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An Experimental and Analytical Study of Blade Tip-Clearance Effects on an Axial-Flow Turbine Performance

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CONTENTS

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ABSTRACT

In order to investigate the effects of rotor tip-clearance on a highly-loaded axial-flow turbine, for application to high-temperature engines, a cold-air test on a single-stage axial-flow turbine was conducted with three rotor tip-clearance to rotor blade height ratios in the range of 70 ~ 110% turbine equivalent speeds and 1.4 ~ 2.2 turbine equivalent total-to-total pressure ratios. The radial tip-clearance of the rotor was changed by increasing the diameter of the rotor shroud ring, while keeping the rotor blade tip diameter unchanged.

The main results of the test are as follows;

1) As the ratio of the rotor tip-clearance increased, the turbine inlet mass flow increased and the turbine adiabatic efficiency, based on the turbine torque measured by the 1600 kW-electric dynamometer, decreased. Turbine efficiency of 85.5%, 84.2% and 83.0% was obtained for 1.5%, 2.6% and 4.2% rotor tip-clearance ratios, at the designed equivalent turbine speed and equivalent turbine pressure ratio. Corresponding maximum turbine efficiency obtained at the designed mean wheel speed-to-isentropic velocity ratio was 86.5%, 84.7% and 83.8% respectively.

2) Fine measurements of the gas flow state in the turbine stage show that the region of blade tip-leakage flow inefficiency extends to the mid-span of flow passage. The decrease in the relative outlet flow angle from the rotor blades is remarkable in the corresponding inefficiency region.
In a comparison of the present experimental results with the theoretical and empirical equations of other authors, the following equation proved to be most appropriate in describing the rotor blade tip-clearance effect on the turbine efficiency;

$$\Delta\eta_T = \eta_T - \eta_{T0} = -B\frac{\eta_T\,[C_L/(s/c)]_R^2 \sec^3\beta_{m,R}}{(V_{ad}/U_m)^2(U_m/V_{a3})^2}\left(\frac{k}{h}\right)_R$$

where,

$B = 0.5$

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** Aeroengine Division

For multi-stage turbo-machinery with any degree of reaction, the above equation can be expressed in general form as follows;

$$\Delta\eta_T = \frac{-\eta_T}{(V_{ad}/U_m)^2} \sum_{i=1}^{i} \left\{ \frac{B_N [C_L/(s/c)]_N^2 \sec^3 \alpha_{m,N}}{(U_m/V_{a2})^2} \left(\frac{k}{h}\right)_N + \frac{B_R [C_L/(s/c)]_R^2 \sec^3 \beta_{m,R}}{(U_m/V_{a3})^2} \left(\frac{k}{h}\right)_R \right\}_i$$

where i is the stage number.

概　　要

空冷式高温タービン用の高負荷の一段軸流タービンについて，その空力性能に及ぼす動翼先端間隙の影響を調べるため常温空気による試験を行った。寸法の異なる三種の動翼シュラウドリングを用いて1.5％，2.6％，4.2％の先端間隙の場合について，本試験タービンの全体性能及び内部流動を詳細に測定した。その結果，動翼先端間隙はタービンの断熱効率等に大きく影響を与えるが，高負荷タービンという特殊性によると思われる異常な効率の低下はないことが判った。また，内部流動の結果から，その影響は翼の先端付近に限らず，翼の平均径近傍にまで達していることが判明した。また，翼先端間隙の効率に及ぼす影響を試算する各種の式を系統的に整理し本報の試験結果とも比較すると共に，最適と思われる計算式を提示した。

SYMBOLS

A	: Area
A_a	: Annular area
A_k, A_k'	: Annular area of blade tip-clearance, effective area of blade tip-clearance
B	: Coefficient of function of blade drag coefficient in formula (18) for blade tip-clearance effect
C_D, D_{DO}	: Drag coefficient of blade; formula (14), C_D when $k = 0$
C_{Dk}	: Blade drag coefficient due to blade tip-clearance
C_L	: Lift coefficient of blade; formula (15)
c	: Blade chord length
c_p	: Constant pressure specific heat
D	: Diameter
D_{sh}	: Shroud ring inner diameter; see Figure 9
d_L, d_T	: Leading edge, Trailing edge blade diameters
G	: Inlet mass flow of turbine
g	: Gravitational acceleration
H_t	: Total enthalphy
ΔH_T	: Turbine thermal head
h	: Blade height
i	: i stage or i number
J	: Work equivalent of heat; 426.9 kg-m/kcal
k, k/h	: Blade tip-clearance, blade tip-clearance ratio
$I\!K$	: Flow coefficient through blade tip-clearance; formula (B-4)
K	: Ratio of lift maintained at blade tip versus two-dimensional value of blade lift [10]
L_T	: Turbine output
ΔL	: Size of overlapping part; see Appendix Figure 2
l	: Arm length of dynamometer; 1.4606 m

M : Mach number

N : Turbine revolutions per minute

N^* : Ratio of actual corrected revolutions versus design turbine coerrected revolutions;
$N^* = (N/\sqrt{\theta_{cr}}) / (N/\sqrt{\theta_{cr}})_{des}$

n : Number of blades

O : Blade throat width

P_s, P_t : Static pressure, total pressure

P_L, P_R, P_C : Pressures measured at left, right and center of three-hole pitot tube

R : Gas constant

r : Radius

$I\!R$: Drag coefficient of flow through blade tip-clearance

s : Blade pitch

T_s, T_t : Static temperature, total temperature

U : Peripheral velocity

U_m : Mean peripheral velocity

V : Absolute velocity

V_{a2}, V_{a3} : Axial velocity at stator outlet, axial velocity at rotor outlet; see Appendix Figure 1

V_{ad} : Adiabatic theoretical velocity corresponding to total stage expansion ratio π_T of turbine

V_0 : Velocity corresponding to turbine thermal head; $V_0^2 = \eta_T V_{ad}^2$

W : Relative velocity

W_T : Dynamometer load

Y_t : Total pressure loss coefficient of blade

Greek letters:

α : Absolute outflow angle

α_m : Mean value based on formula (7) of absolute inlet flow angle α_1 and absolute outlet flow angle α_2 of stator blade row

β : Relative outflow angle

β_m : Mean value based on formula (7) of relative inlet flow angle β_2 and relative outlet flow angle β_3 of rotor blade row

γ : Specific weight

δ : Ratio of total turbine inlet pressure versus standard pressure

δ^* (δ_s^*, δ_p^*): Boundary layer displacement thickness (blade suction side δ_s^*, blade pressure side δ_p^*)

ϵ_{cr} : Mass correction coefficient; refer to section 3.4

η : Turbine adiabatic efficiency

η_T : Turbine adiabatic efficiency based on ratio of total stage expansion and total enthalpy drop of turbine; formula (6)

η_{To} : η_T when blade tip-clearance $k = 0$

$\Delta\eta_T$: Difference between η_T and η_{To} ;
$\Delta\eta_T = \eta_T - \eta_{To}$

η_t : Turbine adiabatic efficiency calculated from dynamometer output and total turbine stage expansion ratio π_T

η_{1-3} : Adiabatic (local) temperature efficiency based on total temperature ratio, total pressure ratio; see section 3.4

θ_{cr} : Velocity ratio; see section 3.4

κ : Specific heat ratio

λ : Ratio of $-\Delta\eta_T$ to (k/h);
$\lambda = -\Delta\eta_T/(k/h)$

λ_R : λ of rotor blade; formula (1)

ξ : Stagger angle of blade row

π_T : Total expansion ratio of turbine (total inlet pressure/total outlet pressure)

ρ_R : Degree of reaction;
$\rho_R = 1 - (W_2/W_1)^2$

τ : Turbine torque

φ : Velocity coefficient of blade

ϕ : Axial flow velocity ratio;
$\phi = V_a/U_m$

$\phi_{cr}\pi_T$: Total stage corrected expansion ratio

ψ : Blade load coefficient

Subscripts

a : Axial direction or annular

ad	: Adiabatic
cr	: Value at $M = 1.0$
des	: Design
i	: *i*-th number
k	: Blade tip-clerance; see Figure 9 Fig-
m	: Mean
M	: Mean blade value
max	: Maximum
N	: Stator blade
R	: Rotor blade
r	: Relative
s	: Static or suction
st	: Standard state
t	: Stagnant (total) or turbine
thr	: Blade throat
Tip	: Blade tip
u	: Peripheral velocity or peripheral direction
w	: due to flow in blade height direction
0	: Value when $k = 0$ or orifice position (section 3.4, Figure 4)
1	: before stator blade (turbine inlet position)
2	: behind stator blade or before rotor blade
3	: behind rotor blade (turbine outlet position)
–	: Arithmetic mean

1. INTRODUCTION

In order to improve the cycle performance of jet engines used in aircraft, for example, the cycle maximum temperature, that is the inlet gas temperature of the high pressure turbine, should be as high as possible. The energy of this high temperature gas is converted into work by the turbine, and it is desirable that this be accomplished by turbines with as few stages as possible. For this reason, the amount of work per turbine stage should be as great as possible; each blade of the turbine should operate under a high loading condition. The turbine of this study is a relatively high loaded turbine, which has been designed for the operation in such conditions that the turbine blades are characterized as follows: I) The surface pressure difference between pressure side and suction side surfaces of the blade is great. II) Gas expansion through the blade row is large. III) The gas flow is greatly deflected by the blade row. IV) The flow through the blade row is of high density, low volume at the high pressure operational condition and the turbine flow path, therefore, becomes small, so that the blade dimension should be fairly small. V) The blade chord length should be large, while the blade height should be low, because of the requirement for the high loading and large gas deflection, so that the blade should have a small aspect ratio. All of these characteristics required for the high temperature turbine blades would result in a greater efficiency decline due to the existence of blade tip-clearance.

It is important to note, in determining the turbine performance, that the thermal expansion of the turbine parts, that is blades, disk, inner and outer casings and so on, would result in different change in size of the rotor tip-clearance between in the performance test condition using unheated compressed air and in the actual turbine operating condition of high temperature burning gas. Moreover, care must be taken so that the rotor blade tip does not contact the outer shroud casing under high temperature operational condition and also at the starting condition, taking into consideration the thermal expansion of turbine parts. This leads to that the rotor blade tip-clearance should originally be kept in the design stage. In general, the blade pressure loss due to the blade tip-clearance is a major portion of the overall pressure loss and, especially in the case of a high loaded turbine with the characteristics indicated previously, a greater efficiency decline due to the blade tip-clearance would be brought about.

In order to investigate this problem, a single stage high loaded turbine for aeroengines were tested, changing the tip clearance of rotor blades in three manners. The experimental results were divided into overall performance results and measurements of the internal gas flow conditions at the rotor outlet. In addition, various calculation methods were consolidated which would predict the decline in turbine efficiency due to the rotor blade tip-clearance, and these were compared with the present experimental results. The overall performance results indicated a decline in turbine efficiency due to increased rotor tip-clearance. However, abnormal declines in efficiency due to the high blade loading were not evident. The internal flow results indicated that the blade tip-clearance affected the flow near the blade tip, and the greater the blade tip-clearance, the greater the local efficiency declined, especially near the blade tip. Those effects were found to extend to the center of the flow path.

2. DESIGN VELOCITY TRIANGLE AND BLADE PROFILE

Table 1 shows the major design specifications of this turbine [1]. Figure 1 shows the design velocity triangle at the radius positions of the TIP, MEAN, and ROOT. Figure 2 shows the blade arrangements of stator and rotor blades at the MEAN section. As Figures 1 and 2 indicate, the stator and rotor blade rows were designed to have a great deflection angles. See [1] for details on the turbine and the blade design.

3. EXPERIMENTAL METHOD AND ANALYSIS METHOD

3.1 Experimental Equipment and Measuring Device

Figure 3 shows a cross section of the main body of the aerodynamic test rig used for the present aerodynamic performance test.

Non-heated compressed air (about 100°C at the turbine inlet) from a 3700 kW compressor was used as the operational gas of the turbine. A JIS standard disc type orifice was installed in the piping (495.2 mm inner diameter) before the turbine inlet for measurement of the inlet mass flow rate. See Figure 4.

The torque and the revolutions of the turbine were measured by a 1600 kW direct current dynamometer.

The gas state before and after the turbine stage was measured by the thermocouples and 3-hole pitot tubes shown in Figure 5. The instruments in front of the turbine inlet were fixed to the center of the flow path, while the instruments behind the turbine stage were traversed in the radial direction. Figure 6 shows the positions of each measuring devices.

TABLE 1. MAJOR DESIGN SPECIFICATIONS

Item	Notation	Design value	Unit
Gas flow rate	G	3.95	kg/sec
Inlet total pressure	P_{t1}	25000.	kg/m^2
Inlet total temperature	T_{t1}	1423.15	K
Adiabatic efficiency	η_T	0.85	
Adiabatic heat drop	ΔH_{ad}	63.25	kcal/kg
Revolutions per minute	N	13300	rpm
Peripheral velocity	U_M	366.7	m/sec
Expansion ratio	η_T	2.02	
Theoretical velocity	V_{ad}	727.5	m/sec
Velocity ratio	U_M/V_{ad}	0.504	
Degree of reaction	$\rho_{R,M}$	0.464	

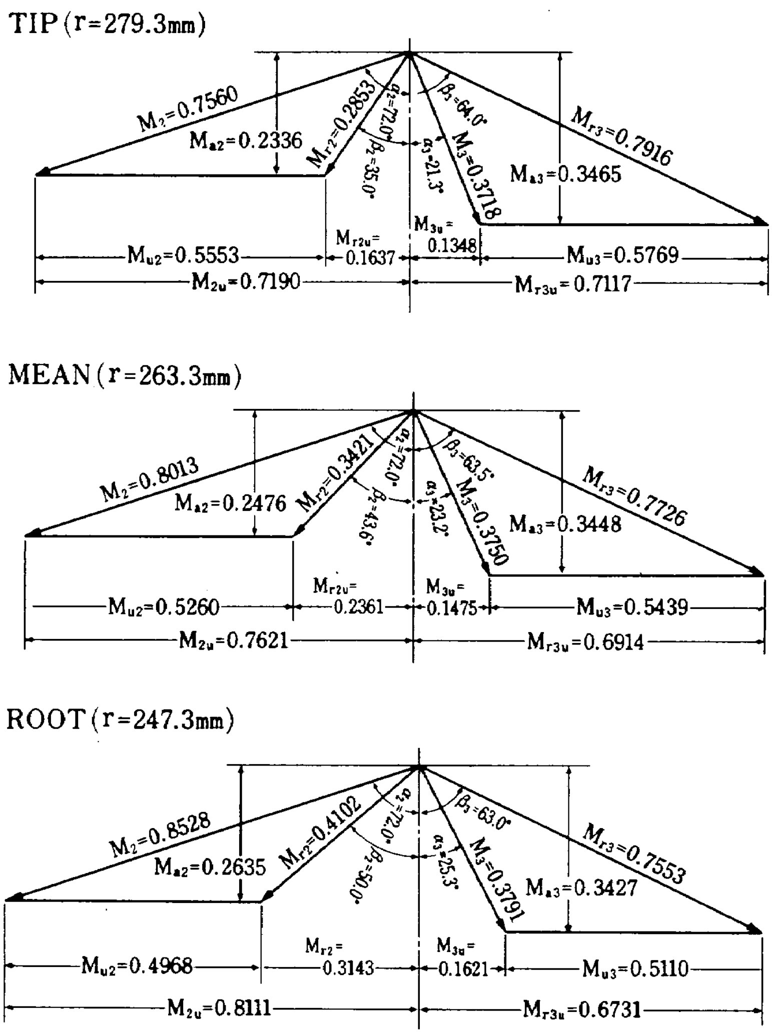
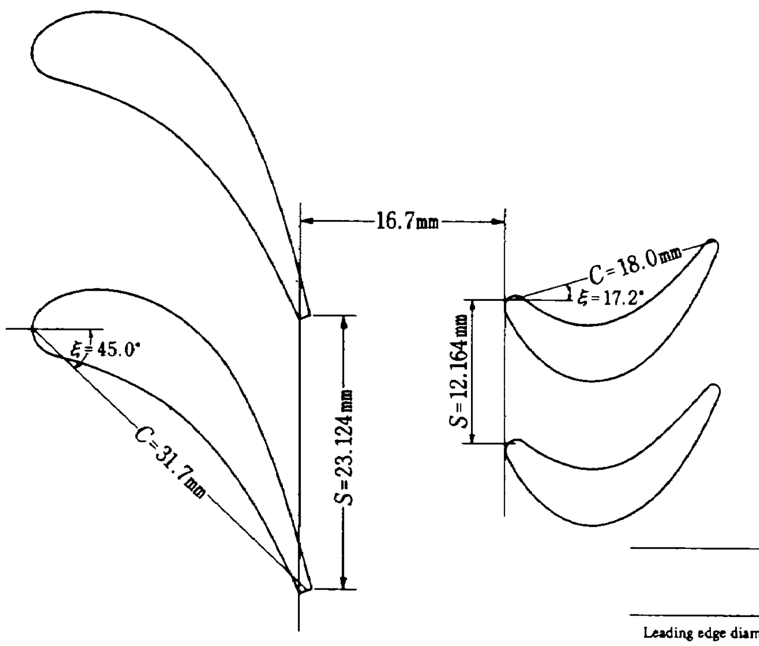


