Simple Ideas for the Accuracy and Efficiency Improvement of the Compressible Flow Simulation Methods

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1. Introduction

Both accuracy and efficiency are the key issues of the simulation methods in Computational Fluid Dynamics. Sometimes, simple ideas improve these key features. Two topics are focused and discussed in this paper.

2. Accuracy Enhancement by the Grid Movement

First, accuracy improvement by the usage of the moving grid system is discussed. One interesting feature of the moving grid flow simulations was found through the simulation of the blast wave propagation[1]. As shown in Fig. 1, the local fine grid region was prepared and overlaid onto the base grid that covers the whole computational region. The local find grid moves with the blast wave. Figure 2 shows the ground surface pressure distributions computed by the moving grid system. Only the blast wave region is closed up in the figure. The single-zone solutions with the very fine (4001 points along the ground) and fine grids (801 points) are plotted for comparison. The moving-grid solution shows the highest pressure peak at the shock wave. The interesting observation here is that the spatial grid resolution of the locally moving grid in the zonal solution is the same as the grid resolution of the 801 grid points. Even with the spatial resolution of 801 grid points, the solution obtained captures the frontal shock wave crisply than the solution by the stationary 4001 grid points. The fact that the grid moves at the blast wave speed obviously improved the solution accuracy.

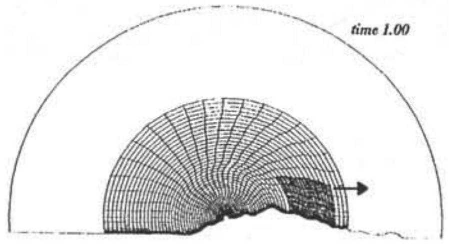

Fig. 1 Overset moving grid system for the blast wave simulation

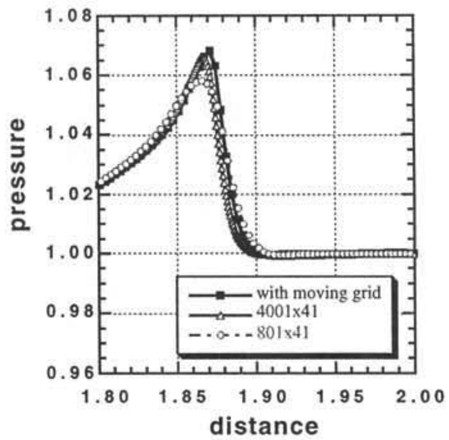

Fig. 2 Computed surface pressure distributions

The moving grid system has a nice nature for the physical phenomenon transporting in the flow field like the example above. The linear scalar equation for convection, when formulated in the frame of moving grids, all the truncation error terms have the coefficient of $(c-u_g)$ where c is the speed of convection and u_g is the grid speed. Therefore, setting the grid speed to be close to the speed of convection, the truncation errors will be reduced. When $u_g = c$, all the truncation errors will disappear and the solution is perfect. In the real computation, the solution at each grid point does not change in time and no errors will appear. When considering, for instance, the flux difference splitting, the truncation errors may be reduced when the grid speed is set to be close to one of the eigenvalues of the flux Jacobian. The solution accuracy improvement observed above can be explained by this idea.

To confirm it, a one-dimensional shock tube problem is formulated. Figure 3 shows the density plots in one of the computed results. The computational grid moves at 90 % of the shock speed. The improvement is clearly seen when compared to the stationary grid solution (dash line). On the other hand, the solution accuracy is degraded in the expansion region since the rarefaction wave proceed to the opposite direction to the grid motion. To enhance the accuracy in the whole region, we have to set three overset grids, each of which moves at the speed of shock wave, contact discontinuity and rarefaction wave, respectively.

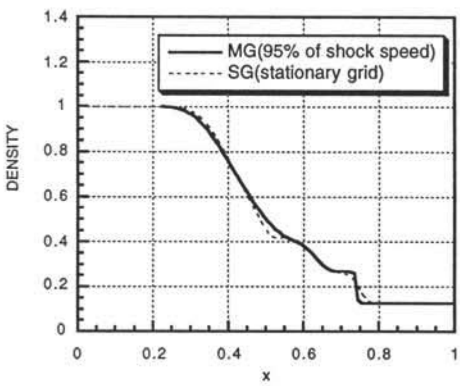

Fig. 3 Shock tube problem
- effect of moving grid

Next, the transport phenomenon of the isolated vortex is simulated. An isolated vortex is placed in the freestream and transported downstream. The grid moves at the freestream speed. Since this is an Euler computation, the vortex strength would not essentially change. One of the results is shown in Fig. 4. The time history of the minimum density and maximum vorticity in the center of the vortex core is plotted. Both the moving grid and stationary grid solutions are plotted. Due to the discretization errors, both the results show vortex decay but the density is lower for the moving grid and the vorticity is higher in the moving grid solution. The discretization errors are clearly reduced for the solution with the moving grid. Although we need to investigate the dependency of the vortex decay to the grid resolution, freestream speed and other factors, the result is promising.

