Aeronautical Research Institute, University of Tokyo
Report No. 351, February 1960

FUNDAMENTALS OF ACOUSTICAL SILENCERS
(III) Attenuation characteristics studied by an electric simulator

By

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Summary. Attenuation characteristics of the acoustic filters have been studied by an acoustic method [*1*].

Analytically, they can be obtained by calculating from the equivalent circuits and the method of measurement of the four-terminal constants of the black boxes was reported in Part II [*2*].

The present paper includes the method of measuring the attenuation characteristics by the electric simulator. Several acoustic elements were simulated by the electric elements and the results obtained agreed well to those by the acoustic method. Without constructing any real models of the silencers we can get the attenuation characteristics by proper selection of the electric elements. Effect of length of the pipe which connects the engine and the muffler was also studied by this method.

1. INTRODUCTION

The characteristics of the acoustic filters have been analyzed by an electric circuit, which is equivalent to the acoustical circuit. If the acoustic system is expressed by a proper electric circuit, we could simulate it by the electric elements.

By preparing several electric elements we can simulate various types of the acoustical circuits. Several acoustic elements mentioned in Part I have been simulated and their attenuation characteristics were obtained. The cavity penetrated by the perforated tube exhibits particular attenuation, though it can not be calculated easily, but by the simulator, the effects of positions of holes can be studied for various conditions. Even the two-dimensional phenomena in the cavity can be simulated by ladder connections of L, C elements.

2. ELECTRO-ACOUSTICAL ANALOGY

Acoustical elements such as straight tubes, resonators, can be expressed by electric equivalent circuits, which have been treated in several papers [*3*] [*4*] [*5*]. We reported on the physical meaning of the equivalent circuit in another paper [*6*]. That is, if the cross section of a tube is smaller than a wave length, then the straight

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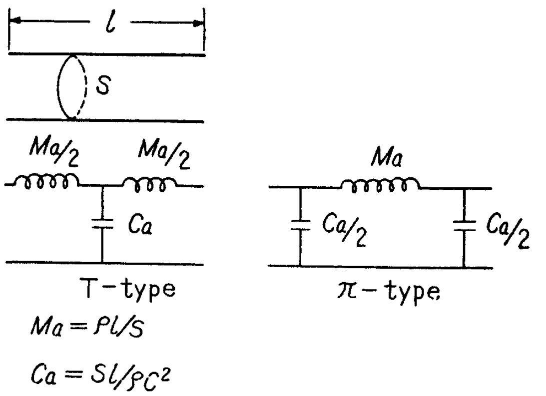

FIGURE 1. Acoustical element and its equivalent circuit.

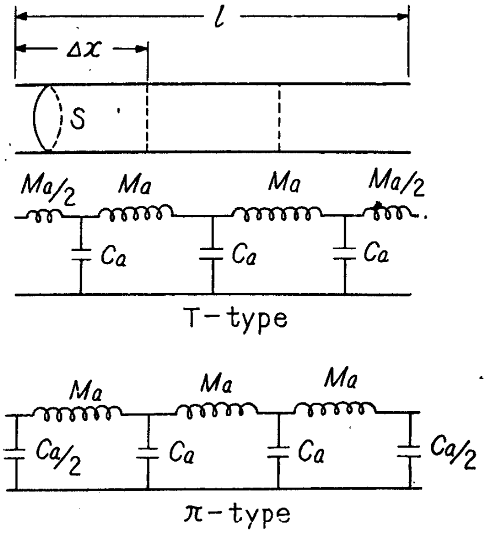

FIGURE 2. The expression of the distributed circuit.

tube is simulated by T or π net-work as shown in Fig. 1. If the tube is devided into several segments as is shown in Fig. 2, the equivalent circuit is expressed as series connection of T or π net-works, and wave phenomena in the tube can be simulated up to the high frequencies. Smaller the tube is devided, the higher the simulated frequency range is extended. If we choose Δx as the length of the acoustical element, $\Delta x/c$ is the time required to transmit sound through Δx. There are some relations between the delay time, the characteristic impedance, the acoustic mass and the acoustic compliance, that is,

$$\tau_a=\sqrt{M_aC_a}=\sqrt{(\rho\Delta x/S)(V/\rho c^2)}=\Delta x/c, \qquad R_{a0}=\sqrt{M_a/C_a}=\rho c/S$$

where $M_a=\rho\Delta x/S$, $C_a=V/\rho c^2=S\Delta x/\rho c^2$, τ_a: delay time in ms., c: velocity of sound, ρ: density of medium, then, $M_a=R_{a0}\tau_a$, $C_a=\tau_a/R_{a0}$.

In the simulated electric circuit we can choose two of them arbitrarily. If we set $\tau_a=\tau$ ms, and $R_{a0}=R_0=1\ \mathrm{k\Omega}$, then the corresponding L and C of the electric circuit can be determined. For example, $\tau_a=\tau=\Delta x/c=0.1$ ms (it corresponds to the length of the acoustic element 3.4 cm), and $R_{a0}\rightarrow R_0=1\ \mathrm{k\Omega}$, $M_a\rightarrow L=R\tau=0.1$ H, $C_a\rightarrow C=\tau/R=0.1\ \mu$F.