Fig. 1 Design velocity triangle



	Stator blade, r=261.3mm	Rotor blade, r=263.3mm
Leading edge diameter ratio (d_L/C)	0.126	0.111
Trailing edge thickness ratio (d_T/C)	0.032	0.056
Max. blade thickness ratio (d_{max}/C)	0.22	0.26
Solidity (C/S)	1.37	1.48
Aspect ratio (h/C)	1.03	1.86

Blade nondimensional values based on MEAN

Fig. 2 Blade arrangements (MEAN)

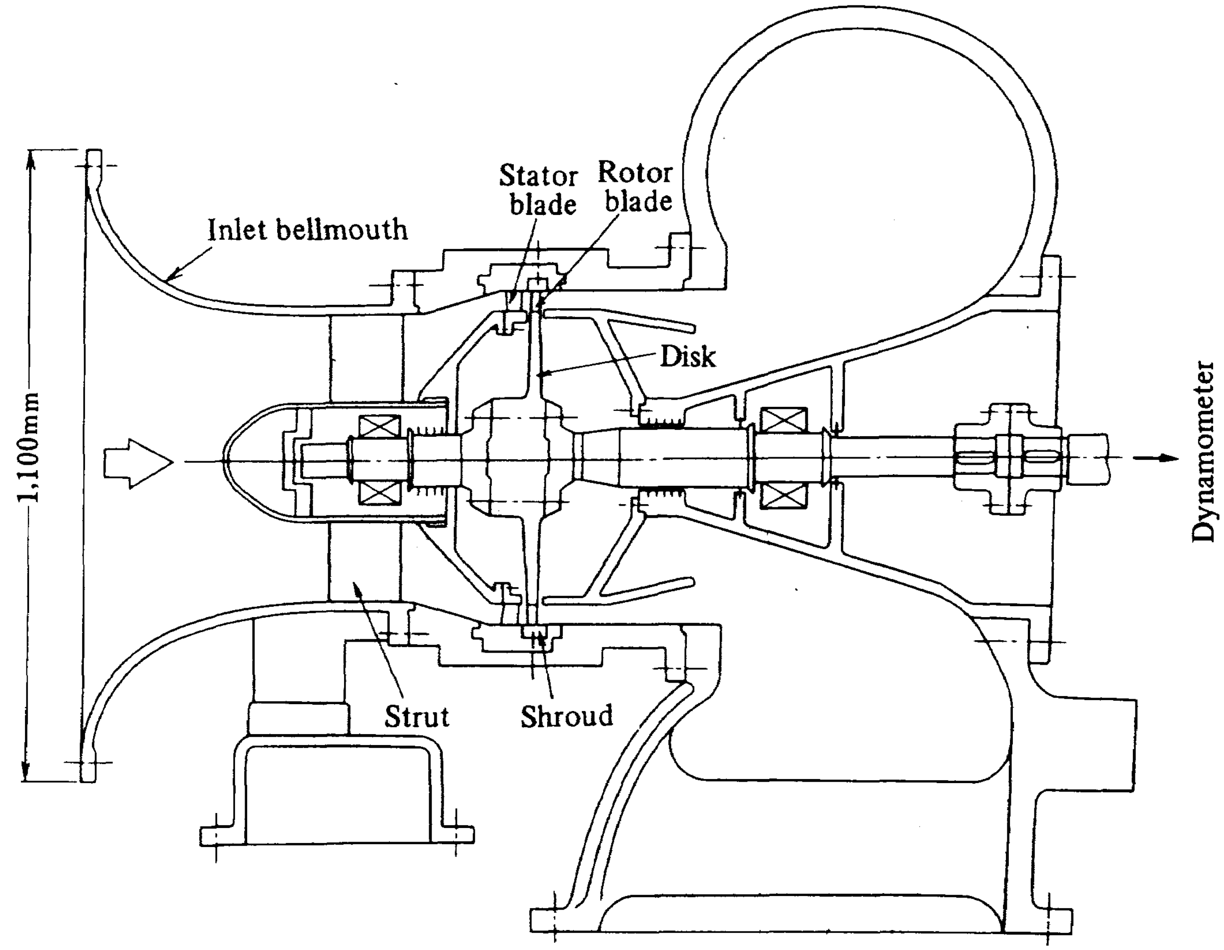


Fig. 3 Main body of aerodynamic test rig

Figure 7 shows the measuring and data processing system. Figure 8 illustrates the several main devices.

3. 2 Rotor Tip Shroud Rings

Three types of rotor blade shroud rings (termed S0, S1, and S2) with different inner diameters were manufactured to change the rotor blade tip-clearance in this experiment. Figure 9 illustrates those dimensions. Experiments were conducted with these three different rotor blade tip-clearances. Excluding the S0 (nominal) shroud ring, the measured values of the tip-clearances shown in Figure 9 were obtained by the following way; After raising the upper lid of the main body of the turbine test rig, then putting several lead lines of suitable thickness in several spots on the surface of the blade tip, and closing the lid, we measured the thickness of the broken lines by a point micrometer and calculated the average of the measured values. The calculated average values include two measured values (by a clearance gauge) of the gaps between the inner surface of the shroud and the rotor blade tip in the horizontal plane, where measurement is possible when the lid is open. However, the lower half of the rotor blade tip-clearance below the axle of rotation was not measured. The average outer diameter of the rotor blade tip is $D_{TIP} = 555.73$mm, which was used to determine k/h for the shroud ring S0.

Figure 10 presents photographs of the rotor blade rotating part and the three types of rotor blade shroud rings tested.

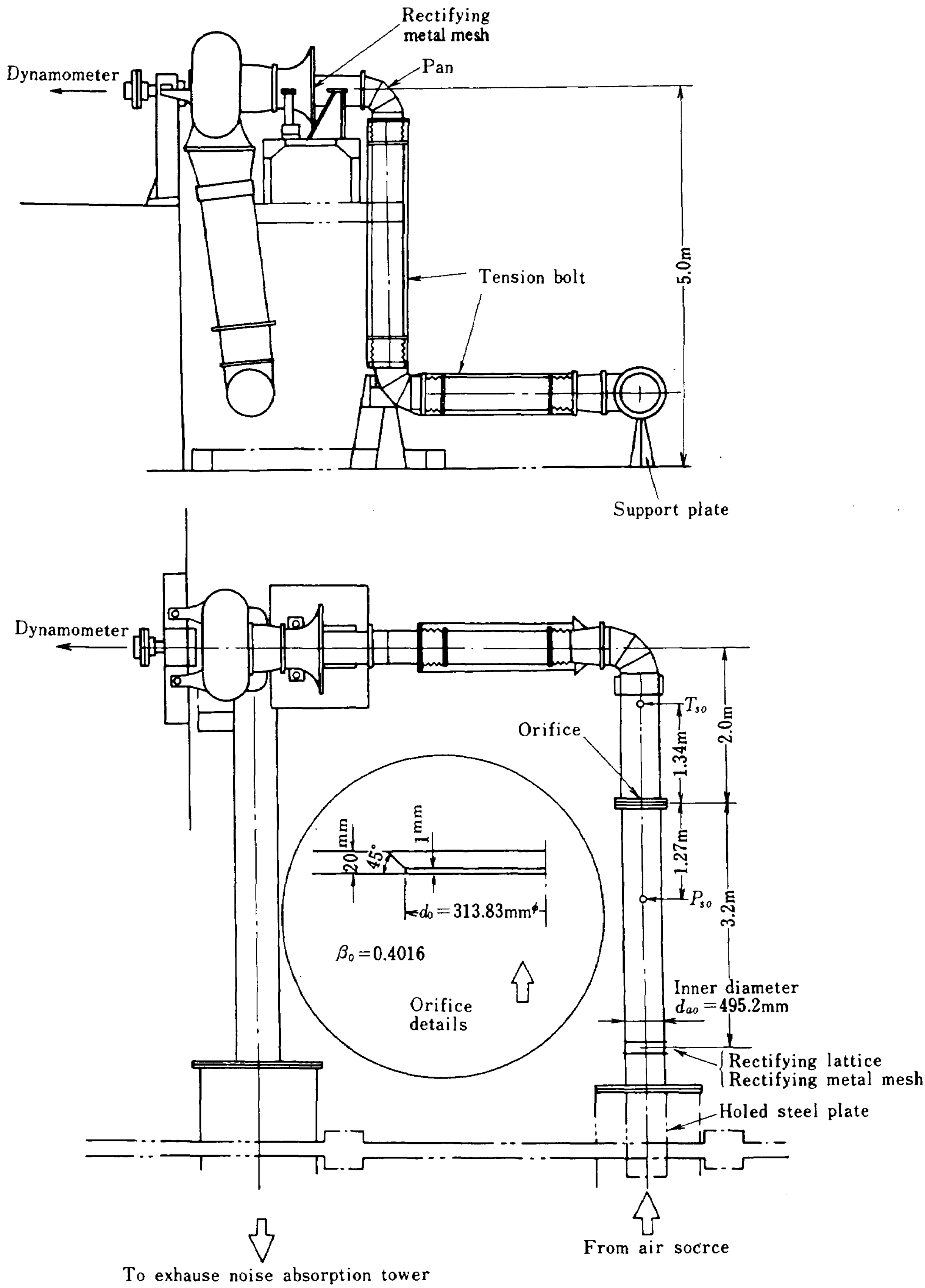
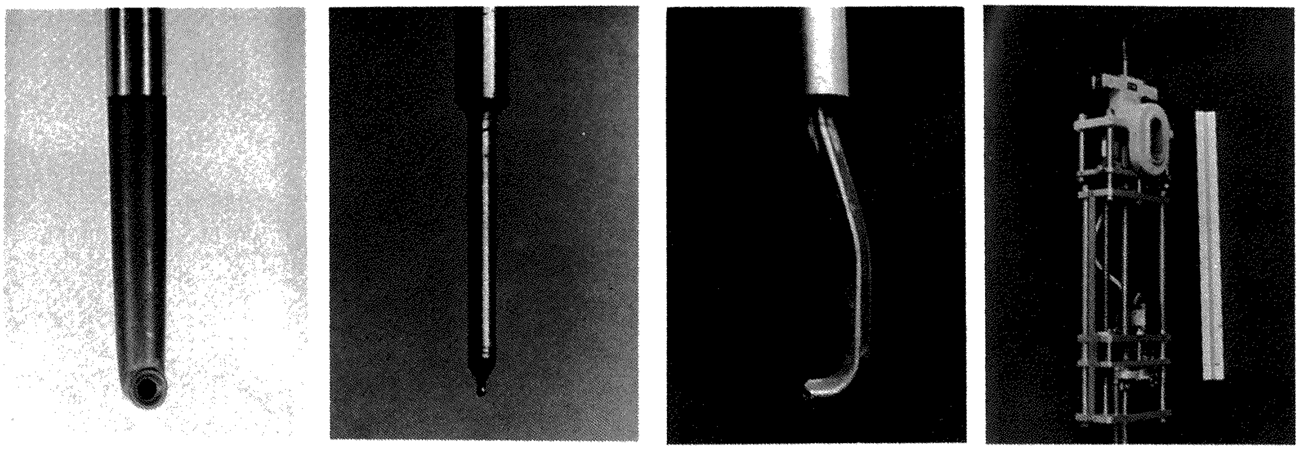


Fig. 4 Arrangement of the aerodynamic test rig



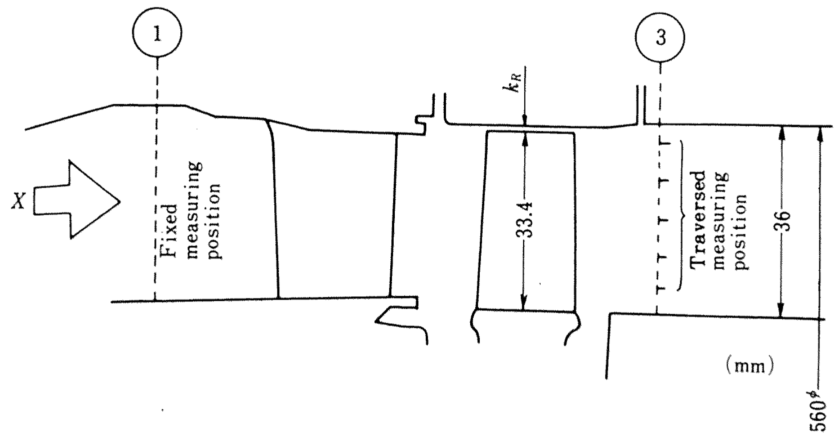
(a) Stagnation type CA thermocouple thermometer (T_1-1, T_1-2)

(b) Tip exposed type CA thermocouple thermometer (T_3-1, T_3-2)

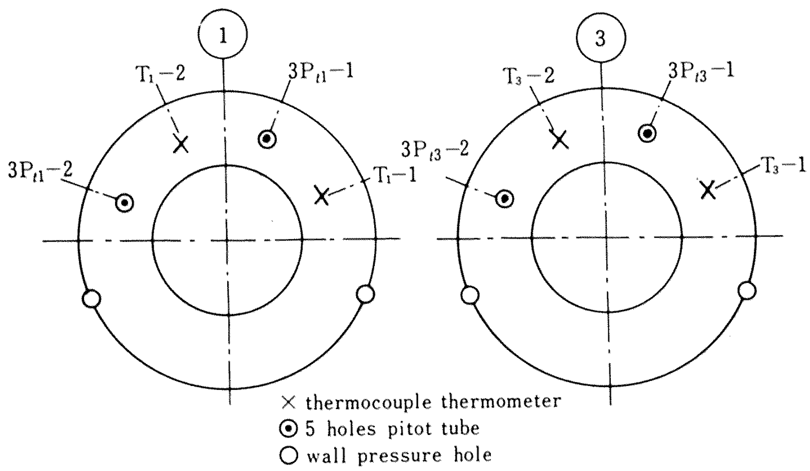
(c) 3 holes pitot tube ($3P_{t1}-1$, $3P_{t1}-2$, $3P_{t3}-1$, $3P_{t3}-2$)

(d) Moving part of electro-traverse apparatus

Fig. 5 Pictures of measuring sensors and moving part apparatus



(a) Axial position



(b) Peripheral position (front view of the test rig)

Fig. 6 Arrangement of various measurement devices

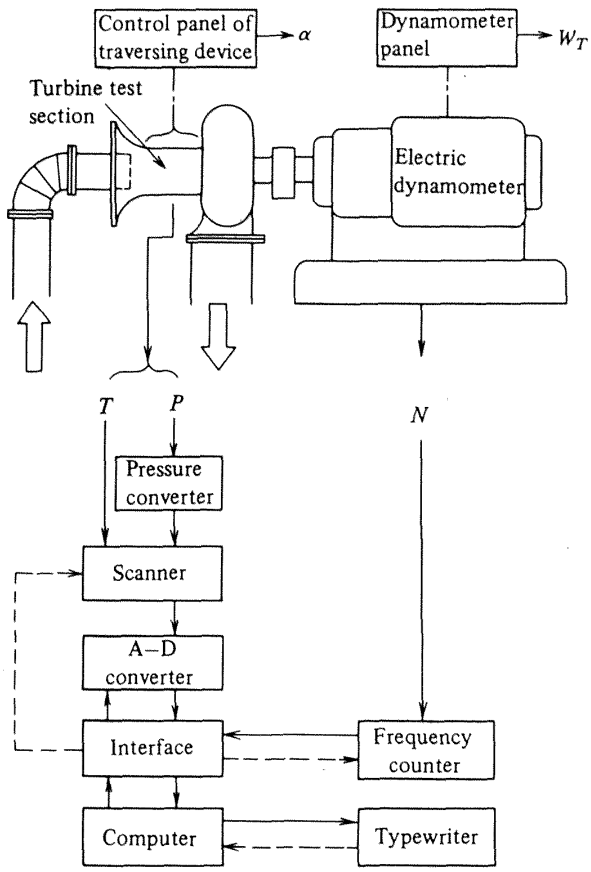
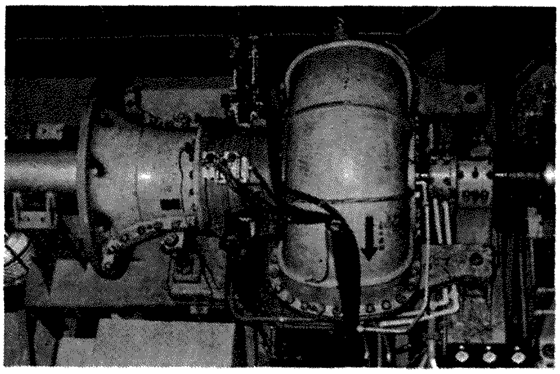
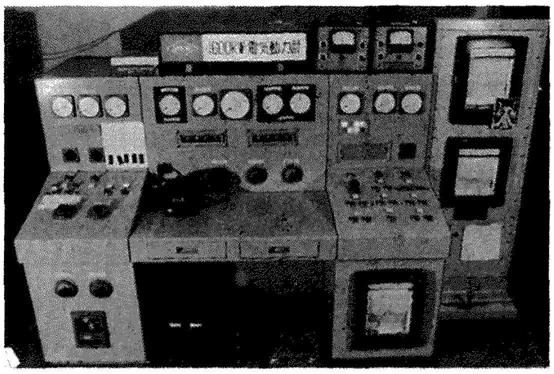


