Propagation of Light Ion Beam
"Micro- and Macrostabilities of Rotating Ion Layer"

By

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1. Introduction

In the investigation of the inertial confinement fusion by a light ion beam (LIB), it is the most important problem to propagate the LIB stably through the chamber to the target. Many authors [1, 2] have investigated the propagation of LIB through a fusion target chamber by the method of plasma channels. Neverthless in the plasma channels, it is known that such a plasma is unstable in the rarefied background plasma [3].

In this paper, we propose the method of rotating ion layer as a possible driver for inertial confinement fusion for the purpose of obtaining more stable ion beam for various micro- and macroinstabilities. The analysis is carried out within the frameworks of the Vlasov and fluid models.

2. Vlasov Equilibrium and Microstability Analysis

We consider a rotating ion layer propagating in the z-direction. The beam is described by a distribution function $f(\boldsymbol{r}, \boldsymbol{v}, t)$ which satisfies the Vlasov equation

$$\frac{\partial f(\boldsymbol{r}, \boldsymbol{v}, t)}{\partial t}+\boldsymbol{v}\cdot\frac{\partial f(\boldsymbol{r}, \boldsymbol{v}, t)}{\partial \boldsymbol{r}}+\frac{e_b}{m_b}\left(\boldsymbol{E}+\frac{\boldsymbol{v}}{c}\times\boldsymbol{B}\right)\cdot\frac{\partial f(\boldsymbol{r}, \boldsymbol{v}, t)}{\partial \boldsymbol{v}}=0 \tag{1}$$

where $\boldsymbol{r}$ stands for the cylindrical coordinates, the velocity $\boldsymbol{v}$ for v_r, v_θ, v_z, e_b and m_b, respectively, denote the charge and mass of the ion beam. To obtain a solution to the steady state ($\partial/\partial t=0$) ion Vlasov equation, we take the following assumptions.

(a) The equilibrium is independent of z ($\partial/\partial z=0$) and θ ($\partial/\partial\theta=0$).

(b) The radial thickness of the ion layer is much smaller than its major sadius, i.e.,

$$a\ll R. \tag{2}$$

(c) The square of the thermal velocity of the beam ion is much smaller than the square of the mean velocity.

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Using the above assumptions, we can describe the distribution function for an ion layer in equilibrium as follows:

$$f_b^0(H, P_\theta, P_z)=n_b\left(\frac{m_b}{2\pi T}\right)^{3/2}\exp\left[-\frac{1}{T}(H-\omega_c P_\theta-V_z P_z)\right],$$

where the total energy

$$H=\frac{m_b}{2}(v_r^2+v_\theta^2+v_z^2) \tag{4}$$

the canonical angular momentum

$$P_\theta=m_b r v_\theta+e_b\frac{r}{c}A_\theta(r) \tag{5}$$

and the axial canonical momentum

$$P_z=m_b v_z+\frac{e_b}{c}A_z(r) \tag{6}$$

are the three single particle constants of the motion. Here, $A_\theta(r)$ and $A_z(r)$ are the θ and z components of the vector potential of magnetic fields. The θ and z components of the Maxwell equation can be expressed as

$$\frac{\partial}{\partial r}\frac{1}{r}\frac{\partial}{\partial r}rA_\theta(r)=-\frac{4\pi e_b}{c}\int d^3v\, v_\theta f_b^0(H, P_\theta, P_z) \tag{7}$$

$$\frac{1}{r}\frac{\partial}{\partial r}r\frac{\partial}{\partial r}A_z(r)=-\frac{4\pi e_b}{c}\int d^3v\, v_z f_b^0(H, P_\theta, P_z). \tag{8}$$

Eq. (3) can be rewritten as

$$f_b^0(H, P_\theta, P_z)=n_b\left(\frac{m_b}{2\pi T}\right)^{3/2}\exp\left(\frac{\psi(r)}{T}\right)\exp\left[-\frac{m_b(v_r^2+(v_\theta-r\omega_c)^2+(v_z-V_z)^2)}{2T}\right] \tag{9}$$

where

$$\psi(r)=r\omega_c\frac{e_b}{c}A_\theta(r)+V_z\frac{e_b}{c}A_z(r)+\frac{m_b}{2}r^2\omega_c^2+\frac{m_b}{2}V_z^2. \tag{10}$$