If we devide the tube, length l, into longer unit such as $2\Delta x$, then the delay time $\tau_a'=2\Delta x/c=2\tau_a$, and the acoustic mass and the acoustic compliance must be taken as $2M_a$, $2C_a$. For larger cross section such as $2S$ the characteristic impedance is $(1/2)R_{a0}$ and the acoustic mass and the acoustic compliance become $(1/2)M_a$ and $2C_a$ respectively.

In practice, acoustic elements are terminated by a source or a load. For a piston source, it can be regarded as a constant current source. At the closed end of the tube, the volume velocity is zero, and it corresponds to the open end of the electric circuit. The pressure at the open end of the tube is not zero, but it depends on the radiation impedance, the equivalent circuits of an open end and a closed end are shown in Fig. 3 [7].

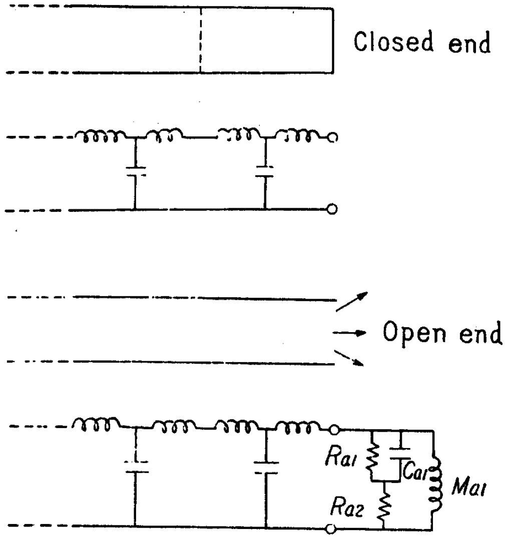

FIGURE 3. Equivalent circuits of a closed and an open end.

3. ERROR CAUSED BY THE APPROXIMATE EQUIVALENT CIRCUIT

The equivalent circuit is the approximation of the acoustic system and the error must be taken into consideration. Four terminal constants of the n-section T or π net-work which correspond to the tube of length l, and the cross section S, are, [8]

$$\begin{pmatrix} \cos\beta^* & jR^*_{a0}\sin\beta^* \\ j(\sin\beta^*)/R^*_{a0} & \cos\beta^* \end{pmatrix}.$$

In this expression, phase angle β^*, and characteristic impedance R^*_{a0} relate to each other in the range of $\beta \leq 2n$ (i.e. $\beta^* \leq n\pi$)

$$\beta^* = 2n\sin^{-1}(\beta/2n) \qquad \beta = \omega l/c \qquad n\text{: number of segments}$$

$$R^*_{a0} = R_{a0}\sqrt{1-(\beta/2n)^2} \qquad \text{for T net-work}$$

$$= R_{a0}/\sqrt{1-(\beta/2n)^2} \qquad \pi \text{ net-work.}$$

Error of phase angle caused by such approximation is;

$$\Delta\beta/\beta = (\beta^*-\beta)/\beta = [\{\sin^{-1}(\beta/2n)\}/(\beta/2n)] - 1 \approx (\beta/n)^2/24.$$

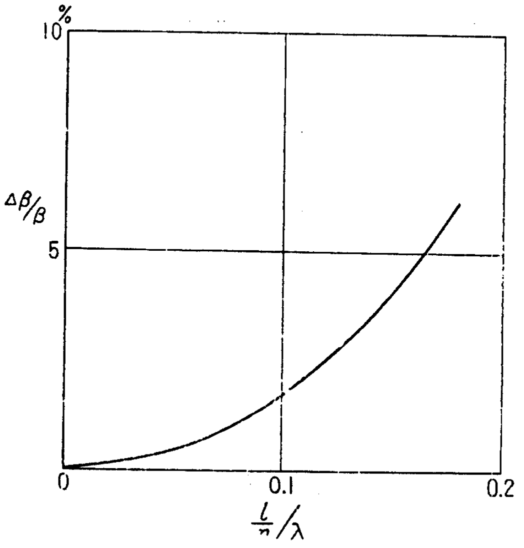

FIGURE 4. Error of the phase angle vers. $(l/n)/\lambda$

As $\beta=2\pi l/\lambda$, the relation between $\Delta\beta/\beta$ and $l/n\lambda$ can be obtained (Fig. 4). For the error 5 percents, we can estimate $l/n\lambda=0.16$, then

$$f_c=nc/2\pi l \quad \text{or} \quad \Delta x=l/n=c/2\pi f_c, \ (f_c\text{: critical frequency})$$

f_c can be extended two times when the numbers of divisions are doubled.