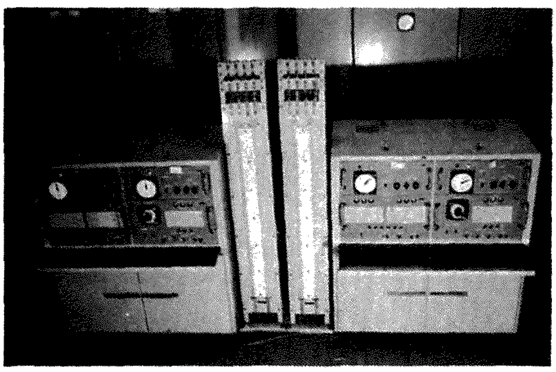
Fig. 7 Measuring-processing system



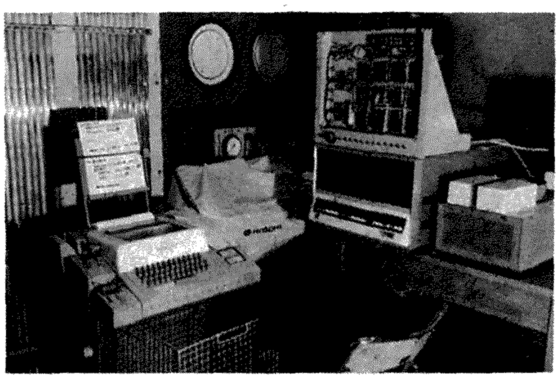
(a) Main body of turbine test rig operation



(b) 1600kW DC electro-dynamometer control table



(c) Electro-traverse apparatus



(d) Data processing conputer

Fig. 8 Photographs of experimental apparatus

Name of shroud ring	Ring inner diameter D_{sh} (design value, mm)	Rotor blade tip-clearance k_R		Ratio over blade height h (=33.4) $(k/h)_R$ (actual measurement)	Remarks k_R measurement method
		Design value, mm	Actual meas., mm		
S 0	556.7	0.4	0.49	1.5%	Actual D_{Tip} measurement
S 1	557.5	0.8	0.88	2.6	Lead wire
S 2	558.3	1.2	1.39	4.2	〃

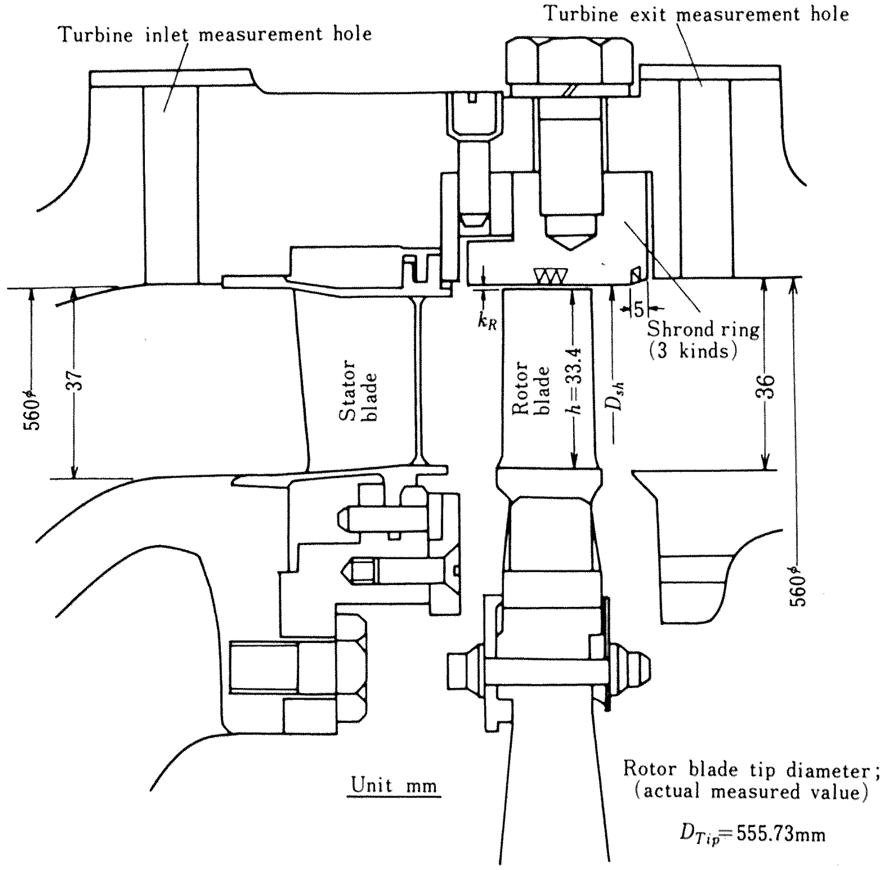
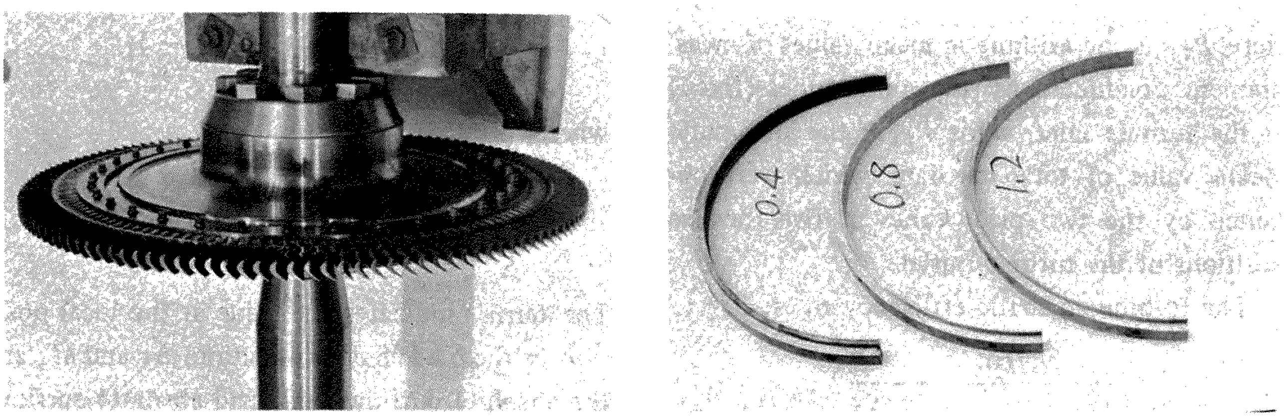


Fig. 9 Different rotor blade shroud rings and dimensions of blade tip-clearance



(a) Rotor blade rotating part

(b) Three types of rotor blade shroud rings (photograph shows the half portion)

Fig. 10 Rotor blade rotating part and rotor blade shroud rings

3. 3 Experimental Method

In this experiment, the turbine expansion ratio and the turbine speed were set at several target values; The experimental points were every 0.2 at an expansion ratio of 1.4 – 2.1, and at every 10% for 70 – 110% of the design turbine speed. After the inlet turbine gas temperature was virtually constant, the gas temperature and pressure were measured in several traverse positions in the radial direction. The turbine inlet gas temperature was about 90 – 110°C during the experiment. It took about ten minutes to obtain a performance at a target point by five traverse measurements.

3. 4 Analysis Method

The turbine inlet gas flow G was measured by the orifice flow meter. The orifice Reynolds number in the test range, $R_{e,0}$ ($\equiv Vd_{a0}/\nu$), was about $(3 \sim 4) \times 10^5$ (d_{a0} is the piping inner diameter (Figure 4) and ν is the dynamic viscosity coefficient).

The calculated formulas used in analysis are shown below:

Turbine torque τ : $\tau = lW_T$

Turbine output L_T : $L_T = \dfrac{2\pi}{60J}\tau N$

Turbine specific output ΔH_T : $\Delta H_T = \dfrac{L_T}{G}$

Turbine stage expansion ratio π_T : $\pi_T = \dfrac{\overline{P}_{t1}}{\overline{P}_{t3}}$

Here, $\overline{P}_{t1}$ is the arithmetic mean values of measurement conducted by two pitot tubes (fixed) at the turbine inlet, while $\overline{P}_{t3}$ is the arithmetic means value of total ten data, which were obtained by the two pitot tubes at five traverse positions of the turbine outlet.

The turbine adiabatic efficiency η_t is:

$$\eta_t = \frac{\Delta H_T}{c_p \overline{T}_{t1} \{1 - (\frac{1}{\pi_T})^{\frac{\kappa-1}{\kappa}}\}}$$

Here, c_p and κ are the constant pressure specific heat and the specific heat ratio corresponding to the arithmetic means temperature of the turbine inlet temperature and outlet temperature. $\overline{T}_{t1}$ is the mean total gas temperature at the turbine inlet. In the experimental condition, we assumed $c_p = 0.240$, and $\kappa = 1.40$.

The local adiabatic temperature efficiency η_{1-3} in the radius direction (blade height) of the turbine flow path is:

$$\eta_{1-3} = \frac{1 - \dfrac{\overline{T}_{t3,i}}{\overline{T}_{t1}}}{1 - (\dfrac{\overline{P}_{t3,i}}{\overline{P}_{t1}})^{\frac{\kappa-1}{\kappa}}}$$

$\overline{T}_{t1}$, $\overline{P}_{t1}$ are the arithmetic mean values of total temperature and total pressure measured at two locations in the peripheral direction of the turbine inlet. $\overline{T}_{t3,i}$ and $\overline{P}_{t3,i}$ are the total temperature and total pressure arithmetic mean values measured at the same radial positions of the turbine outlet by two thermocouples and by two pitot tubes, respectively. The total temperatures were obtained by correcting the measured values with tested temperature recovery coefficients of the sensors (which are functions of Mach number).

The local value in the blade height direction of the gas relative outflow angle at the rotor outlet was calculated from the local values of absolute outflow angle, absolute outflow Mach number, local peripheral velocity and local total temperature as follows:

$$\beta_{3,i} = \tan^{-1} (\tan\alpha_3 + \frac{M_{u3}}{M_3 \cos\alpha_3})_i$$

where,

$$M_{u3,i} = \frac{\pi N}{60\sqrt{k.gR}} (\frac{D}{\sqrt{\overline{T}_{s3}}})_i$$

The term i indicates the value at the local position in the blade height direction. α_3 and M_3 are the absolute outflow angle and absolute outflow Mach number at the rotor blade outlet, respectively. $\overline{T}_{s3,i}$ is the arithmetic mean local static temperature measured by the two thermocouples at the rotor outlet.

The turbine velocity ratio U_M/V_{ad} is:

$$\frac{U_M}{V_{ad}} = \frac{\pi D_M N}{60\sqrt{2\,gJ\,\Delta H_{T,ad}}}$$

where,

$$\Delta H_{T,ad} = c_p T_{t1}\{1 - (\frac{1}{\pi_T})^{\frac{\kappa-1}{\kappa}}\}$$

The Mach number is found by the following method for three-hole pitot tubes [3]. The test variable H of the pitot tube is defined as:

$$H = \frac{P_R + P_L}{P_C}$$

P_R, P_L, and P_C are the pressures measured at the right, left, and center of the three-hole pitot tube. When the pitot tube faces the direction of flow ($P_R = P_L$), H is a function of the Mach number and Reynolds number. If the Reynolds numbers at the time of pitot tube testing and at the time of the present experiment are virtually equal, H would be a function of the Mach number only; the H-M curve was obtained by testing and used.

The experimental values to express the turbine performances were corrected into the values at the standard state in the following manner:

flow G → corrected flow $\epsilon_{cr} G\sqrt{\theta_{cr}}/\delta$

number of revolutions N → corrected revolutions $N/\sqrt{\theta_{cr}}$

specific output ΔH_T → corrected specific output $\Delta H_T/\theta_{cr}$

torque τ → corrected torque $\epsilon_{cr}\,\tau/\delta$

expansion ratio π_T → corrected expansion ratio $\phi_{cr}\pi_T$

where θ_{cr}, δ, ϵ_{cr} and ϕ_{cr} are correction coefficients as follow:

$$\left.\begin{aligned}
\theta_{cr} &= (\frac{k}{k+1} R\bar{T}_{t1})/(\frac{k_{st}}{k_{st}+1} R_{st}\, T_{t,st}) \\
\delta &= \bar{P}_{t1}/P_{t,st} \\
\epsilon_{cr} &= \{k_{st}(\frac{k+1}{2})^{\frac{k}{k-1}}\}/\{k(\frac{k_{st}+1}{2})^{\frac{k_{st}}{k_{st}-1}}\} \\
\phi_{cr} &= \frac{1}{\pi_T}\{1 - (\frac{k_{st}-1}{k_{st}+1})(\frac{k+1}{k-1}) \\
&\quad (1 - \pi_T^{\frac{1-k}{k}})\}^{\frac{k_{st}}{1-k_{st}}}
\end{aligned}\right\}$$

The sufix [st] indicates standard states.

The adiabatic efficiency η_t can be expressed as follows with these correction values:

$$\eta_t = \frac{\Delta H_T/\theta_{cr}}{c_{p,st}\, T_{t,st}\{1 - (\frac{1}{\phi_{cr}\pi_T})^{\frac{k_{st}-1}{k_{st}}}\}}$$

The standard state in this report is of the following values:

$$\left.\begin{aligned}
T_{t,st} &= 288.2\ \mathrm{K} \\
P_{t,st} &= 10332\ \mathrm{kg/m^2} \\
R_{st} &= 29.27\ \mathrm{kg \cdot m/K \cdot kg} \\
K_{st} &= 1.401
\end{aligned}\right\}$$

In this case, $\epsilon_{cr} = \theta_{cr} = 1.0$ can be used for the correction of experimental values, while $\epsilon_{cr} = 1.038$, $\theta_{cr} = 1.037$ for the design values, since $k_{des} = 1.31$. Table 2 shows the major design specifications and their corrected values by the above method.

TABLE 2. DESIGN VALUES AND THEIR CORRECTED VALUES ($k_{des} = 1.31$)

		Design values	Corrected values
Inlet total temperature	T_{t1} (K)	1423.2	288.2
Inlet total pressure	P_{t1} (kg/m²)	25000	10332
Turbine output	ΔH_T (kcal/kg)	53.8	11.2
Turbine mass flow	G (kg/sec)	3.95	3.71
Revolutions	N (rpm)	13300	6071
Expansion ratio	π_T	2.02	2.09

4. EXPERIMENTAL RESULTS DISCUSSION

4.1 Overall Performance

(a) Turbine inlet mass flow characteristics

Figure 11 shows the effects of rotor blade tip-clearnce on turbine inlet flow characteristics, the flow at a constant expansion ratio and constant turbine speed increases as the rotor blade tip-clearance increases; Generally speaking, the mass flow characteristic curves of each turbine speed were sifted in parallel in the vertical direction

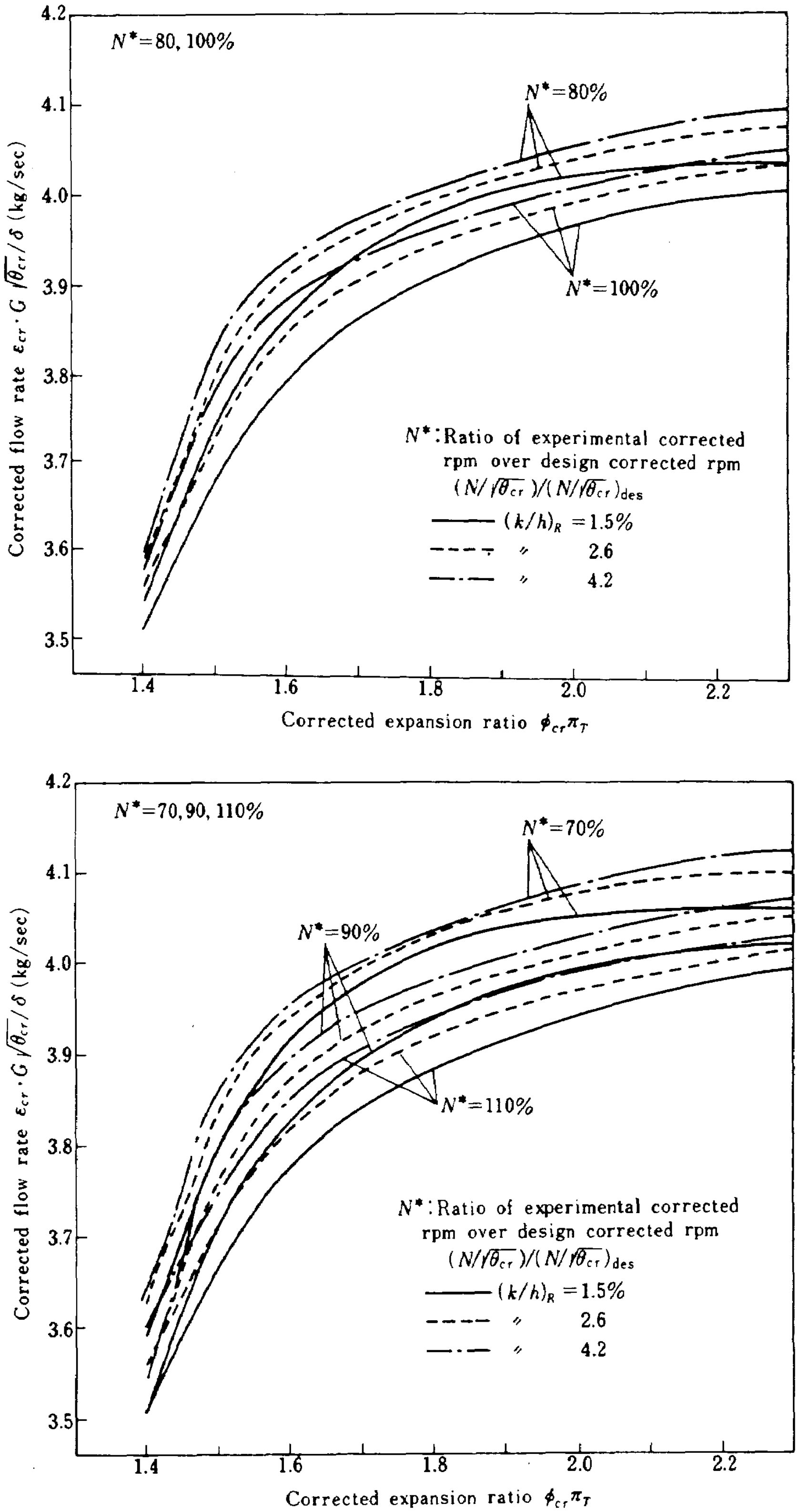


Fig. 11 Characteristics of turbine inlet flow rate

due to the different rotor tip-clearances. The mass flow rate at the design turbine speed ($N^* =$ 100%) and at the design expansion ratio ($\phi_{cr}\pi_T$ = 2.09) in the case of rotor blade tip-clearance of $(k/h)_R$ = 1.2% is 1.0% greater than that in the case of $(k/h)_R$ = 1.5%.