We transform the Vlasov equation from the independent variables $(\boldsymbol{r}, \boldsymbol{v}, t)$ in the laboratory frame to the independent variables $(\boldsymbol{r}', \boldsymbol{v}', t')$ in the rotating frame, where

$$r'=r,\ \theta'=\theta-\omega_c t,\ z'=z,\ v_r'=v_r,\ v_\theta'=v_\theta-r\omega_c,\ v_z'=v_z,\ t'=t. \tag{11}$$

The linearized Vlasov equation in variables appropriate to the rotating frame can be expressed as

$$\left(\frac{\partial}{\partial t'}+\boldsymbol{v}'\cdot\frac{\partial}{\partial \boldsymbol{r}'}+2\omega_c\boldsymbol{v}'\times\boldsymbol{e}'_z\cdot\frac{\partial}{\partial \boldsymbol{v}'}\right)\delta f(\boldsymbol{r}',\boldsymbol{v}',t')=-\frac{e_b}{m_b}\left(\delta\boldsymbol{E}+\frac{\boldsymbol{v}'}{c}\times\delta\boldsymbol{B}\right)\cdot\frac{\partial}{\partial \boldsymbol{v}'}f_b^0, \tag{12}$$

where $\boldsymbol{e}'_z$ is the unit vector parallel to the z-direction. The dispersion relation of filamentation instability can be expressed as

$$\omega^2-c^2k^2-\omega_b^2\sum_{n=-\infty}^{\infty}d\boldsymbol{v}\left[-v_z\frac{\partial f_b^0}{\partial v_z}+\frac{n\omega_c v_z^2}{v_\perp}\frac{\partial f_b^0}{\partial v_\perp}\frac{J_n^2(kv_\perp/\omega_c)}{n\omega_c-\omega}\right]=0, \tag{13}$$

where $v_\perp$ is the velocity component perpendicular to the z-direction and $J_n(x)$ is the Vessel function of n-th order. Assuming $kv_\perp/\omega_c \ll 1$, we expand $J_n(kv_\perp/\omega_c)$ in a power series of the argument $kv_\perp/\omega_c$. Eq. (13) can be solved as

$$\begin{aligned}\omega^2=\tfrac{1}{2}[4\omega_c^2+\omega_b^2+k^2c^2\pm\{(4\omega_c^2+\omega_b^2+k^2c^2)^2\\ -4(4k^2c^2\omega_c^2+4\omega_b^2\omega_c^2-k^2\omega_b^2(u_b^2+V_z^2))\}^{1/2}],\end{aligned} \tag{14}$$

where $u_b=(2T/m_b)^{1/2}$ and $\omega_b^2=4\pi e_b^2 n_b/m_b$. From eq. (14), the stability condition which requires eq. (14) to have positive real roots for ω^2 results in

$$\omega_c^2>\frac{u_b^2+V_z^2}{4c^2}\omega_b^2. \tag{15}$$

In the cases of $V_z \gg u_b$ and $V_\theta=R\omega_c$, eq. (15) can be rewritten as

$$\left(\frac{V_\theta}{V_z}\right)^2>\frac{1}{4}\left(\frac{R\omega_b}{c}\right)^2. \tag{16}$$