The transmission loss of the acoustic tube terminated by the characteristic impedance R_{a0} is;

$$\text{T.L.}=\cos\beta+j\sin\beta\,.$$

For the simulated circuit,

$$\text{T.L.}=\cos\beta^*+j(1/2)\{\sqrt{1-(\beta/2n)^2}+1/\sqrt{1-(\beta/2n)^2}\}\sin\beta^*.$$

If the acoustic system is terminated by mR, (m corresponds to the ratio of the cross sections of the two tubes connected in series), the transmission loss becomes,

$$\text{T.L.}=\cos\beta+j(1/2)\{m+(1/m)\}\sin\beta\,.$$

The transmission loss is zero for $f_N=N_c/2l$ (N: integer).
For the simulated circuit (π-type)

$$\text{T.L.}=\cos\beta^*+j(1/2)\{m\sqrt{1-(\beta/2n)^2}+(1/m)/\sqrt{1-(\beta/2n)^2}\}\sin\beta^*.$$

In this case T.L. becomes zero for $\beta^*=N\pi$, accordingly, $f_N^*=(nc/\pi l)(\sin N\pi/2n)$ then,

$$\Delta f_N/f_N=(f_N^*-f_N)/f_N=[\{\sin(N\pi/2n)\}/(N\pi/2n)]-1\approx-(1/24)(N\pi/n)^2\,.$$

The frequency of the loss free point at the simulated circuit shifts to the lower side by an amount of $(1/24)(N\pi/n)^2$.

4. CONSTRUCTION OF THE SIMULATOR

To obtain the attenuation characteristics of the acoustic filter the direct simulator was constructed. It is composed of sets of electric elements such as inductance, capacitance and resistance. The minimum values of them were selected as 0.01 H,

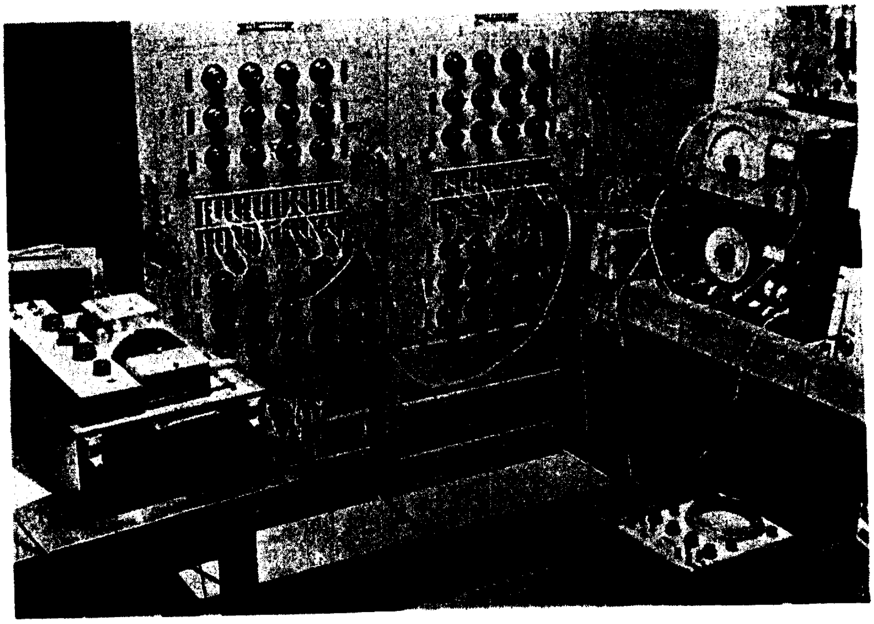

FIGURE 5. Photograph of the simulator.

0.01 μF and 0.1 kΩ, the maximum values 1 H, 1 μF and 10 kΩ respectively. They are variable by 0.01 H, 0.01 μF and 0.1 kΩ steps. Eight sets of variable elements and twenty fixed value L (0.05 H) and C (0.05 μF) have been prepared (Fig. 5).

The accuracy of values of L, C and R is below one percent and the Q-value of L is above 50 at 1 kc/s. The inductane is made of molibden permalloy dust core and the capacitance is polystyrol and the resistance is wounded by manganin wire. The electronic sweep oscillator and the high speed level recorder were used as the measuring instruments. L, C and R are set at the predetermined values, and connected by plug-in cords, following to the equivalent circuit. It takes about one minute to sweep over the frequency range such as 3 kc/s to 100 c/s. The accuracy of the recorded response is within ± 0.3 db.

5. VARIOUS TYPES OF THE ACOUSTIC ELEMENTS SIMULATED BY THE ELECTRIC SIMULATOR

(1) *Cavity type.* The cylindrical cavity of radius a and length l can be simulated by T or π net-work of the electric transmission line. If we choose, arbitrarily, the characteristic impedance of the cylinder as 1 kΩ, then the inductance and the capacitance of the transmission line can be determined as follows,

$$L=\tau \text{ H}, \quad C=\tau\ \mu\text{F}, \quad \tau\text{: delay time in ms.}$$

