B12

微小デブリ軌道上観測データの統計的解析手法

Statistical Analysis on In-situ Measurements of Small Space Debris

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大きさが 2 mm に満たないような微小なスペースデブリは地上からの観測が不可能であるが，電源ハーネスを切断するなどして衛星を機能喪失させ得る．この問題に対し現在，衝突センサを搭載した衛星により微小デブリを軌道上で観測する計画が各国で進められており，その観測データの効果的な利活用が期待される．そこで本発表では，空間的・時間的に高解像度が期待される軌道上観測データに対し統計的手法を活用することで様々な解析が可能であることを示す．まず，空間的分布に関しては，逐次モンテカルロフィルタを適用して微小デブリの軌道面分布を逐次的に推定する手法について，概略とシミュレーション結果を示す．また，時間的分布について，環境変動の有無を数学的根拠を伴って判断しその規模を推定するための手法としてカイ二乗検定と赤池情報量基準を導入し，シミュレーションにより検証する．将来の微小デブリの軌道上観測におけるこれらの手法の活用により，宇宙開発の安全と持続可能性の向上が期待される．

The impact of small pieces of space debris (less than 2 mm in size) that ground-based observations are unable to detect can cause fatal damage to a spacecraft. Accordingly, to monitor the environment of these small pieces, in-situ measurements that utilize on-orbit impact sensors capable of detecting small objects are being advanced. This study proposes statistical methods to analyze the in-situ data of small debris, which has high spacial and temporal resolution. First, the sequential Monte Carlo filter is applied to estimate the spacial distribution of small debris. Second, this presentation describes that temporal changes of the debris environment can be detected and assessed utilizing the Chi-squared testing and Akaike Information Criterion. These proposed methods are evaluated by simulations with an engineering models to simulate impacts with small debris. The methods proposed by this study can contribute the sustainable space development and utilization by providing better knowledge of small debris utilizing the in-situ measurements.

9th Space Debris Workshop
B12

Statistical Analysis on In-situ Measurements of Small Space Debris
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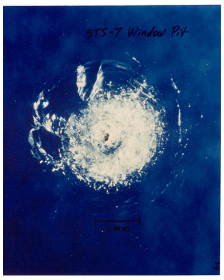
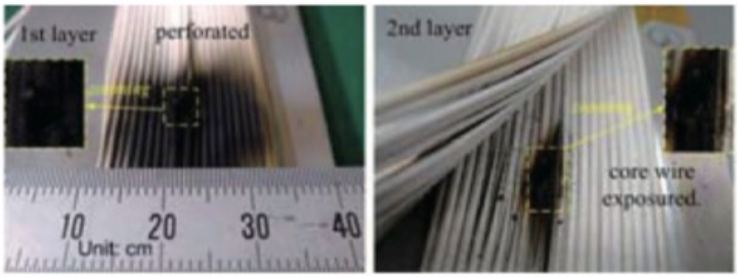
Background
Sub-millimeter-size Debris

- Small debris (< 2 mm) can neither be tracked nor detected by the ground-based observations
- There is a difference between the existing models (MASTER / ORDEM)
- Impact of small debris can cause a fatal damage on a spacecraft



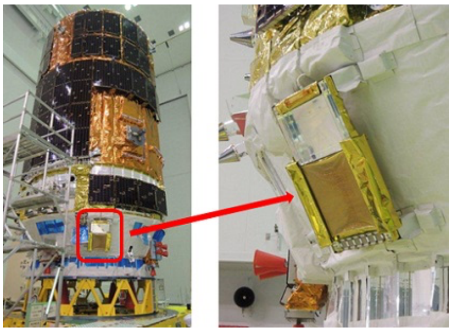
Small debris impact
on shuttle window (NASA)



Cables severed by an impact with simulated debris (0.3 mm)
Nitta et. al., 2010

Background

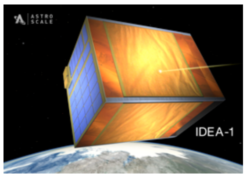
In-situ Measurement



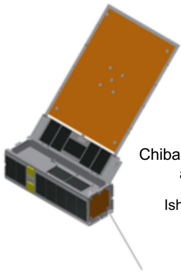
Space Debris Monitor on HTV-5
(image credit: JAXA)

Space Debris Sensor on ISS
(image credit: NASA)

Small Satellites



▲ IDEA by Kyushu Univ. and Astroscale
(Launch Failed)



◀ ASTERISC by
Chiba Institute of Tech.
and Tohoku Univ.

