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Shifts of Spectral Lines emitted from Stars

By

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Abstract The origin of shift of spectral lines that are emitted from stars is generally assumed to be due to either of the following causes or combination of them: (1) Doppler effect, (2) plasma shift, (3) Einstein's general-relativistic redshift. Of these, (1) and (2) are discussed in detail. In connection with the Doppler effect, the reason of the fact that Hubble's constant can take a variety of values is explained. There are cases in which Doppler effect is negligible and plasma shift dominates. As an example, the shifts of spectral lines emitted from the symbiotic CI Cygni in quiescent state and tabulated in the literature (reporting the spectroscopic observation done during the period between July 1980 and October 1980) are ascribed to plasma shifts alone.

Keywords: cosmology—plasma shift—CI Cygni

1. Introduction

Let λ denote the wavelength of a spectral line emitted from a star, and λ_0 the corresponding wavelength of the spectral line when it is emitted from a light source which is at rest and is unperturbed. We call $\Delta\lambda=\lambda-\lambda_0$ the shift of the spectral line emitted from the star.

It is generally assumed that the shift is due to either of the following three causes or combination of them:

(1) Doppler effect when the star is moving away from or nearer to the observer,

(2) plasma shift,

(3) the light emitted from the star passes by a heavy object (for example the sun); this is the so-called Einstein's general-relativistic redshift.

We will discuss the causes (1) and (2) separately.

2. Doppler Effect

In Fig. 1 let P denote the emitter of the spectral line, and O the observer. Assume that the emitter is in motion with velocity $\boldsymbol{v}$. Let $\boldsymbol{a}$ denote the unit vector in the direction $\overrightarrow{\mathrm{PO}}$, and θ the angle between the two vectors $\boldsymbol{v}$ and $\boldsymbol{a}$. Then the Doppler effect is expressed by the well-known formula

$$\left.\begin{aligned} \lambda &= \lambda_0 \frac{1-(\boldsymbol{v}/c)\cdot\boldsymbol{a}}{\sqrt{1-(v/c)^2}} \\ \frac{\boldsymbol{v}}{c}\cdot\boldsymbol{a} &= \frac{v}{c}\cos\theta, \end{aligned}\right\} \qquad (1)$$

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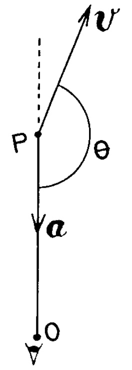

Fig. 1. Emitter of the spectral line (P) which is in motion and the observer (O).

in which c is the velocity of light.

In case $\theta = 180°$, the formula (1) can be written

$$\lambda = \lambda_0 \frac{1-(v/c)}{\sqrt{1-(v/c)^2}} = \lambda_0 \sqrt{\frac{1-(v/c)}{1+(v/c)}} \tag{1$'$}$$

If we put $z = \Delta\lambda/\lambda$, the formula (1)′ can be written

$$1+z = \sqrt{\frac{1+(v/c)}{1-(v/c)}} \tag{1$''$}$$

$$\frac{v}{c} = \frac{z(2+z)}{2+2z+z^2} \tag{1$'''$}$$

In case $v \ll c$, we have the simple relation

$$\frac{v}{c} = z \tag{2}$$

It is clear from (1)‴, that if a star is moving away from the earth, the spectral line shifts to the red ($z > 0$), and the quantity z is called the redshift.

In the numerical calculations of astrophysics, the following units and constant are often used.

pc (parsec) $= 3.0856 \times 10^{18}$ cm $= 3.2615$ light year
ly (light year) $= 9.4605 \times 10^{17}$ cm
$c = 3 \times 10^{10}$ cm/sec $= 3 \times 10^5$ km/sec.

If a spectral line exhibits self-reversed redshift, the recession velocity of the emitting star and that of the absorbing gas can be found by analysis of the whole self-reversed profile. Such a procedure is often used in a variety of astrophysical analysis.

