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Optimal Problem for Delay-Differential Control Systems
(On the Existence and Uniqueness of Optimal Solution)

Shigeki HATAYAMA

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Optimal Problem for Delay-Differential Control Sytems
(On the Existence and Uniqueness of Optimal Solution)*

By
Shigeki HATAYAMA**

ABSTRACT

General delay-differential control systems dependent on both the previous history of the state and of the control are modelled. The optimal control problem for such systems with the cost functional, the state constraint condition and the time-varying target set is formulated. This formulation generalizes the optimal problem examined by M.N. Oguztoreli in [13]. An existence theorem of an optimal policy in the class of absolutely continuous initial state functions in the C space and measurable control functions in the L_2 space is proved. Further, the uniqueness of an optimal control in the case of linear delay-differential systems is considered.

概　　要

本報告では，状態と制御両者の過去の履歴に依存した非線系遅れ微分方程式

$$x(t)=\phi(t), \qquad \alpha\leq t\leq t_0$$
$$\dot{x}(t)=V(x_t(\cdot), u_t(\cdot), t), \qquad t_0<t\leq\gamma$$

で記述される制御系に対して，状態拘束条件を満たしつつ，積分評価基準

$$J(\phi, u)=\int_{t_0}^{t_f} V_0(x_t(\cdot), u_t(\cdot), t)dt, \quad t_0\leq t_f\leq\gamma$$

を最小にして，初期状態函数を時変標的集合に到達させる最適制御問題を定式化し，最適政策 (ϕ^*, u^*) の存在を，$V(x_t(\cdot), u_t(\cdot), t)$ の u に関する弱連続性および $V_0(x_t(\cdot), u_t(\cdot), t)$ の u に関する弱下半連続性という条件のもとで証明する。これは M.N. Oguztoreli によって定式化された遅れ微分制御系についての最適問題を一般的にした場合における最適政策の存在定理を与える。さらに，汎函数の弱連続性および弱下半連続性に関する十分条件を与え，線形遅れ微分方程式に対する解の一般的性質を示して，線形な遅れ微分制御系についての最適問題を取り上げ，初期状態函数を固定した場合における最適制御の一意性を，$J(u)$ が u の空間の上で厳密に凸であるという条件のもとに証明する。

1. INTRODUCTION

It is very usual to describe the mathematical model for a control process by a system of ordinary differential equations, under such the assumption that the future behavior of the physical process depends only upon the present state and control, the influence of which is instantaneous. However, there are not a few control systems of which evolution depends not only upon the present state and control, but also upon their past history. In general, the delay phenomena may occur in the case that the transmission of informations is related to the transfer of a material or a field. This kind of control processes should be formulated by a system of delay-differential equations.

Retarded actions are present in modern control problems for aerospace vehicles, and give rise to the following effect. In remote control of distant spacecrafts, the communication delay can adversely affect the stability of the overall control system [1]. Time delays in the engine response of large jet transports can seriously affect the handling qualities of aircrafts [2]. In manned systems, lagging commands caused by slow human response can bring a normally stable system to become unstable [3]. Hypervelocity entry vehicles can lose aerodynamic stability as a result of flowfield lags caused by the spacecraft motions and ablation [4]. Combution delays in rocket engines can lead to intermittant running, possibly terminating with an explosion of the engines [5]. Also in various branches of technology, economics, biology and medical science, the importance of studies for such the aftereffect has been recognized recently.

An optimal problem for delay-differential control systems has been considered first of all by G. L. Haratisvili [6], who obtained necessary conditions,

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the generalized maximum principle, in order to minimize the cost

$$\int_{t_0}^{t_f} f_0(x(t),\ x(t-\tau),\ u(t))dt,$$

to the case of control processes involving time-lag τ

$$(1.1) \qquad \dot{x}(t)=f(x(t),\ x(t-\tau),\ u(t)).$$

Thereafter, N.N. Krasovskii [7], M.N. Oguztoreli [8], E. Shimemura, et al [9]-[11], D.H. Chyung and E.B. Lee [12] have discussed in detail fundamental problems for linear differential-difference control systems

$$(1.2) \quad \dot{x}(t)=A(t)x(t)+B(t)x(t-\tau)+C(t)u(t).$$

Also, M.N. Oguztoreli [13] have given an existence theorem of an optimal policy such that it minimizes the cost

$$\int_{t_0}^{t_f} V_0(x(t),\ u(t),\ t)dt,$$

to the case of control processes generalized Eq. (1.1)

$$(1.3) \qquad \dot{x}(t)=V(x_t(\cdot),\ u(t),\ t).$$

On the other hand, A. T. Fuller [14] and R.E. Foerster [15] have considered an optimal problem for linear systems with a time delay in the control function

$$(1.4) \qquad \dot{x}(t)=Ax(t)+Bu(t-\tau).$$

Moreover, D.H. Chyung and E.B. Lee [16] have studied the properties of various sets of attainability and given the geometrical proof of the maximum principle, to the case of linear processes dependent on both the previous history of the state and of the control

$$(1.5) \quad \dot{x}(t)=\int_{-\tau}^{0} A_1(t,s)x(t+s)ds+A_2(t)x(t-\tau) + \int_{-\tau}^{0} B_1(t,s)u(t+s)ds+B_2(t)u(t-\tau).$$

As clarified in the works quoted above, the following difficulties take rise in optimal problems for delay-differential control systems, contrary to the case of ordinary differential systems. Firstly, in order to prescribe the state of such the systems as Eqs. (1.1)-(1.3) and (1.5), we have to know all the values of $x(s)$ on $t-\tau \leq s \leq t$, and hence the state space becomes infinite dimensional. Secondly, even in the stationary case, the characteristic equation of Eq. (1.2) is a transcendental equation, and hence there are in general infinitely many characteristic roots. Thirdly, as M.N. Oguztoreli pointed out in the optimal problem for Eq. (1.3), the value of the cost depends upon the choise not only of the control function, but also of the initial state function, and hence we should seek their optimal pair. Fourthly, such the systems with control delays as Eq. (1.4) are inherently uncontrollable in the initial time interval $[t_0, t_0+\tau]$, and also a kind of the optimal feedback control law becomes a function of the predicted state $x(t+\tau)$. Due to such the difficulties, the optimal problem for delay-differential control systems generalized Eq. (1.5)

$$(1.6) \qquad \dot{x}(t)=V(x_t(\cdot),\ u_t(\cdot),\ t)$$

has not ever been considered.