Ishimaru et al., 2020

Advantages of in-situ measurements:
High resolution of time and location of impacts

Objectives

Data from in-situ measurements
- Location Identification
- High Temporal Resolution

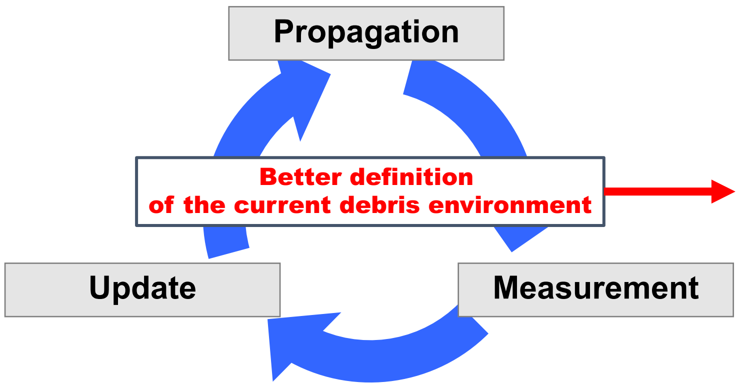
\+ **Statistical Analysis**

→

1. Dynamic environmental estimation
   Distribution model of debris that can be updated sequentialy based on latest data
2. Detection of environmental change
   Quantitative identification of environmental changes posed by breakups or influence of the solar activity

Dynamic Environmental Estimation

Concept of the dynamic modeling



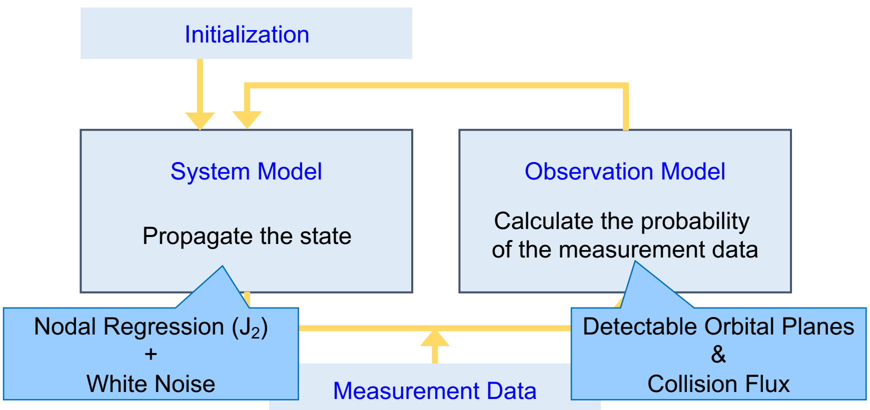
- Optimal Protection Design
- "Collision Alleviation"

Dynamic Environmental Estimation

Estimation Algorithm

(Furumoto, M. and Hanada, T., Advances in Space Research, Vol. 62, No. 3, Pages 533-541, 2018.)