For example, Surdej and Swings [1] studied the resonance doublet ($^2S_{1/2}$—$^2P^0_{1/2,3/2}$) profiles of C IV and Si IV of PHL 5200 and determined the velocities corresponding to the locations v_i of the P Cygni absorption component that is the most displaced towards the short wavelengths, v_c of the sharp cut-off of the red wing of the emission line, and v_t of the abrupt transition between the P Cygni absorption and emission components.

3. Expansion of the Universe

In the year 1929 Hubble [2] discovered a relation between distance (r) and radial velocity (v) among extragalactic nebulae. The relation can be expressed by

$$v = H_0 r, \tag{3}$$

where H_0 is nearly a constant, and is often called Hubble's constant.

Peebles [3] discussed about the distance and redshift of the Virgo cluster and obtained

$$H_0 = 85,\ 51,\ 50,\ 72,\ 51,\ 67\ \mathrm{km \cdot sec^{-1}\ Mpc^{-1}}. \tag{4}$$

He argues that "These numbers are not consistent within estimated random errors: some certainly are bad and should be eliminated. However, I have not seen an argument that positively identifies the bad ones. The straight arithmetic mean of the six numbers is 63, the geometric mean is 61, and the individual standard deviation from the mean is 14 $\mathrm{km \cdot sec^{-1} \cdot Mpc^{-1}}$". He also quotes that Lynden-Bell [4] has given a very clever argument based on a model for expanding compact radio sources that requires $H_0 = 110 \pm 10\ \mathrm{km \cdot sec^{-1} \cdot Mpc^{-1}}$.

Hubble's law suggested that the universe is expanding, and later Gamow [5] assumed that the universe started from a big bang of small size and that the age of the universe today is about 10^{10} yrs.

Hubble's law can be explained by Friedmann model of the universe [6] in the following way.

Assume that the universe is uniform and isotropic. Let $a(t)$ denote the parameter that expresses the size of the universe (a is often called the scale factor of the universe). Then the universe can be expressed by

$$\left(\frac{da}{dt}\right)^2 - \frac{8}{3}\pi a^2 \rho G - \frac{\Lambda}{3} + \frac{k}{a^2} = 0, \tag{5}$$

in which G is the gravitational constant, ρ is the total energy density of the universe, namely ρ is the energy density of all particles (radiation, hadrons, electrons, neutrinos, etc) that fill the universe. Λ is cosmological constant introduced by Einstein, and is assumed to be very small in ordinary theories. But Davies and Unwin [7] state that "In the absence of a fundamental physical reason why Λ should be so small, a possible anthropic explanation suggests itself. Perhaps the excessive smallness of Λ is a feature that only characterizes our particular region of the universe. In other regions this fine-tuning fails and Λ assumes much greater values".

k is a parameter that determines the geometry of the universe.

$$k = \begin{cases} +1: & \text{positive curvature; closed universe.} \\ \ \ 0: & \text{zero curvature; flat universe.} \\ -1: & \text{negative curvature; open universe.} \end{cases}$$

If the universe is assumed to be spherical, $a(t)$ represents the radius of the sphere.

$(4/3)\pi a^3$ is the volume of the universe.

For the universe today it can be assumed that $k=0$, so the equation (5) can be written

$$\begin{aligned} \frac{1}{a^2}\left(\frac{da}{dt}\right)^2 &= \frac{8}{3}\pi a^3 \xi G \frac{1}{a^3} + \frac{\Lambda}{3a^2}, \\ \frac{1}{\mathrm{a}}\frac{da}{dt} &= \left(\frac{8}{3}\pi a^3 \xi G \frac{1}{a^3} + \frac{\Lambda}{3a^2}\right)^{1/2}, \end{aligned} \qquad (5)'$$

The left-hand quantity of (5)′ namely $(1/a)(da/dt)$ is equivalent to v/r in (3) namely H_0. From the argument of Davies and Unwin it follows that the right hand quantity of (5)′ is different for different regions in the universe, so that H_0 need not be constant for different regions in the universe. This is in agreement with observed facts pointed out by Peebles.

4. Plasma Shift

In case when the Doppler effect is very small, we have to take the plasma shift into account. As an example the spectroscopic observations of CI Cygni published by Fehrenbach and Chun [8] will be quoted and explanation will be given here in some detail.