In this fashion, the turbine mass flow increases as the blade tip-clearance increases. This seems to come from the increase in blade throat area due to the increase of the shroud ring diameter. Assuming an increase in the annular area of the rotor outlet only by the amount of increase of the shroud ring inner diameter, one-dimensional calculation of off-design turbine performance was conducted. In this calculation, the relative outflow angle of the rotor blade row was assumed equal to the design value of MEAN position even under the off-design condition. The calculation, as indicated in Figure 12, predicted a 1.3% increase in the inlet flow at $(k/h)_R$ = 4.2% comparing to the case of $(k/h)_R$ = 1.5%. This is virtually equal to the 1.0% experimental value.

Moreover, the changes in turbine inlet mass flow due to changes in the throat area of the stator blade and rotor blade of a single stage turbine ($A_{thr,N}$ and $A_{thr,R}$) are expressed by the following formula based on the small deviation method [4] at a constant turbine expansion ratio ($\Delta\pi_T = 0$):

$$\Delta(\epsilon_{cr}G\sqrt{\theta_{cr}}/\delta) = a_1\Delta A_{thr,N} + (1-a_1)\Delta A_{thr,R}$$

a_1 is the coefficient which expresses the influence of changes in the stator blade throat area $\Delta A_{thr,N}$, on the corrected inlet mass flow $\epsilon_{cr}G\sqrt{\theta_{cr}}/\delta$. In the case of this turbine, a_1 = 0.70 is found when calculated by the method of [4] using the experimental velocity triangle at the design speed and the design expansion ratio in the case of $(k/h)_R$ = 1.5% [1]. Thus,

$$\Delta(\epsilon_{cr}G\sqrt{\theta_{cr}}/\delta) = 0.7\Delta(A_{thr,N}) + 0.3\Delta(A_{thr,R})$$

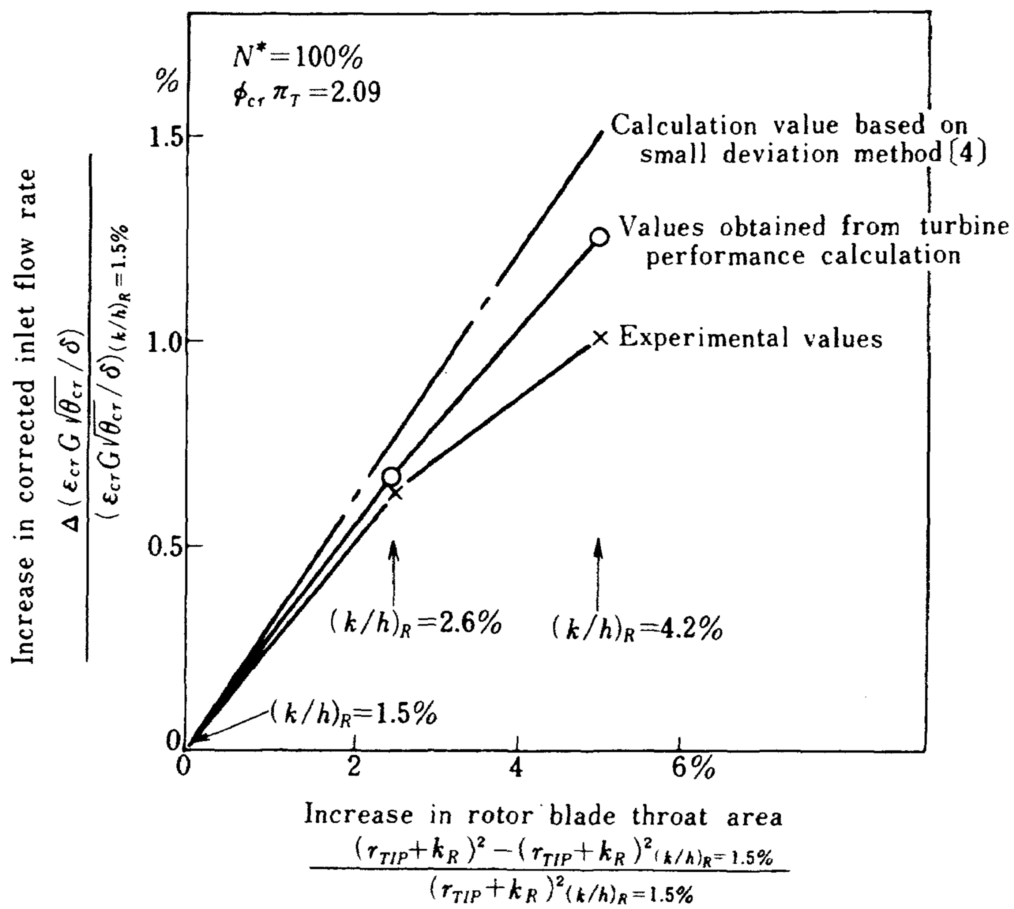


Fig. 12 Relationship between change in blade tip-clearance and change in inlet corrected flow rate

30% of the increased rate of rotor blade throat area, $\Delta(A_{thr,R})$, would therefore become the turbine inlet flow increase ratio. This result is also shown in Figure 12. In comparison to the case of $(k/h)_R = 1.5\%$, this predicts in the case of $(k/h)_R = 4.2\%$, a 1.5% increase in turbine inlet corrected flow.*

The experimental increase in the turbine mass flow rate was smaller than the calculated. This is because the calculations were conducted at the MEAN position or by using MEAN experimental triangle, while the actual flow near the blade tip-clearance includes the flow in the boundary layer which develops on the outer wall of the flow path, so that the actual flow velocity in that vicinity will be slower than the mean axial flow velocity of the main stream.

* The literature [8, 10, 11, 14, 20, 22, 23, 29, 39, 40] on blade tip-clearance, refer to calculations of gas flow passing through the blade tip-clearance.

(b) Turbine torque characteristics

Figure 13 illustrates the effect of rotor blade tip-clearance on turbine torque. Examination of the values of each turbine speed indicates that the general discussion of the effect of $(k/h)_R$ is not possible all over the range of total expansion ratios tested, but in general, in the regions of higher expansion ratio – that is, where the torque tends to decrease slightly with increase in $(k/h)_R$.

Figure 14 also shows the above results of the effect of rotor blade tip-clearance on the turbine inlet corrected mass flow and the corrected torque, as well as the results in the literatures [5, 6].

In contrast to the previous experiments [5, 6] in which the rotor tip-clearance was increased by

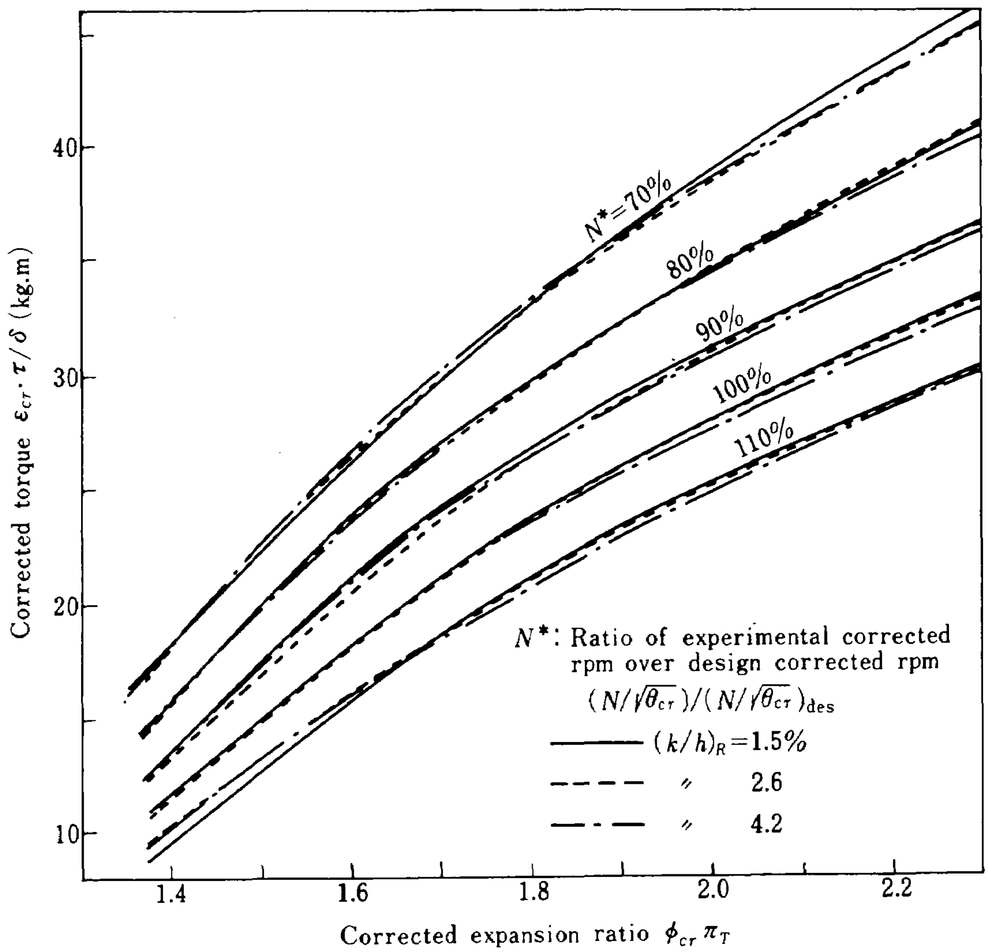


Fig. 13 Turbine torque characteristics

$$\frac{\Delta\eta}{\eta_{ref}} = \frac{\Delta(\varepsilon_{cr}\cdot\tau/\delta)}{(\varepsilon_{cr}\cdot\tau/\delta)_{ref}} - \frac{\Delta(\varepsilon_{cr}\cdot G\sqrt{\theta_{cr}}/\delta)}{(\varepsilon_{cr}\cdot G\sqrt{\theta_{cr}}/\delta)_{ref}}$$

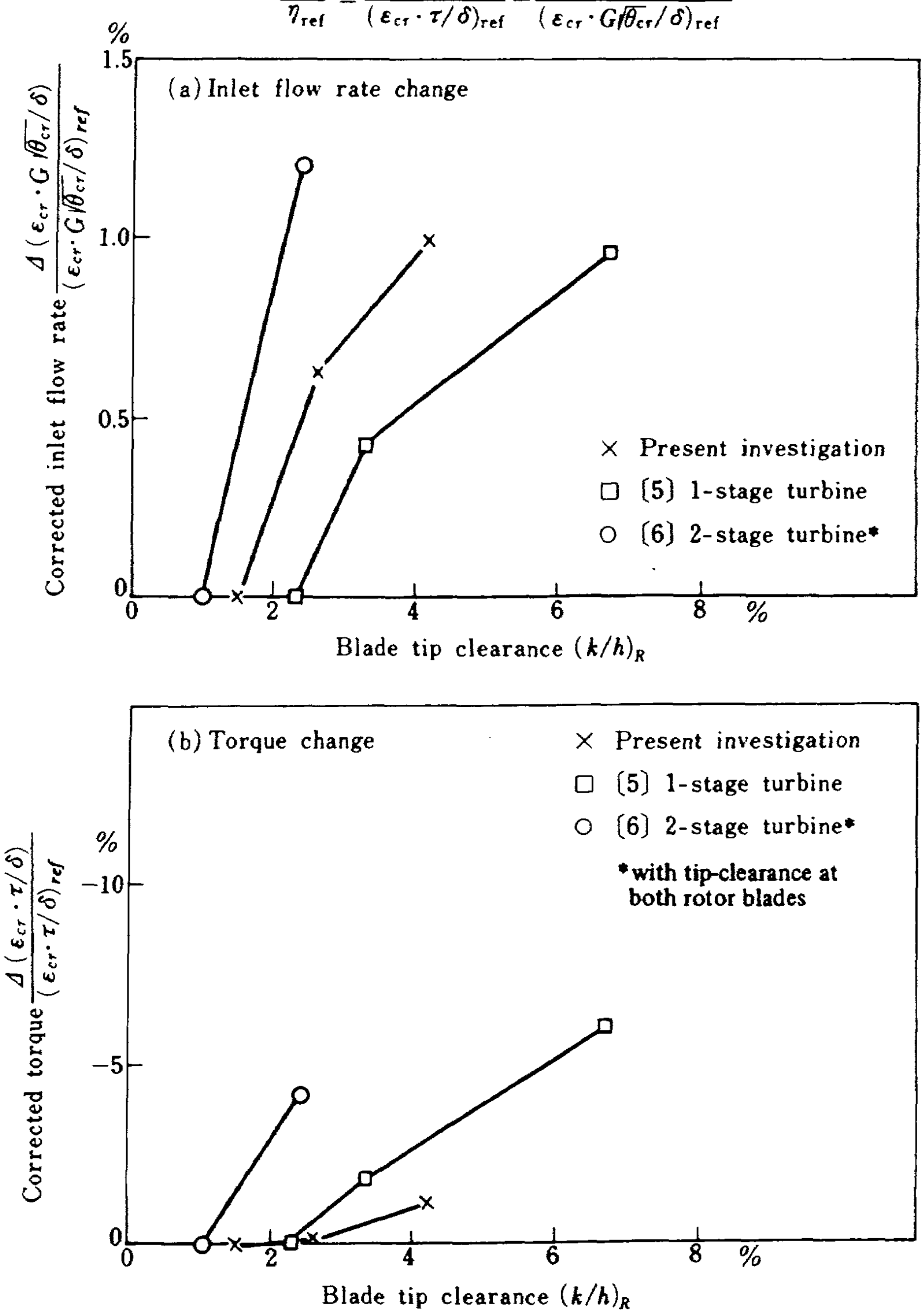


Fig. 14 Changes in turbine inlet flow rate and torque by different blade tip-clearances

scraping off the tip of the rotor blade [5 ~ 7], in this experiment the tip-clearance was increased by increasing the shroud ring inner diameter of the rotor blade. Accordingly, the blade load, that is, the turbine torque did not so much decrease as seen in the previous literatures; Figure 14 (b) shows that the rate of torque decrease was far smaller than the increase rate in blade tip-clearance in the present experiment.