❖ Sequential Monte Carlo (SMC) filter
: A statistical method to estimate the state incorporating simulation and measurement data



Dynamic Environmental Estimation

System Model

◆ State Vector

- Represents the orbital plane distribution of small debris in circular orbits at same altitude of the detector
- Approximate the environment by an ensemble of orbits that represent a huge number of debris

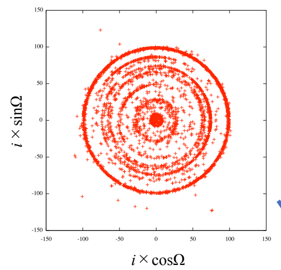
$$\boldsymbol{x}^{(i)} = (N_d, \Omega_1, i_1, \Omega_2, i_2, \dots, \Omega_n, i_n)$$

N_d : Number of debris that each orbit includes
n : Number of orbital planes

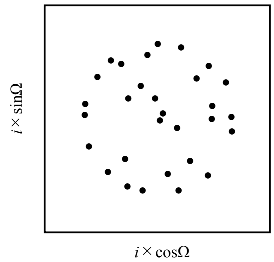
◆ Propagation

- Nodal regression (J_2)
- Random Noize

$$\begin{cases} N_d(t+\Delta t) = N_d(t) + \Delta t v_N \\ \Omega(t+\Delta t) = \Omega(t) + \Delta t \left(-\frac{3}{2}\frac{J_2 a_e^2}{p^2} n \cos i + v_\Omega \right) \\ i(t+\Delta t) = i(t) + \Delta t v_i \end{cases}$$



Actual Distribution



Ensemble Approximation

Dynamic Environmental Estimation

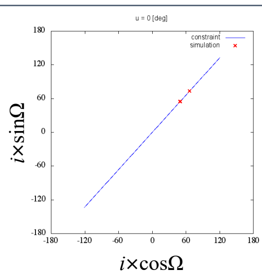
Observation Model

$$\log l(\boldsymbol{y}_1, \dots, \boldsymbol{y}_n) = \sum_{i=1}^{n} \log \lambda(\boldsymbol{y}_i) - \int_A \lambda(\boldsymbol{y}) d\boldsymbol{y}$$: Likelihood based on Inhomogeneous-Poisson process

◆Orbital plane constraint

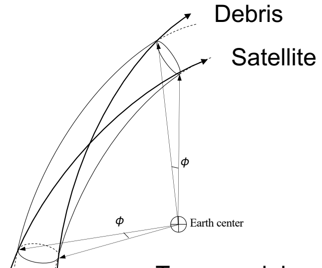
The orbital plane on which debris is detected by the satellite is constrained by the position of the satellite

$$\frac{x}{r}\sin\Omega' \sin i' - \frac{y}{r}\cos\Omega' \sin i' + \frac{z}{r}\cos i' = 0$$



◆Collision Flux

The collision flux **λ** between two circular orbits is approximately determined by the torus model