All the strong lines listed by the above-mentioned authors are given in Table 1, but very faint lines are omitted here. Attribution to element for each line by the authors and the transition classification due to Moore [9] are given in Table 1.

It will be noted that some lines are shifted to the violet and still others are shifted to the red, so that Doppler effect is negligible here, and the plasma shift is dominating.

According to Table 1 the strong line $H\gamma$ is shifted to the red by the amount $-0.90\ \mathrm{cm}^{-1}$. Comparison with the result of laboratory experiment (Table 2) shows that the electron density N_e can be estimated to be 10^{16}/cc. The shift of $H\delta$ has also the right order of magnitude and right sign. The shift of $H\beta$ has the right sign but the magnitude is somewhat larger than expected. The shift of $H\alpha$ could not be measured with good accuracy, but the shift given in Table 1 has magnitude that is far larger than that expected from Table 2.

The plasma-shift of He II λ4686 has been measured in the laboratory by the author [10] and the result is shown in Fig. 2. The shift depends on the electron density N_e and electron temperature T_e and, in addition, it also depends on the pressure. Fig. 2 shows that He II λ4686 at $N_e=10^{16}$/cc is expected to shift slightly to the red, but the shift given in Table 1 is shift to the violet. This discrepancy cannot be explained so clearly at present, but the shift is small and there is probably left some room for assuming the existence of neighboring weak impurity line on the violet side (in the star) which makes the apparent shift ($\Delta\nu$) negative.

The plasma-shift of isolated line at $N_e=10^{16}$/cc is listed in Table 3. The first seven lines are all expected to shift to the violet, and in each case the sign of the actual shift (Table 1) is in agreement with the expectation.

Table 1. List of spectral lines emitted from CI Cygni according to ref. 8

element	classification	wavelength unperturbed λ_0 (A)	wavelength observed λ (A)	intensity	wavelength shift $\Delta\lambda$ (A)	wave number shift $\Delta\nu$ (cm^{-1})
H 13	n2—n′13	3734. 37	3734. 16	tr	−0. 21±44%	+1. 50
H 12	n2—n′12	3750. 15	3750. 00	4	−0. 15±58%	+1. 07
O III	$3s^3P_1$—$3p^3D_2$	3754. 67	3754. 05	4	−0. 62±1%	+4. 40
O III	$3s^3P_0$—$3p^3D_1$	3757. 21	3756. 38	1	−0. 83±?	+5. 88
O III	$3s^3P_2$—$3p^3D_3$	3759. 87	3759. 62	16	−0. 25±29%	+1. 77
H 11	n2—n′11	3770. 63	2770. 38	6	−0. 25±21%	+1. 75
O III Fe VI	$3s^3P_1$—$3p^3D_1$	3774. 05	3773. 60	tr	−0. 45±?	+3. 16
O III	$3s^3P_2$—$3p^3D_2$	3791. 26	3790. 87	1	−0. 39±2%	+2. 71
H 10	n2—n′10	3797. 90	3797. 89	10	−0. 01±333%	+0. 07
O III	$3s^3P_2$—$3p^3D_1$	3810. 96	3810. 36	tr	−0. 60±?	+4. 13
H 9	n2—n′9	3835. 39	3835. 24	12	−0. 15±31%	+1. 02
Si II	$3p^2\,{}^2D_{5/2}$—$4p^2P_{3/2}$	3856. 02	3855. 76	2	−0. 26±14%	+1. 75
Si II	$3p^2\,{}^2D_{3/2}$—$4p^2P_{1/2}$	3862. 59	3862. 43	3	−0. 16±13%	+1, 07
H 8 He I	n2—n′8	3889. 05	3889. 11	16	+0. 06±168%	−0. 40
H϶	n2—n′7	3970. 07	3970. 17	20	+0. 10±41%	−0. 63
Hδ	n2—n′6	4101. 74	4102. 00	30	+0. 26±37%	−1. 54
Hγ	n2—n′5	4340. 47	4340. 76	50	+0. 29±33%	−1. 01
N III	$3^2P_{1/2}$—$3^2D_{3/2}$	4634. 16	4634. 02	6	−0. 14±61%	+0. 66
N III	$3^2P_{3/2}$—$3^2D_{5/2}$	4640. 64	4640. 62	12	−0. 02±313%	+0. 09
He II	n3—n′4	4685. 68	4685. 56	80	−0. 12±33%	+0. 55
Hβ	n2—n′4	4861. 33	4861. 86	100	+0. 53±2%	−2. 24
He I	2^1S_0—3^1P_1	5015. 68	5015. 41	10	−0. 27±32%	+1. 07
He I	2^3P—3^3D	5875. 63	5875. 86	30	+0. 23±?	−0. 33
Hα	n2—n′3	6562. 82	6563. 98	300	+1. 16±?	−2. 70