For the parameters $n_b=10^{16}\,\mathrm{cm}^{-3}$ and $R=0.1\,\mathrm{cm}$, the stability condition becomes

$$\frac{V_\theta}{V_z}>0.22. \tag{17}$$

3. Equilibrium and Macrostability Analysis

For the propagation of ion layer, to determine the properties of equilibrium state from the macroscopic standpoint, we start from the following equations

$$\frac{\partial n_b}{\partial t}+\nabla\cdot(n_b\boldsymbol{v})=0, \tag{18}$$

$$m_bn_b\left[\frac{\partial \boldsymbol{v}}{\partial t}+\nabla\left(\frac{1}{2}\boldsymbol{v}^2\right)-\boldsymbol{v}\times(\nabla\times\boldsymbol{v})\right]=-\nabla p+e_bn_b(\boldsymbol{E}+\boldsymbol{v}\times\boldsymbol{B}), \tag{19}$$

$$\nabla\times\boldsymbol{E}+\frac{\partial \boldsymbol{B}}{\partial t}=0, \tag{20}$$

$$e_bn_b\boldsymbol{v}=\frac{1}{\mu}\nabla\times\boldsymbol{B}-\varepsilon\frac{\partial \boldsymbol{E}}{\partial t} \tag{21}$$

where eq. (18) is the equation of continuity of ion beams, eq. (19) the equation of motion and eqs. (20) and (21) are the Maxwell equations. The followings are the main assumptions pertaining to the equilibrium configuration:

(a) Equilibrium properties are azimuthally symmetric ($\partial/\partial\theta=0$) about the z axis.

(b) Ion beams are rigidly rotating about the axis of symmetry with constant angular velocity.

(c) The velocity of ion beam parallel to the z-direction is constant and the velocity parallel to the r-direction is zero.

(d) Electromagnetic fields are $\boldsymbol{E}_0=(0, 0, 0)$ and $\boldsymbol{B}_0=(0, B_{\theta 0}(r), B_{z0}(r))$.

(e) The particle number density depends only on the r coordinate.

With these assumptions, the equations for $n_{b0}(r)$, $B_{\theta 0}(r)$ and $B_{z0}(r)$ are

$$\frac{dn_{b0}}{dr}=\frac{e_b n_{b0}}{\kappa T_0}(r\omega_c B_{z0}-v_{z0}B_{\theta 0})+\frac{m_b n_{b0} r\omega_c^2}{\kappa T_0} \tag{22}$$

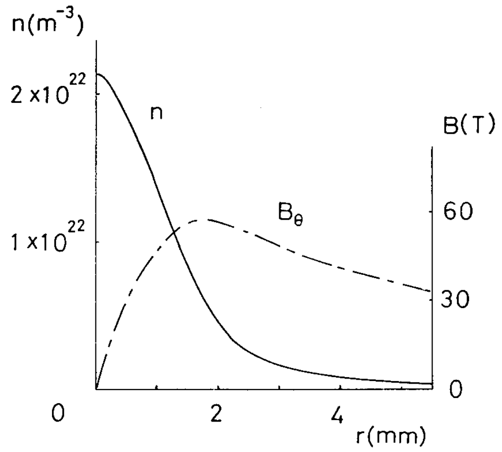

Fig. 1 Plot of number density n and magnetic field B_θ for the choice of equilibrium parameters: $\Omega=0$ and $T_0=1.68\times10^{10}$(K).

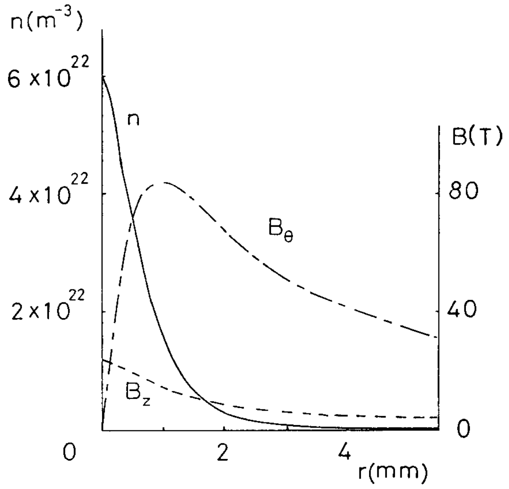

Fig. 2 Plot of number density n and magnetic fields B_θ and B_z for the choice of equilibrium parameters: $\Omega=0.4$ and $T_0=1.306\times10^{10}$(K).