(c) Turbine adiabatic efficiency η_t based on dynamometer

Figure 15 illustrates the turbine adiabatic efficiency based on the turbine torque and the turbine inlet mass flow. At lower expansion ratios, the effect of rotor tip-clearance at each turbine speed can not be generally discussed, but at higher expansion ratios, the adiabatic efficiency is clearly shown to decrease with increase in

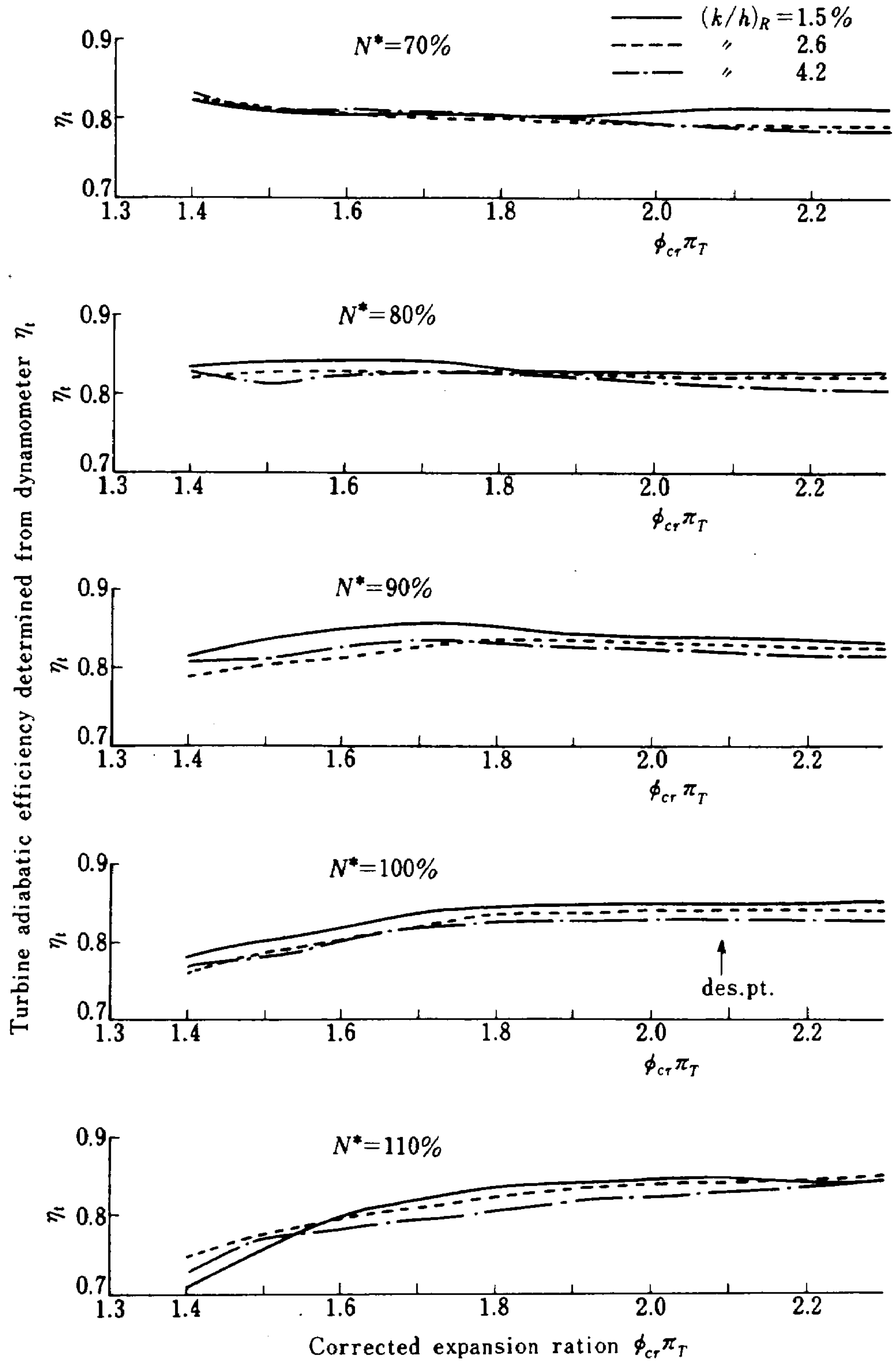


Fig. 15 Adiabatic efficiency characteristics

$(k/h)_R$ except in the case of $N^* = 110\%$.

The indication [des. pt.] in the figure means the point of the turbine design speed and design expansion ratio, where η_t decreases about 2% when the blade tip-clearance increases from $(k/h)_R = 1.5\%$ to $(k/h)_R = 4.2\%$.

Figure 16 plots the experimental adiabatic efficiencies at $(k/h)_R = 2.5\%$, 2.6% and 4.2% in relation to the velocity ratio U_M/V_{ad}. The points include all experimental η_t at $N^* = 70\% \sim 110\%$. In order to illustrate the effect of $(k/h)_R$ clearly, all efficiencies obtained at same $(k/h)_R$ are illustrated by same marks, regardless of N^*. The three kinds of lines show the lines which envelop the points of peak efficiency $\eta_{t,max}$ of η_t $(k/h)_R = 1.5\%$, 2.6%, and 4.2%. As these envelope lines indicate, the effect of k/h is fairly evident. For example, at the design velocity ratio ($U_M/V_{ad} = 0.504$), $\eta_{t,max}$ decreased about 2% at $(k/h)_R = 4.2\%$ in comparison to at $(k/h)_R$ = 1.5%.

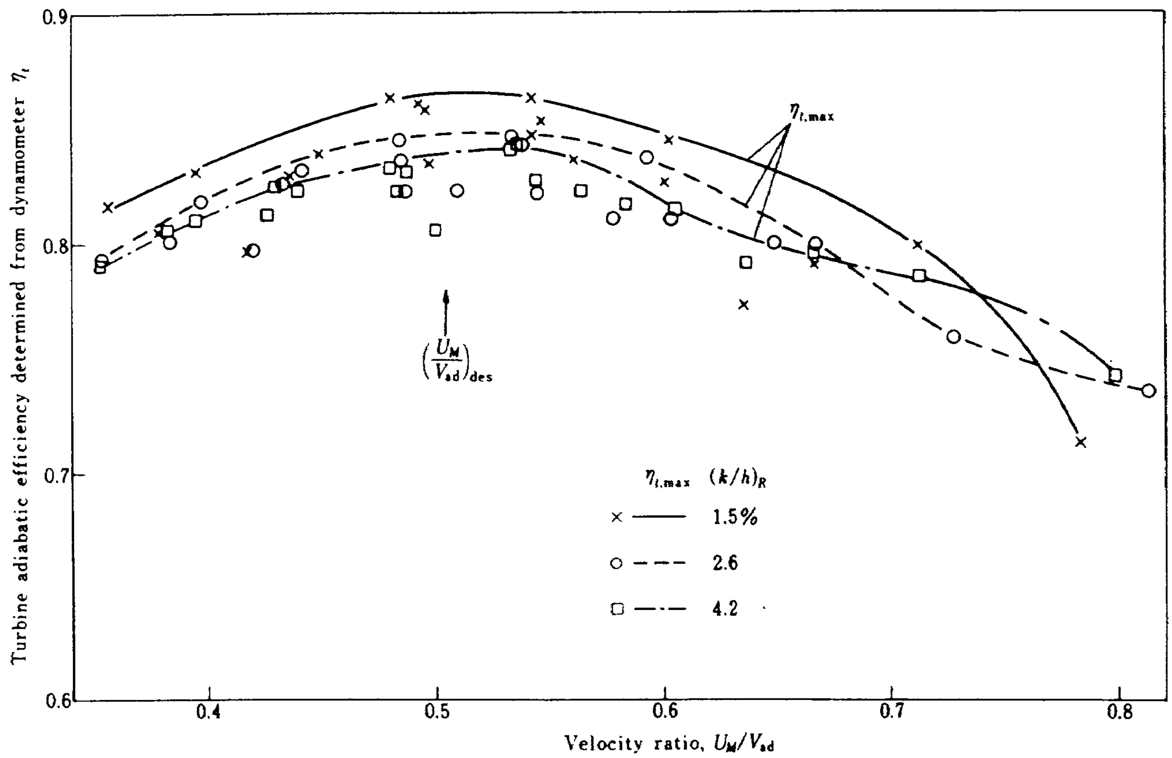


Fig. 16 Adiabatic efficiency-velocity ratio characteristics

4. 2 Internal Flow

(a) Spanwise distribution of adiabatic temperaature efficiency

Figures 17 (a) – (c) show the local adiabatic temperature efficiency distribution in the rotor blade height direction for each experiment of $(k/h)_R$ = 1.5%, 2.6%, 4.2%. The parameters are turbine expansion ratio and turbine speed.

In general, the efficiency, espeically near the tip region, tended to decrease with increase in the rotor tip-clearance. This is the case in all experimental range. The effect of the tip-cleance is not restricted to the blade tip vicinity and it extends to the mean radial position of the flow path. However, the effect vanishes near the root.

(b) Spanwise distribution of rotor outlet relative gas flow angle

The relative outflow angle β_3 at the rotor outlet is one of the important factors of the internal flow measurements; Since this is a result of deflection of the gas flow by the rotor blade, it is directly involved in turbine work. Figure 18 illustrates the radial distribution of the relative gas flow angle β_3 at the rotor blade outlet obtained by two pitot tubes. Differences in β_3 between the two pitot tubes were evident, indicating difference in flow due to the different measuring positions in the circumferential direction. Examining the data by the same pitot tube, however, one can see that β_3 would clearly become smaller, especially near the tip, as $(k/h)_R$ becomes larger. This seems to be a result of the gas leaking through the rotor tip-clearance from the blade pressure side to the blade suction side. In this manner, the rotor blade deflection angle becomes fairly smaller near the tip as the clearance increases and the work of the rotor blade near the tip, therefore, decreases. In addition, a total pressure loss would increase due to the mixture of the main gas stream with the leaking gas from the tip-clearance. These results seem to bring about the decline in the local turbine adiabatic temperature efficiency η_{1-3} and in the overall turbine adiabatic efficiency η_t discussed previously.

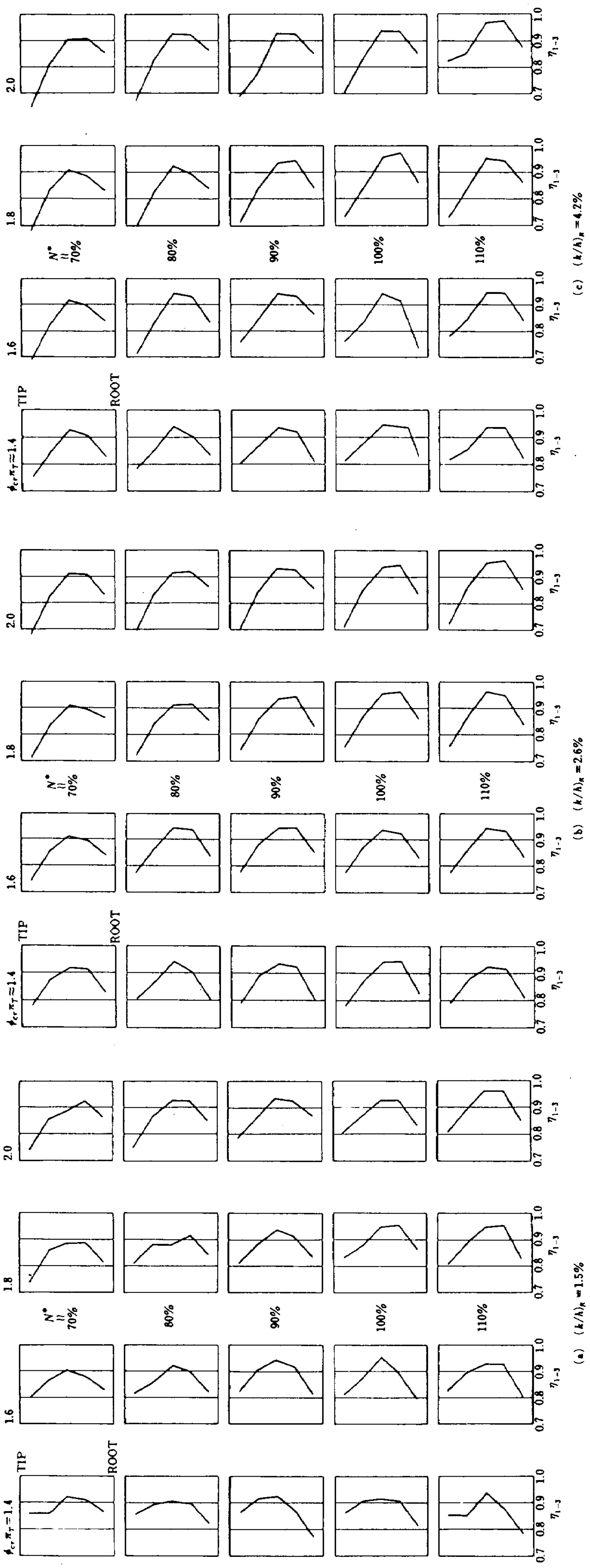


Fig. 17 Spanwise distribution of adiabatic temperature efficiency

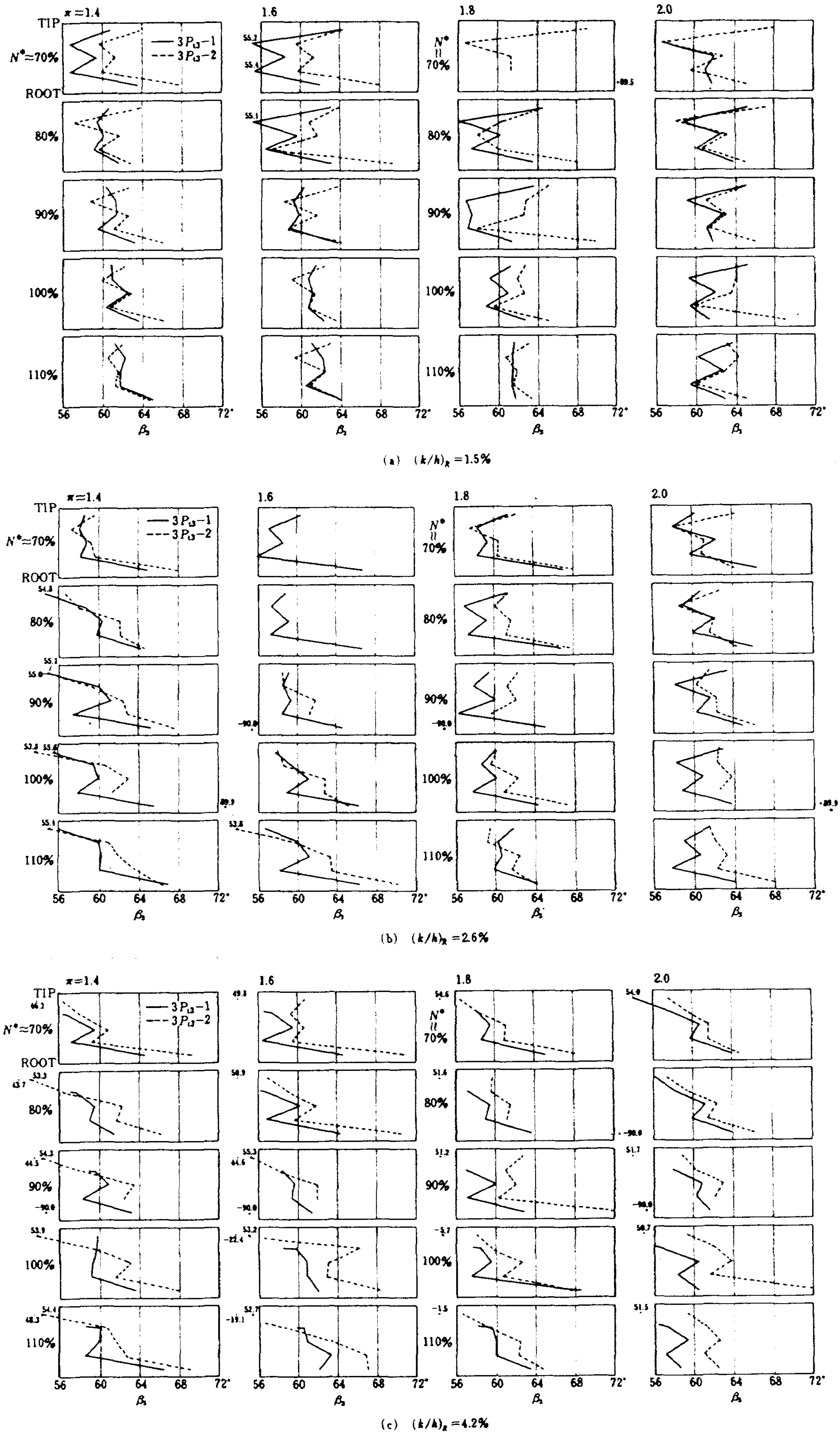


Fig. 18 Spanwise distribution of relative gas flow angle at turbine exit

5. COMPARISON OF VARIOUS PREDICTION METHODS OF THE INFLUENCE OF ROTOR BLADE TIP-CLEARANCE ON TURBINE EFFICIENCY WITH THE PRESENT EXPERIMENTAL RESULTS

5. 1 Consolidation of Prediction Methods

Various methods have been presented for predicting the effect of blade tip-clearance on the turbomachinery aerodynamic performances, especially on the overall adiabatic efficiency. We will classify and consolidate those methods here. All the methods can be simply expressed by (for the rotor):

$$\Delta\eta_T = -\lambda_R \left(\frac{k}{h}\right)_R$$

and we will divide the methods into the following three types depending how to determine the value of the proportional constant or function, λ_R;

A) The first is empirical calculation methods which determine experimentally the proportional constant λ_R by assuming that the adiabatic efficiency decline $\Delta\eta_T$ is proportional to only the blade tip-clearance ratio. Various empirical formulas are presented based on different values of the constant.

