Numerical Simulations for Intense Light-Ion Beam Propagation in Channel under Influence of Plasma Inertia

By

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(February 1, 1983)

Summary: The intense light-ion beam (LIB) propagation through a plasma channel is numerically investigated by using a two-dimensional simulation code. Analyses are given for the LIB propagation which couples with the motion of channel plasma. Although the electron back current neutralizes the LIB current under a typical beam and channel plasma condition, the Lorentz force by the electron back current expands the LIB radially. The expansion of the LIB depends strongly on the density of the channel plasma. A strong current of the order of 1 MA is shown to propagate stably in the argon plasma channel.

1. Introduction

Electrical technology at the present stage can construct the power source from which a strong enough light-ion beam (LIB) of the practical order of several mega joules as an energy driver of inertial confinement fusion (ICF) is able to be extracted (Suzuki & Kitagawa, 1982).

On the other hand, LIB has many advantages as an energy driver of ICF with respect to the interaction with the target plasma. Owing to the finite stopping range, LIB is suitable as an energy driver for a cannon ball type implosion of a hollow shell target which has a high hydrodynamic efficiency and hence a high target gain (Tamba, Nagata, Kawata & Niu, 1983). In order to achieve a LIB-ICF power reactor, one of the most crucial problem to be solved is to focus the LIB on a target or to propagate the LIB in a reactor chamber.

In order that LIB is transported being confined in a radius of about 0.5 cm, the plasma channel must satisfy the following conditions:

1. The magnetic field in the azimuthal direction induced by the z-discharged channel current must be strong enough to confine the LIB in a small radius.
2. The density of the channel plasma must be so high as to neutralize the beam current and beam charge.
3. On the contrary, the density of the channel plasma must be low so that the collision loss of the beam in the channel can be small.
4. The channel must be stable during the LIB propagation.

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It has been shown numerically that the plasma channel which satisfies the required conditions described above can be formed by the z-discharge (Kawata, Niu & Murakami, 1983; Freeman, Baker & Cook, 1982), and during the formation the channel remains stable with respect to the macro instabilities owing to the action of the radial velocity of the channel (Murakami, Kawata & Niu, 1983). It has also become clear that the channel parameters with which the beam does not induce the serious micro (electrostatic and electromagnetic) instabilities in the channel during the beam propagation can be found. (Okada & Niu, 1981; Ottinger, Mosher & Goldstein, 1979).

The purpose of this paper is to investigate the macroscopic behaviour of the LIB propagation through a well-formed plasma channel. For this purpose, two dimensional (in space) numerical calculations are carried out. MHD equations are applied to analyze the behaviour of the channel plasma. Motions of the proton beam treated as a fluid couple with those of the channel plasma through collisions and electro-magnetic fields.

2. Numerical Model

The fundamental equations used in this paper are as follows:

\[
\frac{\partial \rho}{\partial t} + \nabla \cdot \rho \cdot v = 0, \tag{1}
\]

\[
\rho \left[ \frac{\partial v}{\partial t} + (v \cdot \nabla) v \right] = -\nabla P + j \times B/c + S v_n |v_n|, \tag{2}
\]

\[
\rho \left[ \frac{\partial U}{\partial t} + (v \cdot \nabla) U \right] = -PF \cdot v + SV_n (v_n - v)/|v_n| \]
\[
+ \frac{1}{\sigma} + \nabla K_e \cdot \nabla T - P_a, \tag{3}
\]

\[
\frac{\partial \rho_s}{\partial t} + \nabla \cdot \rho_s \cdot v_s = 0, \tag{4}
\]

\[
\rho_s \left[ \frac{\partial v_s}{\partial t} + (v_s \cdot \nabla) v_s \right] = -\nabla P_s + n_e e E + j_s \times B/c - S v_n |v_n|, \tag{5}
\]

\[
T_s = \text{constant}, \tag{6}
\]

\[
\frac{\partial B}{\partial t} = -\nabla \times E, \tag{7}
\]

\[
j + j_s = \frac{c}{4\pi} \nabla \times B, \tag{8}
\]

\[
E = -v \times B/c + j/\sigma. \tag{9}
\]

In the above equations, the following notations are employed for the channel plasma: \(\rho\) is the mass density, \(v = (v_r, 0, v_z)\) is the velocity, \(U\) is the internal
energy per unit mass, \( P \) is the pressure, \( S \) is the stopping power for the proton beam, \( \sigma \) is the electric conductivity, \( j \) is the current density, \( K_e \) is the electron thermal conductivity, \( T \) is the temperature and \( P_B \) is the Bremsstrahlung radiation energy loss. The corresponding variables with respect to the beam are attached by the suffix \( b \) and \( n_b \) is the number density of the beam. The notations \( E = (E_r, 0, E_z) \) and \( B = (0, B_r, 0) \) stand for the electric and magnetic field, respectively. The equation of state is approximated by the ideal one and the Saha equation decides the charge state of the channel plasma. The Braginskii formulae (Braginskii, 1965) which include the magnetic effect are used for the electric and thermal conductivities.