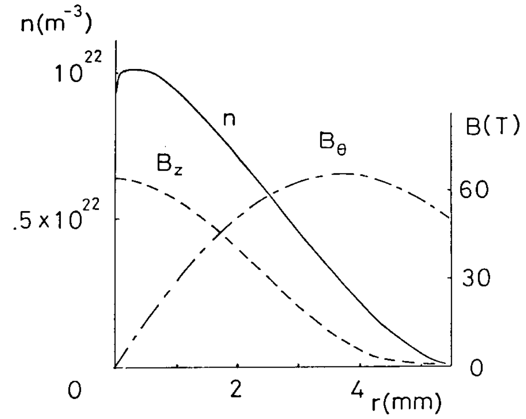

Fig. 3 Plot of number density n and magnetic fields B_θ and B_z for the choice of equilibrium parameters: $\Omega=0.8$ and $T_0=2\times10^6$(K).

$$\frac{dB_{z0}}{dr} = -\mu e_b n_{b0} r \omega_c \tag{23}$$

$$\frac{dB_{\theta 0}}{dr} = \mu e_b n_{b0} v_{z0} - \frac{B_{\theta 0}}{r}. \tag{24}$$

The stationary solutions can be obtained numerically from eqs. (22)–(24). The results are summarized in Figs. 1–3.

We assume perturbations of all quantities are proportional to $\exp\left[i\left(\int k_r dr + k_z z\right) + \gamma t\right]$ where k and γ are the wave number and angular frequency and get the linearized equations. From linearized equations, we obtain the dispersion equation

$$\begin{vmatrix}
\Gamma & 0 & 0 & 0 & -iK_z & 0 & 0 & 0 & 0 & 0 \\
0 & \Gamma & 0 & iK_z & 0 & -2\pi iK_r & 0 & 0 & 0 & 0 \\
0 & 0 & \Gamma & 0 & 2\pi iK_r + \frac{1}{R} & 0 & 0 & 0 & 0 & 0 \\
0 & iK_z & 0 & 0 & 0 & 0 & 0 & 2N & 0 & 0 \\
-iK_z & 0 & 2\pi iK_r & 0 & 0 & 0 & EU_\theta & 0 & EN & 0 \\
0 & -\left(2\pi iK_r + \frac{1}{R}\right) & 0 & 0 & 0 & 0 & E & 0 & 0 & E \\
0 & 0 & 0 & 0 & 0 & 0 & \bar{\bar{\Gamma}} & N(2\pi iK_r + N_R) & 0 & iK_z \\
0 & CN & -DNU_\theta & -CN & 0 & 0 & \bar{\bar{C}}_\omega & N\bar{\bar{\Gamma}} & -NG & CB_T \\
-D & 0 & 0 & 0 & -D & 0 & 0 & \bar{U}_\theta + D\bar{B}_z & \bar{\bar{\Gamma}} & 0 \\
DNU_\theta & 0 & 0 & 0 & 0 & -CN & iK_z V_T^2 & -CNB_T & 0 & \bar{\bar{\Gamma}}
\end{vmatrix} = 0. \tag{25}$$

In eq. (25),

$$\Gamma = \frac{\gamma}{1/\tau_0}, \quad K_r = \frac{k_r}{2\pi/r_0}, \quad K_z = \frac{k_z}{1/r_0}, \quad R = \frac{r}{r_0}, \quad N = \frac{n}{n_0}, \quad V_T = \frac{\sqrt{kT_0/m}}{u_{z0}}$$

$$\bar{B}_z = \frac{B_z}{B_{z0}}, \quad B_T = \frac{B_\theta}{B_\theta(r')}, \quad N_R = \frac{1}{R} + \frac{1}{N}\frac{\partial N}{\partial R}, \quad U_\theta = \begin{cases} \frac{r_0\omega_c}{u_{z0}} R = \Omega R & (r < Ar_0) \\ AR & (r \geqq Ar_0) \end{cases}$$