B) The second is theoretical calculation methods derived from the theoretical formula of adiabatic efficiency and the semi-theoretical formula of blade drag coefficient due to the blade tip-clearance. In this case, the proportional constant λ_R includes an empirical value of B and various factors concerning the performance of turbomachinery. B may be a constant, or a function of several parameters representing the blade drag coefficient. Various formulas can be derived according to different values of B. We will call this group

TABLE 3. THE INFLUENCE OF ROTOR BLADE TIP-CLEARANCE RATIO ON TURBINE ADIABATIC EFFICIENCY PREDICTED BY THE REFERENCES WITH DIFFERENT C_{Dk} ($\Delta\eta_T = -\lambda_R (k/h)_R$)

Classification	Formula	Cited reference	λ_R value or λ_R calculation formula	Remarks	λ_R for present test turbine
Empirical formula (λ_R is constant)	A1	Stodola (1925)[8]	1.55*	Reaction Turbine	1.55
	A2	Meldahl (1941)[8]	1.75*	Reaction Turbine	1.75
	A3	Ainley (1955)[8]	1.30*	Reaction Turbine	1.30
	A4	Kofskey (1967)[6]	1.67*	2-stg. Reaction 〃	1.67
	A5	Szanca (1974)[5]	1.39	Reaction Turbine	1.39
Theoretical formula I (based on expression of C_{Dk})	B1	Carter I (1948)[8,22]	$B = 0.5$, Eq. (20)	50% Reaction	2.29
		Carter II (1948)[8,22]	$B = 0.5$, Eq. (22)	Arbitrary Reaction	2.02
	B2	Ainley I (1955)[9]	$B = 0.5$, Eq. (22)	With shroud: $B = 0.25$	2.02
		Ainley II (1955)[9]	$B = 0.5$, Eq. (24)		1.72
	B3	Meldahl (1941)[10]	$B = \frac{1}{4\cos\beta_3}$, Eq. (22)		2.26
	B4	Vavra I (1960)[11]	$B = f\sqrt{C_L}\,(s/c)$, Eq. (22)	$f = 0.29$ used[11]	0.42
		Vavra II (1960)[11]	$B = f\sqrt{C_L}\,(s/c)$, Eq. (24)	$f = 0.29$ used[11]	0.35
	B5	Lakshminarayana I (1963)[14,15]	$B = 0.7$, Eq. (22)		2.83
	B6	Lakshminarayana II (1970)[15]	$B = 0.7 + B_w$, Eq. (32)	$(\frac{k}{h})_R = 1.5\%$でのλ_R→	4.89
	B7	Lakshminarayana III (1970)[15]	$B = 0.7\,\eta_T^2$, Eq. (22)		2.04
Theoretical formula II	C1	Soderberg (1953)[16] Amann (1963)[17] Rogo (1968)[18] (1968)[19]	$\frac{\pi D_{Tip}\,h}{A_{thr}}\,\eta_{To}$		2.12
	C2	Craig (1970)[20]	$1.5\,F_k\,\frac{\pi D_{Tip}\,h}{A_{thr}}\,\eta_{To}$, For F_k: See Appendix Fig. 2.	Consideration of Ov. Ratio ΔL	1.59

* Taking 1/2 time of the original coefficient, see Appendix A for the reasons.

'theoretical formula I'.

C) The third is another theoretical methods in which the decline of adiabatic efficiency $\Delta\eta_T$ is considered to be proportional to the area ratio of tip-clearance versus blade throat area. There are several formulas presented following to the correction coefficient of the theoretical formula. We call this group 'theoretical formula II'.

Details of A), B), and C) are presented in Appendices A and B.

Table 3 consolidates these methods and the column to the far right shows the calculated value λ_R corresponding to the present turbine. In the case of A, the formula is generally $\Delta\eta_T \approx -1.5\,(\frac{k}{h})_R$. In the case of B, λ_R is somewhat greater than in A. In the case of C, in spite of the very simple theory, appropriate λ_R values are obtained compared to others. Among these three, however, the B methods seem to be most logical.

5. 2 Comparison of various predicted values with the present experimental data

Figure 19 shows the various predictions of the turbine adiabatic efficiency by the above methods based on the design efficiency. The experimental data of the present study are also included for comparison; The sign 'o' indicates the overall adiabatic efficiency η_t obtained at the design expansion ratio and design turbine speed, while the sign 'Δ' indicates the value at the design velocity ratio of the peak efficiency line $\eta_{t,max}$ shown in Figure 16. The experimental decline shows $\lambda_R \approx 1.30$ approximately. This coincides closely with the predicted values with smaller values for λ_R, especially with the Ainley method (A3) and the Szanca formula (A5) among the

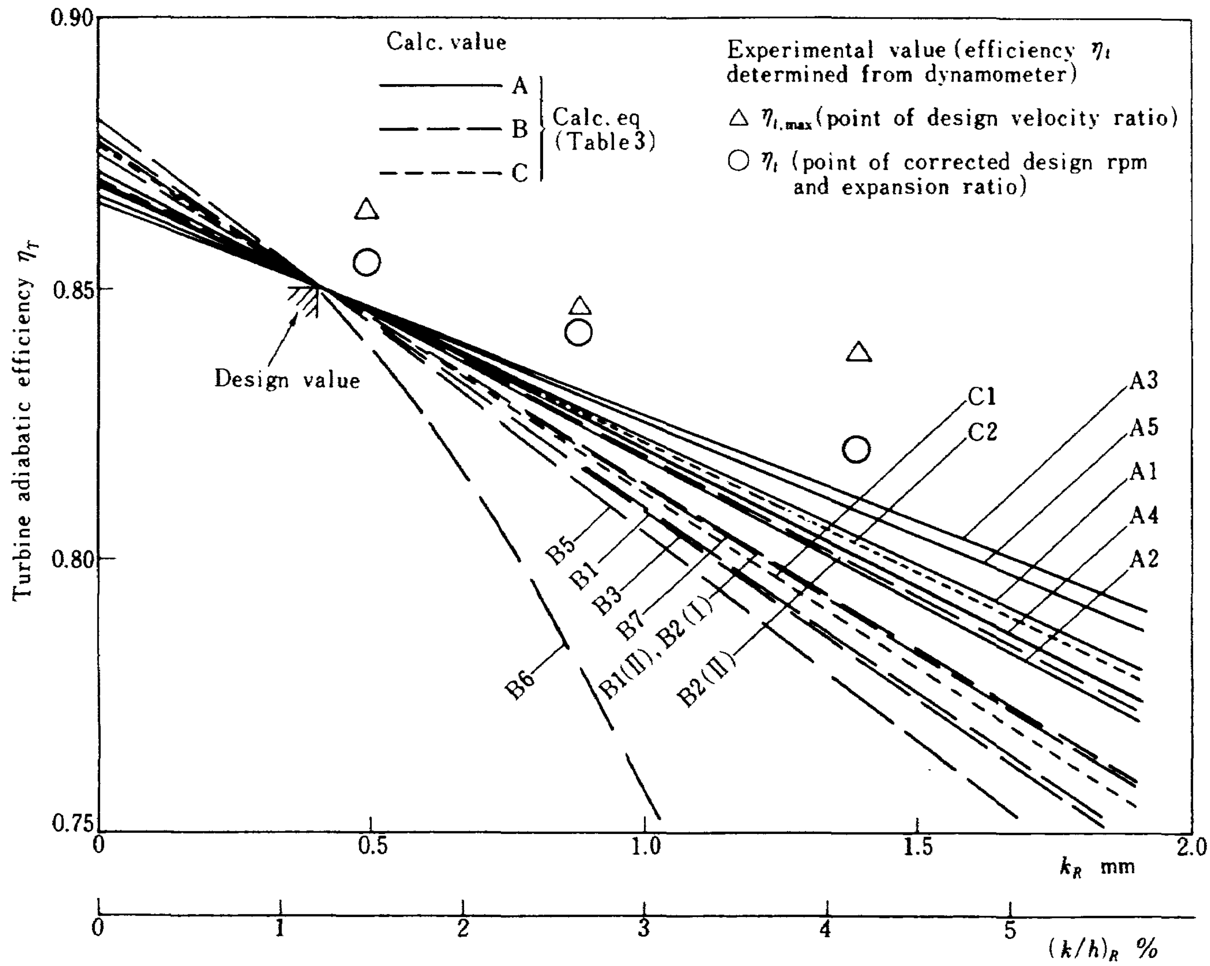


Fig. 19 Comparison between the results of the present experimental investigation and calculated values by various equations

empirical formula I, and with the Craig method (C2) among the theoretical formula II. In the case of the theoretical formula I in which the turbine performance parameters such as the blade lift coefficient C_L can be logically taken into account, the Ainley's value of B ($B = 0.5$) with formula (24) was the best prediction method; that is

$$\left.\begin{aligned} \Delta\eta_T &= -B\frac{\eta_T\,[C_L/(s/c)]_R^2 \sec^3\beta_{m,R}}{(\frac{V_{ad}}{U_m})^2\,(\frac{U_m}{V_{a3}})^2}\,(\frac{k}{h})_R \\ &\text{with} \\ B &= 0.5 \end{aligned}\right\} \quad \text{B-2 (II)}$$

The above formula can be rewritten for an axial flow turbo-machinery with an arbitrary number of stages as follows (Refer to note 5 in Appendix A for multi-stage turbine);

$$\Delta\eta_T = \frac{-\eta_T}{(\frac{V_{ad}}{U_m})^2}\sum_{i=1}^{i}\left\{\frac{B_N\,[C_L/(s/c)]_N^2 \sec^3\alpha_{m,N}}{(\frac{U_m}{V_{a2}})^2}(\frac{k}{h})_N + \frac{B_R\,[C_L/(s/c)]_R^2 \sec^3\beta_{m,R}}{(\frac{U_m}{V_{a3}})^2}(\frac{k}{h})_R\right\}_i$$

Here, i is the number of stages. This is valid also for the trubines with an arbitrary degree of reaction.

6. CONCLUSION

Experiments were conducted to determine the effect of rotor blade tip-clearance in a single-stage, high loaded axial-flow turbine for air-cooled high temperature turbine use by altering the dimensions of the rotor shroud rings. The main results are as follows;

1) Experiments were conducted at corrected turbine speeds of $N^* \approx 70\% \sim 110\%$ with corrected expansion ratios of $\phi_{cr}\pi_T \approx 1.4 \sim 2.2$, and the overall performance including the inlet mass flow rate and the turbine adiabatic efficiency were affected by the size of the rotor blade tip-clearance;
The turbine inlet mass flow at a constant turbine speed and constant expansion ratio increased as the rotor tip-clearance increased. When the turbine inlet flow characteristics are illustrated on the virtical axis against the expansion ratio on the horizontal axis with a parameter of the turbine speed, the characteristic curve of each turbine speed was sifted in parallel in the virtical direction due to the change of rotor tip-clearance size.
The turbine adiabatic efficiency decreases as the rotor tip-clearance increased. The efficiencies at the design turbine speed and design expansion ratio were 85.5%, 84.2% and 83.0% when the tip-clearances were 1.5%, 2.6% and 4.2%, respectively. Moreover, the peak efficiencies at the design velocity ratio were 86.5%, 84.7% and 83.3%, respectively.
2) The internal flow at the turbine outlet was obtained throughout the experimental range. From the distribution in the radial direction of the adiabatic temperature efficiency, the efficiency near the rotor tip was found to decrease substantially as the rotor tip-clearance increased and the influence extended to the vicinity of the rotor blade mean radius. From the spanwise distribution of the relative outflow angle of the rotor, the outflow angle was found to decrease as the tip-clearance increased; The defelection in the vicinity of the rotor blade tip decreased very much. This reduces the amount of turbine work done near the blade tip and it results in the efficiency decline.
3) The various methods of predicting the effect of blade tip-clearance on the turbine adiabatic efficiency were systematically classified and consolidated, resulting in a division of those methods into A) empirical formula, B) theoretical formula I and C) theoretical formula II. A comparison of the predicted results with the present experimental data indicated that the most promissing formula for the prediction could be expressed as follows:

$$\Delta\eta_T = \eta_T - \eta_{T0}$$

$$= -B \frac{\eta_T [C_L/(s/c)]_R^2 \sec^3 \beta_{m,R}}{(\frac{V_{ad}}{U_m})^2 (\frac{U_m}{V_{a3}})^2} (\frac{k}{h})_R$$

Here,

$$B = 0.5$$

In addition, the general formula for predicting $\Delta\eta_T$ with both rotor and stator blade tip-clearances in multi-stage axial-flow turbomachinery could be expressed by:

$$\Delta\eta_T = \frac{-\eta_T}{(\frac{V_{ad}}{U_m})^2} \sum_{i=1}^{i} \{ \frac{B_N [C_L/(s/c)]_N^2 \sec^3 \alpha_{m,N}}{(\frac{U_m}{V_{a2}})^2} (\frac{k}{h})_N$$

$$+ \frac{B_R [C_L/(s/c)]_R^2 \sec^3 \beta_{m,R}}{(\frac{U_m}{V_{a3}})^2} (\frac{k}{h})_R \}_i$$

Here, i is the number of stages.

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APPENDIX A: PREDICTION METHODS OF THE INFLUENCE OF BLADE TIP-CLEARANCE ON ADIABATIC EFFICIENCY

Various calculation methods have been suggested for estimating quantitatively the influence of rotor or stator blade tip-clearance on performance of turbomachinery, espeically on efficiency. Here, various calculation methods have been consolidated by the authors. The calculation methods were divided into three groups, (A), (B), and (C). Among these three, (A) is based on empirical correlations while (B) and (C) are theoretical formulas with or without an empirical constant or empirical function.

The adiabatic efficiency η_T used here is defined by the total temperature and total pressure at the inlet and outlet of turbomachinery, the so-called total to total state. The influence on the adiabatic efficiency η_T of the rotor blade tip-clearance ratio $(k/h)_R$ (k: blade tip-clearance, h: blade height) can be expressed as follows:

$$\Delta\eta_T = -\lambda_R \left(\frac{k}{h}\right)_R \quad (1)$$

Here, $\Delta\eta_T = \eta_T - \eta_{T0}$ (η_{T0} indicates η_T when $k = 0$) and the value of λ_R or the formula expressing λ_R differs depending on the researchers.

(A) **Empirical formula** (λ_R = constant)

From the reaction steam turbine tests in which both the rotor and stator blades had blade tip-clearances, Stodola [8] presented the following formula;

$$\Delta\eta_T \approx -3.1 \frac{k}{h} \text{ (for both stator and rotor blades with clearance)} \quad (2)$$

and here if there is clearance only for rotor blade, the following is assumed to hold by reducing the above constant 3.1 to 1/2 time:

$$\Delta\eta_T \approx \underbrace{-1.55}_{\lambda_R} \left(\frac{k}{h}\right)_R \quad \text{(A-1) Stodola}$$

Similarly, Meldahl presented the following formula:

$$\Delta\eta \approx -3.5 \frac{k}{h} \text{ (for both stator and rotor blades with clearance)} \quad (3)$$

in a single stage reaction turbine, and developed the following formula in the same manner as (A-1) for the clearance of a rotor blade only:

$$\Delta\eta_T \approx \underbrace{-1.75}_{\lambda_R} \left(\frac{k}{h}\right)_R \quad \text{(A-2) Meldahl}$$

The above Stodola-Meldahl formula has been reviewed by Ainley [8]. Moreover, Ainley achieved the following formula:

$$\Delta\eta_T \approx -2.6 \frac{k}{h} \text{ (for both stator and rotor blades with clearance)} \quad (4)$$

from experimental results on turbines with 50% reaction. Accordingly, the following develops for rotor blade tip-clearance:

$$\Delta\eta_T \approx \underbrace{-1.3}_{\lambda_R} \left(\frac{k}{h}\right)_R \quad \text{(A-3) Ainley}$$

Kofskey et. al. [6] showed, in their experiments with changing the rotor blade tip-clearances of both stages of two-stage reaction turbine in the same extent,

$$\Delta\eta_T \approx -3.20 \left(\frac{k}{h}\right) \text{ (for rotor blades of both stages having same clearance)} \quad (5)$$

Accordingly, the decline in efficiency at each stage of the rotor blade tip-clearance considering the form of formula (21) shown later would be expressed by the following:

$$\Delta\eta_T \approx \underbrace{-1.67}_{\lambda_R} \left(\frac{k}{h}\right)_R \quad \text{(A-4)* Kofskey}$$

*Note 1: Kofskey et al. [6, 7] and Szanca et al. [5] express the experimental values in the form $\Delta\eta/\eta_{ref} = -\text{const.}\left(\frac{k}{h}\right)$, where η_{ref} is η of the minimum k tested.