Torus model

Dynamic Environmental Estimation

Simulation

MASTER 2009

↓

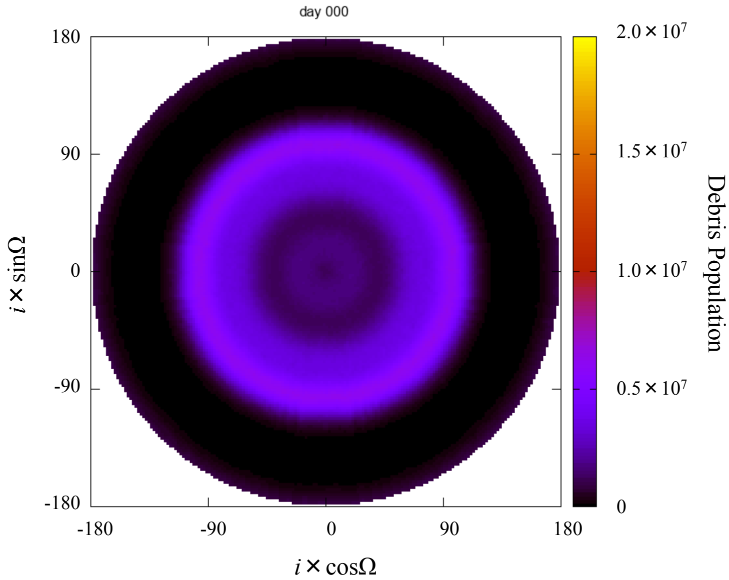
Simulated impact data

↓

Estimation Model

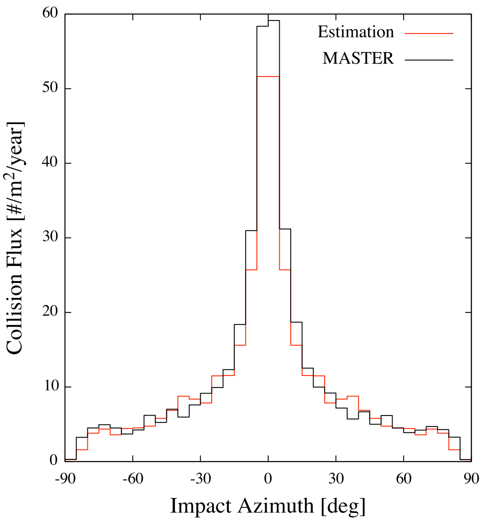
↓

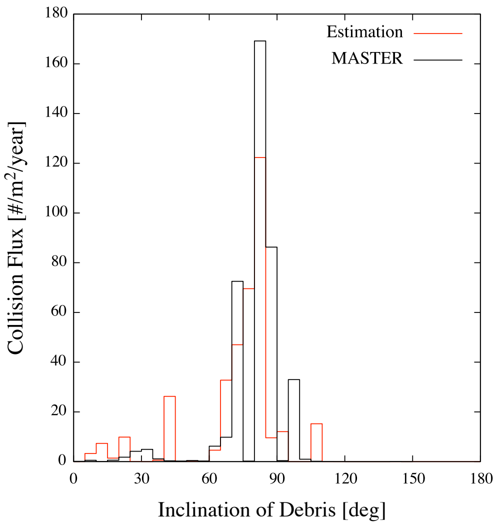
Estimated Distribution



Dynamic Environmental Estimation

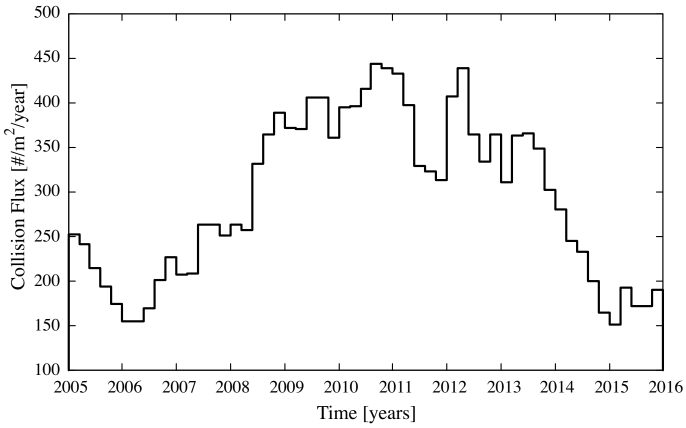
Simulation Results





Detection of Environmental Change

(Furumoto, M. and Sahara, H., Acta Astronautica, Vol. 177, Pages 666-672, 2020.)

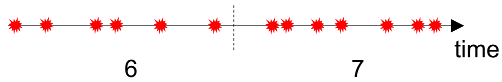


Collision Flux History in MASTER-8 at 865 km SSO

- Debris environment changes continuously / drastically

  influenced by
  - Solar activity
  - Ejection
  - Breakup

- Sampled data must include randomness

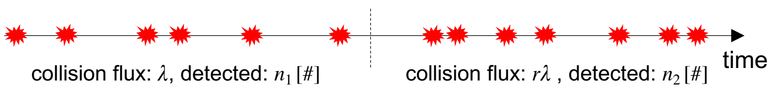


→ Statistical evaluation is important

Detection of Environmental Change

Methods 1/2

- Chi-Squared Testing



Null hypothesis: The density of the Poisson process, which is followed by the impact of fragments, is constant throughout the entire period of measurement.

Alternative hypothesis: The Poisson process with a uniform density cannot explain the measured quantity of impacts.

