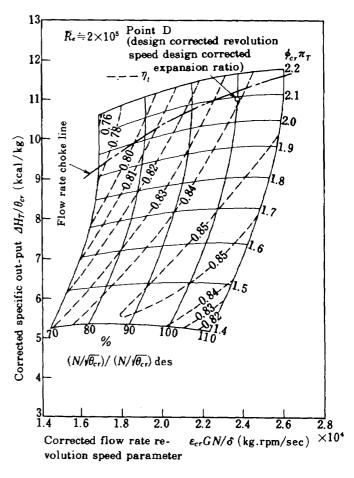


Fig. 17. Comparison of velocity triangle (comparison with calculations) [10]

blade mounting angle is also assumed from the stator blade outflow angle in the diagram of the velocity triangle (Fig. 14) obtained in the experiments. Furthermore, the velocity triangle at point D in Fig. 16, which was calculated using the measured throat area, agrees well with that of the experiments, as is shown in Fig. 17. It may be concluded from the preceding that the main cause of the excessive flow rate in the experiments is the following. That is, since the setting angle for mounting the stator blades deviated about 1°, the throat area became excessively large as a result. In this manner, in stator blades of an air-cooled turbine, in which it is necessary to select a large outflow angle, a slight deviation in the mounting angle will have a great influence on the flow rate. Therefore, it is especially necessary to pay careful attention to this point.

In the experimental results for the internal flow, the various experimental values had a considerable distribution in the radial direction, and locally they displayed values which differed from the design values. In particular, it is thought that the complicated shapes of the relative outflow angles of the rotor blade were probably caused by the secondary flow, the flow from the blade tip clearance and so on. When these results for the internal flow are considered in connection with the results for the overall performance as described above, the following may be said concerning the design. That is, the hypotheses (or the designing methods) used in designing this turbine locally do not agree with the actual flow, but when viewed in terms of the overall performance they may be called hypotheses appropriate (or designing methods) in the average sense. Nevertheless, in future designing it will be necessary to make improvements while taking this actual flow into consideration.

#### 4. CONCLUSION

An air-cooled turbine was designed and manufactured with a turbine inlet total temperature of 1,150°C as the target, and experiments were performed here without

supplying blade cooling air in order to examine its aerodynamic performance. Studies were made of the overall performance and of the internal flow. The ranges of the experiments were a revolution speed of approximately 70–110% and a turbine expansion ratio of 1.4–2.1.

The conclusions concerning the design obtained from these experimental results were the following:

(1) It was learned that even air-cooled turbines which are subject to various restrictions in their blade shape or other items because of the necessity of passing cooling air inside the blades can be designed and manufactured, without any impariment of their performance, by applying the design techniques used in the past in non-cooled turbines, under a number of basic pollicies which are required in air-cooled blades.

The chief experimental results were the following:

- (2) Within the range of the experiments, this air-cooled turbine displayed an adiabatic efficiency of 0.7-0.865. The maximum adiabatic efficiency of 0.865 was obtained at a revolution speed in the vicinity of 90% and at an expansion ratio in the vicinity of 1.72.
- (3) In this air-cooled turbine, the specific output obtained at the design revolution speed and the design expansion ratio was approximately the same as that in the design. The adiabatic efficiency at that time was 0.865, and the adiabatic temperature efficiency (mass flow rate mean value) was 0.877.
- (4) The flow rate of this air-cooled turbine was choked by the rotor blade, and the expansion ratio at which it was thought that choking was performed for all revolution speeds in the experiments was approximately 2.3.
- (5) The turbine flow rate at the design revolution speed and the design expansion ratio was approximately 7.3% greater than the design value. It is believed that this is chiefly because the throat area of the stator blades was greater than the design value. In blades with a rather great outflow angle such as these stator blades,

careful attention must be paid when setting the throat part.

- (6) The torque limit of the turbine does not yet occur within the range of operations in the experiments.
- (7) As for the work distribution in the blade height direction at the design revolution speed and the design expansion ratio, it is small near the TIP and the ROOT and great near the MEAN. However, it was learned that the designed work is obtained as a whole.

The following problems are left for the future:

- (8) The hypothesis for the flow inside the turbine used in this design coincides on the average with the actual flow, but it cannot be said to coincide locally. In the future, it will be necessary to make improvements in the design, taking into consideration these real flow phenomena.
- (9) Since it is thought that the cooling air has an influence on the turbine flow, and consequently on the aerodynamic performance, it will be necessary to perform designing and experimentation in which this influence is also taken into consideration.
- (10) In the actual machinery, it is anticipated that there will be considerable thermal deformations, not only in the blades, but also in the various turbine parts, and it is believed that, in short blades such as these air-cooled blades, they would have a relatively great influence on the performance properties of the rotor blade tip clearance. More detailed experiments are necessary for individual blades in order to clarify the various performance properties of the air-cooled turbine and air-cooled blades, including these questions.

#### 5. POSTSCRIPT

The co-operation of the following persons was obtained in this research. The writers take this opportunity to express their gratitude to them.

Director Matsuki, Aeroengine Division; Director Torisaki, Aerial Pollution Research Group; Department Chief, K. Nishio; technical official Koshinuma (measuring system); technical officials Ogura, Shimodaira, and Yamada (air sources); and the Technical Development Laboratory, Aircraft Engines Department, Ishikawajima-Harima Heavy Industries C., Ltdo. (designing, manufacturing).

#### REFERENCES

- 1. Yamamoto, A., and K. Takahara: "A consideration of the flow rate of turbine cooling air and of the performance properties of turbofan ergines," Dai-11-kai koku gendoki ni kan-suru koenkai koen shu [Lectures delivered at the 11th lecture meeting concerning aircraft prime movers], 1971.
- Whitney, W. J., E. M. Szanca, T. P. Moffitt and D. E. Monreo: Cold-Air Investigation of a Turbine for High-Temperature-Engine Application. I. Turbine Design and Overall Stator Performance, NASA TN D-3751, 1966.
- Prust, H. W. Jr., H. J. Schum, and F. P. Behning: Cold-Air Investigation of a Turbine for High-Temperature-Engine Application. II. Detailed Analytical and Experimental Investigation of Stator Performance, NASA TN D-4418, 1967.
- Whitney, W. J., E. M. Szanca, B. Bider, and D. E. Monreo: Cold-Air Investigation of a Turbine for High-Temperature-Engine Application. III. Overall Stage Performance, NASA TN D-4389, 1967.
- Hatta, K., M. Matsuki and T. Torisaki ed.: Kitai kikai handobukku [Gas machinery handbook], Asakura Shoten, 1968, p. 603.
- 6. Glawe, G. E., F. S. Simmons, and T. M. Stickney: Recovery Corrections and Time Constants of Several Chromel-Alumel Thermocouple Problems in High-Temperature, High-Velocity Gas Streams, NASA TN 3766, 1956.
- 7. Fujii, S., H. Nishiwaki, A. Yoshida, M. Gomi, K. Takeda and N. Sugawara: The Aerodynamic Performance of a Single-Stage Axial-Flow Compressor with Double Circular-Arc Blades, Technical Report of National Aerospace

- Laboratory TR-134, 1967.
- Matsuki, M., T. Torisaki, K. Nishio, K. Takahara et. al.: Research and Development on a Forced Balance Type Pressure Tranducer for Computer Aided Measurements, Technical Report of National Aerospace Laboratory TR-382, 1974.
- Torisaki, T., H. Nouse, M. Morita, S. Inoue and S. Sekine: An Investigation of a Transonic Axial-Flow Turbine
   (I) —A Cold Air Test of the Annular Turbine Nozzle Cascade—, National Aerospace Laboratory TR-272, 1972.
- Ainley, D. G. and G. C. R. Mathieson: A Method of Perfoormance Estimation for Axial Flow Turbines, ARC R & M. No. 2974, 1951.
- Takahara, K., H. Nouse, T. Torisaki, Nashio, and K. Suzuki,: Koon tabin shiken setsubi High temperature turbine testing equipment], preprint for the 1969 National Aerospace Laboratory research meeting, 1969.
- 12. Oba, J.: Joki oyobi gasu tabin [Steam and gas turbines], Nikkan Kogyo Shimbun Sha, 1958, pp. 85-87.
- 13. Chappel, M. S. and E. P. Cockshutt: Gas Turbine Cycle Calculations: Thermodynamic Data Tables for Air and Combustion Products. National Research Council of Canada. Aeronautical Report R-5, 1969.
- Fielding, D. and J. E. C. Topps: Thermodynamic Data for the Calculation of Gas Turbine Performance, ARC R &. No. 3099, 1959.
- Ainley, D. G. and G. C. R. Mathieson: Examination of the Flow and Pressure Losses in Blade Row of Axial-Flow Trubines, ARC R & M. No. 2891, 1951.
- For example, Katto, Y.: Dennetsu gairon [Introduction to heat transfer], third ed., Yokendo, 1967, pp. 394-416.
- Miser, J. W., W. L. Stewart, and D. E. Monreo: Effect of High Rotor Pressure-Surface Diffusion on Performance of a Transonic Turbine, NACA RM E 55 H 29 a. 1955.
- Shapiro, A. H.: The Dynamics and Thermodynamics of Compressible Fluid. Vol. 1, The Ronald Press Company,