The present study is concerned with the optimal problem with the following data; (a) the control system which can be described by Eq. (1.6); (b) the cost

$$\int_{t_0}^{t_f} V_0(x_t(\cdot),\ u_t(\cdot),\ t)dt;$$

(c) the state constraint condition; (d) the time-varying target set. For this problem, we shall obtain an existence theorem of an optimal policy, an optimal pair of the initial state function and the control function. Also, to the case that V of Eq. (1.6) is linear with respect to x and u, we shall consider on the uniqueness of an optimal control such that it transfers a given initial state to the target point with minimum cost, satisfying the state constraint condition. The construction of an optimal policy shall be discussed in another article.

2. NOTATION, DEFINITIONS AND ASSUMPTIONS

Let α, β, γ and t_0 be fixed numbers such that

$$-\infty<\alpha<t_0\leq\beta<\gamma<\infty,$$

where $|\alpha|$ is sufficiently large. For any vector $z=(z_1, z_2, \cdots, z_p)\in E^p$, the norm is defined by

$$\|z\|=\left\{\sum_{j=1}^{p}|z_j|^2\right\}^{1/2}.$$

Also the norm of a functional space Y is denoted by $\|\cdot\|_Y$. Let G and R be sets as follows:

(a) G is a compact subset of E^n;

(b) R is a compact convex subset of E^r which contains the origin as an interior point.

We denote by Φ the set of all absolutely continuous functions $\phi\in C([\alpha, t_0], G)$, with derivatives $\dot{\phi}(t)$ such that

$$\|\dot{\phi}(t)\|\leq n(t), \quad \text{for a.e. } t\in[\alpha, t_0],$$

where $n(t)$ is a square Lebesgue integrable function for $t\in[\alpha, t_0]$. X denotes the set of all continuous functions $x\in C([\alpha,\gamma], E^n)$, and U the set of all measurable functions $u\in L_2([\alpha,\gamma], R)$. Moreover, for any $t\in[t_0,\gamma]$, let us represent the restriction of $x\in X$ and $u\in U$ on the interval $[\alpha, t]$ by $x_t(\cdot)$ and $u_t(\cdot)$, and let sets of all such $x_t(\cdot)$

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and $u_t(\cdot)$ denote by X_t and U_t, respectively.

We denote by S the set of all ordered triples (y, v, t) for $y \in X_t$, $v \in U_t$ and $t \in [t_0, \gamma]$. Let $V(y, v, t)$ and $V_0(y, v, t)$ be two Volterra functionals of S into E^n and of S into E^1, respectively, such that the following assumptions are satisfied:

(c) $V(y, v, t)$ is bounded on S, and $V(x_t(\cdot), u_t(\cdot), t)$ is measurable in t for each fixed $x \in X$ and $u \in U$ [13];

(d) $V(y, v, t)$ is strongly continuous in y for each fixed v and t; i.e., for any sequence $\{y^{(n)}\}$ of X_t which converges strongly to $y \in X_t$, the relation

$$\lim_{n\to\infty} \| V(y^{(n)}, v, t) - V(y, v, t) \| = 0$$

is valid [17];

(e) $V(y, v, t)$ is weakly continuous in v for each fixed y and t; i.e., for any sequence $\{v^{(n)}\}$ of U_t which converges weakly to $v \in U_t$, the relation

$$\lim_{n\to\infty} \| V(y, v^{(n)}, t) - V(y, v, t) \| = 0$$

is valid [17];

(f) There exists a square Lebesgue integrable function $m(t)$ for $t \in [t_0, \gamma]$ such that

$$\| V(y, v, t) \| \leq m(t),$$

uniformly with respect to y and v;

(g) $V(y, v, t)$ satisfies the Osgood condition with respect to y for each fixed v and t [13];

(h) $V_0(y, v, t)$ is nonnegative on S, and $V_0(x_t(\cdot), u_t(\cdot), t)$ is measurable in t for each fixed $x \in X$ and $u \in U$;

(i) $V_0(y, v, t)$ is strongly continuous in y for each fixed v and t;

(j) $V_0(y, v, t)$ is weakly lower semi-continuous in v for each fixed y and t; i.e., for any sequence $\{v^{(n)}\}$ of U_t which converges weakly to $v \in U_t$, the relation

$$\liminf_{n\to\infty} V_0(y, v^{(n)}, t) \geq V_0(y, v, t)$$

is valid [17];

(k) There exists a Lebesgue integrable function $m_0(t)$ for $t \in [t_0, \gamma]$ such that

$$V_0(y, v, t) \leq m_0(t),$$

uniformly with respect to y and v.

We note that weak continuity of a functional implies strong continuity and weak lower semi-continuity. The converse is not true in general. We also remark that the Osgood condition for a functional is satisfied only if it is locally Lipschitzian, and hence only if it is Gateaux differentiable [13].

3. FORMULATION OF OPTIMAL CONTROL PROBLEM

We consider the control system which can be described by the delay-differential equation

$$\dot{x}(t) = V(x_t(\cdot), u_t(\cdot), t), \quad \text{for } a.e.\ t \in (t_0, \gamma], \tag{3.1}$$

where V is a Volterra functional defined in §2.

Let an interval I be such that $[\alpha, t_0] \subseteq I \subseteq [\alpha, \gamma]$, and let $u(t)$ on I be a given control function. A state function $x(t)$ is called a solution of Eq. (3.1) on I corresponding to an initial function $\phi \in \Phi$, if

(i) $x(t) = \phi(t)$, for all $t \in [\alpha, t_0]$;

(ii) $x(t)$ is absolutely continuous on $I \cap [t_0, \gamma]$;

(iii) $x(t)$ satisfies Eq. (3.1) on $I \cap (t_0, \gamma]$.

Under the assumptions (a)-(d), (f) and (g), it is known in [13] that Eq. (3.1) has a unique solution $x(t) = x(t, t_0; \phi, u)$ on $[\alpha, \gamma]$ corresponding to each fixed $\phi \in \Phi$ and $u \in U$.

Let us denote by $D \subseteq E^n$ the state constraint region, and by $T(t)$ the time-varying target set which belongs to the interior of D for each $t \in [t_0, \gamma]$. When there exists $t_f \in [\beta, \gamma]$ corresponding to any choise of $\phi \in \Phi$ and $u \in U$, we say that a pair (ϕ, u) is an admissible policy, if

(i) corresponding to $\phi \in \Phi$ and $u \in U_{tf}$, the solution $x(t) = x(t, t_0; \phi, u)$ of Eq. (3.1) is defined on $[\alpha, t_f]$;

(ii) $x(t, t_0; \phi, u) \in D$, for all $t \in [\alpha, t_f]$;

(iii) $x(t_f, t_0; \phi, u) \in T(t_f)$.

We denote by P the set of all such pairs.