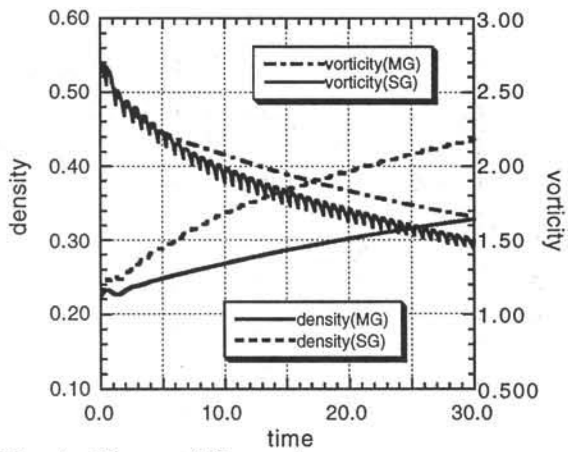

Fig. 4 Decay of the vortex:
- minimum density and vorticity of the center of the vortex core -

Moving grid systems introduce the "Lagrangean" effect into the Eulerian computations. The truncation error analysis briefly presented here shows that it can be explained as the change of the coefficients of the truncation errors. The examples above clearly showed the improvement of the solution accuracy by the moving grid systems. There are many applications where the solution can be improved by using the moving grid system. The computational overhead is simply the computation of the time metrics term (which is constant in the examples above) and existing computer codes can be easily modified. The computation of the low speed flow over a cylinder is currently underway, where the grid moves at the speed of Karman vortex shedding to improve the accuracy of vortex streets although the result is not available yet.

3. Efficiency Improvement by the Approximation of Flux Jacobians

Second, efficiency improvement is discussed. Implicit time integration methods have not made remarkable progress since ADI-like AF was introduced by Beam & Warming[2]. However, the number of operations was reduce by introducing Diagonal scheme[3], LU-ADI scheme[4] and else. On the other hand, LU factorization was introduced as another implicit scheme, and LU-SGS scheme[5] developed by Yoon and Jameson successfully reduced the number of operations of the implicit time integration schemes and was extended to be used for the unstructured grid computations[6]. Although the LU-SGS scheme is very efficient, its vectorization for the supercomputer requires the use of the hyperplane and the computer program becomes very different from the code for the existing ADI type factorization. The author has proposed FF-SGS scheme[7] which uses LU-SGS scheme in the streamwise direction and LU-ADI scheme in the other two directions. Similarly to the FF-SGS scheme, implicit operations in any ADI-type schemes can easily be replaced by the one-dimensional LU-SGS scheme. Using the approximate LDU decomposition, the ADI operator in the ξ direction becomes,

$$(I+\Delta t\delta_\xi \hat{A}) = (I+\Delta t\delta_\xi^b \hat{A}^+ - \frac{\Delta t}{\Delta\xi}\hat{A}_j^-)(I+\Delta t(\hat{A}^+ - \hat{A}^-))^{-1}(I+\Delta t\delta_\xi^f \hat{A}^- + \frac{\Delta t}{\Delta\xi}\hat{A}_j^+) \quad (1)$$

The forward and backward sweeps for the operators in Eq. (1) become,

forward sweep: $(I+\frac{\Delta t}{\Delta\xi}(\hat{A}^+ - \hat{A}^-))\Delta\hat{Q}_j^* = -(s.s.) + \frac{\Delta t}{\Delta\xi}\hat{A}_{j-1}^+\Delta\hat{Q}_{j-1}^*$

backward sweep: $(I+\frac{\Delta t}{\Delta\xi}(\hat{A}^+ - \hat{A}^-))\Delta\hat{Q}_j^n = (I+\frac{\Delta t}{\Delta\xi}(\hat{A}^+ - \hat{A}^-))\Delta\hat{Q}_j^* - \frac{\Delta t}{\Delta\xi}\hat{A}_{j+1}^-\Delta\hat{Q}_{j+1}^n$

The backward sweep includes the second matrix operation. Suppose we introduce the approximation of the positive and negative flux Jacobians, $\hat{A}^\pm = \frac{\hat{A}\pm\sigma_\xi}{2}$, the operation becomes simple divisions instead of block matrix inversions. It allows us to keep the original code structure and it is very easy to obtain it by modifying the existing ADI-type coputer code. The vector length can be kept long and constant. Simple modification like this will reduce the number of implicit operations without the loss of convergence. We call this scheme as ADI-SGS as SGS is introduced after ADI factorization. The convergence history for the two dimensional supersonic blunt body problem is shown in Fig. 5. The convergence history for the LU-ADI scheme is also shown for comparison. Both the schemes show almost the same convergence trend although the LU-ADI scheme shows oscillatory behavior. The approximation introduced does not degrade the convergence.