$$\bar{U}_\theta = \frac{\partial U_\theta}{\partial R} + \frac{U_\theta}{R} \begin{bmatrix} 2\Omega & (r < Ar_0) \\ A\Omega/R & (r \geqq Ar_0) \end{bmatrix}, \quad C = \frac{eB_\theta(r')r_0}{mu_{z0}}, \quad \bar{\bar{C}}_\omega = 2\pi iK_r V_T^2 + \bar{C}_\omega$$

$$D = \frac{eB_z(0)r_0}{mu_{z0}}, \quad E = \frac{\mu e n_0 u_{z0} r_0}{B_z(0)}, \quad H = \frac{r_0}{\tau_0 u_{z0}}, \quad G = D\bar{B}_z + 2U_\theta \frac{1}{R}$$

$$B_z(0) = \tfrac{1}{2}\mu e r_0 u_{z0} n_0 \Omega, \quad B_\theta(r') = \tfrac{1}{2}\mu e r_0 u_{z0} n_0, \quad \bar{\bar{\Gamma}} = H\Gamma + iK_z, \quad \bar{C}_\omega = CB_T - DU_\theta \bar{B}_z - \frac{1}{R}U_\theta^2$$

τ_0 is the propagation time from the entrance of the chamber to the target (in this paper, the distance is taken as five meters), n_0 is the number density at the axis of the beam and r_0 is the radius of the beam. Assuming $|\Gamma| < 1$, $K_r < 1$, $\Gamma = \Gamma' + i\bar{\Gamma}$, we approximately obtain the growth rate Γ' of the macroinstability from eq. (25)

$$\Gamma' = \frac{\int K_r \, dR}{\int \frac{F_1 F_4 - F_2 F_3}{2\pi K_z F_2^2} H \, dR} \tag{26}$$

$$\int K_r\, dR=\pi\left(n'+\frac{1}{2}\right),\ (n':\ \text{mode number}) \tag{27}$$

$$F_1=N\frac{1}{R}\left[2(K_z^2+CN)+DEN+CB_T\frac{1}{R}-G(\bar{U}_\theta+D\bar{B}_z)\right]$$
$$+\frac{\partial N}{\partial R}\left[\frac{1}{R}(\bar{C}_\omega-CB_T)-2(K_z^2+DEN)\right] \tag{28}$$

$$F_2=\frac{\partial N}{\partial R}\left[\frac{1}{R}(\bar{C}_\omega-CB_T)-(K_z^2+DEN)\right] \tag{29}$$

$$F_3=DEN\left(\bar{C}_\omega-CB_T+GU_\theta-U_\theta^2\frac{1}{R}\right)\frac{\partial N}{\partial R}-DEN^2U_\theta\frac{1}{R}(\bar{U}_\theta+D\bar{B}_z)$$
$$+K_z^2N\left[K_z^2+DEN-G(\bar{U}_\theta+D\bar{B}_z)+(\bar{C}_\omega-CB_T)\frac{1}{N}\frac{\partial N}{\partial R}\right]$$
$$+N(K_z^2+DEN)\left(2CN+CB_T\frac{1}{R}\right) \tag{30}$$

$$F_4=\frac{\partial N}{\partial R}\left[DENU_\theta\left(G-U_\theta\frac{1}{R}\right)+(K_z^2+DEN)(\bar{C}_\omega-CB_T)\right]. \tag{31}$$

From eq. (26), the macroinstability (sausage instability) occurs for the equilibrium configurations as in Figs. 1, 2. The analysis shows that there is no instability for the equilibrium configuration as in Fig. 3.

4. Conclusions

In Sec. 2, the equilibriums and microinstabilities were discussed. The filamentation instability was suppressed by the magnetic fields due to the rotation of ion beams. In Sec. 3, the equilibriums and macroinstabilities were studied. It was shown that the macroinstabilities were stabilized by the magnetic field $B_{z,\max}$ which was the same order of $B_{\theta,\max}$. Consequently, it was clarified that the most dangerous instabilities for the problem of the propagation of ion beams could be stabilized by using the rotating ion layer.

References

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