Then the optimal control problem can be formulated as follows, which will be refered to as problem (Q_1); find an optimal policy (ϕ^*, u^*) from the set P so as to minimize the cost functional defined on P by

$$J(\phi, u) = \int_{t_0}^{t_f} V_0(x_t(\cdot), u_t(\cdot), t)\,dt, \tag{3.2}$$

where V_0 is a Volterra functional defined in §2. That is, problem (Q_1) is to seek a pair $(\phi^*, u^*) \in P$ such that

$$J(\phi^*, u^*) \leq J(\phi, u), \text{ for each } (\phi, u) \in P.$$

4. EXISTENCE THEOREM

Let us consider the question on the existence of an optimal policy for problem (Q_1). Note that if P is empty, problem (Q_1) has no meaning. We need the following lemmas to obtain the main result.

Lemma 4.1 *Under the assumption* (a), Φ *is a compact subset of* $C([\alpha, t_0], G)$.

proof. Suppose that $\{\phi^{(i)}\}$ is any sequence of Φ. By the assumption (a), it is obvious that $\{\phi^{(i)}\}$ is uniformly bounded. Also since each $\phi^{(i)}(t)$ is absolutely continuous on $[\alpha, t_0]$ and the derivative $\dot{\phi}^{(i)}(t)$ satisfies the relation

$$\| \dot{\phi}^{(i)}(t) \| \leq n(t),$$

for a square integrable function $n(t)$ on $[\alpha, t_0]$, we

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have by Hölder's inequality

$$\|\phi^{(i)}(t_2)-\phi^{(i)}(t_1)\|=\left\|\int_{t_1}^{t_2}\dot\phi^{(i)}(t)dt\right\|$$
$$\leq|t_2-t_1|^{1/2}\left\{\int_\alpha^{t_0}\|\dot\phi^{(i)}(t)\|^2dt\right\}^{1/2}$$
$$\leq|t_2-t_1|^{1/2}\left\{\int_\alpha^{t_0}|n(t)|^2dt\right\}^{1/2},$$

for any t_1 and t_2, $\alpha\leq t_1$, $t_2\leq t_0$. This means that $\{\phi^{(i)}\}$ is equi-continuous. Hence by Ascoli-Arzela's theorem [18], Φ is relatively compact with respect to the uniform norm.

Next, suppose that $\{\phi^{(i)}\}$ is any sequence of Φ which converges strongly to $\phi\in C([\alpha,t_0],G)$. Since $\left\{\int_\alpha^{t_0}\|\dot\phi^{(i)}(t)\|^2dt\right\}$ is a bounded sequence of real numbers, and since $L_2[\alpha,t_0]$ is weakly complete, we can choose a subsequence $\{\dot\phi^{(i_k)}\}$ of $\{\dot\phi^{(i)}\}$ such that it converges weakly to $\psi\in L_2[\alpha,t_0]$. Thus we have for any $t\in[\alpha,t_0]$

$$\int_\alpha^t\psi(s)ds=\lim_{k\to\infty}\int_\alpha^t\dot\phi^{(i_k)}(s)ds$$
$$=\lim_{k\to\infty}(\phi^{(i_k)}(t)-\phi^{(i_k)}(\alpha))$$
$$=\phi(t)-\phi(\alpha).$$

Hence, ϕ is contained in Φ. This completes the proof.

Lemma 4.2 *Under the assumption* (b), U_t *is a weakly compact subset of* $L_2([\alpha,t],E^r)$ *for any* $t\in[t_0,\gamma]$.

proof. According to the definition mentioned in §2, U_t is a subset of $L_2([\alpha,t_0],E^r)$. Now, let t be any fixed number such that $t\in[t_0,\gamma]$. It follows from the assumption (b) that R is a closed, bounded and convex subset of E^r. Hence, it is clear that U_t is convex. Also by boundedness of R, there exists a positive number c for any $v\in U_t$ such that

$$\|v(\tau)\|\leq c,\quad \text{for a.e. } \tau\in[\alpha,t].$$

Thus according to the definition of the L_2 norm, it follows that

$$\|v\|_{L_2}=\left\{\int_\alpha^t\|v(\tau)\|^2d\tau\right\}^{1/2}\leq c|t-\alpha|^{1/2}.$$

This implies that U_t is bounded. Further U_t is strongly closed. Indeed, choose any sequence $\{v^{(i)}\}$ of U_t which converges strongly to $v\in L_2([\alpha,t],E^r)$. Then there exists a subsequence $\{v^{(i_k)}\}$ of $\{v^{(i)}\}$ such that

$$\lim_{k\to\infty}v^{(i_k)}(t)=v(t),$$

for almost all $t\in[\alpha,t]$ [19]. Since R is closed, $v(t)\in R$ for almost all $t\in[\alpha,t]$. Hence, v is a measurable function defined on $[\alpha,t]$ to R, so that $v\in L_2([\alpha,t],R)$.

It is well-known that a strongly closed, bounded and convex subset of $L_2([\alpha,t],E^r)$ is weakly compact [20]. U_t has these properties as shown above. This completes the proof.

Theorem 4.1 *In addition to the assumptions* (a) *to* (k) *in* §2, *suppose that the following conditions hold:*

(l) *D is a compact subset of E^n which contains G;*

(m) *$T(t)$ is a closed set which is upper semi-continuous with respect to inclusion* [21]; *i.e., for any $\varepsilon>0$ and $t\in[t_0,\gamma]$, there exists $\delta(\varepsilon,t)>0$ such that $T(t')$ belongs to an ε-neighborhood of the closed set $T(t)$, whenever $|t'-t|<\delta$.*

If P is nonempty, then there exists at least an optimal policy for problem (Q_1).

proof. (i) Let M denote the infimum over P of $J(\phi,u)$. M is finite. In fact, by the assumptions (h) and (k), we have on P

$$0\leq J(\phi,u)=\int_{t_0}^{t_f}V_0(x_t(\cdot),\ u_t(\cdot),\ t)dt$$
$$\leq\int_{t_0}^{t_f}m_0(t)dt.$$

Hence, since P is nonempty, we can choose a sequence $\{\phi^{(i)},u^{(i)}\}$ of P such that

$$M=\lim_{i\to\infty}\int_{t_0}^{t_f^{(i)}}V_0(x_t^{(i)}(\cdot),u_t^{(i)}(\cdot),t)\,dt, \tag{4.1}$$

where each $u^{(i)}$ is defined on $[\alpha,t_f^{(i)}]$ and each $x^{(i)}$ is the solution $x^{(i)}(t)=x(t,t_0;\phi^{(i)},u^{(i)})$ of Eq. (3.1) on $[\alpha,t_f^{(i)}]$.