The beam which consists of proton particles is treated as a fluid with a constant temperature \( T_b \). In other words, the betatron-like motions of the proton particles in the plasma channel are characterized by the LIB temperature \( T_b \).

The condition of the channel plasma, when the beam is injected, depends on the way how the channel is formed. Here the following initial conditions are chosen for the numerical calculations: The number density of the channel is \( 10^{9} \text{ cm}^{-3} \) and is homogeneous. The channel radius \( r_0 \) is 0.5 cm, where the magnetic field has the peak value. The azimuthal magnetic field as a function of \( r \) is decided by \( 20 \text{ kGauss} \times 2r/[r_0(r_0^2 + r^2)] \). This magnetic field is strong enough to confine beam particles inside the channel whose radius is \( r_0 \) (Ottinger, Mosher & Goldstein, 1980). The channel plasma has no velocity. The channel temperature (shown in figure 2-a) is decided by the force balance between the pressure gradient and the Lorentz force. Regarding the proton beam, the energy is 5 MeV, the beam current within the channel radius of 0.5 cm is 1 MA and the beam temperature is 100 keV. The beam number density is decided by the force balance and is shown in Fig. 3-a.

To ascertain the validity of two-dimensional numerical program, the code was applied to obtaining the linear growth rate of the sausage instability. The numerical results had good agreement with the analytical ones with the discrepancy of 2.7%. The implicit method, using iterations, is used in our numerical calculations. As the space differencing method, the center differencing one is employed. For example, the equation

\[
\frac{\partial f}{\partial t} = \frac{\partial g(x, t)}{\partial x}
\]

is replaced by the difference equation

\[
\frac{f(t+dt) - f(t)}{dt} = \frac{1}{2} \left[ \frac{g(x + dx, t + dt) - g(x - dx, t + dt)}{2dx} \right. \\
\left. + \frac{g(x + dx, t) - g(x - dx, t)}{2dx} \right].
\]

In the \( z \)-direction 20 meshes are employed, and in the \( r \)-direction 30 meshes are employed. The mesh widths in both the \( r \)- and \( z \)-directions are the same and are 0.05 cm. These numbers of meshes are sufficient to investigate the sausage
instability, because the most dangerous wave length coincides with the channel radius (Rukhadze & Triger, 1971). The periodic boundary conditions are used in the z-direction, contrary to the diffuse boundary in the r-direction.

3. Beam Transportation

Figures 1–3 explain changes in parameters when the proton beam propagates through the argon plasma channel. At the initial time, the channel plasma and the proton beam are assumed to be homogeneous in the z-direction. As shown in figure 1, the channel plasma slightly expands in the radial direction as the time passes. The driving force of this expansion comes mainly from the Lorentz force due to the azimuthal magnetic fields $B_\phi$ acting on the electron back current. This expansion of the channel plasma leads to the expansion of the magnetic field. Through this magnetic field expansion, the beam expands. The radial expansion depends strongly on the mass density of the channel plasma. This dependence becomes clear if figure 4 for the hydrogen channel is compared with figure 3 for the argon channel. The parameters used to obtain curves in figure 1–3 are the same with those in figure 1–3 except particle mass. It is clear that the radial expansion is extremely large in the hydrogen channel in comparison with the argon channel. As figures 1–3 indicate, the expansion is quite small in the channel consisting of the gas with a high mass density such as argon. The beam propagation is not affected by the expansion of the argon channel, as is seen in figure 3. Figure 1–3 also show that the macro (sausage) instability does not grow heavily and not prevent the propagation of beam. As seen in figure 4, the macro instability does not grow either in the hydrogen channel plasma.

Figure 5 shows that the time sequences of the total energies of the proton beams constrained within the hydrogen and the argon plasma channels of the radius of 0.5 cm. In the first period ($\leq 20$ nsec), the beam energy losses originate collisions with the plasma particles for both the channels. At the later stage in the hydrogen plasma channel, however, the beam energy within the radius of 0.5 cm decreases because a part of the beam expands radially out of the radius of 0.5 cm. Such a decrease does not appear in the argon channel.