- New York.
- Dunavant, J. C. and J. R. Erwin: Investigation of a Related Series of Turbine-Blade Profiles in Cascade, NACA TN 3802.
- Stechkin: Jetto enjin riron, tabo kikai [Jet engine theory, turbo machinery], tr. by Hamashima, M. Korona Co., 1969, pp. 271-283.
- Berkey, W. E.: Overall Performance of the J 71 Three-State Turbine, NACA RM E 52B29, 1952.
- 22. Ainley, D. G.: Estimation of the Change in Performance Characteristics of a Turbine Resulting from Changes in the Gas Thermodynamic Properties, ARC R & M. No. 2973, 1951.
- Dunham, J. and P. M. Came: Improvements to the Ainley-Mathieson Method of Turbine Performance Prediction, ASME Paper No. 70-GT-2, 1970.
- 24. Takahara, K., A. Yamamoto, S. Inoue, H. Usui, and F. Mimura: "Aerodynamic performance of air-cooled turbines (Part I)," Dai-12-kai koku gendoki ni kan-sura koenkai koen shu [Lectures delivered at the 12th lecture meeting concerning aircraft prime movers], 1972.
- 30. Yamamoto, T., K. Takahara, H. Nose, S. Inoue, H. Usui and F. Mimura: Aerodynamic Investigation of an Air-Cooled Axial Flow Turbine, Part I Turbine Design and Overall Stage Performance Technical Report of National Aerospace Laboratory TR-321 and NASA TT F-16083, 1973.

#### APPENDIX A.

#### DETAILS OF THE DESIGN

Details of the aerodynamic design of this turbine are shown below. The hypotheses in the calculations used here as well as the fundamental policies for determining the blade profiles and blade arrangements are those described in the main text.

- 1. Calculating Turbine Performance Properties (Single-Dimensional Calculations)
  The fundamental equations used in the calculations are the following:
- (1) equation for the relationship between the total temperature and the static temperature at the same point:

$$\frac{T_t}{T_s} = 1 + \frac{\kappa - 1}{2} M^2 \tag{1.1}$$

(2) equation for the relationship between the total pressure and the static pressure at the same point:

$$\frac{P_t}{P_s} = \left(1 + \frac{\kappa - 1}{2} M^2\right)^{\epsilon/\epsilon - 1} \tag{1.2}$$

(3) equation for sound velocity a:

$$a = \sqrt{\kappa \cdot g \cdot R \cdot T_s} \tag{1.3}$$

(4) equation defining velocity coefficient of stator blade  $\varphi_N$ :

$$\varphi_N = \frac{V_2}{V_{2ih}} \tag{1.4}$$

Here,  $V_2$  is the actual outflow velocity of the stator blade, and  $V_{2ih}$  is the theoretical outflow velocity due to the adiabatic thermal head.

(5) equation defining total pressure loss coefficient of rotor blade  $Y_{lR}$ :

$$Y_{tR} = \frac{P_{r2} - P_{r3}}{P_{r2} - P_{r3}} \tag{1.5}$$

Here, the subscript r (used for the temperature and pressure) indicates a state of relative stagnation to the blades. That is,  $P_{\tau 2}$ ,  $P_{\tau 3}$  and  $P_{s3}$  are the relative total pressure at the rotor blade inlet, the relative total pressure at the rotor blade outlet,

and the static pressure at the rotor blade outlet, respectively.

(6) equation for flow rate: If a singledimensional flow is assumed, the turbine flow rate will be obtained by the following equation:

$$G = \gamma \cdot V \cdot A \tag{1.6}$$

Here, A is the channel sectional area orthogonal to flow velocity V.

The chief equations which are derived from the above fundamental equations and which are used in the calculations are the following. When equation (1.6) is written in the form of a dimensionless flow rate using equations (1.1)-(1.3), the following equation will be obtained:

$$\frac{G\sqrt{T_t}}{A\cdot P_t} = \sqrt{\frac{\kappa \cdot g}{R}} M / \left(1 + \frac{\kappa - 1}{2} M_2\right)^{\kappa + 1/2(\kappa - 1)}$$
(1.7)

When the relative outflow Mach number of the rotor blade  $M_{r3}$  is calculated, the necessity of seeking it numerically arises. That is, using the continuous conditions  $G_2 = G_3$  and the adiabatic hypothesis  $T_{r2} = T_{r3}$ , we obtain the following from equation (1.7):

$$\frac{\left(1 + \frac{\kappa - 1}{2} M_{\tau 3}^{2}\right)^{\epsilon + 1/2(\epsilon - 1)}}{M_{\tau 3}} \cdot \frac{P_{\tau 2}}{P_{\tau 3}} = \frac{A_{3}}{A_{2}}$$

$$\times \frac{\left(1 + \frac{\kappa - 1}{2} M_{\tau 2}^{2}\right)^{\epsilon + 1/2(\epsilon - 1)}}{M_{3}} \tag{1.8}$$

Here,  $A_2$  and  $A_3$  are the actual channel areas normal to the flow at the rotor blade inlet and outlet, respectively. In this case, since the annular areas are identical in both places, the area ratio  $A_3/A_2$  is expressed by the following equation:

$$\frac{A_3}{A_2} = \frac{\cos \beta_3}{\cos \beta_2} \tag{1.9}$$

On the other hand, the ratio  $P_{rs}/P_{rs}$  of the relative total pressure before and after the rotor blade is expressed by the following equation, using equations (1.2) and (1.5):

$$\frac{P_{r_2}}{P_{r_3}} = 1 + Y_{tR} \left\{ 1 - \left( 1 + \frac{\kappa - 1}{2} M_{r_3}^2 \right)^{-\frac{(\kappa/\kappa - 1)}{2}} \right\}$$
(1.10)

Consequently, the following equation will be obtained if we eliminate  $A_3/A_2$  and  $P_{r2}/P_{r3}$  from equations (1.8)-(1.10).

$$\frac{\left(1 + \frac{\kappa - 1}{2} M_{r3}^{2}\right)^{\epsilon + 1/2(\epsilon - 1)}}{M_{r3}} \times \left[1 + Y_{tR} \left\{1 - \left(1 + \frac{\kappa - 1}{2} M_{r3}^{2}\right)\right\}^{-(\epsilon/\epsilon - 1)}\right] \\
= \frac{\cos \beta_{3}}{\cos \beta_{2}} \cdot \frac{\left(1 + \frac{\kappa - 1}{2} M_{r2}^{2}\right)^{\epsilon + 1/2(\epsilon - 1)}}{M_{r2}} \tag{1.11}$$

 $M_{r3}$  is sought from this equation by finding a numerical solution.

Furthermore, the total temperature ratio before and after the turbine is expressed by the following equation on the basis of equation (1.1), taking into consideration the adiabatic hypothesis.

$$\frac{T_{t3}}{T_{t1}} = \frac{\left(1 + \frac{\kappa - 1}{2} M_{3}^{2}\right) \left(1 + \frac{\kappa - 1}{2} M_{\tau 2}^{2}\right)}{\left(1 + \frac{\kappa - 1}{2} M_{\tau 3}^{2}\right) \left(1 + \frac{\kappa - 1}{2} M_{2}^{2}\right)}$$
(1.12)

The (total pressure/static pressure) ratio  $P_{t1}/P_{s2}$  before and after the stator blades, on the basis of equations (1.1)-(1.4), will be the following, taking into consideration the fact that  $\varphi_N$  is close to 1:

$$\frac{P_{t1}}{P_{s2}} = \left\{ 1 + \frac{\frac{\kappa - 1}{2} \left(\frac{M}{\varphi_N}\right)^2}{1 + \frac{\kappa - 1}{2} M^2 \left(1 - \frac{1}{\varphi_N^2}\right)} \right\}^{\epsilon/\epsilon - 1}$$

$$\doteq \left\{ 1 + \frac{\kappa - 1}{2} \left(\frac{M}{\varphi_N}\right)^2 \right\}^{\epsilon/\epsilon - 1} \tag{1.13}$$

The total pressure ratio before and after the turbine may be expressed by the following equation, utilizing this equation as well as equations (1.2) and (1.10);

$$\frac{P_{t3}}{P_{t1}} = \left\{ \frac{\left(1 + \frac{\kappa - 1}{2} M_{3}^{2}\right) \left(1 + \frac{\kappa - 1}{2} M_{r2}^{2}\right)}{\left(1 + \frac{\kappa - 1}{2} M_{r3}^{2}\right) \left(1 + \frac{\kappa - 1}{2} \frac{M_{2}^{2}}{\varphi_{N}^{2}}\right)} \right\}^{\kappa/\kappa - 1} \times \left[1 + Y_{tR} \left\{1 - \left(1 + \frac{\kappa - 1}{2} M_{r3}^{2}\right)^{-(\kappa/\kappa - 1)}\right\}\right]^{-1} \tag{1.14}$$

Using the total temperature ratio and the total pressure ratio before and after the turbine, the adiabatic temperature efficiency of the turbine  $\eta_{1-3}$  is defined by the following equation:

$$\eta_{1-3} = \frac{1 - (T_{t3}/T_{t1})}{1 - (P_{t3}/P_{t1})^{s-1/s}}$$
(1.15)