(ii) Each $x^{(i)}(t)$ equivalently satisfies the following equation

$$x^{(i)}(t)=\begin{cases}\phi^{(i)}(t), & \text{for } t\in[\alpha,t_0];\\ \phi^{(i)}(t_0)+\int_{t_0}^t V(x_s^{(i)}(\cdot), u_s^{(i)}(\cdot),s)ds, & \text{for } t\in(t_0,t_f^{(i)}].\end{cases} \tag{4.2}$$

We now extend the definition of each $x^{(i)}(t)$ to $[\alpha,\gamma]$ by

$$V(x_t^{(i)}(\cdot),\ u_t^{(i)}(\cdot),\ t)=V(x_{t_f^{(i)}}^{(i)}(\cdot),\ u_{t_f^{(i)}}^{(i)}(\cdot),\ t_f^{(i)}),\ \text{for } t\in[t_f^{(i)},\gamma]. \tag{4.3}$$

Then, by the assumption (f) and Eq. (4.3), there exists a square integrable function $m(t)$ for $t\in[t_0,\gamma]$ such that

$$\|V(x_t^{(i)}(\cdot),u_t^{(i)}(\cdot),t)\|\leq m(t), \tag{4.4}$$

uniformly with respect to x and u. Since a sequence $\{\phi^{(i)}\}$ of Φ is uniformly bounded as shown in the proof of Lemma 4.1, there is a positive number c independent of i such that

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(4.5) $$\|\phi^{(i)}\|_C=\sup_{\alpha\le t\le t_0}\|\phi^{(i)}(t)\|\le c.$$

It follows from Eqs. (4.2)-(4.5) that for any i,

$$\|x^{(i)}(t)\|\le\begin{cases}c, & \text{for } t\in[\alpha,t_0];\\ c+\int_{t_0}^{t} m(s)ds\le c+\int_{t_0}^{\gamma} m(s)ds, & \text{for } t\in(t_0,\gamma],\end{cases}$$

and hence that by Hölder's inequality

$$\|x^{(i)}(t)\|\le\begin{cases}c, & \text{for } t\in[\alpha,t_0];\\ c+|\gamma-t_0|^{1/2}\left\{\int_{t_0}^{\gamma}|m(s)|^2ds\right\}^{1/2}, & \text{for } t\in(t_0,\gamma].\end{cases}$$

This inequality means the uniform boundedness of $\{x^{(i)}\}$ on the extended interval $[\alpha,\gamma]$. Further, we have for any i and for any t_1, t_2, $\alpha\le t_1$, $t_2\le\gamma$,

$$\|x^{(i)}(t_2)-x^{(i)}(t_1)\|=\left\|\int_{t_1}^{t_2}V(x_s^{(i)}(\cdot),u_s^{(i)}(\cdot),s)ds\right\|$$
$$\le\left|\int_{t_1}^{t_2}m(s)dt\right|\le|t_2-t_1|^{1/2}\left\{\int_{\alpha}^{\gamma}|m(s)|^2ds\right\}^{1/2},$$

which implies the equi-continuity of $\{x^{(i)}\}$ on $[\alpha,\gamma]$. Hence by Ascoli-Arzela's theorem, $\{x^{(i)}\}$ defined on the extended interval $[\alpha,\gamma]$ contains a subsequence such that it converges strongly to $x^*\in X$.

(iii) Let us extend the definition of each $u^{(i)}(t)$ to $[\alpha,\gamma]$ by

(4.6) $$u^{(i)}(t)=0,\quad \text{for } t\in(t_f^{(i)},\gamma].$$

Then by Lemma 4.2, we can extract a subsequence of $\{u^{(i)}\}$ on the extended interval $[\alpha,\gamma]$, which we will call $\{u^{(j)}\}$, such that it converges weakly to a measurable function $u^*\in U$. Since each $t_f^{(j)}$ is contains in the compact interval $[\beta,\gamma]$, there exists a subsequence of $\{t_f^{(j)}\}$, which we will call $\{t_f^{(k)}\}$, such that

$$\liminf_{k\to\infty} t_f^{(k)}=t_f^*,$$

where t_f^* belongs to $[\beta,\gamma]$. Further by Lemma 4.1, we can choose a subsequence of $\{\phi^{(k)}\}$, which will be called $\{\phi^{(l)}\}$, such that it converges strongly to $\phi^*\in\Phi$. Since $\{\phi^{(l)},u^{(l)}\}$ is still admissible, it follows from the step (ii) of this proof that there exists a subsequence of $\{x^{(l)}\}$, which we will call $\{x^{(m)}\}$, such that

(4.7) $$\lim_{m\to\infty}x^{(m)}(\cdot)=x^*(\cdot),\quad \text{strongly in X.}$$

It is obvious that the following limiting processes are still valid;

(4.8) $$\lim_{m\to\infty}u^{(m)}(\cdot)=u^*(\cdot),\quad \text{weakly in } U;$$

(4.9) $$\lim_{m\to\infty}\phi^{(m)}(\cdot)=\phi^*(\cdot),\quad \text{strongly in } \Phi;$$

(4.10) $$\liminf_{m\to\infty} t_f^{(m)}=t_f^*\in[\beta,\gamma].$$

It follows from the assumptions (d) and (e) that Eqs. (4.7) and (4.8) imply

(4.11) $$\lim_{m\to\infty}\|V(x_t^{(m)}(\cdot),u_t^{(m)}(\cdot),t)-V(x_t^*(\cdot),u_t^*(\cdot),t)\|=0,$$

for almost all $t\in[t_0,\gamma]$, according to definitions of strong and weak continuity mentioned in §2. Hence, by Lebesgue's dominated convergence theorem [20], we obtain

(4.12) $$\lim_{m\to\infty}\int_{t_0}^{t}V(x_s^{(m)}(\cdot),u_s^{(m)}(\cdot),s)ds=\int_{t_0}^{t}V(x_s^*(\cdot),u_s^*(\cdot),s)ds,$$

for any $t\in[t_0,\gamma]$. By Eqs. (4.2)-(4.3), (4.6)-(4.9) and (4.12), we have

(4.13) $$x^*(t)=\begin{cases}\phi^*(t), & \text{for } t\in[\alpha,t_0];\\ \phi^*(t_0)+\int_{t_0}^{t}V(x_s^*(\cdot),u_s^*(\cdot),s)ds, & \text{for } t\in(t_0,\gamma].\end{cases}$$

By Eqs. (4.2) and (4.13), we also see that the right-hand side of the inequality

$$\|x^{(m)}(t_f^{(m)})-x^*(t_f^*)\|\le\|\phi^{(m)}(t_0)-\phi^*(t_0)\|$$
$$+\int_{t_0}^{t_f^*}\|V(x_s^{(m)}(\cdot),u_s^{(m)}(\cdot),s)-V(x_s^*(\cdot),u_s^*(\cdot),s)\|ds$$
$$+\int_{t_f^*}^{t_f^{(m)}}\|V(x_s^{(m)}(\cdot),u_s^{(m)}(\cdot),s)\|ds$$

tends to zero as $m\to\infty$, from which follows that

(4.14) $$\lim_{m\to\infty}x^{(m)}(t_f^{(m)})=x^*(t_f^*).$$

Eqs. (4.13) and (4.14) mean that $x^*(t)=x(t,t_0;\phi^*,u^*)$ is the solution of Eq. (3.1) defined on $[\alpha,t_f^*]$ corresponding to $\phi^*\in\Phi$ and $u^*\in U_{t_f^*}$. Hence, $x^*(t)$ satisfies the condition (i) to the admissible policy.