If the radial acceleration induced by the Lorentz force of $jB_\phi/c$ is assumed to be constant, the radial expansion $\Delta r_0$ can be estimated as follows:

$$\Delta r_0 = \frac{j_B x_0 \tau_0^2}{2\rho c}$$  \hspace{1cm} (10)

where $\tau_0$ shows the beam duration time. The radial expansion calculated by (10) has a good agreement with the numerical value. If the radial expansion of 10% of the initial radius is acceptable, the following condition for the channel mass density must be satisfied:

$$\rho \leq \frac{5B_\phi x_0^2 l}{\pi c r_0^3}.$$  \hspace{1cm} (11)
Fig. 1. (a~d) The time sequences of the number density for the argon channel plasma. The radial expansion slightly occurs but is not severe. The sausage instability does not grow so heavily.
Fig. 2. (a~d) The time sequences of the temperature for the argon channel plasma.
Fig. 3. (a–d) The time sequences of the temperature for the argon channel plasma.
Fig. 4. (a~d) The time sequences of the number density ((a) and (b)) and the beam number density ((c) and (d)) through the hydrogen plasma channel.
Fig. 5. The diagram for the contained berm energies within the channel radius of 0.5 cm versus the time. The contained energy is normalized by the initially contained one within the channel radius. In the case of the argon channel, the contained energy decreases only by the collisional loss. On the other hand, in the case of the hydrogen plasma channel, the contained energy decreases much mainly by the radial expansion.

Here the beam current density $j_e$ is assumed to be homogeneous inside the plasma channel of the radius of $r_0$ and hence $I = \pi r_0^2 j_e$, where $I$ is the beam total current. On the other hand, the collision loss $\Delta E$ (eV/cm) is estimated by the Bethe equation.

$$\Delta E = \frac{1.24 \times 10^{-11} n_p z_p}{E_b} \Delta Z$$

(12)

where $\Delta Z$ (cm) is the beam-propagation length, $E_b$ (MeV) is the beam energy and $Z_p$ is the atomic number of the channel plasma. Here the following restriction must be imposed on (12),

$$\Delta E/E_b \leq \alpha, \quad (0 < \alpha < 1),$$

(13)

for a constant $\alpha$. Through this inequality, another condition for the channel number density is obtained:

$$n_p \leq \frac{8.06 \times 10^{19} \alpha E_b^2}{Z_p \Delta Z}.$$  

(14)

In addition to the two conditions described above, the following condition,

$$n_p \gg n_b$$

(15)

must be satisfied in order that the channel plasma (whose number density is $n_p$) neutralizes the beam charge and current. Inequality (11) together with (14) yields

$$\frac{8.1 \times 10^{19} \alpha E_b^2}{Z_p \Delta Z} \geq n_p \geq \frac{3.0 \times 10^{19} IB_s \tau_s^2}{\pi e r_0^3 A_p},$$

(16)

where $A_p$ is the atomic weight of the channel plasma. Inequalities (16) lead to

$$\frac{A_p}{Z_p} \geq \frac{3.9 \times 10^{-8} IB_s \tau_s^2}{\sigma e E_b^2}.$$  

(17)

Any atomic species which satisfies (17) can be used as a species of the channel plasma provided that $n_p$ is chosen as satisfying (15) and (16). As a typical example, the following parameters are employed: $I = 1$ MA, $B_s = 2 \times 10^4$ Gauss, $\tau_s = 30$ nsec, $r_0 = 0.5$ cm, $\Delta Z = 500$ cm and $E_b = 5$ MeV. These values are in accord with the
parameters employed in the numerical calculations. For these parameters, (17) becomes $A_p/Z_p > 0.35/\alpha$. If $\alpha$ is taken to be smaller than 0.35, the hydrogen cannot satisfy (17) and is not employed as a channel species. The argon channel used in numerical calculations satisfies (15), (16) and (17).

From these results, the intense proton beam of the current 1 MA and radius 0.5 cm has been shown to be able to propagate through the argon plasma channel with the number density of $10^{18}$ cm$^{-3}$.

4. Summary

The proton beam propagation through the argon plasma channel has been studied by the numerical simulations. Analyses in this paper are based on the previous researches about the formation (Kawata, Niu & Murakami, 1983; Freeman, Baker & Cook, 1982), the response (Colombant, Mosher & Goldstein, 1980; Freeman, Baker & Cook, 1982) and the macro instability (Murakami, Kawata & Niu, 1983) of the plasma channel. These researches, however, were performed in the scope of the one-dimensional coordinate (in space) and the beam motion was not coupled with the channel behaviour. On the other hand, it was shown that the micro instabilities did not give a large influence on the beam transportation (Okada & Niu, 1981; Ottinger, Mosher & Goldstain, 1979). Therefore the two-dimensional analyses were developed in this paper in order to investigate the coupling macroscopic behaviours of the channel plasma and the proton beam.

Figures 1–3 show that the macro instability does not grow strongly in the channel and the radial expansion of the channel by the Lorentz force can be suppressed by the inertia of the argon channel plasma. Thus the proton beam of the current of 1 MA can be confined stably in a small radius of 0.5 cm during the propagation through the argon plasma channel of the number density of $10^{18}$ cm$^{-3}$, if the beam extracted from a diode can be focused on the channel radius of 0.5 cm to be injected into the plasma channel.

Acknowledgement

This work was partly supported by the Scientific Research Fund of the Ministry of Education, Science and Culture in Japan.

References