The delay time of the transmission line must be selected according to the frequency range considered. If the length of the cylinder is devided 3.4 cm each, which corresponds to the delay time 0.1 ms, then L and C become 0.1 H and 0.1 μF respectively. Moreover, when this cylinder is connected at both ends by infinitely long tubes and the cross sections of which are one tenth of that of the cylinder, then the connecting tubes can be simulated by the characteristic impedance ten folds as much as that of the cavity cylinder. The simulated circuit for measuring attenuation can be expressed as Fig. 6(a). Now, if the π net-work is terminated by the characteristic impedance at both sides, the frequency response should be flat throughout the range considered. The attenuation characteristics of the transmission lines which contain three sections of π or T net-work are illustrated in Fig. 6(b). At high frequencies above 1000 c/s, discrepancy is observed, which has

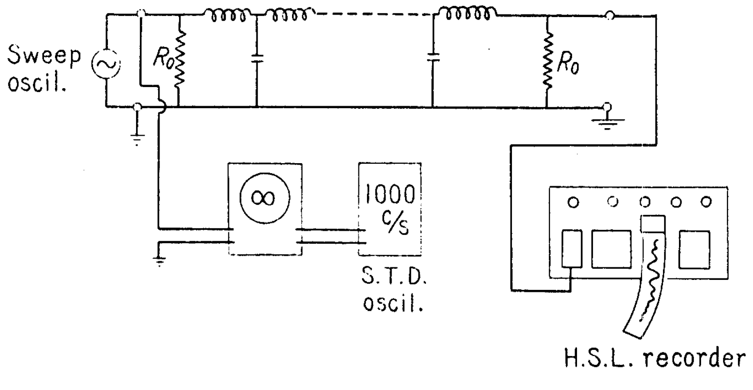

FIGURE 6(a). Block diagram of the measuring system.

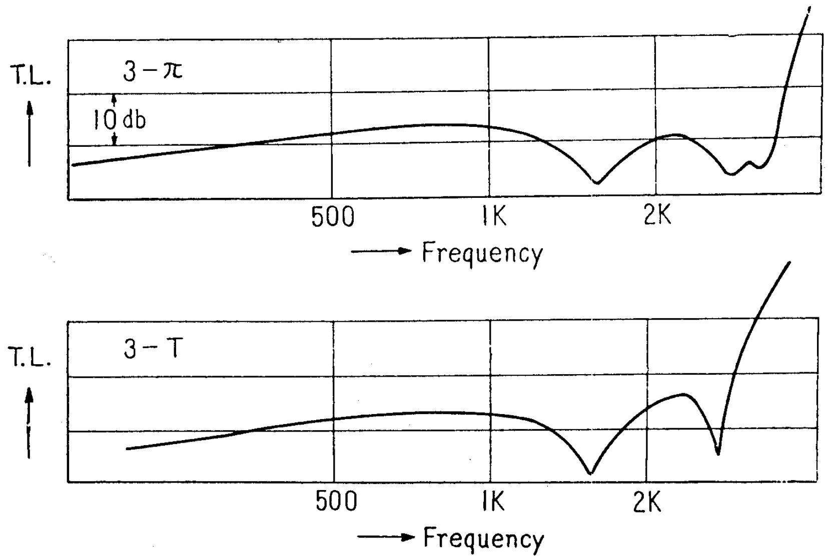

FIGURE 6(b). Transmission loss measured by three sections of π and T net-works.

been discussed in Section 3. Then the length of a segment should be chosen as 1/8 ~ 1/6 of the wave length of the upper frequency considered.

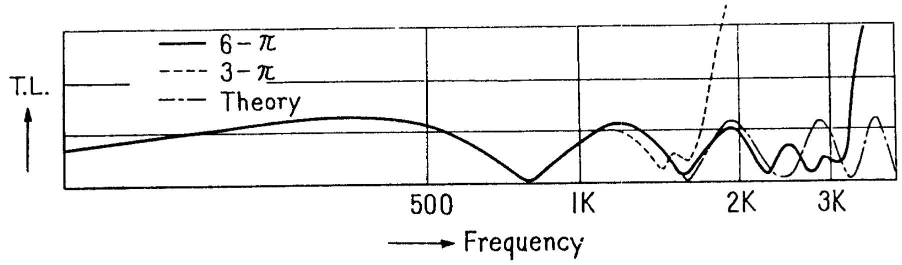

FIGURE 7. T.L. measured by 3-π and 6-π net works for the same cavity cylinder, the theoretical curve is also illustrated. 'The length of the cavity is 20.8 cm.

In Fig. 7, the transmission loss of the same cylinder is illustrated, when it is simulated by six or three sections of π net-works. The delay times in these cases are 0.1 ms and 0.2 ms respectively, acoustically the cylinder (20.8 cm) is devided 3.4 cm or 6.8 cm each. The theoretical curve is also exhibited by dotted line in the same figure.

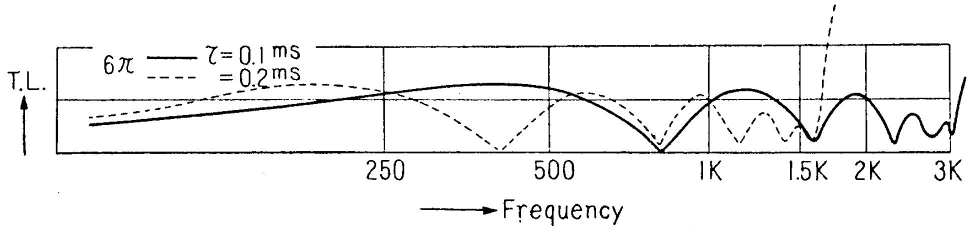

FIGURE 8. T.L. by 6-π net-works, delay times of which are 0.1 and 0.2 ms.