Table 2. Plasma-shift* of hydrogen-like lines at $N_e=10^{16}$/cc and pressure ≈70 torr.

wavelength (A)	classification		shift (cm^{-1})	shift (A)
4686	He II	n3—n′4	negative, see Fig. 2	positive
4102	Hδ	n2—n′6	−1. 08±0. 27	+0. 19
4340	Hγ	n2—n′5	−0. 90±0. 25	+0. 18
4861	Hβ	n2—n′4	−0. 13±0. 21	+0. 03
6563	Hα	n2—n′3	0±1	0

* measured by the present author.

The shift of He I λ5015 at $N_e=10^{16}$/cc is measured [11] to be +1.15 cm^{-1} whereas the observed shift is +1.07±32% cm^{-1}. The estimated $N_e=10^{16}$/cc is here perfectly correct.

The shift of He I λ5876 was measured with a high resolution apparatus (Fabry-

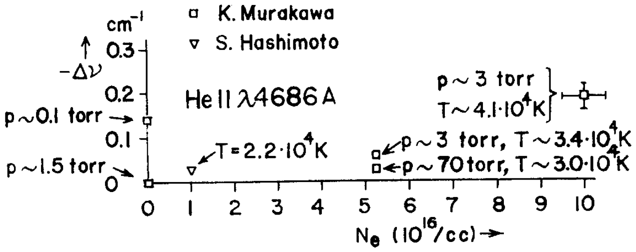

Fig. 2. Plot of plasma-shift of He II λ4686A versus the electron density N_e. Electron temperature T_e and pressure p are parameters.

Table 3. Plasma-shift of isolated lines at $N_e=10^{16}$/cc.

wavelength (A)	classification		shift (cm⁻¹)	shift (A)	ref.
3754.67	O III	$3s^3P_1-3p^3D_2$	positive (calculated)	negative (calc.)	
3757.21	O III	$3s^3P_0-3p^3D_1$			
3759.87	O III	$3s^3P_2-3p^3D_3$			
3856.02	Si II	$3p^2\,{}^2D_{5/2}-4p^2P_{3/2}$			
3863.59	Si II	$3p^2\,{}^2D_{3/2}-4p^2P_{1/2}$			
4644.16	N III	$3^2P_{1/2}-3^2D_{3/2}$			
4640.64	N III	$3^3P_{3/2}-3^2D_{5/2}$			
5015.68	He I	$2^1S_0-3^1P_1$	+..15	−0.29	[11]
5875.618	He I	$2^3P_2-3^3D$	+0.38±0.05	−0.13	*
5875.650	He I	$2^3P_1-3^3D$			
5875.989	He I	$2^2P_0-3^3D$	+0.38±0.05	−0.13	*

* measured by the present author.

Pérot etalon) by the author and the result is given in Table 3. The shift given in Table 1 is opposite in sign, but this discrepancy is not fatal to our discussion, because He I λ5876A consists in reality of three close lying lines and this seems to have lowered the accuracy of the measurement of the shift given in Table 1.

We are thus allowed to conclude that as far as the plasma-shifts of isolated lines in Table 1 are concerned Doppler shift can be neglected, and the electron density N_e can be estimated to be about 10^{16}/cc.