The thermal head inside the turbine  $\Delta H_T$  is defined and sought by the following equation:

$$\Delta H_T = \frac{(W_{2u} + W_{3u})U}{g \cdot J} = \frac{(V_{2u} + V_{3u})U}{g \cdot J}$$
(1.16)

Furthermore, the turbine flow rate is calculated by the following equation. That is, if we take into consideration that  $T_{t1} = T_{t2}$  and  $A_2 = A_{a2} \cos \alpha_2$ , the flow rate G will be as follows on the basis of equation (1.7):

$$G = \frac{\frac{P_{t2}}{\sqrt{T_{t2}}} A_{a2} \cos \alpha_2 \sqrt{\frac{\kappa \cdot g}{R}} M_2}{\left(1 + \frac{\kappa - 1}{2} M_2^2\right)^{\epsilon + 1/2(\epsilon - 1)}}$$
(1.17)

Here,  $P_{t2}$  is sought from the following equation. On the basis of equations (1.1)–(1.4), the total pressure ratio before and after the stator blades will be:

$$\frac{P_{t2}}{P_{t1}} = \left[1 + \frac{\kappa - 1}{2} M_2^2 \left\{1 - \left(\frac{1}{\omega_N}\right)^2\right\}\right]^{\kappa/\kappa - 1}$$
 (1.18)

The sequence and results of these calculations are shown in Appendix Table 1.

Appendix Table 1. Sequence and Results of Calculations of Turbine Performance.

#### Calculating Conditions:

 $T_{11}=1423.2$  °K  $B_{LR}=46.0$  deg. N=1300 r.p.m.  $\varphi_N=0.97$   $M_2=0.80$   $Y_{LR}=0.23$   $\alpha_2=72.0$  deg.  $r_M=0.2633$  m  $\beta_3=63.5$  deg. r=1.315

No.	Item	Equation etc.	Calculated value	Unit
1	U	$2\pi r_M \cdot N/60$	3.667179+02	m/sec
2	$T_{t2}$	$T_{t1}$	1.423200+03	°K
3	$T_{s2}$	②, $M_2 \rightarrow \text{Eq } (1 \cdot 1)$	1.292878 + 03	°K
4	$a_2$	③→Eq (1·3)	6.933386 + 02	m/sec
5	$V_2$	$M_2 \times 4$	5.586709 + 02	m/sec
6	$M_{u2}$	U/@	5.251291 - 01	
7	$M_{r2u}$	$M_2$ · sin $a_2$ —6	2.357159 - 01	
8	$M_{a2}$	$M_1 \cdot \cos a_2$	2.472142 - 01	
9	$M_{r2}$	$\sqrt{\overline{\mathbb{O}^2 + \widehat{\mathbb{S}^2}}}$	3.415799 - 01	
10	$\beta_2$	tan <sup>-1</sup> (⑦/⑧)	4.363619 + 01	deg.
11	$A_3/A_2$	cos β <sub>3</sub> /cos ⑩	6.165203 - 01	
12	$M_{r3}$	(a), (b), $Y_{tR} \rightarrow \text{Eq } (1.11)$	7.732300 - 01	
13	$T_{r3}$	③,	1.316637 + 03	°K
14	$T_{s3}$	①, ③→Eq (1·1)	1.203323+03	°K
15	$a_3$	(4)→Eq (1·3)	6.737185 + 02	m/sec
16	$W_3$	13×15	5.209393+02	m/sec
17	$W_{3u}$	(6)·sin β₃	4.662063 + 02	m/sec
18	$W_2$	<b>4</b> × <b>9</b>	2.385384 + 02	m/sec
19	$W_{2u}$	180 · sin (10)	1.646095 + 02	m/sec
<b>2</b> 0	$\Delta W_u$	17 + 19	6.308158 + 02	m/səc
21	$\Delta H_T$	$\bigcirc \times \bigcirc /g \cdot J \rightarrow \text{Eq } (1 \cdot 16)$	5.529457 + 01	kcal/kg
22	M 43	①/⑯	5.443192 - 01	
<b>2</b> 3	$M_{3u}$	(2)·sin β₃-22	1.476705 - 01	İ
24	M a3	(12)·cos β <sub>3</sub>	3.450143 - 01	ļ
25	<i>M</i> <sub>3</sub>	$\sqrt{2^2+2^2}$	3.752885 - 01	
26	$T_{t3}/T_{t1}$	(a), (a), (b), $M_2 \rightarrow \text{Eq } (1 \cdot 12)$	8.642610-01	
27	$P_{t3}/P_{t1}$	$(\mathfrak{D},\mathfrak{D},\mathfrak{B},M_2,\varphi_N,Y_{tR}\rightarrow \text{Eq }(1\cdot 14))$	4.951582 - 01	
28	η1-3	(a) ∞, ∞→Eq (1·15)	8.759670 - 01	
<b>2</b> 9	$i_R$	$\bigcirc B_{LR}$	-2.363808+00	deg.
<b>3</b> 0	$V_{a3}$	(3)×29	2.324425+01	m/sec

#### 2. Calculation of Velocity Triangles (Two-Dimensional Calculations)

The fundamental equations for a single streamline are as follows. From hypothesis (1), it is assumed that the interior of the turbine is adiabatically insulated from the outside. Therefore,

$$H_{t1} = H_{t2} = H_{t3} + \Delta H_T \tag{2.1}$$

Furthermore, since the gas is treated as a perfect gas,

$$P_s = \gamma \cdot R \cdot T_s \tag{2.2}$$

$$a = \sqrt{\kappa \cdot g \cdot R \cdot T_s} \tag{2.3}$$

$$M = V/a \tag{2.4}$$

$$H_t = H_s + \frac{V^2}{2g \cdot J} \tag{2.5}$$

$$dH = c_p \cdot dT \tag{2.6}$$

$$dS = c_p \frac{dT_s}{T_s} - \frac{R}{J} \frac{dP_s}{P_s} \qquad (2.7)$$

$$c_p = \frac{\kappa}{\kappa - 1} \frac{R}{J} \tag{2.8}$$

A fundamental equation being valid between streamlines is the following conditional equation for the radial balance [12]:

$$\frac{1}{\rho} \cdot \frac{dP_s}{dr} = \frac{V_{u^2}}{r} - \frac{dV_{rad}}{dt} \qquad (2.9)$$

The chief equations which are derived from the above fundamental equations and from hypothesis 2.2 in the main text and which are used in calculations are the following.

On the basis of hypothesis (2), the radial position  $(r_1, r_2, r_3)$  of the streamlines be tween the turbine stages can be written as follows:

$$r_1 = r_2 = r_3 = r.$$
 (2.10)

If the radial direction velocity component is regarded as minute and neglected, equation (2.9) will assume the following from (simple radial balance):

$$\frac{g}{\gamma} \cdot \frac{dP_s}{dr} = \frac{V_{u^2}}{r} \tag{2.11}$$

On the basis of hypothesis (3), we may use equations (2.1) and (2.5):

$$\frac{dH_{t1}}{dr_1} = \frac{dH_{t2}}{dr_2} = \frac{dH_{s2}}{dr_2} + \frac{1}{2gJ} \cdot \frac{dV_{2}^2}{dr_2} = 0$$
(2.12)

On the basis of hypothesis (4), we may use equations (2.2), (2.6) and (2.7):

$$T_{s2}\frac{dS_2}{dr_2} = \frac{dH_{s2}}{dr_2} - \frac{1}{J \cdot \gamma_2} \cdot \frac{dP_{s2}}{dr_2} = 0 \quad (2.13)$$

On the basis of hypothesis (5),

$$\frac{d\alpha_2}{dr_2} = 0 \tag{2.14}$$

On the basis of hypothesis (6), we may use equation (2.11):

$$\frac{g}{r_2} \cdot \frac{dP_{s2}}{dr_2} = \frac{V_{2u^2}}{r_2} \tag{2.15}$$

Therefore, from equations (2.12), (2.13), and (2.15), we obtain:

$$-V_2 \frac{dV_2}{dr_2} = \frac{V_{2u^2}}{r_2} \tag{2.16}$$

If the above equation is transformed using equation (2.14) and  $V_{2u} = V_2 \sin \alpha_2$ , we will obtain the equation for the radial direction distribution of the flow velocity at the stator blade outlet. That is,

$$V_2 r_2^m = {
m const.}$$
 $V_{2u} r_2^m = k_1 ({
m const.})$ 
 $V_{2a} r_2^m = {
m const.}$  (2.17)

Here,

$$m = \sin^2 \alpha_2$$

Furthermore, if hypothesis (7) and equation (2.10) are taken into consideration, the following equation will apply.

$$(V_{2u} + V_{3u})r = \frac{g \cdot J \cdot \Delta H_T}{w} = k_2 \text{(const.)}$$
 (2.18)

Here  $\omega$  is the angular velocity and  $\omega = 2\pi N/60$  (N: rpm).