In Fig. 8, the delay time of one section is chosen as 0.1 and 0.2 ms, and the numbers of sections are 6-π each. That is, the lengths of the cavities are 1:2 in these cases. Namely, from the attenuation curve observed, we can deduce the transmission loss of the arbitrary length of the cavity, if we take the frequency range properly.

In Part I, the frequency characteristics of the cavities connected in series, were reported, the examples observed by the simulator show, of course, the similar results. The effects of the length of the connecting tube can be obtained easily by adding the equivalent net-work of the tube (Figs. 9 and 10).

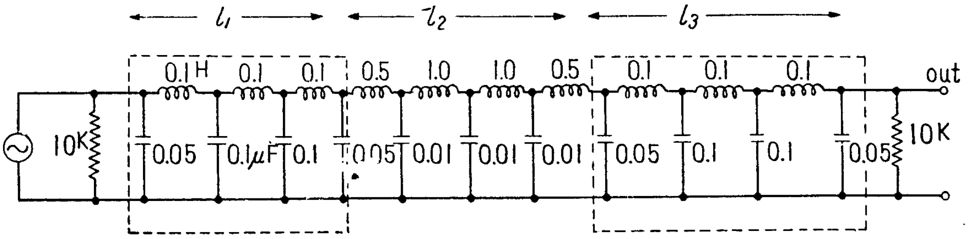

FIGURE 9. The equivalent circuit of the cavities connected in series by a conducting tube.

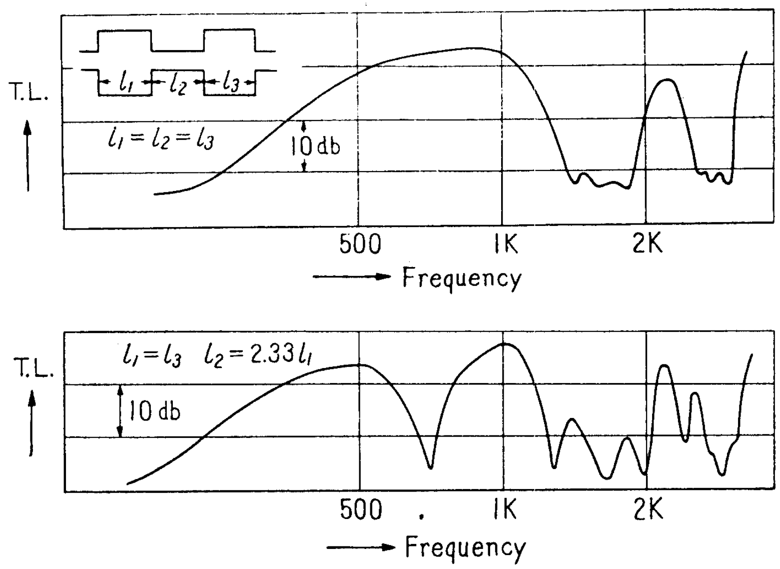

FIGURE 10. T.L. measured by the equivalent circuit which is shown in Fig. 9.

(2) *The side branch tube.* The side branch tube connected to the conducting tube exhibits the resonance characteristics in the attenuation curve. The acoustic model and the equivalent circuit are shown in Fig. 11. The characteristic impedance is taken as 1 kΩ in this case. The resonances at the frequencies to which the length of the side branch becomes a quarter wave length and three-fourth wave length are observed.

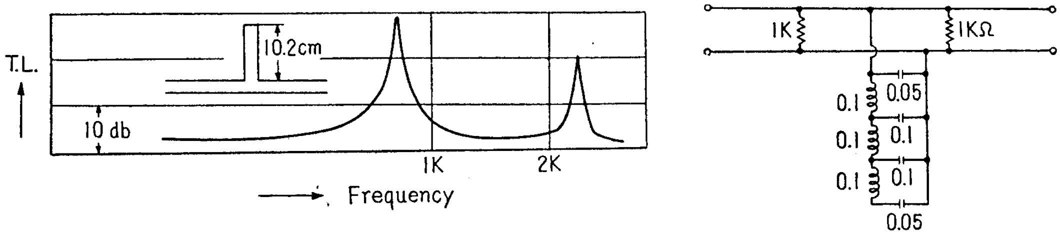

FIGURE 11. T.L. of the side branch tube.