Since $\{\phi^{(m)},u^{(m)}\}$ is admissible, each corresponding solution $x^{(m)}(t)$ of Eq. (3.1) belongs to D for any $t\in[\alpha,t_f^{(m)}]$. Also, Eq. (4.10) implies that

$$[\alpha,t_f^*]\subseteq[\alpha,t_f^{(m)}],$$

for all m sufficiently large. Hence it follows from the assumption (1) that $x^*(t)$ lies in D, or on the boundary of D for $t\in[\alpha,t_f^*]$, that is, $x^*(t)$ satisfies the condition (ii) to the admissible policy.

Further, the assumption (m) implies that the set Λ

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$$\Lambda=\{(t, z) : t \in [t_f^*, \gamma],\ z \in T(t)\}$$

is closed in E^{n+1} [21]. By the facts that $x^{(m)}(t_f^{(m)}) \in T(t_f^{(m)})$ and that $t_f^{(m)} \in [t_f^*, \gamma]$ for all m sufficiently large, we can choose $\{t_f^{(m)}, x^{(m)}(t_f^{(m)})\}$ as a sequence of Λ, which converges to $(t_f^*, x^*(t_f^*))$ by Eqs. (4.10) and (4.14). This limiting point belongs to Λ by closedness of Λ. That is,

$$x^*(t_f^*) \in T(t_f^*),$$

so that $x^*(t)$ satisfies the condition (iii) to the admissible policy.

Thus we have shown that (ϕ^*, u^*) is an admissible policy.

(iv) We must finally verify that $(\phi^*, u^*) \in P$ actually attains the infimum of $J(\phi, u)$. It follows from the assumptions (i) and (j) that Eqs. (4.7) and (4.8) imply

$$\begin{aligned}(4.15)\quad &\liminf_{m\to\infty} V_0(x_t^{(m)}(\cdot), u_t^{(m)}(\cdot), t)\\ &\geq V_0(x_t^*(\cdot), u_t^*(\cdot), t),\end{aligned}$$

for almost all $t \in [t_0, t_f^*]$, according to definitions of strong continuity and weak lower semi-continuity mentioned in §2. Since $[t_0, t_f^*] \subseteq [t_0, t_f^{(m)}]$ for all m sufficiently large, by the assumption (h) and Fatou's lemma [20], we obtain from Eq. (4.15) the relation

$$\begin{aligned}&\int_{t_0}^{t_f^*} V_0(x_t^*(\cdot), u_t^*(\cdot), t)dt\\ &\leq \int_{t_0}^{t_f^*} \liminf_{m\to\infty} V_0(x_t^{(m)}(\cdot), u_t^{(m)}(\cdot), t)dt\\ (4.16)\quad &\leq \liminf_{m\to\infty} \int_{t_0}^{t_f^*} V_0(x_t^{(m)}(\cdot), u_t^{(m)}(\cdot), t)dt\\ &\leq \lim_{m\to\infty} \int_{t_0}^{t_f^{(m)}} V_0(x_t^{(m)}(\cdot), u_t^{(m)}(\cdot), t)dt = M.\end{aligned}$$

On the other hand, it is obvious from Eq. (4.1) that

$$(4.17)\qquad M \leq \int_{t_0}^{t_f^*} V_0(x_t^*(\cdot), u_t^*(\cdot), t)dt.$$

Hence by Eqs. (4.16) and (4.17), we conclude that

$$\begin{aligned}M &= \lim_{m\to\infty} \int_{t_0}^{t_f^{(m)}} V_0(x_t^{(m)}(\cdot), u_t^{(m)}(\cdot), t)dt\\ &= \int_{t_0}^{t_f^*} V_0(x_t^*(\cdot), u_t^*(\cdot), t)dt\\ &= J(\phi^*, u^*).\end{aligned}$$

Since (ϕ^*, u^*) is an admissible policy as shown in the step (iii) of this proof, (ϕ^*, u^*) actually attains the infimum of $J(\phi, u)$ on P. That is, (ϕ^*, u^*) is an optimal policy for problem (Q_1). This completes the proof.

5. OPTIMAL PROBLEM FOR LINEAR DELAY-DIFFERENTIAL CONTROL SYSTEMS

We now consider an optimal problem for linear delay-differential control systems as an application of Theorem 4.1. Such the systems appear in not a few control problems for aerospace vehicles. We need the following preliminaries before taking up the main subject.

A functional $f(y)$ defined on a linear normed space Y is said to be linear, if for any pair of arbitrary real constants λ and μ,

$$f(\lambda y+\mu y')=\lambda f(y)+\mu f(y'),$$

whenever $y, y' \in Y$, and bounded if there exists a real constant $M>0$ such that for all $y \in Y$

$$\|f(y)\| \leq M\|y\|_Y.$$

Also, a functional $f(y)$ defined on a convex subset C of Y is said to be convex, if for any λ, $0\leq\lambda\leq1$,

$$(5.1)\qquad f(\lambda y+(1-\lambda)y')\leq\lambda f(y)+(1-\lambda)f(y'),$$

whenever $y, y' \in C$, and strictly convex if Eq. (5.1) satisfies the equality when and only when $y=y'$. Further, let a functional $f(y)$ be defined on $C'\subset Y$. If for any $y \in C'$ there exists a $y^0 \in C'$ such that

$$f(y^0)\leq f(y),$$

y^0 is said to attain a global minimum of $f(y)$ on C'.

Strong continuity of linear functionals is equivalent to boundedness. If $f(y)$ is a bounded linear functional, there exists an $M>0$ such that for any $y, y' \in Y$

$$\|f(y)-f(y')\| = \|f(y-y')\| \leq M\|y-y'\|_Y,$$

which means that $f(y)$ satisfies the Lipchitz condition. Hence, we have the following lemma.

