

# Analytical Design Method for a Gridded Helicon Ion Thruster

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## Abstract:

We propose the concept of a gridded helicon ion thruster, aiming at combining the high plasma density produced by helicon discharges with the high thrust efficiency obtained by electrostatic acceleration. The concept is investigated from a theoretical point of view. A method to design helicon plasma sources producing a selected plasma density is proposed.

## 1. Introduction

Helicon sources have been largely characterized theoretically and experimentally, although their application for propulsion is still limited, as their beam acceleration performance is relatively limited. On the contrary, helicon ion sources exhibit a very high propellant and electrical efficiency, both reported to easily reach values in the 80-100% range [1]. This is due to the transition from standard RF heating (same as the RIT class ion thrusters) to the more efficient helicon mode in presence of B field and appropriate  $P_{RF}$ .

The concept of a gridded helicon ion thruster has first been proposed by Williams and Walker of Georgia institute of technology [4]. The concept, while having high potential as a design, shows a series of flaws that can be identified as:

- Disproportionally big/powerful plasma source (helicon discharge producing plasma that can't be accelerated due to the Child-Langmuir equation)
- Unnecessary solenoids use electric power to produce a magnetic field
- Low quality grids (pointed out by the authors)

In our concept, we tried to address the issues found in previous research.

## 2. Gridded Helicon Ion Thruster Concept

The first issue can be addressed by designing the helicon ion source based on the maximum ion current that can be accelerated through the grids, based on the Child-Langmuir law. This is likely to lead to a smaller and less powerful ion source (demonstrated by D. Pavarin [6]), needing an expansion chamber to connect to the grids (using the  $\mu 10$  setup to solve issue number 3).

The design can be further improved by introducing a permanent magnet design for the helicon discharge, which various authors point out to achieve equivalent performance to the solenoids version while reducing complexity and power consumption (design proposed by F. Chen [2]).

Finally, this can be coupled with a permanent magnet helicon neutralizer. This can be derived from the model demonstrated by B. Longmier [5], who created a solenoid confined very high current (over 10A) helicon electron source.

A schematic of the concept is shown in Fig. 1.

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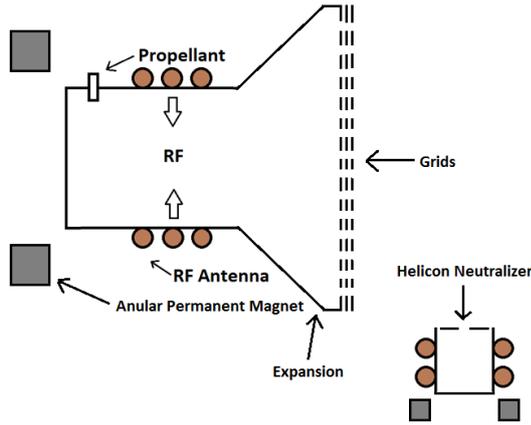


Figure 1: Gridded Helicon Ion Thruster Concept Schematic

### 3. Parametric Design Method for the Helicon Source

To develop an appropriate ion source for the gridded helicon ion thruster concept, we developed an analytical model based on the formulations utilized by F. F. Chen [1] and M. A. Lieberman [3].

We will study a set of conditions to observe how these affect the resulting shape of the helicon source. The model will be explained in combination with the MATLAB results obtained.

We start by defining the resonant energy  $E_r$ , at which electrons will be accelerated, and we derive the phase velocity  $v_\phi$  from:

$$E_r = \frac{1}{2} m_e v_\phi^2 = 30 - 50 \text{ eV}$$

We can now calculate the desired total wave number (we demonstrate this with a frequency of 50MHz):

$$k = \frac{2\pi f}{v_\phi} = \sqrt{k_z^2 + k_\perp^2}$$

The boundary condition for  $E_\theta=0$  leads to:

$$\frac{k_z}{k} = -m \frac{J_m(k_\perp R)}{J'_m(k_\perp R)}$$

Where  $m$  is the mode (we will solve for  $m=+1$ , as Chen reports it leads to a better distributed plasma) and  $J_m$  the Bessel function:

$$J_m(k_\perp R) = \sum_{n=0}^{\infty} \frac{-1^n \left(\frac{k_\perp R}{2}\right)^{2n+m}}{n! (n+m)!}$$