Therefore, if we take into consideration

$$\frac{d(\Delta H_T)}{dr} = 0 \tag{2.19}$$

we obtain the following equation from equations (2.1), (2.5), (2.10) and (2.12):

$$\frac{dH_{t3}}{dr_3} = \frac{dH_{t3}}{dr_3} + \frac{1}{2g \cdot J} \cdot \frac{dV_{3^2}}{dr_3} = 0 \quad (2.20)$$

On the basis of hypothesis (8), we obtain the following equation using (2.1) (2.6), (2.7) and (2.12):

$$T_{s3} \frac{dS_3}{dr_3} = \frac{dH_{s3}}{dr_3} - \frac{1}{J \cdot \gamma_3} \cdot \frac{dP_{s3}}{dr_3} = 0 \qquad (2.21)$$

On the basis of hypothesis (9), we obtain the following equation using (2.11):

$$\frac{g}{r_3} \cdot \frac{dP_{s3}}{dr_3} = \frac{V_{3u^2}}{r_3}$$
 (2.22)

Therefore, on the basis of equations (2.20), (2.21) and (2.22), a similar equation to equation (2.16) can be obtained for the rotor blade outlet as well:

$$-V_3 \frac{dV_3}{dr_3} = \frac{V_{3u^2}}{r_3} \tag{2.23}$$

If we use  $V_3^2 = V_{3a}^2 + V_{3u}^2$ , the above equation will be

$$\frac{dV_{3a^2}}{dr_2} = -\frac{1}{r_2^2} \cdot \frac{d(r_3V_{3u})^2}{dr_2} \qquad (2.24)$$

Consequently, if we use equations (2.10), (2.17), and (2.18) and transform equation (2.24), we obtain:

$$\frac{d(V_{3a})^2}{dr} = 2k_1^2(m-1)r^{-(2m+1)} -2k_1k_2(m-1)r^{-(m+2)}$$
 (2.25)

If we integrate this using the initial conditions that  $V_{2a} = V_{2aM}$  at  $r = r_M$  (the subscript M indicates the value at MEAN), we can obtain the equation for the radial direction distribution of the axial flow velocity at the rotor blade outlet. That is, if we taken into consideration that  $k_1 = V_{2uM}r_M{}^m$  and that  $k_2 = (V_{2uM} + V_{3uM})\gamma_M$ , we obtain

$$V_{3a}^{2} - V_{3aM}^{2}$$

$$= \frac{1-m}{m} \cdot V_{2uM}^{2} \left\{ \frac{1}{(r/r_{M})^{2m}} - 1 \right\}$$

$$- \frac{2(1-m)}{1+m} V_{2uM} (V_{2uM} + V_{3uM})$$

$$\times \left\{ \frac{1}{(r/r_{M})^{m+1}} - 1 \right\}$$
(2.26)

Furthermore, considering that the gas inside the turbine is combustion gas, the specific heat at constant pressure  $c_p$  was regarded as a function of the temperature and of the air-fuel ratio. That is, the specific heat at constant pressure of the air  $c_{p \ air}$  is given as a function of the temperature.

$$c_{n,q,r} = C_0 + C_1 T + C_2 T^2 + \cdots$$
 (2.27)

Here,  $C_0$ ,  $C_1$ , and  $C_2$  are constants. Since the enthalpy H at temperature T is:

$$H = \int_0^T c_p dT \tag{2.28}$$

the enthalpy of the air  $H_{air}$  at temperature T will be expressed by the following equation:

$$H_{air} = C_0 T + \frac{C_1}{2} T^2 + \frac{C_2}{3} T^3 + \dots + CH$$
 (2.29)

Here, CH is a constant.

The pressure or temperature before and after the adiabatic change is sought using the following relative equation:

$$\log_{10}(P_2/P_1) = \phi_2 - \phi_1 \qquad (2.30)$$

Here,  $\phi$  is the entropy function. It is expressed by the following equation:

$$\phi = (\log_{10} e) \frac{1}{R} \int_{0}^{T} \frac{c_{p} \cdot dT}{T}$$
(2.31)

Consequently, the entropy function of the air  $\phi_{air}$  at the temperature T is expressed by the following equation using equation (2.27):

$$\psi_{air} = (\log_{10} e) \frac{1}{R} \left( C_0 \ln T + C_1 T + \frac{C_2}{2} T^2 + \dots + CF \cdot R \right)$$
 (2.32)

Here, CF is a constant.

Thus, for the air one first seeks  $c_{p \ air}$ ,  $H_{air}$  and  $c_{air}$ . Then correction of the airfuel ratio is given, and the  $c_p$ , H and  $\phi$  are sought for the combustion gas [13, 14].

The values obtained by cycle calculations are used as the calculating conditions. On the basis of hypothesis (4), the  $\varphi_{N}'$  defined by the following equation was assumed to be constant in the radial direction.

$$\varphi_N' = \frac{P_{t2}}{P_{t1}} \tag{2.33}$$

Furthermore, if we assume the compressor outlet temperature to be approximately  $500^{\circ}$ C, the air-fuel ratio will have to be approximately 0.015 in order for the turbine inlet temperature to be  $1150^{\circ}$ C. Therefore, calculations were performed with an air-fuel ratio of  $\lambda = 0.015$ . The results of these

Appendix Table 2. Sequence and Results of Calculations of Velocity Triangles.

#### Calculating Conditions

 $T_{t1} = 1423.15^{\circ} \text{ K}$  $P_{t1} = 25000 \text{ kg/m}^2$ 

 $a_2 = 72.0 \deg.$  $V_{2M} = 558.67 \text{ m/sec}$   $\varphi_N'=0.97$  $\lambda = 0.015$ 

N=13300 r.p.m.  $V_{a3M}=232.44 \text{ m/sec}$ 

 $\Delta H_T = 55.29 \text{ kcal/kg}$  $\pi_T = 2.02$ 

No.	Item	Equation etc.	TIP	MEAN	ROOT	Unit
1	r	give	2.793000-01	2.633000-01	2.473000-01	m
2	$r/r_{M}$	①/r <sub>M</sub>	1.060767+00	1.000000+00	9.392328 - 01	
3	$U_{2M}$	$2\pi$ (1) N / 60		3.667179 + 02	<del></del>	m/sec
4	$(V_{2u}+V_{3u})_M$	$g \cdot J \cdot \Delta H_T/3$ , Eq (2·18)	<del></del>	6.307637 + 02		m/sec
5	m	$(\sin \alpha_2)^2$		9.045081 - 01		
6	$V_{a3}$	Eq (2·26)	2.336302+02	2.324400+02	2.309665+02	m/sec
7	U	$2\pi (1)N/60$	3.890023+02	3.667179 + 02	3.444335+02	m/sec
8	V 2	$V_{2M}/@^m$ , Eq (2·17)	5.296412 + 02	5.586700 + 02	5.912650+02	m/sec
9	$V_{a2}$	® cos α <sub>2</sub>	1.636685+02	1.726389 + 02	1.827113 + 02	m/sec
10	$V_{2u}$	® sin α <sub>2</sub>	5.037186+02	5.313266+02	5.623263+02	m/sec
11	$V_{2u}+V_{3u}$	$g \cdot J \cdot \Delta H_T / \mathcal{D}$ , Eq (2.18)	5.946297 + 02	6.307637 + 02	6.715733+02	m/sec
12	$V_{3u}$	(1)-(0)	9.091110+01	9.943708+01	1.092470+02	m/sec
13	$V_3$	$\sqrt{6^2+2^2}$	2.506943+02	2.528163+02	2.555004+02	m/sec
14	$V_{2}^{2}/2g\cdot J$	$\$^2/2g\cdot J$	3.352597 + 01	3.730169+01	4.178131+01	kcal/kg
15	$H_{t1}$	SUB 90 $(T_{t1}, \lambda)$		3.758583 + 02		kcal/kg
16	$P_{t2}$	$\varphi_{N'} \cdot P_{t1}$ , Eq (2·33)		2.425000+04	<del></del>	kg/m²
17	$H_{t2}$	$H_{t1}$		3.758583 + 02		kcal/kg
18	$T_{t2}$	$T_{t1}$		1.423150+03		°K
19 -	$H_{s2}$	17-14	3.423324+02	3.385566+02	3.340770+02	kcal/kg
20	$T_{s2}$	SUB 90 (19, \(\lambda\)	1.308931+03	1.295984+03	1.280599+03	°K
21	$P_{s2}$	SUB 100 (16, 18, 20, 1)	1.694543+04	1.624371 + 04	1.544024 + 04	kg/m²
22	72	②)/R②), Eq (2·2)	4.422961-01	4.282159-01	4.119252 - 01	kg/m³
23	K2	SUB 80 (20), \(\lambda\)	1.307150+00	1.307751+00	1.308481+00	
24	$a_2$	$\sqrt{2gR}$ , Eq (2·3)	7.005601 + 02	6.972472 + 02	6.932869+02	m/sec
25	$W_{2u}$	$8 \sin \alpha_2 - 7$	1.147163+02	1.646087 + 02	2.178928+02	m/sec
26	$W_2$	$\sqrt{9^2+85^2}$	1.998680 + 02	2.385377 + 02	2.843602 + 02	m/sec
27	$eta_2$	sin-1(26/26)	3.502698+01	4.363609+01	5.001899+01	deg.
28	$M_2$	8/24	7.560254-01	8.012509 - 01	8.528398-01	
29	$M_{\tau^2}$	<b>26/29</b>	2.852974 - 01	3.421135-01	4.101607 - 01	
30	$W_{3u}$	(1)-26	4.799134 + 02	4.661550+02	4.536805 + 02	m/sec
31	$W_3$	$\sqrt{6^2+30^2}$	5.332602 + 02	5.208923+02	5.090889+02	m/sec
32	$W_3{}^2/2g\cdot J$	$\mathfrak{P}^2/2g\cdot J$	3.404945+01	3.242752+01	3.097456 + 01	kcal/kg
33	$H_{t3}$	$\bigcirc$		3.205683+02		kcal/kg
34	$T_{t3}$	SUB 90 (3), \(\lambda\)		1.234028+03	<del></del>	°K
35	$P_{t3}$	$P_{t1}/\pi_T$		1.237624+04	<del></del>	kg/m²
36	$V_{\mathfrak{z}^2}/2g\cdot J$	$3^2/2g \cdot J$	7.511185+00	7.638849 + 00	7.801907 + 00	kcal/kg
37	$H_{s3}$	(3)-69	3.130571+02	3.129295+02	3.127664+02	kcal/kg
38	T 53	SUB 90 (69, 1)	1.208016+03	1.207573 + 02	1.207007 + 03	°K
39	$P_{i3}$	SUB 100 (35, 39, 38, \(\lambda\)	1.131391+04	1.129648+04	1.127424+04	kg/m²
40	- «°   γ <sub>3</sub>	⊗/R⊗	3.199761-01	3.196003-01	3.191207 - 01	kg/m³
41	κ <sub>3</sub>	SUB 80 (89, $\lambda$ )	1.312173+00	1.312197 + 00	1.312228+00	_