(3) *The resonator.* The equivalent circuit of the resonator in Fig. 12(a) can be expressed as (b) in which the volume of the cavity and the acoustic mass of the

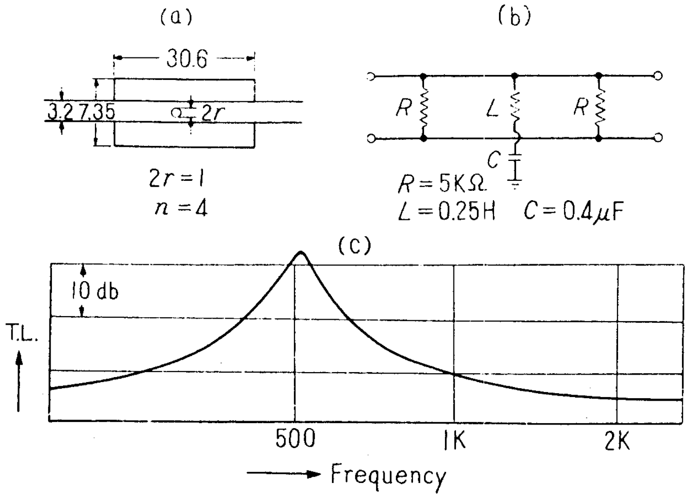

FIGURE 12. Simulation of the resonator; the volume of the cavity and the acoustic mass of holes are taken as lumped impedance.

holes are taken as the lumped impedance of capacitance and inductance. The attenuation characteristics in this cases is shown in Fig. 12(c).

However, if the cavity is devided into proper length, say, four sections, the equivalent circuit and the attenuation curve become somewhat different at the higher frequencies. Wave phenomena in the cavity can be realized by the parallel net-work (Fig. 13). Now, the effects of positions of holes in the cavity were studied.

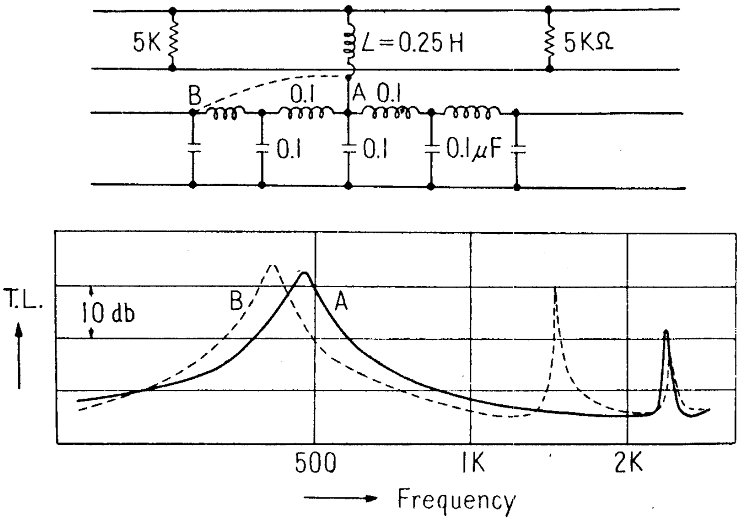

FIGURE 13. The cavity cylinder of the resonator is devided into four sections, the effects of the position of holes are observed.

We assumed in the above mentioned case the diameter of the cavity, 7.35 cm and that of the conducting tube, 3.32 cm. The conducting tube was connected by four holes to the cavity, and the thickness of holes were 1.5 mm, and the diameters of holes were 1 cm each. The characteristic impedance of the cavity was taken as 1 kΩ, then that of the conducting tube could be set as 5 kΩ.

As was already reported in Part I, if we set the four holes at one end of the cavity or at the intermidiate position, the resonance frequency shifts to the lower frequency, compared with the case when they are at the center of the cavity. In Fig. 13, the shift of the resonance frequency can be observed. When the holes are distributed along the conducting tube, the attenuation characteristics can not easily be predicted from calculation, but by connecting the proper position of the simulated net-work of the cavity to the corresponding position of the tube, it can be measured easily. In Fig. 14, a special example is shown, the inductance which connects the cavity and the conducting tube must be doubled if the numbers of holes at one position are halved.

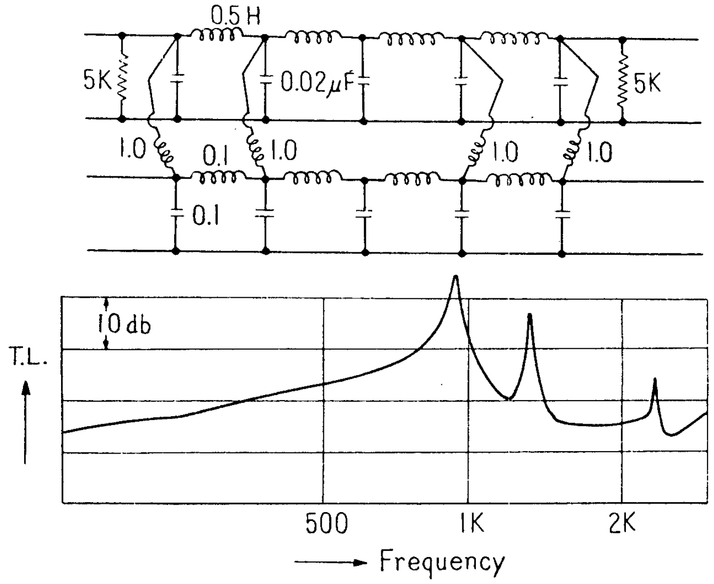

FIGURE 14. The cavity cylinder and the conducting tube are devided into four cells, holes are distributed along the tube.