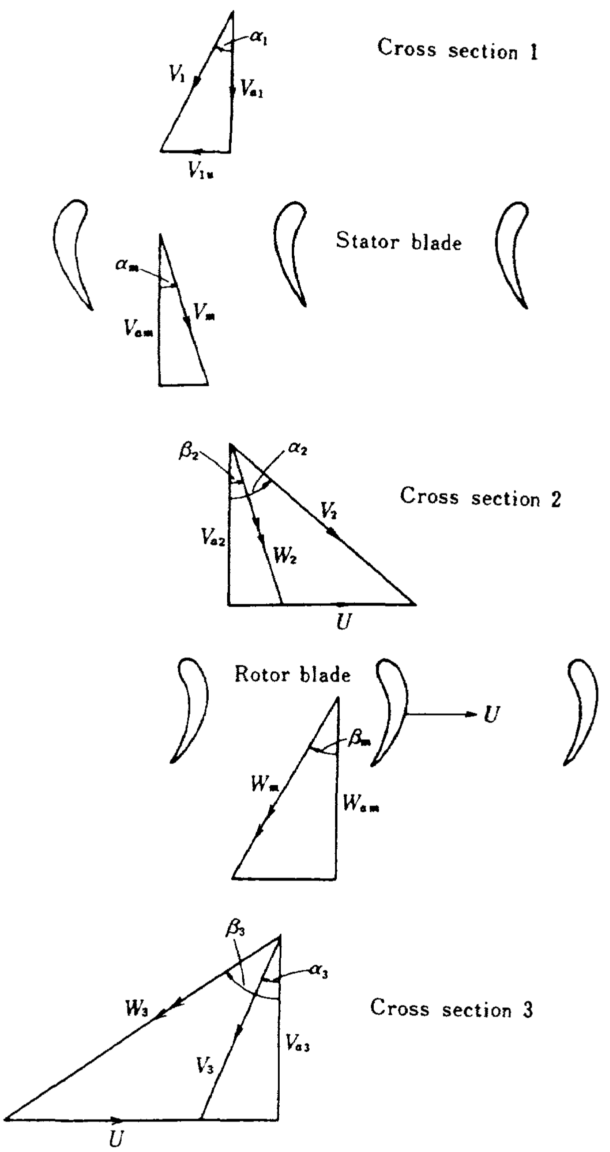
In addition, the following expression follows from the experimental results on rotor blade tip-clearance in axial-flow turbines designed for single-stage high temperature turbines by Szanca [5] (80.5% reaction at tip):

$$\Delta\eta_T = \underbrace{-1.39}_{\lambda_R}\ \left(\frac{k}{h}\right)_R \qquad \text{(A-5)*} \quad \text{Szanca}$$

(B) Theoretical formula I

The calculation formula of this category are derived from the equation of adiabatic efficiency and from the theoretical or semi-theoretical

*See note to Formula (A-4).



Appendix Fig. 1 Velocity triangles and notation

formula expressing the blade row drag coefficient C_{Dk} or blade total pressure loss coefficient Y_{tk}. See Appendix Figure 1.

The adiabatic efficiency is expressed by the following formula [16]:

$$\frac{1}{\eta_T} = \frac{H_{t1} - H_{t3,ad}}{H_{t1} - H_{t3}} = \frac{\Delta H_{T,ad}}{\Delta H_T}$$

$$\approx 1 + \frac{\left\{C_{D,N}\left(\frac{c}{s}\right)_N \frac{V_{a2}^2}{\cos^3\alpha_{m,N}} + C_{D,R}\left(\frac{c}{s}\right)_R \frac{V_{a3}^2}{\cos^3\beta_{m,R}}\right\}}{2\,\Delta H_T} \tag{6}$$

Here, the suffixes N, R represent stator and rotor blades, respectively. C_D is the blade row drag coefficient and c/s is the (chord/pitch) ratio, while V_{a2} and V_{a3} are the axial flow velocities at the outlet of stator and rotor blades, respectively. In addition, α_m and β_m are the mean flow angles of the stator and rotor blades expressed by the following approximate formula (with $V_{a1} = V_{a2} = V_{a3}$):

$$\left.\begin{aligned} \alpha_{m,N} &= \tan^{-1}\left(\frac{\tan\alpha_2 - \tan\alpha_1}{2}\right) \\ \beta_{m,R} &= \tan^{-1}\left(\frac{\tan\beta_3 - \tan\beta_2}{2}\right) \end{aligned}\right. \tag{7}$$

On the other hand, the efficiency when the blade tip-clearance is zero is expressed as follows from formula (6) where H_t is expressed as H_{T0} when the blade tip-clearance is 0**;

$$\frac{1}{\eta_{T0}} = 1 + \frac{C_{D0,N}\left(\frac{c}{s}\right)_N \frac{V_{a2}^2}{\cos^3\alpha_{m,N}} + C_{D0,R}\left(\frac{c}{s}\right)_R \frac{V_{a3}^2}{\cos^3\beta_{m,R}}}{2\,\Delta H_{T0}} \tag{8}$$

where C_{D0} comes from $C_{D,N} = C_{D0,N} + C_{Dk,N}$, $C_{D,R} = C_{D0,R} + C_{Dk,R}$ (where C_{D0} is the blade drag coefficient when the blade tip-clearance k equals zero, while C_{Dk} is the additional blade drag coefficient due to blade tip-clearance). Considering

**Note 2: Here, the changes in V_{a2}, V_{a3}, α_m, β_m due to tip-clearance variation are assumed to be negligible.

$$\Delta H_T = \frac{1}{2} V_{ad}^2 \eta_T, \ \Delta H_{T0} = \frac{1}{2} V_{ad}^2 \eta_{T0}$$

$$(V_{ad} = \sqrt{2\, gJ\, \Delta H_{T,ad}})$$

and from formulas (6) and (8), the value of

$$\Delta \eta_T = \eta_T - \eta_{T0}$$

would be expressed by**

$$\Delta \eta_T = -\left\{ \frac{C_{D,N}\,(\frac{c}{s})_N}{(\frac{V_{ad}}{U_m})^2\,(\frac{U_m}{V_{a2}})^2 \cos^3 \alpha_{m,N}} + \frac{C_{D,R}\,(\frac{c}{s})_R}{(\frac{V_{ad}}{U_m})^2\,(\frac{U_m}{V_{a3}})^2 \cos^3 \beta_{m,R}} \right\} \quad (9)$$

The following holds when the stator blade tip has no clearance, and only the rotor blade has clearance:

**Note 3: On formula (9): Formula (6) would lead, in stead of (9),

$$\Delta \eta_T \approx -\eta_T \left\{ \frac{C_{D,N}\,(\frac{c}{s})_N}{(\frac{V_{ad}}{U_m})^2\,(\frac{U_m}{V_{a2}})^2 \cos^3 \alpha_{m,N}} + \frac{C_{D,R}\,(\frac{c}{s})_R}{(\frac{V_{ad}}{U_m})^2\,(\frac{U_m}{V_{a3}})^2 \cos^3 \beta_{m,R}} \right\} \quad (9a)$$

assuming that the second term denominator on the right side in formula (8), ΔH_{T0}, were approximately $\Delta H_T = \Delta H_{T0}$ ($\eta_T = \eta_{T0}$). It would be η_T time in contrast to the right side of formula (9). This η_T time cannot be ignored since $\eta_T = 0.95 \sim 0.8$. In particular, it is important since it is effective on the slope (λ) of the curve of ($\Delta \eta_T \sim \frac{k}{h}$), directly. The formula corresponding to formula (21), therefore, is the following:

$$\Delta \eta_T \approx \frac{-\eta_T}{(\frac{V_{ad}}{U_m})^2} \sum_{i=1}^{i} \left\{ \frac{B_N\,[C_L/(s/c)]_N^2 \sec^3 \alpha_{m,N}}{(\frac{U_m}{V_{a2}})^2} (\frac{k}{h})_N + \frac{B_R\,[C_L/(s/c)]_R^2 \sec^3 \beta_{m,R}}{(\frac{U_m}{V_{a3}})^2} (\frac{k}{h})_R \right\}_i \quad (21a)$$

Formula (24) is derived from formula (21a).

$$\Delta \eta_T = -\frac{C_{D,N}\,(\frac{c}{s})}{(\frac{V_{ad}}{U_m})^2\,(\frac{U_m}{V_{a3}})^2 \cos^3 \beta_{m,R}} \quad (10)$$

(for rotor blade tip-clearance only)

The above formulas (9) and (10) also hold for the turbines with arbitrary reaction.

In general form, formula (9) can be rewritten as follows for a turbine of arbitrary i-stages:

$$\Delta \eta_T = -\frac{1}{(\frac{V_{ad}}{U_m})^2} \sum_{i=1}^{i} \left\{ \frac{C_{D,N}\,(\frac{c}{s})_N}{(\frac{U_m}{V_{a2}})^2 \cos^3 \alpha_{m,N}} + \frac{C_{D,R}\,(\frac{c}{s})_R}{(\frac{U_m}{V_{a3}})^2 \cos^3 \beta_{m,R}} \right\}_i \quad (11)$$

In the case of 50% reaction, since the two terms within { } in formula (6) are mutually equivalent, and since [16];

$$\Delta H_T \approx U_m\, C_L\, V_a \sec \alpha_m / 2\,(s/h),$$

we have

$$\frac{1}{\eta_T} = 1 + (2\, C_D / C_L) \operatorname{cosec} 2\,\alpha_m \quad (12)$$

Considering that $\eta_T \approx \eta_{T0}$:

$$\therefore \Delta \eta_T \approx -2\eta_T^2\,(C_{Dk}/C_L) \operatorname{cosec} 2\,\alpha_m$$

(for 50% reaction turbines with tip-clearance at both rotor and stator blades) (13)

The blade drag coefficient C_D and the lift coefficient C_L are defined by the following formula:

$$\left. \begin{aligned} C_{D,N} &= \frac{P_{t1} - P_{t2}}{\frac{1}{2}\rho_m V_m^2} (\frac{s}{c})_N \cos \alpha_{m,N} \\ C_{D,R} &= \frac{P_{r2} - P_{r3}}{\frac{1}{2}\rho_m W_m^2} (\frac{s}{c})_R \cos \beta_{m,R} \end{aligned} \right\} \quad (14)$$

$$\left. \begin{aligned} C_{L,N} &= 2\,(\frac{s}{c})_N\,(\tan\alpha_1 + \tan\alpha_2) \cos\alpha_{m,N} + C_{D,N} \tan\alpha_{m,N} \\ C_{L,R} &= 2\,(\frac{s}{c})_R\,(\tan\beta_2 + \tan\beta_3) \cos\beta_{m,R} + C_{D,R} \tan\beta_{m,R} \end{aligned} \right\} \quad (15)$$

In the case of a low Mach number flow, considering the incompressible relation that $\rho_2 = \rho_m$, $V_m \cos\alpha_m = V_2 \cos\alpha_2$, C_D and Y_t would have the following relation:

$$\left.\begin{aligned} C_{D,N} &= Y_{tN} \left(\frac{s}{c}\right)_N \frac{\cos^3 \alpha_{m,N}}{\cos^2 \alpha_2} \\ C_{D,R} &= Y_{tR} \left(\frac{s}{c}\right)_R \frac{\cos^3 \beta_{m,R}}{\cos^2 \beta_3} \end{aligned}\right\} \tag{16}$$

In addition, the following formula for C_L could be approximately used [8]:

$$\left.\begin{aligned} C_{L,N} &\approx 2 \left(\frac{s}{c}\right)_N (\tan\alpha_1 + \tan\alpha_2) \\ C_{L,R} &\approx 2 \left(\frac{s}{c}\right)_R (\tan\beta_2 + \tan\beta_3) \end{aligned}\right\} \tag{17}$$

Formula expressing C_{DkD}

From now, the drag coefficient C_{Dk} due to blade tip-clearance is expressed in the following form:

$$C_{Dk} = BC_L^2 \left(\frac{k}{h}\right) / (s/c) \tag{18}$$

Here, the blade lift coefficient C_L is defined on the basis of V_0^2 $(= \eta_T V_{ad}^2)$ for rotor blade and B is a constant or a function.

The formula for the efficiency decline portion $\Delta\eta_T$ due to blade tip-clearance for machinery with a 50% reaction becomes the following from formulas (13) and (18);

$$\Delta\eta_T \approx -2\eta_T^2 B [C_L/(s/c)] \operatorname{cosec} 2\alpha_m \left(\frac{k}{h}\right) \quad (\text{for } \rho_R = 0.5) \tag{19}$$

Since this assumes the identical blade tip-clearance for both stator and rotor blades, the influence due to the blade tip-clearance of rotor only would be 1/2 time the above decline. Thus,

$$\Delta\eta_T \approx -\eta_T^2 B [C_L/(s/c)]_R \operatorname{cosec} 2\beta_{m,R} \left(\frac{k}{h}\right)_R \tag{20}$$

Moreover, in the case of a multi-stage (i-stage number) with an arbitrary reaction, (11) and (18) would become

$$\Delta\eta_T = \frac{-1}{\left(\frac{V_{ad}}{U_m}\right)^2} \sum_{i=1}^{i} \left\{ \frac{B_N [C_L/(s/c)]_N^2 \sec^3 \alpha_{m,N}}{\left(\frac{U_m}{V_{a2}}\right)^2} \left(\frac{k}{h}\right)_N + \frac{B_R [C_L/(s/c)]_R^2 \sec^3 \beta_{m,R}}{\left(\frac{U_m}{V_{a3}}\right)^2} \left(\frac{k}{h}\right)_R \right\}_i \tag{21}$$

B_N and B_R mean the values of B for stator and rotor blade blades, respectively. Then, if the value of B is known, $\Delta\eta_T$ can be calculated for any multi-stage machinery with any arbitrary blade tip-clearance of rotor and stator blades by the above formula.

In the case of a clearance for only the rotor blade in one stage, the following is, therefore, derived from (21) putting $i = 1$ and $(k/h)_N = 0$;

$$\Delta\eta_T = -\underbrace{\frac{B_R}{\left(\frac{V_{ad}}{U_m}\right)^2} \frac{[C_L/(s/c)]_R^2 \sec^3 \beta_{m,R}}{\left(\frac{U_m}{V_{a3}}\right)^2}}_{\lambda_R} \left(\frac{k}{h}\right)_R \tag{22}$$

Various formulas which express B are given below.

(I) The theoretical formula for C_{Dk} by Carter [22] is:

$$C_{Dk} = \frac{1}{2} C_L^2 \left(\frac{k}{h}\right) / (s/c) \tag{23}$$

In this case, clearly,

$B = 0.5$ (B-1) Carter

(II) Ainley [9] provided the following values as the B value in formula (18);

Without blade tip-shroud: $B = 0.5$ (B-2)
With blade tip-shroud : $B = 0.25$ Ainley

$\Delta\eta_T$ can be calculated by substitution of these B values in (22). However, Ainley [8] provided the following for the absence of shroud, $(B = 0.5)$;

$$\Delta\eta_T = -B\frac{\eta_T\,[C_L/(s/c)]_R^2\,\sec^3\beta_{m,R}}{(\frac{V_{ad}}{U_m})^2\,(\frac{U_m}{V_a})^2}\,(\frac{k}{h})_R \tag{24}$$

In contrast to the results from (22) with $B = 0.5$, here the right term becomes η_T time that of (22). This is the same as the present result achieved by assuming $\Delta H_0 = \Delta H$ (i.e. $\eta_{T0} \approx \eta_T$) and $V_{a3} = V_a$ in the present procedure*.

(III) Meldahl [10] determined

$$C_{Dk} = \frac{1}{4\cos\beta_3}C_L^2\,(\frac{k}{h})\,/\,(s/c) \tag{25}$$

This corresponds to:

$$B = 1/(4\cos\beta_3) \tag{B-3}$$

Meldahl

(IV) Vavra [11] derived the following formula theoretically from the mass flow through the blade tip-clearance and the kinetic energy of the flow;

$$C_{Dk} = f\,(\frac{t}{c},\frac{k}{c})\,C_L^{3/2}\,(\frac{k}{h}) \tag{26}$$

Here, $f(\frac{t}{c},\frac{k}{c})$ is a function of blade thickness and blade tip-clearance and is theoretically derived as $f(\frac{t}{c},\frac{k}{c}) = \frac{4\sqrt{2}}{5}I\!K\cdot I\!R^3$ ($I\!K$ is a flow coefficient of the flow leaking through the clearance, while $I\!R$ is a factor considering the drag of the flow within the clearance). This corresponds to:

$$B = \frac{4\sqrt{2}}{5}I\!K\cdot I\!R^3(s/c)\,/\,\sqrt{C_L} \tag{B-4}$$

i.e., Vavra

As an example, Vavra [11] has given $f = 0.29$ ($I\!K = 0.5$, $I\!R = 0.8$).