No.	Item	Equation etc.	TIP	MEAN	ROOT	Unit
42	a <sub>3</sub>	$\sqrt{@gR}$	6.743048 + 02	6.741873 + 02	6.740372+02	m/sec
<b>4</b> 3	$M_3$	⅓/@	3.717826-01	3.749942 - 01	3.790597-01	
44	$M_{r3}$	<b>(1)</b> / <b>(2)</b>	7.915711 - 01	7.726326-01	7.552831-01	
45	$\beta_3$	sin <sup>-1</sup> 30/30	6.404244+01	6.349773+01	6.301965+01	deg.
<b>4</b> 6	$M_{a3}$	<b>❸</b> · cos <b>④</b>	3.464756-01	3.447706-01	3.426613-01	
47	$\alpha_3$	cos <sup>-1</sup> (⑥/⑬)	2.126236+01	2.316124+01	2.531428+01	deg.
48	M a 2	$\otimes \cdot \cos \alpha_2$	2.336252-01	2.476007 - 01	2.635426-01	
49	$H_{s2}-H_{s3}$	<b>19</b> − <b>3</b>	2.927522+01	2.562716+01	2.131056+01	kcal/kg
<b>5</b> 0	$ ho_R$	$\Theta/\Delta H_T$	5.294848 - 01	4.635045-01	3.854331-01	
51	Ē	(23+(1))/2	1.309661+00	1.309974+00	1.310351+00	
<b>52</b>	Ĉp ∣	$\mathfrak{S}R/J(\mathfrak{S})-1)$	2.899803-01	2.897575-01	2.894859 - 01	kcal/kg /°K
<b>5</b> 3	$P_{t3}/P_{t2}$	$1/\pi_T$		4.950495-01	<del></del>	/ ==
54	$T_{t3}/T_{t1}$	$\mathfrak{G}/T_{t1}$		8.671105-01		
55	$1 - (1/\pi_T)^{(\tilde{s}-1)/\tilde{s}}$	<b>(1)</b> , <b>(3)</b>	1.531597-01	1.532681 - 01	1.534001 - 01	
<b>56</b>	71	$\Delta H_T/ \mathfrak{D} T_{i1} \mathfrak{D}$	8.747481-01	8.748026 - 01	8.748689-01	
57	η <sub>1-3</sub>	(1-(3))/(3)	8.676531-01	8.670391-01	8.662934-01	
58	Ttrad	SUB 110 $(T_{t1}, \pi_T, \lambda)$		1.206500+03		°K
59	H <sub>t2ad</sub>	SUB 90 (🚳, $\lambda$ )		3.126040+02		kcal/kg
60	4Had	17)—59		6.325431+01		kcal/kg
61	$\eta_T$	$\Delta H_T/ \odot$ , Eq (2.34)		8.740906-01		

Appendix Table 2. (Continued)

Reference: Concerning the subroutines used

Suboutine	Input	Output	Reference
SUB 80 " 90 " 100 " 110	$T$ , $\lambda$ $T$ (or $H$ ), $\lambda$ $P_1$ , $T_1$ , $T_2$ , $\lambda$ $P_{t1}$ , $P_{t1}/P_{t2}$ , $\lambda$	κ, c <sub>p</sub> H (or T) P <sub>2</sub> T <sub>t2</sub>	(13), (14)

calculations, together with the sequence of calculation, are shown in Appendix Table 2. Thus, the design velocity triangle will be as shown in Fig. 2 in the main text.

The turbine adiabatic efficiency  $\eta_T$  as defined by the following equation was used.

$$\eta_T = \frac{\Delta H_T}{\Delta H_{ad}} \tag{2.34}$$

Here,  $\Delta H_T$  and  $\Delta H_{ad}$  are the actual thermal head and the adiabatic head inside the turbine from the turbine inlet total pressure to the outlet total pressure. Even though  $\eta_T = 87.4\%$  from the results calculated in No. 61 of Table 2, the adiabatic efficiency

in designing was taken at 85.0%, taking into consideration the effects of the blowout of the cooling air because the turbine is an air-cooled turbine. Consequently, the specific output  $\Delta H_T$  of this turbine had a target of 53.8 kcal/kg (=0.85. $\Delta H_{ad}$ ).

Furthermore, the degree of reaction  $(\rho_R)$  and the theoretical velocity  $(V_{th})$  of this turbine were defined and sought by the following equations:

$$\rho_R = \frac{H_{s2} - H_{s3}}{\Delta H_T} \tag{2.35}$$

$$V_{th} = \sqrt{2gJ \cdot \Delta H_{ad}} \tag{2.36}$$

The chief design specifications of this turbine obtained in this way are shown in Table 1 in the main text.

### 3. Determining the Profile and Arrangement of the Air-Cooled Blades

#### (a) Stator Blades

We used the method to determine the blade profile by "fleshing out" suitably on the camber line. That is, here we approximated the camber line by a cubic expression; we drew a series of circles having their centers on the camber line; and we assumed that their envelopes were the external shape of the blade. In the following, this method is described in detail (see Appendix Fig. 1).

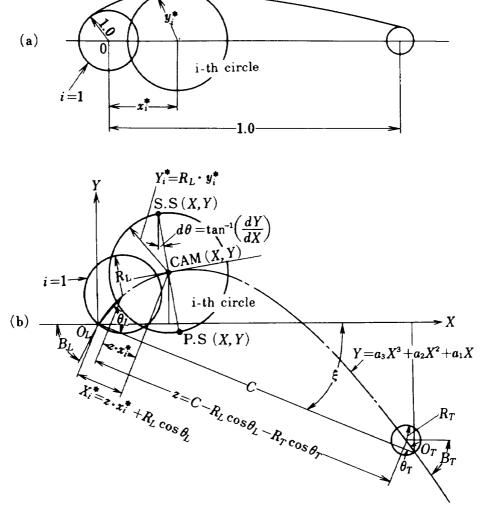
(1) Determining the camber line: We took the blade's leading edge  $O_L$  as the

point of origin, as in Appendix Fig. 1 (b). Then we plotted coordinate systems with the axial direction as the X axis and the circumferential direction as the Y axis. The camber line was expressed by the following cubic expression

$$Y = a_3X^3 + a_2X^2 + a_1X$$

The angles formed by the tangents on this camber line at the blade leading edge  $O_L$  and trailing edge  $C_T$  with the X-axis direction are defined as the blade inlet angle  $B_L$  and the outlet angle  $B_T$ , respectively. The angle formed by the chord connecting  $O_L$  and  $O_T$  with the X-axis direction is defined as the angle of stagger  $\xi$ , and the length of the chord is defined as the chord length C.

If  $B_L$ ,  $B_T$ ,  $\xi$ , and C are assigned suitably



Appendix Fig. 1. Method of culculating stator blade profile and blade arrangements.

with due consideration to the fundamental policies (1), (3), and (4), it is possible to determine the coefficients  $a_1$ ,  $a_2$ , and  $a_3$  of the cubic expression.

(2) Method of "fleshing out": When the shape of the camber line has been determined, the points on it CAM (X, Y) can be represented by coordinate  $X^*$  on the blade chord  $(X^*)$  is the point of orthogonal projection of point CAM (X, Y) on the blade chord). Next we shift this coordinate  $X^*$  to a new coordinate system  $x^*$  (Appendix Fig. 1 (a)) which has been made dimensionless so that the center of the circle i=1 (circle corresponding to the blade leading edge) will correspond to  $x^*=0$  and the center of the circle i=m (circle corresponding to the blade trailing edge) will correspond to  $x^*=1$ . In Appendix Fig. 1 (b), the distance between the  $X^*$  coordinates centering in circle i=1 and i=m will be:

$$z = C - R_L \cos \theta_L - R_T \cos \theta_T$$

Therefore, this will be the divisor for conversion to dimensionless terms in shifting from  $X^*$  to  $x^*$ . Here,  $\theta_L$  is the angle between the blade chord and the line connecting the center of the circle i=1 and  $O_L$ .  $\theta_T$  is the angle between the blade chord and the line connecting the center of i=m and  $O_T$ .