Lemma 5.1 *Bounded linear functionals defined on a linear normed space satisfy the Osgood condition.*

We also prove the following lemmas for weak continuity and weak lower semi-continuity of functionals.

Lemma 5.2 *Bounded linear functionals $f(y)$ defined on a linear normed space Y are weakly continuous.*

proof. Suppose that $\{y^{(n)}\}$ is any sequence of Y which converges weakly to $y \in Y$. Then, we can find a subsequence $\{y^{(k)}\}$ of $\{y^{(n)}\}$ such that the arithmetic mean

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$$\frac{1}{m}\sum_{k=1}^{m} y^{(k)}$$

converges strongly to y [22]. Also, it follows from linearity of $f(y)$ that

$$\frac{1}{m}\sum_{k=1}^{m} f(y^{(k)}) = f\left(\frac{1}{m}\sum_{k=1}^{m} y^{(k)}\right).$$

Hence by strong continuity of $f(y)$, we obtain

$$\lim_{k\to\infty} f(y^{(k)}) = \lim_{m\to\infty} \frac{1}{m}\sum_{k=1}^{m} f(y^{(k)}) = \lim_{m\to\infty} f\left(\frac{1}{m}\sum_{k=1}^{m} y^{(k)}\right) = f(y),$$

from which follows that even for all the n

(5.2) $$\lim_{n\to\infty} f(y^{(n)}) = f(y)$$

is valid. In fact, suppose for contradiction that Eq. (5.2) is not true. Then, we can extract a subsequence $\{y^{(i)}\}$ of $\{y^{(n)}\}$ such that for all the i and for any $\varepsilon > 0$

(5.3) $$\|f(y^{(i)}) - f(y)\| \geq \varepsilon.$$

However, since $\{y^{(i)}\}$ also converges weakly to y, there exists a subsequence $\{y^{(j)}\}$ of $\{y^{(i)}\}$ such that

$$\|f(y^{(j)}) - f(y)\| < \varepsilon,$$

according to the above discussion. This contradicts Eq. (5.3), so that Eq. (5.2) is valid. This completes the proof.

Lemma 5.3 [22] *Functionals $f(y)$ defined on a convex subset C of a linear normed space are weakly lower semi-continuous, if $f(y)$ is strongly continuous and convex.*

proof. Suppose that $\{y^{(n)}\}$ is any sequence of C which converges weakly to $y \in C$. It contains a subsequence $\{y^{(k)}\}$ such that

$$\frac{1}{m}\sum_{k=1}^{m} y^{(k)}$$

converges strongly to y. Also, it follows from convexity of $f(y)$ that

$$\frac{1}{m}\sum_{k=1}^{m} f(y^{(k)}) \geq f\left(\frac{1}{m}\sum_{k=1}^{m} y^{(k)}\right).$$

Hence, by strong continuity of $f(y)$, we have

$$\liminf_{k\to\infty} f(y^{(k)}) = \liminf_{m\to\infty} \frac{1}{m}\sum_{k=1}^{m} f(y^{(k)}) \geq \liminf_{m\to\infty} f\left(\frac{1}{m}\sum_{k=1}^{m} y^{(k)}\right) = f(y),$$

from which follows that even for all the n, the relation

$$\liminf_{n\to\infty} f(y^{(n)}) \geq f(y)$$

is valid. This completes the proof.

Now, let us consider as a particular case of Eq. (3.1) linear delay-differential control systems which can be described by

(5.4) $$\dot{x}(t) = V_1(x_t(\cdot), t) + V_2(u_t(\cdot), t), \text{ for } t \in (t_0, \gamma],$$

where V_1 and V_2 are continuous with respect to t, bounded for any $t \in (t_0, \gamma]$ and linear with respect to x and u, respectively. Further, let us consider the following equation

(5.5) $$\dot{\bar{x}}(t) = V_1(\bar{x}_t(\cdot), t), \text{ for } t \in (t_0, \gamma].$$

We denote by $x(t, t_0;\ \phi, u)$ the solution of Eq. (5.4) with the initial function $\phi \in \Phi$ and the control function $u \in U$, and by $\bar{x}(t, t_0;\ \phi)$ the solution of Eq. (5.5) with the initial function $\phi \in \Phi$. Let us define the function $z(t)$ by

$$z(t) = x(t, t_0;\ \phi, u) - \bar{x}(t, t_0;\ \phi).$$

Since V_1 is linear with respect to x, we have by Eqs. (5.4) and (5.5)

$$\begin{aligned}\dot{z}(t) &= V_1(x_t(\cdot, t_0;\ \phi, u), t) + V_2(u_t(\cdot), t) \\ &\quad - V_1(\bar{x}_t(\cdot, t_0;\ \phi), t) \\ &= V_1(z_t(\cdot), t) + V_2(u_t(\cdot), t).\end{aligned}$$

Therefore, $z(t)$ is a solution of Eq. (5.4) which corresponds to the identically zero initial function $z(t) = 0$ for all $t \in [\alpha, t_0]$ and the control function u, so that we can denote as $z(t) = x(t, t_0;\ 0, u)$. Hence, we obtain the relation

(5.6) $$x(t, t_0;\ \phi, u) = x(t, t_0;\ 0, u) + \bar{x}(t, t_0;\ \phi).$$

Also, by convexity of U_t, we have for any λ, $0 \leq \lambda \leq 1$,

$$u_t(\cdot) = \lambda u_t^1(\cdot) + (1-\lambda) u_t^2(\cdot) \in U_t,$$

whenever $u_t^1, u_t^2 \in U_t$. Since V_1 and V_2 are linear with respect to x and u, respectively, the solution $x(t, t_0;\ 0, u^1)$ of Eq. (5.4) satisfies

$$\begin{aligned}\lambda \dot{x}(t, t_0;\ 0, u^1) &= \lambda V_1(x_t(\cdot, t_0;\ 0, u^1), t) \\ &\quad + \lambda V_2(u_t^1(\cdot), t) \\ &= V_1(\lambda x_t(\cdot, t_0;\ 0, u^1), t) + V_2(\lambda u_t^1(\cdot), t),\end{aligned}$$

from which follows that

(5.7) $$\lambda x(t, t_0;\ 0, u^1) = x(t, t_0;\ 0, \lambda u^1).$$

As the same way, the solution $x(t, t_0;\ 0, u^2)$ of

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Eq. (5.4) satisfies

(5.8) $(1-\lambda)x(t, t_0; 0, u^2)=x(t, t_0; 0, (1-\lambda)u^2)$.