We can solve the Bessel function numerically, and obtain from it the correlation between  $k_z/k$  and  $k_\perp R$ . As Lieberman suggests that  $k_z$  should be smaller than  $k_\perp$  [3], we select:

$$\frac{1}{10} < \frac{k_z}{k} < \frac{1}{\sqrt{2}}$$

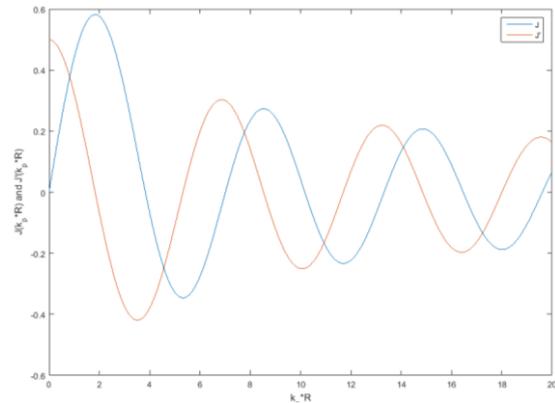


Figure 2: Bessel Function and its Derivative

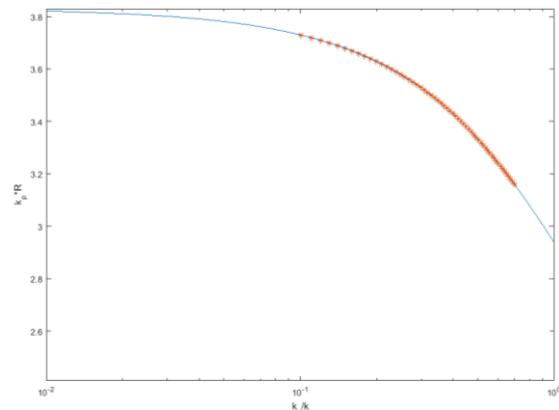


Figure 3: Solution of the Boundary Condition

Notice how the condition imposed for  $k_z/k$  is reflected in the values obtained.

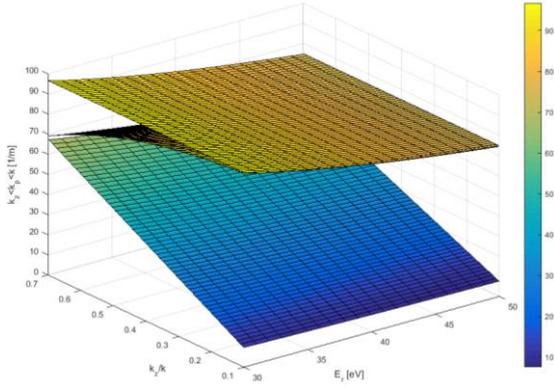


Figure 4:  $k$  (top),  $k_L$  (middle) and  $k_z$  (bottom) for different  $E_r$  and  $k_z/k$

We can now obtain the antenna radius  $R$  (knowing  $k$  and  $k_L$ ) and the antenna length  $l_a$  as:

$$l_a = \frac{\pi}{k_z}$$

Antennas couple well with the helicon mode for this and every odd multiple of this solution:

$$l_a = \frac{\pi}{k_z} = \frac{3\pi}{k_z} = \frac{5\pi}{k_z} = \dots$$

So we choose the first to limit plasma losses (we will see Chen's formulation soon).

As we see from Fig5,  $R$  increases moderately with  $E_r$  and  $k_z/k$ , while the  $l_a$  has a strong increase at low  $k_z/k$ , with an upper limit at  $R$  as set by our conditions.

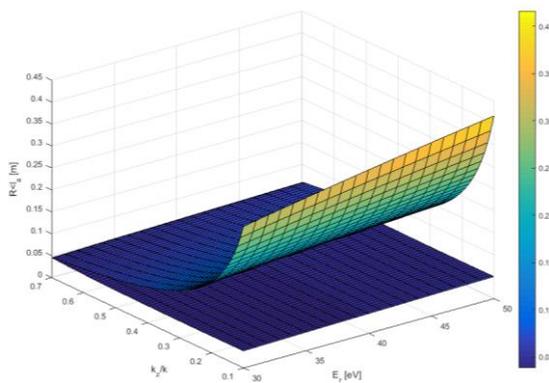


Figure 5: Antenna radius  $R$  and length  $l_a$

Assuming our gridded helicon ion thruster will have a different diameter at the antenna and at the grids, we must set a condition for which the density multiplied by the area remains constant, limited by the Child-Langmuir

equation (from which we obtain the maximum plasma density we can accelerate).

$$n_H \pi R_H^2 = n_{gr} \pi R_{gr}^2$$

From which, knowing  $R$  at the helicon antenna position, we can obtain the corresponding  $n$ .