Thus, the correspondence between  $x^*$  and the coordinate CAM (X, Y) on the camber line will be valid. On the other hand, if we divide the radius  $Y^*$  of the circle determining the blade thickness at CAM (X, Y) by the blade leading edge radius  $R_L$   $(Y^*$  of the circle i=1) to render it dimensionless and represent it as  $y^*$ , then plot a circle with a radius  $y^*$  at point  $x^*$ , the results will be as shown in Appendix Fig. 1 (a). This will form a symmetrical blade profile, and the blade thickness will be given in this shape so that there will be a good shape from the aerodynamical standpoint.

On the contrary, if we supply the blade thickness distribution on the  $(x^*, y^*)$  plane as shown in Appendix Fig. 1 (a) and also supply the blade leading edge radius according to fundamental policy (3), the blade trailing edge radius  $R_T$  will be determined

from the ratio between the radius  $y^*$  of the circle i=m and  $R_L$ . Furthermore,  $\theta_L$  and  $\theta_T$  will be determined if we use a known camber line equation. Therefore, z can be calculated from the equation given above. Consequently, with respect to a certain  $x_i^*$  in Appendix Fig. 1 (a), one performs conversion

$$X_i^* = zx_i^* + R_L \cos \theta_L$$

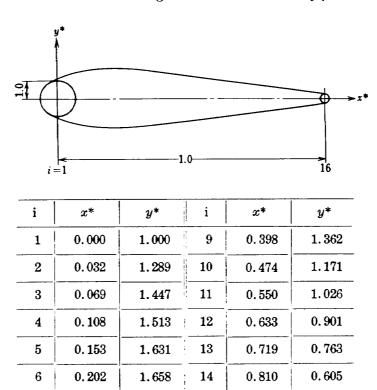
The position  $X_i^*$  on the blade chord on the physical plane in Appendix Fig. 1 (b) will thus be determined, and the coordinate CAM (X, Y) on the camber line will also be determined. On the other hand, the blade thickness  $Y_i^*$  will a so be determined using the relationship

$$Y_i^* = R_L \cdot v_i^*$$

on the basis of the relationship of  $y_i^*$  to  $x_i^*$  in Appendix Fig. 1 (a). In the final analysis, if one draws a circle with a radius  $Y_i^*$  centering around point CAM (X, Y) in Appendix Fig. 1 (b) and finds the two points of intersection between it and the normal lines established on the camber line at this point, they will be the coordinates on the upper and lower surfaces of the blade, S.S (X, Y) and P.S (X, Y).

(3) Since the blade profile has been determined by the above, the blade arrangement can next be determined if the number of blades n is given. Therefore, the pitch S and the throad width O will be determined, and  $\cos^{-1}(O/S)$  can be calculated.

In this manner, the blade profiles and blade arrangements were calculated by means of a computer for several values of  $B_L$ ,  $B_T$ ,  $\xi$ , C,  $R_L$ , and n, and the blades to be adopted were determined in accordance with fundamental policies (1) and (2). That is, taking into consideration the influence of the Mach number and the curvature of the blade surface from the throat to the trailing edge, we adopted a design in which the calculated value of  $\cos^{-1}(O/S)$ , mentioned above, would be slightly smaller [15] than the stator blade absolute outflow angle  $(\alpha_2)$  of 72.0° of the design velocity triangle, and in which the number of blades would be as small as possible.



15

16

0.907

1.000

1.572

1.461

Appendix Table 3. Blade Thickness Distribution used in Calculating Stator Blade Profiles [2]

As for the blade thickness distribution, the blade thicknesses of air-cooled blades [2] having a design velocity triangle relatively similar to the present air-cooled blades were calculated in reverse in the form shown in Appendix Fig. 1 (a) and used. They are shown in Appendix Table 3. The same blade thickness distribution was used in every section, the TIP, the MEAN, and the ROOT (the chord length etc. were different).

7

8

0.264

0.329

#### (b) Rotor Blades

Several blade profiles and blade arrangements were determined for the TIP, the MEAN, and the ROOT following the fundamental policies, and the blade surface velocities were sought experimentally for each by the electrical analog method using electro-conductive paper [16]. The blade loss and blade inlet critical Mach number were estimated from the results, and we adopted the design which had the smallest blade loss and in which the blade inlet critical Mach number had an adequate

margin and was considerably larger than the design inlet Mach number of the rotor blades, In the following this method is described in stages.

0.428

0.237

- (1) Following fundamental policy (1), the solidity giving the least profile loss (optimum solidity) was determined by the method [15] in the bibliography, taking into consideration the rotor blade outflow angle and the degree of reaction for each section. It was determined so that the solidity would be slightly smaller than the optimum solidity because of fundamental policy (2), considering the fact that the blades are air-cooled blades.
- (2) Next, following fundamental policy (4), it was assumed that there are almost no changes of the chord length in the blade span direction, and the chord length and number of blades were determined, taking into consideration the solidity determined in (1), so that the aspect ratio would not be too small.
  - (3) The blade leading edge and trailing

edge diameters were determined in accordance with fundamental policy (3).

- (4) On the other hand, the throat width O was determined from the rotor blade relative outflow angle  $\beta_3$  in the velocity triangle. In other words, the difference between  $\beta_3$  and  $\cos^{-1}(O/S)$  was determined by the method [15] in the bibliography, taking into consideration the influence of the design outflow Mach number.
- (5) Next, the incidence at the design point was determined slightly on the minus side from the zero incidence, taking into consideration the operating range of the turbine. This decision was made on account of the general tendency of the characteristic curves of the profile loss so that the efficiency would not drop even at partial loads. The inlet angle of the blades  $B_L$  was determined from this incidence and from the  $\beta_2$  of the velocity triangle.
- (6) Next, the angle of stagger was determined, and a blade profile was drawn so as to satisfy the previously determined pitch S, the leading edge diameter  $d_L$ , the trailing edge diameter  $d_T$ , the throat width O, and the blade inlet angle  $B_L$ . At this time, the curvature of the back surface of the blade between the throat and the trailing edge will have an influence on the outflow angle [15]. Therefore, this was taken into consideration when making the decision. Since there is still a certain degree of freedom in the blade profile and blade arrangements even under these conditions, several blade profiles were drawn. The designs for the rotor blades, unlike those for the stator blades, were drawn manually without using a computer.
- (7) In this way, three types of blade profiles and blade arrangements were compiled for the TIP, the MEAN, and the ROOT, making up a total of nine designs. Since the goal was to select blade which would have the optimum pressure distribution and which would also have a low heat transfer rate outside the blades, in accordance with fundamental policy (5), the surface velocity distribution was sought by the electrical analog method for each of the blades. The following equations were used to calculate blade diffusion

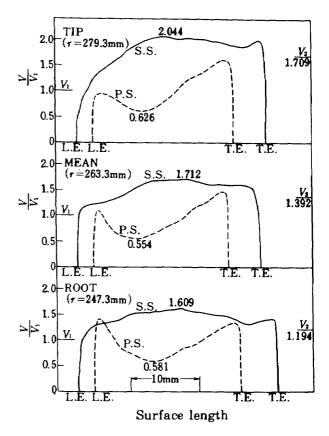
parameters (D) from these results.

$$D_{s}=1-\frac{V_{s}}{V_{s}}$$

$$D_{p}=1-\frac{V_{p}}{V_{1}}$$

$$D_{t}=D_{p}+D_{s}$$
(3.1)

Here,  $V_L$  is the blade inlet velocity  $V_1$ is the blade outlet velocity, V<sub>s</sub> is the maximum surface velocity on the suction side surface of the blade, and  $V_p$  is the maximum surface velocity on the pressure side surface of the blade. According to [17] in the bibliography, the speciac blade loss L(L= $(1-\eta_t n)/\sigma_N$ ; here  $\eta_t$  is the adiabatic efficiency calculated from the turbine output, and  $\sigma_M$  is the solidity at the MEAN) is particularly influenced by D: but generally declines together with the decline of  $D_{\iota}$ . Therefore, we selected the design with the smallest  $D_s$  and  $D_t$ . The results of the surface velocity distribution of the rotor blades adopted in this turbine are shown in Appendix Fig. 2, and the results of calculations of the  $D_s$ ,  $D_p$ , and  $D_t$  are shown



Appendix Fig. 2. Rotor blade surface velocity distribution by electrical analog method.

in Appendix Table 4.