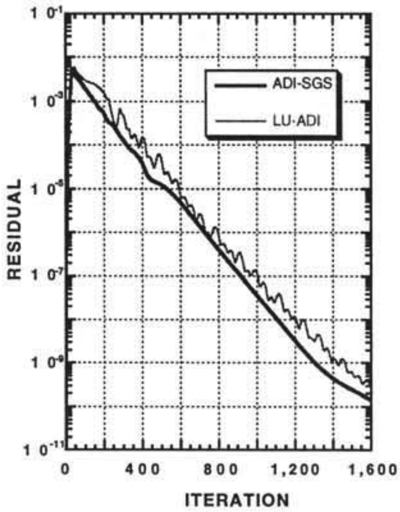

Fig. 5 Residual history for the 2-D blunt body problem ($M_\infty = 2.0$, $\alpha = 0$ deg., Euler)

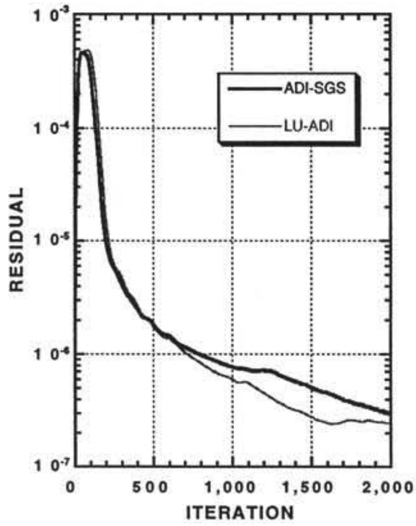

Fig. 6 Residual history for the 3-D blunt body problem ($M_\infty = 2.0$, $\alpha = 5$ deg., Euler)

The convergence history for the three-dimensional blunt body problem is shown in Fig. 6. The convergence history for the ADI-SGS scheme shows almost the same trend as that of the LU-ADI scheme shown in the same figure. Remember that the approximation of the flux Jacobians reduces the matrix operation for the diagonarization and the number of the operations becomes much smaller. The computer time reduction for the left-hand side implicit operations is roughly 50 %. The computed density contour plots are shown in Fig. 7.

Final example is the computation of the leading-edge vortex flow field over the delta wing. The Mach number is set 0.3 and Navier-Stokes simulation at the Reynolds number 1.0×10^6 is conducted. The convergence history for the computation is shown in Fig. 8. The trend is the same as the two Euler examples above.

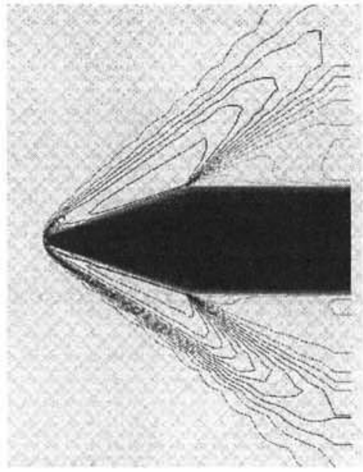

Fig. 7 Density contour plots for the 3-D blunt body problem ($M_\infty = 2.0$, $\alpha = 5$ deg., Euler)

Conclusions

Two simple ideas are presented, each of which improves the accuracy and efficiency of the compressible flow simulations, respectively. Both the idea are simple and easily implemented into the existing computer code without much effort. The computed results indicate that these two approaches are useful to try and they help the accuracy and efficiency of the simulations.

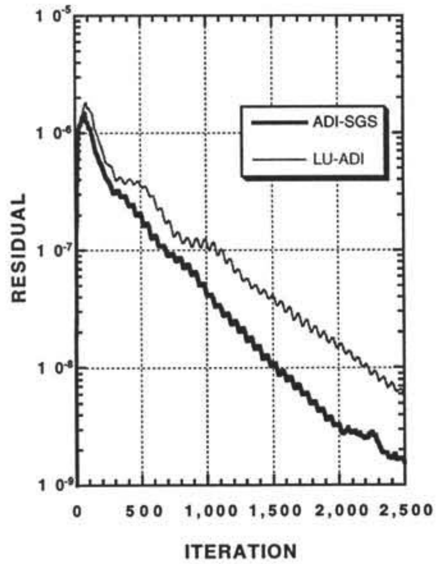

Fig. 7 Density contour plots for the 3-D blunt body problem ($M_\infty = 2.0$, $\alpha = 5$ deg., Euler)

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