Let us define the function $w(t)$ by

$$w(t)=x(t, t_0; 0, \lambda u^1)+x(t, t_0; 0, (1-\lambda)u^2).$$

Since V_1 and V_2 are linear with respect to x and u, respectively, we have by Eq. (5.4)

$$\begin{aligned}\dot{w}(t)= &V_1(x_t(\cdot, t_0; 0, \lambda u^1), t)+V_2(\lambda u_t^1(\cdot), t)\\ &+V_1(x_t(\cdot, t_0; 0, (1-\lambda)u^2), t)\\ &+V_2((1-\lambda)u_t^2(\cdot), t)\\ =&V_1(w_t(\cdot), t)+V_2(u_t(\cdot), t).\end{aligned}$$

Therefore, $w(t)$ is a solution of Eq. (5.4) which corresponds to the identically zero initial function $w(t)=0$ for all $t\in[\alpha, t_0]$ and the control function $u=\lambda u^1+(1-\lambda)u^2$, so that we can denote as $w(t)=x(t, t_0; 0, u)$. Hence, we obtain the relation

$$(5.9)\qquad x(t, t_0; 0, u)=x(t, t_0; 0, \lambda u^1)+x(t, t_0; 0, (1-\lambda)u^2).$$

Easily from Eqs. (5.6)-(5.9), we have the following lemma.

Lemma 5.4 *Let us assume that V_1 and V_2 of Eq.* (5.4) *are continuous with respect to t, bounded for any $t\in(t_0, \gamma]$ and linear with respect to x and u, respectively. Then, for any u^1, $u^2\in U$ and for any λ, $0\le\lambda\le1$, the following relation is valid, among the solutions of Eq.* (5.4) *with the initial function $\phi\in\Phi$ which correspond to the control functions u^1, u^2 and $u=\lambda u^1+(1-\lambda)u^2$;*

$$x(t, t_0; \phi, u)=\lambda x(t, t_0; \phi, u^1)+(1-\lambda)x(t, t_0; \phi, u^2).$$

Under the preliminaries mentioned above, we consider the optimal control problem (Q_2) with the following data:

(D_1) a given initial function $\phi\in\Phi$;

(D_2) the terminal condition $x(t_f)=x_f$, where t_f and x_f are given;

(D_3) the state constraint region $D\subseteq E^n$;

(D_4) the state equation described by Eq. (5.4);

(D_5) the cost functional

$$(5.10)\qquad J(u)=\int_{t_0}^{t_f} V_0(x_t(\cdot), u_t(\cdot), t)dt.$$

We say that $u\in U$ is an admissible control for problem (Q_2), if the solution $x(t)=x(t, t_0; \phi, u)$ of Eq. (5.4) belongs to D for all $t\in[t_0, t_f]$ and satisfies $x(t_f)=x_f$. Let us denote by P all the admissible controls. If there exists a $u^*\in P$ such that for all $u\in P$

$$J(u^*)\le J(u),$$

then u^* attains the global minimum cost for problem (Q_2). When the set of all such u^* consists of only one element, the optimal control is decided uniquely. We now prove the following theorem on the existence and uniqueness of an optimal control for problem (Q_2).

Theorem 5.1 *In addition to the assumptions* (a)-(b), (h)-(i) *and* (k) *in* §2, *suppose that the following conditions hold:*

(A) *V_1 and V_2 of Eq.* (5.4) *are continuous with respect to t, bounded for any $t\in(t_0, \gamma]$ and linear with respect to x and u, respectively;*

(B) *V_0 of Eq.* (5.10) *is strongly continuous and convex with respect to u;*

(C) *D is a compact convex subset of E^n;*

(D) *$J(u)$ of Eq.* (5.10) *is strictly convex on U.*

If P is nonempty, then there exists a unique optimal control for problem (Q_2).

proof. By Lemmas 5.1 and 5.2, the assumptions (c)-(g) in §2 follow from the condition (A), and by Lemma 5.3, the assumption (j) follows from the condition (B). Hence, all the assumptions of Theorem 4.1 are satisfied under the conditions of this theorem. Thus there exists at least an optimal control $u^*\in P$ for problem (Q_2) such that for all $u\in P$

$$J(u^*)\le J(u).$$

Let us denote by Ω the set of all such u^*. For the proof of the uniqueness, it will suffice only to show that Ω actually consists of only one element.

Firstly for this, we see that $J(u)$ of Eq. (5.10) can be defined on the convex hull of Ω. Let us define the function $v=\lambda v^1+(1-\lambda)v^2$, for any v^1, $v^2\in\Omega$ and for any λ, $0\le\lambda\le1$. Then it is clear that $v\in U$. Hence by Lemma 5.4, the following relation is valid, among the solutions of Eq. (5.4) with the initial function ϕ which correspond to the control functions v^1, v^2 and v;

$$(5.11)\qquad x(t, t_0; \phi, v)=\lambda x(t, t_0; \phi, v^1)+(1-\lambda)x(t, t_0; \phi, v^2).$$

Since v^1 and v^2 are admissible, both the corresponding solutions $x(t, t_0; \phi, v^1)$ and $x(t, t_0; \phi, v^2)$ belong to D for all $t\in[t_0, t_f]$. Hence by the condition (C) and Eq. (5.11), the solution $x(t, t_0; \phi, v)$ of Eq. (5.4) corresponding to v also belongs to D for all $t\in[t_0, t_f]$. Further, from Eq. (5.11) follows that at $t=t_f$,

$$x(t_f, t_0; \phi, v)=\lambda x(t_f, t_0; \phi, v^1)+(1-\lambda)x(t_f, t_0; \phi, v^2)=x_f$$

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is valid, so that the solution $x(t, t_0; \phi, v)$ of Eq. (5.4) satisfies the terminal condition (D_2). Thus the function represented by $v=\lambda v^1+(1-\lambda)v^2$ is an admissible control for any λ, $0\leq\lambda\leq 1$. This means that $J(u)$ is defined on the convex hull of Ω.

We next show that Ω consists of only one element. Suppose for contradiction that there exist distinct elements v^1 and v^2 in Ω. Then by the condition (D), the relation

$$J(\lambda v^1+(1-\lambda)v^2)<J(v^1)=J(v^2)$$

is satisfied for any λ, $0<\lambda<1$. On the other hand, it is obvious according to the definition of Ω that for any λ, $0<\lambda<1$,

$$J(v^1)=J(v^2)\leq J(\lambda v^1+(1-\lambda)v^2),$$

which is contradiction. This completes the proof.