Lastly let us examine the electron density that can be estimated by Inglis-Teller relation [12]. This relation is expressed by

$$\log_{10} N_e = 23.26 - 7.5 \log_{10} n_m, \tag{6}$$

where n_m is the highest number of n' (principal quantum number of the upper level of the Balmer series lines) that can be recognized as discrete line.

If we assume that $n_m=13$, and by putting this into the equation (6), we get $N_e \approx 10^{14}$/cc. In view of some assumptions in deriving the relation (6), we can consider that the electron density obtained here ($N_e \approx 10^{14\pm2}$/cc) is not in disagreement with the previously obtained estimation ($N_e=10^{16}$/cc), and we can conclude that the shifts listed in Table 1 can be explained by the assumption that the Doppler effect in this case is negligible.

In the above discussions the spectroscopic shift data of CI Cygni were taken from the publication of Fehrenbach and Chun who took the spectroscopic pictures of CI Cygni during the period between July 1980 and October 1980. In the present article a simplified assumption was made that CI Cygni was in a quiescent state during that period, and in addition to this, the expanding velocity of the envelope was assumed to be negligible. There is no definite proof that the latter assumption is justified. It is therefore possible that the above discussions require some small corrections. Thus it is possible that the shift of Hα to the red ($\Delta\nu = -2.70\ \mathrm{cm}^{-1}$) and shift of He II λ4686A to the violet ($\Delta\nu = -0.55\ \mathrm{cm}^{-1}$) are connected with this correction.

Lastly some spectroscopic literature concerning CI Cygni may be mentioned. Iijima [13] carried out spectrophotometric observations of CI Cygni in quiescent state from May to December 1979 with the 122 cm telescope of the Astrophysical Observatory of Asiago, Italy. The spectral range covered λ3800—8000A. From the intensity ratio [0 III] λ4363 to λ5007+λ4959 an electron density $\sim 10^7/\mathrm{cm}^3$ was derived.

This value is considered by Iijima to be about 5.10^{-2} times lower than $5.10^9/\mathrm{cm}^3$ estimated from the IUE observation by the intensity ratio of the ultraviolet semi-forbidden lines C III]/N III].

The following is the introductory chapter of the article of Iijima to the previous literature: "CI Cyg is a well known symbiotic star. Results of the photometric observations are collected in the review papers of Boyarchuk (1968) [14] and Belyakina (1975) [15] from which two kinds of light variations are known. In general, the magnitude of the star changes between $m_{pq} \sim 12$ and 13 with a period of about 855d. Nova-like outbursts have also been observed, superposed to the slow variation, in 1911, 1937, 1971, and 1973 (Belyakina, 1975 [15]; Tempesti, 1976 [16]). Spectra taken during the outbursts in 1971 and 1973 were published by Mammano et al (1975) [17]".

Spectroscopic study of the outburst phase of CI Cygni has been published by Audouze, Bouchet, Fehrenbach and Wosczyck [18]. Their article is accompanied by good spectrograms, and it is at once noted that some lines show self-reversal close to an emission part. This can be treated by the procedure similar to that of Surdej and Swings [1].

According to Bath [19, 20]: "symbiotic variables are characterized by the presence of spectral features of a cool M giant and of emission lines normally indicative of high excitation temperatures. Thus both TiO absorption bands and H, He I, Mg II, He II, [O III], [Ne III] in emission may be present simultaneously. Symbiotic stars generally exhibit irregular brightness variations of 2—3 mag on timescales of the order of months/years and may suffer occasional nova-like outbursts of larger amplitude. In general the relative strengths of the lines, both in absorption and emission, are correlated with the brightness of the continuum. The degree of ionization of the emission lines increases as the luminosity decreases. The TiO bands reappear at lower luminosities, basically unaffected by the continuum variations. References are too numerous to give in full, but, apart [from the most recent work, these may be found in the review paper of Boyarchuk (1968)".

Purpose of the present paper is not to discuss about the outburst phase of CI Cygni in detail, but the author wishes to do this in connection with the spectroscopic analysis of ref. 18 in the near future.

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