The maximum pressure coefficient on the suction side surface of the blade  $C_{pi,\max}$  was sought from the results in Appendix Fig. 2 by means of

$$C_{pi, \max} = 1 - \left(\frac{V_2}{V_2}\right)^2$$
 (3.2)

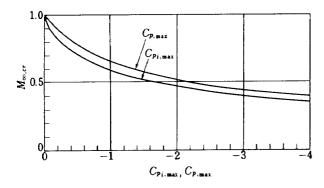
The pressure coefficient and the critical inflow Mach number at the blade inlet were determined using the Karman-Tsien method of correcting the effect compressibilit [18, 19]. That is, according to the Karman-Tsien method, the relationship between the pressure coefficient  $C_p$  considering the compressibility and the pressure coefficient of an incompressible fluid  $C_{pi}$  is expressed as follows:

$$C_{p} = \frac{C_{pi}}{\sqrt{1 - M_{\infty}^{2}} + \frac{M_{\infty}^{2}}{1 + \sqrt{1 - M_{\omega}^{2}}} \cdot \frac{C_{pi}}{2}}$$
(3.3)

Furthermore, from the definition of the pressure coefficient  $C_p$ , we have

$$C_{p} = \frac{2}{\kappa M_{\infty}^{2}} \left\{ \left( \frac{1 + \frac{\kappa - 1}{2} M_{\infty}^{2}}{1 + \frac{\kappa - 1}{2} M^{2}} \right)^{\kappa/\kappa - 1} - 1 \right\}$$
(3.4)

Here,  $M_{\infty}$  is the mainstream Mach number at an infinite distance, and M is the local Mach number on the blade surface. If we use  $C_{pi, \max}$ ,  $C_{p, \max}$ , and  $M_{\infty, cr}$  to represent the  $C_{pi}$ ,  $C_p$  and  $M_{\infty}$  when M=1.0, their relationships will be as shown in Appendix Fig. 3, using equations (3.3) and



Appendix Fig. 3. Relationship between maximum pressure coefficient and critical Mach number ( $\kappa=1.315$ ).

- (3.4). Let us assume that the Mach number  $M_1$  of the mainstream at the blade inlet is identical with the  $M_{\infty}$  in the Karman-Tsien equation. Then we can seek the critical inflow Mach number  $M_{1,cr}$  $(=M_{\infty,cr})$  at the blade inlet and the maximum pressure coefficient  $C_{p,\max}$  max of the blade taking into consideration the compressibility for the TIP, MEAN, and ROOT sections from the values of  $C_{pi, \max}$  sought previously from equation (3.2) and from Appendix Fig. 3. These results are shown in Appendix Table 4. As a result, it was possible to ascertain that the blade inlet critical Mach number  $M_{1,cr}$  is considerably greater than the rotor blade inlet Mach number  $M_{r2}$  in the design velocity triangle, and that the blade profile and blade arrangements were satisfactory.
- (8) Finally, some modifications incorporated so that the blade shapes would be smooth together with the stacking of the blade profiles having different sections sought inaccordance with fundamental policy (4). On account of this, the throat width O was finally aligned at the MEAN, while it was slightly smaller at the TIP and slightly larger at the ROOT. Consequently, the condition in the design policity that the work distribution in the rotor blades must be constant in the radial direction (hypothesis (7) in Section 2.2 of the main text) was not completely satisfied, there being excessive work at the TIP and insufficient work at the ROOT, while the

Appendix Table 4. Values of Rotor Blade Diffusion Parameters, Pressure Coefficients, and Critidal Inflow mach Numbers

	TIP (r=277.3 mm)	$\begin{array}{c} \texttt{MEAN} \\ (r = 261.3 \\ \texttt{mm}) \end{array}$	$\begin{array}{c} \text{ROOT} \\ (r = 245.3 \\ \text{mm}) \end{array}$
$D_s$	0.164	0.187	0.258 (0.267)
$D_p$	(0.240)	0.251)	0.419
$\frac{P}{D_t}$	0.538	0.633	0.677
$C_{pi}$ , max	-3.178	-1.931	-1.589
$C_p$ , max	-4.0	-2.4	-2.1
$M_1$ , cr	0.39	0.48	0.51

design specifications were satisfied at the MEAN.

#### APPENDIX B.

#### EQUIVALENT CONDITIONS OF TURBINE OPERATING STATES

#### (a) Hypotheses

The equivalent conditions of the turbine operating states are sought under the following hypotheses.

- 1) It is assumed that, when the Mach number M of the flow inside the blade and the Mach number  $M_u$  of the peripheral velocity are identical in two flows, the flows will be similar. The similarity conditions are to be determined while ignoring the effects of the Reynolds number.
- 2) The fluid inside the turbine is assumed to be a perfect gas, and the flow is assumed to be an adiabatic single-dimensional flow.
- 3) The specific heat ratio inside the turbine is assumed to be constant.

#### (b) Fundamental Equations

Equation for the relationship between the total temperature and the static temperature:

$$\frac{T_t}{T_c} = 1 + \frac{\kappa - 1}{2} M^2 \tag{4.1}$$

Equation for the relationship between the total pressure and the static pressure:

$$\frac{P_t}{P_s} = \left(1 + \frac{\kappa - 1}{2}M^2\right)^{s/\epsilon - 1} \tag{4.2}$$

Equation for the perfect gas:

$$\gamma = \frac{P_s}{R \cdot T_s} \tag{4.3}$$

Equation for the relationship between the velocity and the Mach number:

$$V = M\sqrt{\kappa \cdot g \cdot R \cdot T_s} \tag{4.4}$$

Equation for the flow rate: Assuming that the flow is a one-dimensional flow:

$$G = \gamma \cdot A \cdot V \tag{4.5}$$

#### (c) Derivation of the Equivalent Conditions. Corrected Flow Rate

The following equation is obtained by using equations (4.1)-(4.4) in the continuous equation (4.5).

$$M^{2} = \frac{G \cdot V}{P_{t}} \cdot \frac{\left(1 + \frac{\kappa - 1}{2} M^{2}\right)^{\kappa/\kappa - 1}}{\kappa} \frac{1}{A \cdot g}$$

The above equation is valid also at the standard state. In other words, it can be written in the following manner if the subscript st is used to indicate the standard state

$$M_{st}^{2} = \frac{G_{st} \cdot V_{st}}{P_{t,st}} \cdot \frac{\left(1 + \frac{\kappa_{st} - 1}{2} M_{st}^{2}\right)^{\epsilon_{st}/\epsilon_{st} - 1}}{\kappa_{st}} \frac{1}{A \cdot g}$$

If we assume that the flow is similar to the flow at the standard state, because  $M = M_{st}$ , the flow rate at the standard state  $G_{st}$  can be expressed in the following terms from the two equations given above

$$G_{st} = \frac{\varepsilon \cdot G \cdot \sqrt{\theta}}{\delta}$$
Here,
$$\theta = \left(\frac{V}{V_{st}}\right)^2 = \frac{\left(\frac{\kappa \cdot T_t}{1 + \frac{\kappa - 1}{2}M^2} \cdot R\right)}{\left(\frac{\kappa_{st} \cdot T_{t,st}}{1 + \frac{\kappa_{st} - 1}{2}M^2} \cdot R_{st}\right)}$$

$$\gamma = \frac{P_t}{P_{t,st}}$$

$$\varepsilon = \frac{\kappa_{st}}{\kappa} \cdot \frac{\left(1 + \frac{\kappa - 1}{2}M^2\right)^{\epsilon/\epsilon - 1}}{\left(1 + \frac{\kappa_{st} - 1}{2}M^2\right)^{\epsilon_{st}/\kappa_{st} - 1}}$$

In other words, when the flow rate at a certain state is G, the flow rate at the standard state, where the flow is similar with that state, can be written as  $\varepsilon \cdot G \cdot \sqrt{\theta}/\delta$ . Because of this, the equivalent conditions of the flow rate are that  $\varepsilon \cdot G \cdot \sqrt{\theta}/\sigma$  is identical.

Corrected Revolution Speed

The relationship between the revolution speed and the peripheral velocity is expressed by the following equation

$$U = \frac{\pi \cdot D \cdot N_{st}}{60} \tag{4.7}$$

Here, D is the diameter of revolution and N is the revolutions per minute. If we use equation (4.4) to express the peripheral velocity in terms of the Mach numbers, since  $M_u = U/\sqrt{\kappa \cdot g \cdot R \cdot T_s}$ , the following will be obtained from equation (4.7)

$$M_u = \frac{\pi \cdot D \cdot N}{60} \cdot \frac{M}{V}$$

This equation will be equally valid in the standard state as well.

$$M_{u,st} = \frac{\pi \cdot D \cdot N_{st}}{60} \cdot \frac{M_{st}}{V_{st}}$$

If we assume that the flow is similar to the flow in the standard state, since  $M = M_{st}$  and  $M_u = M_{u,st}$ , the revolution speed at the standard state  $N_{st}$  can be expressed as follows on the basis of the two preceding equations.

$$N_{st} = \frac{N}{\left(\frac{V}{V_{st}}\right)} = \frac{N}{\sqrt{\theta}} \tag{4.8}$$

Consequently, the equivalent conditions for the revolution speed are that  $N/\sqrt{\theta}$  is identical.