When holes are distributed along the cavity separately, resonances are quite different from above mentioned case. Interference between holes will be the cause of these phenomena.

(4) *Internal tube type cavity.* The particular attenuation caused by the internal tube in the cavity was shown in Figs. 19 and 20 in Part I. Several experiments by a simulator showed similar results, except little difference which will be attributed to the small difference of the effective length of the internal tube. As shown in Fig. 15, the conducting tubes are simulated by the characteristic impedance and the cavity is simulated as previous manner, but for the part with internal tube, it has different cross section, then the simulated net-work is somewhat different from the cavity itself. The internal tubes of $l/2$ and $l/4$ at both sides of the cavity showed good attenuation when measured by the acoustic model, it was observed by the simulator too (Fig. 16). In the same figure, the attenuation of the cavity itself is also illustrated for comparison.

The effects of the internal tube can be known by setting the proper number of corresponding elements. As a special case when the internal tube is five eighths of that of the cavity is shown in Fig. 17.

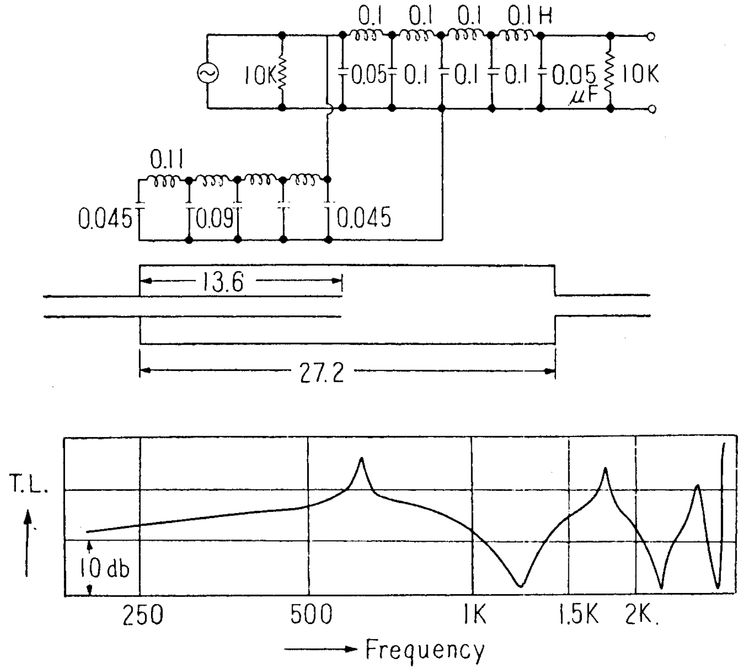

FIGURE 15. T.L. of the internal tube type cavity.

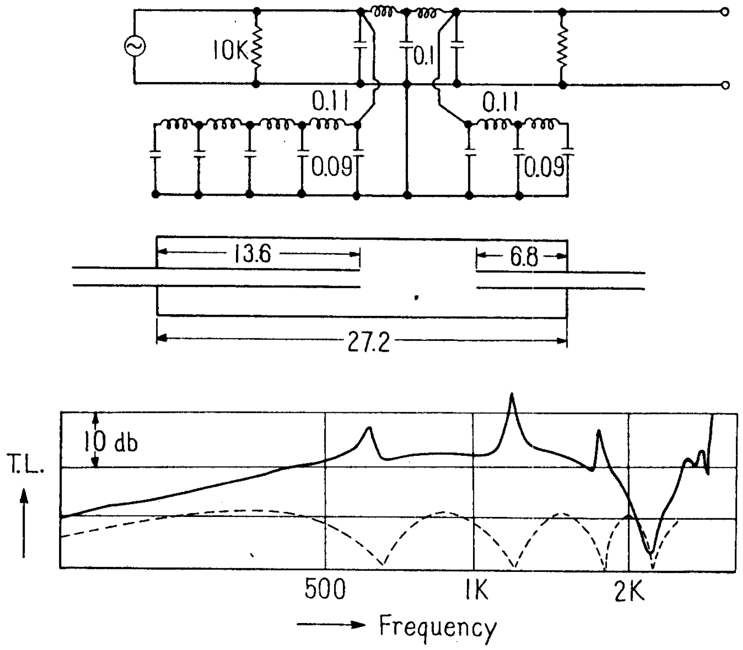

FIGURE 16. The lengths of the internal tubes are $l/2$ and $l/4$, the theoretical curve of the cavity itself is also illustrated by a dotted line.

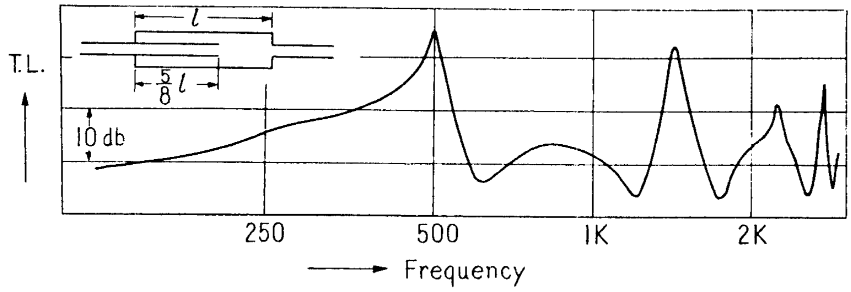

FIGURE 17. The length of the internal tube is (5/8) to that of the cavity.