Based on Chi-squared testing, the null hypothesis is rejected when (= change confirmed)

$$\chi^2 = \frac{(n_1 - n_2)^2}{n_1 + n_2} \geq 3.84$$

(significance level 5%)

Substitute $n_1 = \frac{\lambda}{2}, n_2 = \frac{r\lambda}{2}$ (Ideal measurement data) → $r = 1 + \frac{\chi^2}{\lambda} \pm \sqrt{\frac{\chi^2}{\lambda}\left(\frac{\chi^2}{\lambda} + 4\right)}$ Detectability Equation

(χ^2 = 3.84)

Detection of Environmental Change

Methods 2/2

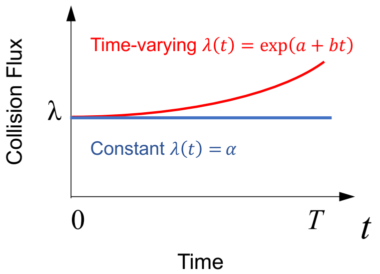
- AIC Comparison

AIC (Akaike Information Criterion): Criterion to select the model that explains the data (smaller is better)

$$\mathrm{AIC} = -2\left(\log\hat{L} - N_{param}\right)$$

N_{param}: Number of unknown parameters in the model

$\hat{L}$: Maximum likelihood



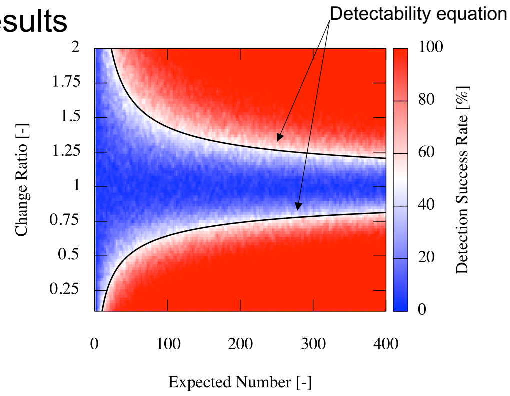
1. Calculate maximum likelihood and AIC
2. Environmental change is confirmed if the AIC of the time-varying model is smaller (also extent of the change can be estimated)

Detection of Environmental Change

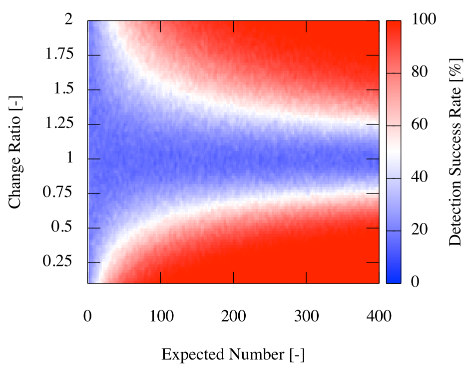
Simplified Simulation

1. Randomly generate impacts following Poisson process with an expected number of impact and its change ratio
2. Detect the environmental change using proposed methods
3. Repeat 1. and 2. in 200 times and calculate the percentage of successfully detected case

Results



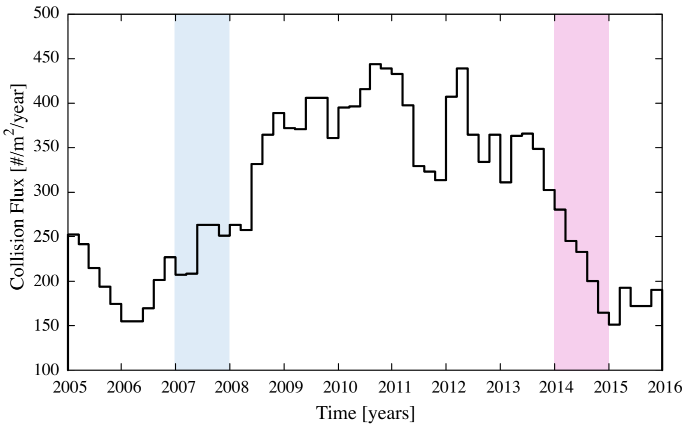
Chi-squared testing



AIC comparison

Detection of Environmental Change

Precise Simulation



Start epoch	Collision flux [#/m²/year] Initial	Final	Change Ratio (r)
Jan 2007	207.2	263.4	1.271
Jan 2014	280.1	152.0	0.542