Corrected Specific Output

The enthalpy drop is expressed by the following equation

$$\Delta H = \frac{V^2}{2 \cdot g \cdot J} \tag{4.9}$$

This equation is valid similarly also at the standard state.

$$\Delta H_{st} = \frac{V_{st}^2}{2 \cdot g \cdot J}$$

If we assume that the flow is similar to the flow at the standard state, since  $M = M_{st}$ , the enthalpy drop at the standard state  $\Delta H_{st}$  can be expressed as follows on the basis of the two preceding equations.

$$\Delta H_{st} = \frac{\Delta H}{\left(\frac{V}{V_{st}}\right)} = \frac{\Delta H}{\theta} \tag{4.10}$$

Consequently, the equivalent conditions for the enthalpy drop (specific output) are that  $\Delta H/\theta$  is identical.

Corrected Torque

The torque is expressed by the following equation using the flow rate G, the revolution speed N, and the specific output  $\Delta H$ .

$$\tau = \frac{60 \cdot J}{2\pi} \cdot \frac{G \cdot \Delta H}{N} \tag{4.11}$$

The above equation is valid also at the standard state.

$$\tau_{st} = \frac{60 \cdot J}{2\pi} \cdot \frac{G_{st} \cdot \Delta H_{st}}{N_{st}}$$

If we substitute equations (4.6), (4.8), and (4.10) in this equation, we obtain:

$$\pi_{st} = \frac{60 \cdot J}{2\pi} \frac{\varepsilon \cdot G \cdot \sqrt{\theta}}{\delta} \cdot \frac{\Delta H}{\theta}$$
$$= \frac{\varepsilon}{\delta} \left( \frac{60 \cdot J}{2\pi} \cdot \frac{G \cdot \Delta H}{N} \right) \tag{4.12}$$

Furthermore, if we use equation (4.11), the torque at the standard state  $\tau_{st}$  can be written as follows.

$$\tau_{st} = \frac{\varepsilon \cdot \tau}{\delta} \tag{4.13}$$

Consequently, the equivalent conditions of the torque are that  $\epsilon \cdot \tau/\delta$  is identical.

Corrected Expansion Ratio

The adiabatic efficient  $\eta_t$  of the turbine is defined by the following equation

$$\eta_{t} = \frac{\Delta H}{\frac{\kappa}{\kappa - 1} \cdot \frac{R}{J} \cdot T_{t1} \left\{ 1 - \left( \frac{1}{\pi_{T}} \right)^{\epsilon - 1/\epsilon} \right\}} \quad (4.14)$$

The equation is also valid at the standard state.

$$\eta_{t,st} = \frac{\Delta H_{st}}{\frac{\kappa_{st}}{\kappa_{st} - 1} \cdot \frac{R_{st}}{J} \cdot T_{t,st} \left\{ 1 - \left(\frac{1}{\kappa_{T,st}}\right)^{\kappa_{st} - 1/\kappa_{st}} \right\}}$$

If we assume that  $\eta_i = \eta_{t,st}$  when the flow is similar to the flow at the standard state, the following can be obtained from the two preceding equations.

$$\frac{1 - \left(\frac{1}{\pi_{T,st}}\right)^{\epsilon_{st} - 1/\epsilon_{st}}}{1 - \left(\frac{1}{\pi_{T}}\right)^{\epsilon - 1/\epsilon}} \\
= \left(\frac{\kappa_{st} - 1}{\kappa - 1}\right) \left(\frac{\kappa \cdot R \cdot T_{t1}}{\kappa_{st} \cdot R_{st} \cdot T_{t,st}}\right) \frac{\Delta H_{st}}{\Delta H}$$

Furthermore, if we transform it using equations (4.6) and (4.10), we obtain:

$$\frac{1 - \left(\frac{1}{\pi_{T,st}}\right)^{\epsilon_{st} - 1/\epsilon_{st}}}{1 - \left(\frac{1}{\pi_{T}}\right)^{\epsilon - 1/\epsilon}}$$

$$= \left(\frac{\kappa_{st} - 1}{\kappa - 1}\right) \frac{1 + \frac{\kappa - 1}{2}M^{2}}{1 + \frac{\kappa_{st} - 1}{2}M^{2}}$$

From this, when we solve for the expansion ratio at the standard state  $\pi_{T,st}$ , the following equation is obtained:

$$\begin{aligned}
\pi_{T,st} &= \phi \cdot \pi_T \\
\text{Here,} \\
\phi &= \frac{1}{\pi_T} \left\{ 1 - \left( \frac{\kappa_{st} - 1}{\kappa - 1} \right) \left( \frac{1 + \frac{\kappa - 1}{2} M^2}{1 + \frac{\kappa_{st} - 1}{2} M^2} \right) \right\} \\
&\times (1 - \pi_T^{1 - \epsilon/\epsilon}) \right\}^{\epsilon_{st}/1 - \epsilon_{st}}
\end{aligned} (4.15)$$

Consequently, the equivalent conditions of the expansion ratio are that  $\phi \cdot \pi_T$  is identical.

## (d) Approximate Treatment of the Various Equivalent Condition Equations

As was mentioned above, the equations for the equivalent conditions for the flow rate, the revolution speed, the specific output, the torque, and the expansion ratio can be expressed in terms of equations (4.6), (4.8), (4.10), (4.13), and (4.15), respectively. Since the flow inside the turbine is usually close to Mach 1, if we assume that M is approximately equal to 1.0 in these equations, the equations for  $\theta$ ,  $\varepsilon$  and  $\phi$  containing M will be as follows. Let us use the subscript cr to indicate the values of  $\theta$ ,  $\varepsilon$  and  $\phi$  when M=1.0. In this case,

 $\theta$  will be as follows on the basis of equation (4.6):

$$\theta_{c\tau} = \left(\frac{V_{c\tau}}{V_{c\tau,st}}\right)^2 = \frac{\frac{\kappa}{\kappa+1} \cdot R \cdot T_t}{\frac{\kappa_{st}}{\kappa_{tt}+1} \cdot R_{st} \cdot T_{t,st}}$$
(3.16)

 $\varepsilon$  will also be as follows on the basis of equation (4.6):

$$\varepsilon_{cr} = \frac{\kappa_{st}}{\kappa} \frac{\left(\frac{\kappa+1}{2}\right)^{s/\kappa-1}}{\left(\frac{\kappa_{st}+1}{2}\right)^{s_s^{1/\kappa_{st}-1}}} \tag{4.17}$$

 $\phi$  will be as follows on the basis of equation (4.15):

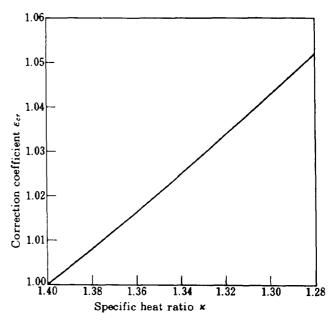
$$\phi_{er} = \frac{1}{\pi_T} \left\{ 1 - \left( \frac{\kappa_{st} - 1}{\kappa - 1} \right) \left( \frac{\kappa + 1}{\kappa_{st} + 1} \right) \right.$$

$$\times \left( 1 - \pi_T^{1 - \kappa/\kappa} \right) \right\}^{\kappa_{st}/1 - \kappa_{st}}$$
(4.18)

There  $\theta_{cr}$ ,  $\epsilon_{cr}$  and  $\phi_{cr}$  may be used approximately in the aforementioned equivalent condition equations.

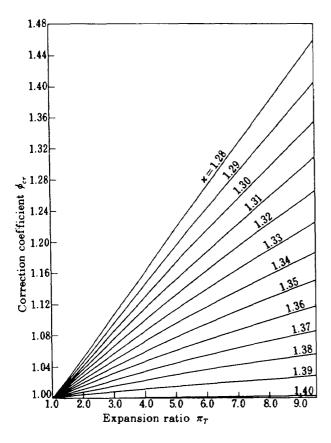
In this report, the following values were adopted for the standard state:

$$T_{t,st} = 288.2^{\circ} \text{K}$$
  
 $P_{t,st} = 10332 \text{ kg/m}^2$   
 $R_{st} = 29.27 \text{ kg} \cdot \text{m/}^{\circ} \text{K} \cdot \text{kg}$   
 $\kappa_{st} = 1.401$ 



Appendix Table 4. Flow rate correction coefficient.  $\epsilon_{cr}(\kappa_{st}=1.401)$ 

In this case,  $\varepsilon_{cr}$  and  $\phi_{cr}$  may be shown graphically in terms of the differences in the specific heat ratio. They will be as shown in Appendix Figs. 4 and 5. It is clear that, when the specific heat ratio differs considerably from the standard state,  $\varepsilon_{cr}$  and  $\phi_{cr}$  will have magnitudes great enought to require consideration.



Appendix Fig. 5. Expansion ratio correction coefficient.  $\phi_{cr}(\kappa_{st}=1.401)$ 

## TECHNICAL REPORT OF NATIONAL AEROSPACE LABORATORY TR-321T

### 航空宇宙技術研究所報告321T号(欧文)

昭和56年1月発行

発行所 航 空 宇 宙 技 術 研 究 所 東京都調布市深大寺町 1880 電話武蔵野三鷹(0422)47-5911(大代表)〒182 印刷所株式会社東京プレス 東京都板橋区桜川 2 - 27 - 12

Published by
NATIONAL AEROSPACE LABORATORY
1,880 Jindaiji, Chōfu, Tokyo
JAPAN