We can now solve the dispersion equation:

$$k_z k = \frac{e \mu_0 n \omega}{B_0}$$

And obtain the required  $B_0$  (central magnetic field) to sustain a helicon mode in the conditions selected.

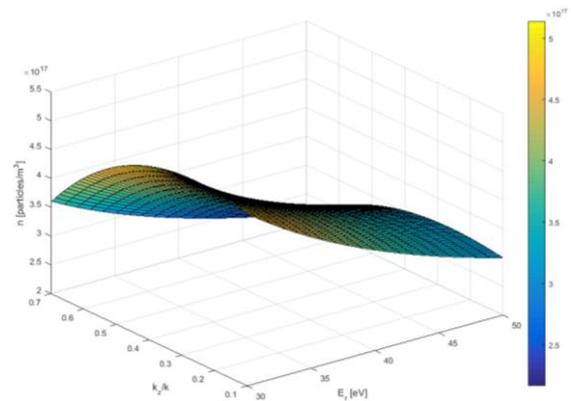


Figure 6: Plasma density for different  $E_r$  and  $k_z/k$

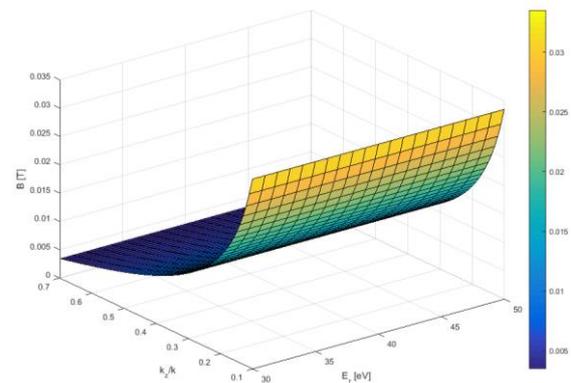


Figure 7: Central  $B$  for different  $E_r$  and  $k_z/k$

We notice how the value of  $B_0$  is not affected by the resonant energy  $E_r$ . Finally, using the semi-empirical for the power loss to the endplates and to the walls (adjusting the units):

$$P_{end} = 3500 \left( \frac{R^2}{l_a} \right) n$$

$$P_{wal} = 2 \left(\frac{n}{B}\right)^2 L$$

Considering one wall to be the grids and the other to be the upstream wall, we obtain the power efficiency:

$$\eta = \frac{1}{2} \frac{1}{1 + \frac{P_{wal}}{P_{end}}}$$

Although small the wall losses are, this would limit our efficiency to 50%.

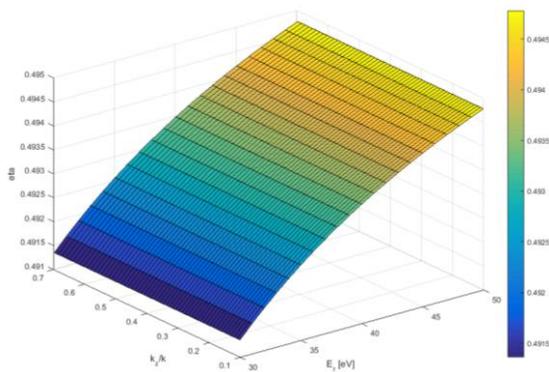


Figure 8:  $\eta$  for different  $E_r$  and  $k_z/k$

#### 4. Small Helicon Discharge Investigation

From the previous model, it was pointed out that the small-scale helicon discharge utilized with a hypothetical 10cm diameter grid would need to operate at unusually high frequency.