- Generate impacts following MASTER-8 definition
- 100 MC cases
- 4 cases of measurement system (with sensor area of A [m²])
  - Microsatellite, $A = 0.25$
  - Legacy satellite, $A = 1$
  - Constellation of microsatellites, $A = 2.5$
  - Constellation of legacy satellites, $A = 10$

Detection of Environmental Change

Precise Simulation

Detection Results (Limit of r: result of detectability equation)

in 2007 (actual change ratio r = 1.271)

A [m²]	λ_0 [#]	Limit of r	Success Rate [%] Chi-squared	AIC
0.25	51.8	1.62	8	1
1	207.2	1.29	19	39
2.5	518	1.18	55	81
10	2072	1.09	99	100

in 2014 (actual change ratio r = 0.542)

A [m²]	λ_0 [#]	Limit of r	Success Rate [%] Chi-squared	AIC
0.25	70.0	0.58	25	18
1	280.1	0.78	67	74
2.5	700.2	0.85	91	100
10	2801	0.92	100	100

Detection of Environmental Change

Precise Simulation

Estimated extent of environmental change

in 2007 (actual change ratio r = 1.271)

A [m^2]	Average	Median	Standard Deviation
0.25	5.08	5.08	N/A
1	1.61	1.51	0.29
2.5	1.40	1.40	0.14
10	1.37	1.38	0.08

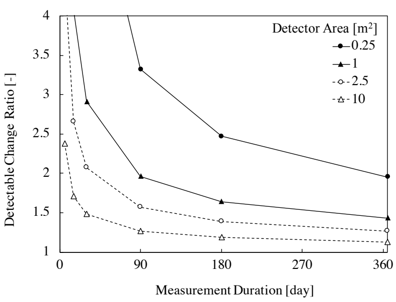
in 2014 (actual change ratio r = 0.542)

A [m^2]	Average	Median	Standard Deviation
0.25	0.36	0.38	0.09
1	0.44	0.44	0.09
2.5	0.55	0.55	0.07
10	0.55	0.55	0.04

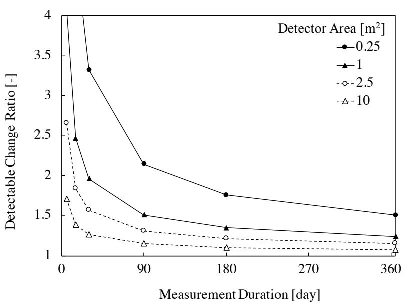
Detection of Environmental Change

Requirements on In-situ Measurement

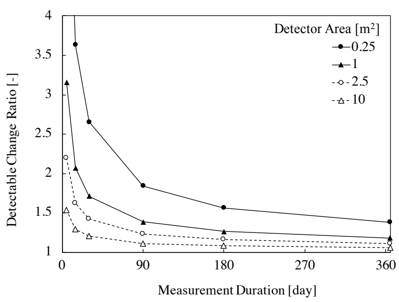
Required mission duration to detect environmental changes
(Calculated by detectability equation)



initial collision flux = 100 [#/m^2/year]



initial collision flux = 300 [#/m^2/year]



initial collision flux = 500 [#/m^2/year]

Conclusion

Statistical analysis for in-situ measurements provides better understanding of the environment of small debris

- Dynamic Environmental Estimation
  - Sequential model based on SMC filter can estimate distribution of sub-millimeter-sized debris utilizing in-situ measurements

- Detection of Environmental change
  - Statistical methods to identify environmental changes were developed
    - Chi-squared testing: Low computation cost, benefit to on-board detection
    - AIC comparison: More reliable and capable to estimate the change extent
  - Detectability equation determines the limit of change ratio of environment
  - Requirement on in-situ measurement systems to detect environmental changes were presented