(V) Meanwhile, Lakshminarayama [10] has theoretically derived the following formula:

$$C_{Dk} = C_L^2\,(1-K)\,f(\frac{s}{c},\frac{k}{s},1-K)\,/\,(h/c)$$

where,

$$f = \frac{1}{4\pi}\ln\frac{(e^{\frac{2\pi h}{s}}-1)\,(2.03+0.03\coth\frac{2\pi k}{s})}{0.03\,[(1-\coth\frac{2\pi k}{s})+(1+\coth\frac{2\pi k}{s})e^{\frac{2\pi h}{s}}]} \tag{27}$$

He gave, theoretically, $f/(k/s) \approx 1.4$, while experimentally, $K = 0.5$ [14]. Accordingly, the following was derived:

$$C_{Dk} = 0.7C_L^2\,(\frac{k}{h})\,/\,(s/c) \tag{28}$$

This clearly is proportionate to:

$$B = 0.7 \tag{B-5}$$

Lakshminarayana

Adding the kinetic energy in the form of pressure loss of the blade spanwise flow due to the blade tip-clearance to the drag (28) induced by the blade tip-clearance, Lakshminarayana [15] provided a formula for a decline in efficiency of turbomachinery due to blade tip-clearance. The same formula as in [15] will be derived by a different manner here; for total pressure loss coefficient $Y_{t,w}$ due to the spanwise flow, the following formula was given [15];

$$Y_{t,w} \approx \frac{\delta_s^* + \delta_p^*}{s}\,\frac{h}{c}\,\frac{C_L^{3/2}\,(\frac{k}{s})^{3/2}\,V_m^2}{W_a\,W_1} \tag{29}$$

This corresponds to the following, considering (16);

$$C_{Dw} \approx B_w\,C_L^2\,(\frac{k}{h})\,/\,(s/c)$$

where, (30)

$$B_w = \frac{\delta_s^* + \delta_p^*}{s}\sqrt{(\frac{h}{c})(\frac{k}{h})\,/\,[C_L/(s/c)]}$$

As a result,

$$C'_{Dk} = C_{Dk}\text{ of (28)} + C_{Dw}\text{ of (30)}$$
$$= (B + B_w)\,C_L^2\,(\frac{k}{h})\,/\,(s/c) \tag{31}$$

where

$$B = 0.7 \tag{B-5}$$

Accordingly, the following may be substituted for the B in formula (22);

$$B = (0.7 + B_w) \tag{B-6}$$

*Cf. footnote page 30, Note 3.

Therefore, the following develops* for rotor tip-clearance only:

$$\Delta\eta_T = -(B+B_w)\frac{[C_L/(s/c)]_R^2 \sec^3\beta_{m,R}}{(\frac{V_{ad}}{U_m})^2(\frac{U_m}{V_{a3}})^2}(\frac{k}{h})_R \tag{32}$$

In (30), Lakshminarayana [15] established $\frac{\delta_s^*+\delta_p^*}{s}=7$ (const.). Accordingly, formula (32) is written as:

$$\Delta\eta_T = -\underbrace{\{0.7+\underbrace{7\sqrt{(\frac{h}{c})(\frac{k}{h})/[C_L/(s/c)]}}_{B_w}\}\frac{[C_L/(s/c)]_R^2 \sec^3\beta_{m,R}}{(\frac{V_{ad}}{U_m})^2(\frac{U_m}{V_{a3}})^2}}_{\lambda_R}\cdot(\frac{k}{h})_R \tag{33}$$

In formula (33), the second term B_w within { } can not generally be ignored [15]. However, since the term $\frac{\delta_s^*+\delta_p^*}{s}$ is as small as few percent [25–28] in the case of turbines, the second term within { } could be ignored.** In this case, formula (33) is identical to (22) with (B-5).

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*Note 4: Using the pressure loss $(\Delta P)_L$, Lakshminarayana [15] has defined the efficiency decline as $\Delta\eta=(\Delta P)_L/(\Delta P_t)_{ad}$, and this is identical to the $\Delta\eta_T$ of this report. Accordingly, formula (15) in [15] can be derived also by the method of this report.
Note that formula (32) is only for the rotor blade tip-clearance per stage. For multi-stage turbines, summation is needed; see the following footnote**.)

**Note 5: Two-stage turbine experiments of Kofskey [6] and examination of B_w: The following follows from the velocity triangles at rotor blade tips of both stages of a two-stage turbine of Kofskey;

$$(\frac{V_{ad}}{U_m})^2 = \frac{2gJ\Delta H_{ad}}{(\pi D_m N)^2}$$

$$= \frac{2\times9.8\times426.9\times0.24\times288.2\times\{1-(\frac{1}{1.225})^{0.286}\}}{(3.14\times0.232\times12000/60)^2}$$

$$=1.54$$

$$(\frac{V_{a3}}{U_m})_{stg.1} = (\frac{V_{a3}}{U_m})_{stg.2} = 0.291$$

$$\beta_{m,R}\ (stg.1) = \beta_{m,R}\ (stg.2) \approx 64.9^\circ$$

$$[C_L/(s/c)]_{stg.1} = [C_L/(s/c)]_{stg.2} = 2.07 \quad (s/c=1.16)$$

so that formula (21) ($i=2$, $B_N=0$) would be:

$$\Delta\eta_T = -2B\times\frac{(2.07)^2\sec^3 64.9^\circ}{1.54/(0.291)^2}(\frac{k}{h})_R$$

$$= -6.18B(\frac{k}{h})_R \tag{21'}$$

The following formula shown in (B-6);

$$B = 0.7+B_w = 0.7+7\sqrt{1.2(\frac{k}{h})_R/2.07}$$

$$= 0.7+5.33\sqrt{(\frac{k}{h})_R}$$

is used, and we get

$$\Delta\eta_T = -\{4.32+32.9\sqrt{(\frac{k}{h})_R}\}(\frac{k}{h})_R \tag{32'}$$

Here, putting that $B_w=0$, then,

$$\Delta\eta_T = -4.32(\frac{k}{h})_R \tag{32''}$$

If (24) were used with $B_w=0$, ($\eta_T=0.845$),

$$\Delta\eta_T = -4.32\,\eta_T(\frac{k}{h})_R = -3.65(\frac{k}{h})_R \tag{24'}$$

In addition, if (21) were used with B = 0.5 instead of 0.7,

$$\Delta\eta_T = -6.18\times0.5(\frac{k}{h})_R = -3.08(\frac{k}{h})_R \tag{21''}$$

Appendix Figure 3 illustrates the calculation results of formula (32'), and the results of (32"), (24') and (21") with $B_w=0$. The calculation results with $B_W=0$ agree with the test data; the results of formula (21) with $B=0.5$ of Ainley of Carter's constant is best. The method of [15] assuming $\frac{\delta_s^*+\delta_p^*}{s}=7$ for B_w predicted an extremely large decline $\Delta\eta_T$ in a turbine. In the case of turbines, therefore, putting $B_w=0$ seems to be better.

(C) Theoretical formula II

Based on some simplified assumptions, Soderberg [16], Amman et al. [17], and Rogo [18] expressed the effect of the rotor blade tip-clearance on adiabatic efficiency by the following formula:

$$\eta_T = \eta_{T\rho} \frac{A_{thr}}{A_{thr} + A_k} \tag{34}$$

Here, A_k is the area of the blade tip-clearance ($A_k = \pi D_{Tip}\ k$, D_{Tip} is the rotor blade tip diameter). A_{thr} is the blade throat area (not including A_k). Thus,

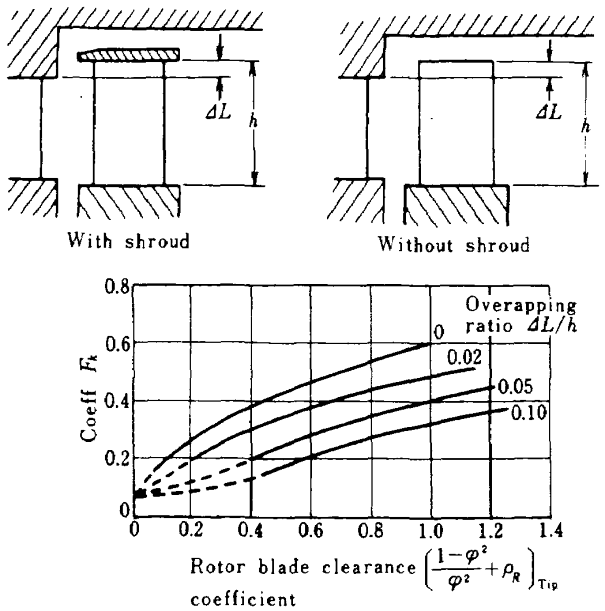
$$\Delta\eta_T = -\underbrace{\frac{\pi D_{Tip}\, h}{A_{thr}}}_{\lambda_R}\ \eta_{T0}\ (\frac{k}{h})_R \tag{C-1}$$

Meanwhile, Craig et al. [20] have suggested the following formula introducing a corrected coefficient F_k into the preceding formula (C-1):

$$\Delta\eta_T = -F_k \frac{A'_k}{A_{thr} + A'_k}\eta_{T0}\ \text{(shroud)} \tag{35}$$

$$\Delta\eta_T = -1.5F_k \frac{A'_k}{A_{thr} + A'_k}\eta_{T0}\ \text{(no shroud)} \tag{36}$$

F_k is a function of reaction ρ_R $(\equiv 1 - \frac{W_2^2}{W_1^2})$,



Appendix Fig. 2[20)]

rotor blade velocity coefficient ψ and of overlapping ratio $\Delta L/h$, as shown in Appendix Figure 2. A'_k is the effective area of the rotor blade tip-clearance. In the absence of blade tip shroud, the following formula is an approximation of formula (36):

$$\Delta\eta \approx -\underbrace{1.5\, F_k \frac{\pi D_{Tip}\, h}{A_{thr}}}_{\lambda_R}\ \eta_{T0}\ (\frac{k}{h})_R \tag{C-2}$$

One of the features of this formula is to be able to take account of overlapping ratio.

APPENDIX B: CALCULATION OF $\Delta\eta_T \sim (k/h)_R$ FOR THE PRESENT TEST TURBINE

Conditions for calculation of the theoretical method I

In this report, the mean diameter values are used;

$\beta_1 = 43.6°$, $\beta_3 = 63.5°$,

$s/c = 1/1.48 = 0.676$, $\frac{h}{c} = 1.86$

From formula (7), we have

$$\beta_{m,R} = \tan^{-1}(\frac{\tan 63.5° - \tan 43.6°}{2}) = 27.8°$$

From formula (17), we have

$C_{L,R} = 2 \times 0.676 \times (\tan 43.6° + \tan 63.5°) = 3.54$

$\eta_T = 0.85$ (design value)

From the design theoretical velocity ratio $U_m/V_{ad} = U_M/V_{ad} = 0.504$,

$$(\frac{V_{ad}}{U_m})^2 = 3.94$$

axial flow velocity ratio

$\phi = (V_{a3}/U_m) = (232.44/366.72) = 0.634$

(load blade coefficient

$$\psi = \frac{1}{2}(\frac{V_0}{U_m})^2 = \frac{1}{2} \times 0.85 \times 3.94 = 1.67)$$

Conditions for calculation of the theoretical method II

TIP diameter D_{Tip} = 0.556m (measured values)

h = 0.0334m,

$$A_{thr} = O \times h \times n = 0.00515 \times 0.0334 \times 136 = 0.0234\text{m}^2$$

$$\Delta L/h = 0.0007/0.0334 = 0.021$$

$$\varphi^2_{Tip} = 1 - \left(\frac{Y_{tR}}{1 + \frac{KM^2}{2}}\right)_{Tip}$$

$$= 1 - 0.23/(1 + \frac{1.31 \times 0.7916^2}{2}) = 0.84$$

$$\rho_{R,Tip} = 1 - (\frac{W_2}{W_1})^2_{Tip} = 1 - (\frac{199.87}{533.76})^2 = 0.86$$

Calculated results

The λ_R of formula (22) is:

$$\Delta\eta_T = \frac{-B}{3.94} \frac{(3.54/0.676)^2 \sec^3 27.8°}{(\frac{1}{0.634})^2} (\frac{k}{h})_R$$

$$= -4.04B(\frac{k}{h})_R \qquad (22')$$

For reaction of 50%, formula (20) leads to

$$\Delta\eta_T \approx -0.85^2 B\, [3.54/0.676]\ \text{cosec}\,(2 \times 27.8°)\ (\frac{k}{h})_R = -4.59B(\frac{k}{h})_R \quad (\text{for } \rho_R = 0.5) \qquad (20')$$

Depending on the various values for B, the above formulas will be as follows;

(B-1) Carter: B = 0.5

From formula (20′),

$\Delta\eta_T = -2.29\,(\frac{k}{h})_R$ .... Carter I

From formula (22′)

$\Delta\eta_T = -2.02\,(\frac{k}{h})_R$ .... Carter II

(B-2) Ainley: B = 0.5

From (22′),

$\Delta\eta_T = -2.02\,(\frac{k}{h})_R$ .... Ainley I

From (24),

$\Delta\eta_T = -1.72\,(\frac{k}{h})_R$ .... Ainley II

(B-3) Meldahl: $B = 1/(4\cos 63.5°) = 0.56$

From (22′),

$\Delta\eta_T = -2.26\,(\frac{k}{h})_R$ .... Meldahl

(B-4) Vavra:

$K = 0.5$, $R = 0.8$ ($f = 0.29$), then $B = 0.10$, thus, from (22′),

$\Delta\eta_T = -0.42\,(\frac{k}{h})_R$ .... Vavra I

In addition, the following develops if (24) is used:

$\Delta\eta_T = -0.42\eta_T(\frac{k}{h})_R = -0.35(\frac{k}{h})_R$ .. Vavra II

(B-5) Lakshminarayana: (B = 0.7)

Formula (B-5): from formula (22′) when $B = 0.7$, we have

$\Delta\eta_T = -2.83\,(\frac{k}{h})_R$ .... Lakshminarayana I

(B-6) Lakshinarayana: $(\frac{\delta^*_s + \delta^*_p}{s} = 7)$

Formula (30) states:

$$B_w = 7 \times \{1.86 \times (\frac{k}{h})_R/(3.54/0.676)\}^{1/2} = 4.17\,(\frac{k}{h})^{1/2}_R$$

accordingly, from formula (32):

$$\Delta\eta_T = -\{0.7 + 4.17\,(\frac{k}{h})^{1/2}_R\}(\frac{k}{h})_R = -2.83\{1 + 5.95\,(\frac{k}{h})^{1/2}_R\}(\frac{k}{h})_R$$

...... Lakshminarayana II

(B-7) Lakshminarayana: ($B = 0.7\eta^2_T$, $B_w = 0$)

From formula (22′),

$\Delta\eta_T = -2.04\,(\frac{k}{h})_R$ .... Lakshminarayana III

(C-1) Soderberg, Amman, Rogo:

From formula (C-1), we have

$\Delta\eta_T = -2.12\,(\frac{k}{h})_R$ .... Soderberg, Amann, Rogo

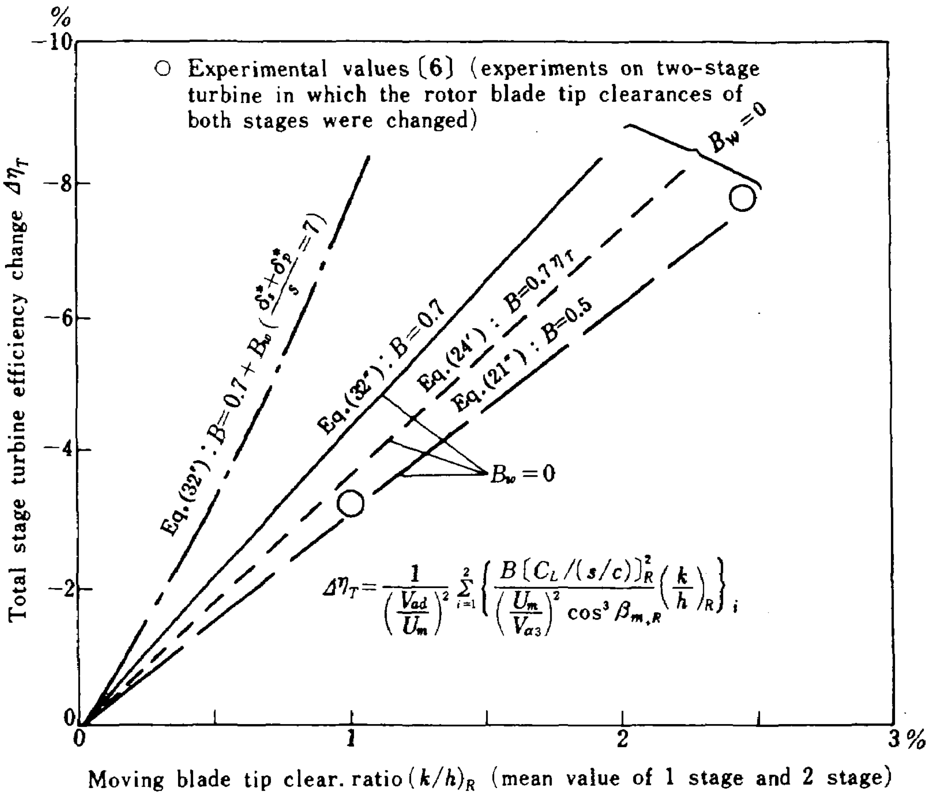
(C-2) Craig:

$$(\frac{1-\varphi^2}{\varphi^2} + \rho_R)_{Tip} = 1.05, \quad \Delta L/h = 0.021$$

as a result of the above, $F_k = 0.5$ from Appendix Figure 2, so that from (36), we have

$$\Delta\eta_T = -1.59\left(\frac{k}{h}\right)_R \quad \ldots\ldots\ldots\ldots\ldots \text{Craig}$$

The above calculation results are shown in Table 3 and Figure 19 in the main text.



Appendix Fig. 3 Two-stage turbine experiments and examination of B_w

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