To verify what was the difference between our design and experiments previously done in other laboratories [2,6], the code for the parametric design of helicon ion sources was rearranged, prioritizing design parameters and later solving boundary condition and dispersion equations. In this version of the model, we select in advance parameters such as  $n$  and  $B$ , and later verify if a helicon wave can occur in these conditions.

We test this approach for a small helicon ion source, with  $R=2-5\text{cm}$ ,  $E_r=50\text{eV}$  and two different frequencies: 13.56MHz (standard helicon source) and 100MHz (our design).

The design parameters are then used to verify if the design satisfies the boundary conditions of a helicon discharge.

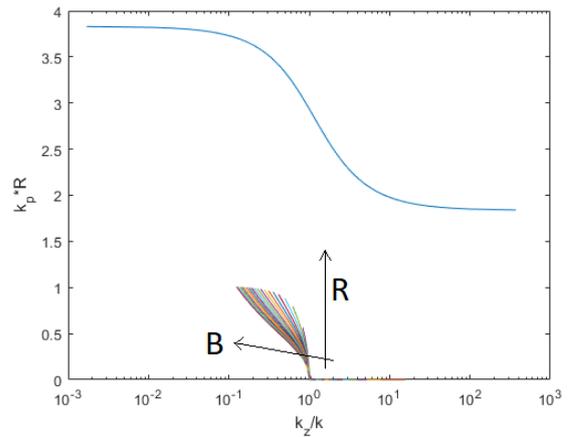


Figure 9: Boundary Conditions for a 13.56MHz Small Helicon

In Fig. 9 it can be seen how the 13.56MHz RF input can't satisfy the boundary conditions for 50eV resonant energy. After multiple tests with the code, it turns out how, to reach the boundary condition line, one has three options:

- Increase  $R$  (however, we reached the 5cm size of  $\mu 10$ )
- Increase  $f$
- Reduce  $E_r$

Increasing  $f$  would meet the specifications previously obtained through our model.

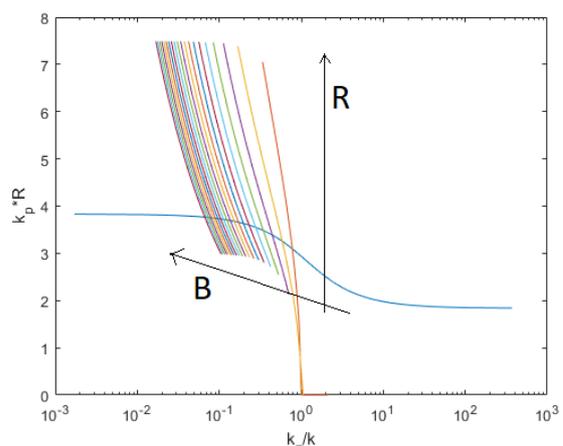


Figure 10: Boundary Conditions for a 100MHz Small Helicon

We see how in this case our design meets the boundary conditions, the lines intercept the solution when  $R=2.5\text{cm}$ .

## 5. Conclusions

A method for designing a gridded helicon ion thruster has been proposed. The approach utilized is also applicable to the design of helicon ion sources with a selected density.

The results presented in paragraph 4 point out some discrepancies between the present design method and experiments done in the past by other laboratories (especially about small thrusters). The possible explanations are:

- The definition of resonant energy  $E_r$  we are starting with, and from which we derive our phase velocity  $v_\phi$ , is incorrect. This parameter seems to be rarely utilized in other papers (except by Chen).
- Presently available small helicon thrusters don't perform as well as their larger counterparts as they don't satisfy the boundary conditions for a helicon discharge. Hence, they might be operating as regular RF thrusters.

The analysis of the helicon ion thruster concept pointed out how, although simplifying assumptions are made, the electrical efficiency of the thruster is capped at 50% due to the symmetry of the discharge:

$$\eta = \frac{1}{2} \frac{1}{1 + \frac{P_{wal}}{P_{end}}}$$

Once the RF power supply efficiency (lower than 50%) and other real-case conditions are considered, it seems very unlikely that the helicon discharge could offer results comparable/superior to the ECR and DC counterparts in term of efficiency.

However, the concept would be useful for the development of high-thrust ion thrusters, as the helicon discharge can reach (and exceed) the density limits imposed by the Child-Langmuir equation at the grids, unlike ECR and DC thrusters.

## References

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