Now, let us give a concrete example of the cost functional $J(u)$ which satisfies the conditions of Theorem 5.1. For any elements y and y' of a linear normed space Y and for any λ, $0\leq\lambda\leq 1$, the following relation is valid;

$$\begin{aligned}\|\lambda y+(1-\lambda)y'\|_Y^2 &\leq \lambda^2\|y\|_Y^2+2\lambda(1-\lambda)\|y\|_Y\|y'\|_Y \\ &\quad +(1-\lambda)^2\|y'\|_Y^2 \\ &=\lambda\|y\|_Y^2+(1-\lambda)\|y'\|_Y^2-\lambda(1-\lambda) \\ &\quad \times(\|y\|_Y-\|y'\|_Y)^2 \\ &\leq \lambda\|y\|_Y^2+(1-\lambda)\|y'\|_Y^2.\end{aligned} \tag{5.12}$$

Hence, $\|\cdot\|_Y^2$ is convex. Thus the functional

$$V_0(x_t(\cdot),\ u_t(\cdot),\ t)=\|x_t(\cdot)\|_{X_t}^2+\|u_t(\cdot)\|_{U_t}^2$$

satisfies the conditions on V_0 of Theorem 5.1. Let us consider the cost functional corresponding to the above

$$J(u)=\int_{t_0}^{t_f}(\|x_t(\cdot)\|_{X_t}^2+\|u_t(\cdot)\|_{U_t}^2)dt. \tag{5.13}$$

From Lemma 5.4 and Eq. (5.12) follows that for any u^1, $u^2\in U$ and for any λ, $0<\lambda<1$

$$\begin{aligned}J(\lambda u^1+(1-\lambda)u^2)&=\int_{t_0}^{t_f}(\|\lambda x_t(\cdot, t_0;\ \phi,\ u^1) \\ &\quad +(1-\lambda)x_t(\cdot, t_0;\ \phi,\ u^2)\|_{X_t}^2 \\ &\quad +\|\lambda u_t^1(\cdot)+(1-\lambda)u_t^2(\cdot)\|_{U_t}^2)dt \\ &\leq\lambda J(u^1)+(1-\lambda)J(u^2),\end{aligned}$$

the last equality of which is satisfied if and only if for all $t\in[t_0, t_f]$ the relations

$$\begin{aligned}&\|\lambda x_t(\cdot, t_0;\ \phi,\ u^1)+(1-\lambda)x_t(\cdot, t_0;\ \phi,\ u^2)\|_{X_t} \\ &\quad =\lambda\|x_t(\cdot, t_0;\ \phi,\ u^1)\|_{X_t} \\ &\qquad +(1-\lambda)\|x_t(\cdot, t_0;\ \phi,\ u^2)\|_{X_t}; \\ &\|\lambda u_t^1(\cdot)+(1-\lambda)u_t^2(\cdot)\|_{U_t}=\lambda\|u_t^1(\cdot)\|_{U_t} \\ &\qquad +(1-\lambda)\|u_t^2(\cdot)\|_{U_t}\end{aligned}$$

are identically valid, that is, when and only when $u^1=u^2$. Hence, $J(u)$ of Eq. (5.13) is strictly convex on U. Thus we have the following;

Theorem 5.2 *Suppose that the assumptions* (a)-(b) *in* § 2 *and the conditions* (A), (C) *hold. If P is nonempty, then there exists a unique optimal control for problem* (Q_2) *with the cost functional of Eq.* (5.13).

Finally, we give somewhat concrete representations for linear delay-differential control systems described by Eq. (5.4). The control system with time delays τ_1 and τ_2

$$\dot{x}(t)=A(t)x(t-\tau_1)+B(t)u(t-\tau_2)$$

is a particular case of Eq. (5.4). Further by Riesz's representation theorem, Eq. (5.4) has under the condition (A) of Theorem 5.1 the form

$$\dot{x}(t)=\int_\alpha^t d_\tau A(\tau, t)x(\tau)+\int_\alpha^t B(\tau, t)u(\tau)d\tau,$$

where $A(\tau, t)$ is an $n\times n$ matrix, all of whose elements are of bounded variation uniquely determined by the functional V_1, and $B(\tau, t)$ is an $n\times r$ matrix, all of whose elements are the functions of L_2 uniquely determined by the functional V_2. The integration is performed in the sense of Stieltjes with respect to τ for fixed t. The equation of the form

$$\begin{aligned}\dot{x}(t)=A_1(t)x(t-\tau_1)&+\int_\alpha^t A_2(t-\tau)x(\tau)d\tau \\ &+\int_\alpha^t B(t-\tau)u(\tau)d\tau\end{aligned}$$

is a special case of the above.

6. CONCLUDING REMARKS

In this paper, we formulated the optimal problem for control processes which can be described by a system of general delay-differential equations, and gave a theorem on the existence of an optimal policy. This problem is a generalization of one formulated by M.N. Oguztoreli [13], since we take into consideration the past history not only of the state, but also of the control. For Oguztoreli's optimal control problem with the state equation

$$\dot{x}(t)=V(x_t(\cdot),\ u(t),\ t)$$

and the cost functional

$$J(\phi, u)=\int_{t_0}^{t_f}V_0(x(t),\ u(t),\ t)dt,$$

there exists an optimal policy, under the condition that the set

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$$\bar{V}(x_t(\cdot), R, t) = \{V_0(x(t), u(t), t),\ V(x_t(\cdot), u(t), t):\ u \in U\}$$

is convex for any $t \in [t_0, \gamma]$ and $x(t) \in D$, which is the straightforward extension of the existence condition of a time optimal control for ordinary differential processes due to A.F. Filippov [23]. On the other hand, for the problem formulated in §3, an optimal policy exists, as shown in §4, under the conditions that V of Eq. (3.1) is weakly continuous with respect to u, and that V_0 of Eq. (3.2) is weakly lower semi-continuous with respect to u.

In §5, we gave the sufficient conditions on weak continuity and weak lower semi-continuity of functionals, and the general property on the solution of linear delay-differential equations. Further, we considered an optimal problem for linear delay-differential control systems, for which the uniqueness of an optimal control is guaranteed under the condition that $J(u)$ of Eq. (5.10) is strictly convex on U.

Finally, we remark distinctly that the present study is concerned not with the state control problem, but with the output control problem. An optimal policy (ϕ^*, u^*) for our problem is such that the control u^* transfers the initial state ϕ^* to the target set T with minimum cost, satisfying the state constraint condition, and the terminal condition $x^*(t_f^*) \in T(t_f^*)$ is finite dimensional. On the other hand, in the case of the state control problem, we have to prescribe the state after transfer, for instance, by such the condition as $x^*(t) \in T(t)$ for all t, $t \geq t_f^*$, which is infinite dimensional. That is where difficulty takes rise in the state control problem, which will become the important subject of studies for delay-differential control systems.

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