(5) *Tail pipe.* In practical use, a muffler is connected to the tail pipe and it opens to the air at the other side. The open end of the tube is expressed by the radiation impedance, the equivalent circuit of which is shown in Fig. 3.

Constants of this circuit have the following relations:

For the open end with infinite baffle

$$R_{a1}=0.44\,R_{a0},\quad R_{a2}=R_{a0},\quad C_{a1}=1.89\,(a/l)C_{a0},\quad M_{a1}=0.85\,(a/l)M_{a0}.$$

Without baffle,

$$R_{a1}=0.51\,R_{a0},\quad R_{a2}=R_{a0},\quad C_{a1}=1.73\,(a/l)C_{a0},\quad M_{a1}=0.61\,(a/l)M_{a0},$$

where $R_{a0}=\rho c/S=\rho c/\pi a^2,\quad M_{a0}=(\rho c/S)(l/c)=\rho l/\pi a^2,$

$$C_{a0}=(S/\rho c)(l/c)=\pi a^2 l/\rho c^2$$

a: radius of the conducting tube, l: length of the tube.

When the tube is connected to the high impedance source and is opened at the other side, the equivalent circuit and the frequency response measured at the source side and the open end are shown in Fig. 18.

(6) *Two-dimensional simulation.* The equivalent circuit of the acoustical element can be extended to the two dimensions. The rectangular thin cavity which is shown in Fig. 19 is devided into twelve cells. They can be simulated by three sets of 4-T net-works, connected at the corresponding points. The constants are chosen as $L=0.1$ H, $C=0.1\,\mu$F and $R=10$ kΩ. The cavity is connected to the

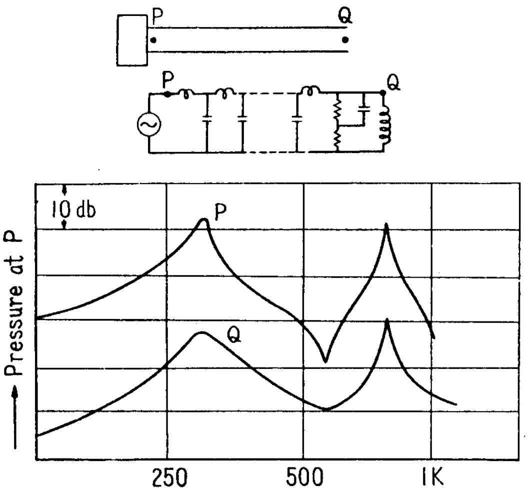

FIGURE 18. The pressure observed at the input and output of the tail pipe.

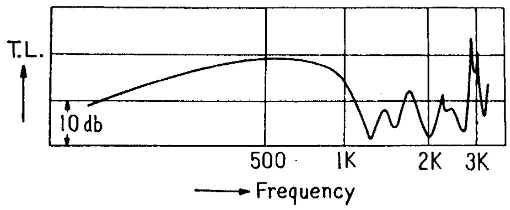

FIGURE 20. T.L. of a two-dimensional acoustic model measured by a simulator.

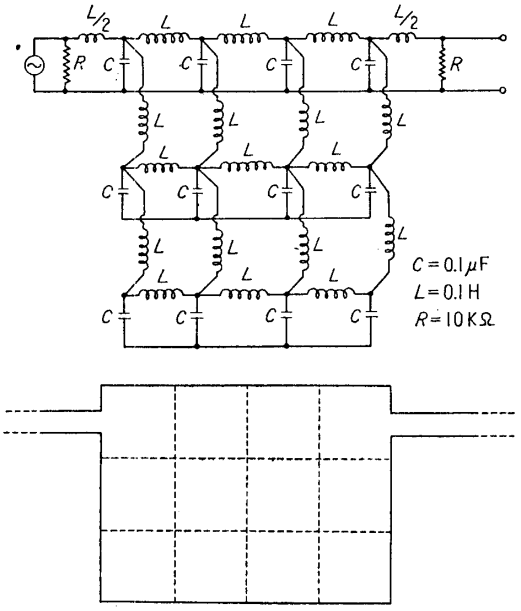

FIGURE 19. The rectangular thin cavity is simulated by sets of one-dimensional circuits.

conducting tube as shown in the figure, the cross section of which is one tenth of that of devided cell. The transmission loss of this systems is shown in Fig. 20.

6. THE EFFECTS OF THE EXHAUST PIPE

The acoustical silencing devices are generally connected to the engine by the exhaust pipe of some length. Sometimes the length of the exhaust pipe affects the efficiency of the engine greatly. When the pressure at the input of the exhaust pipe becomes high, the exhaust gas can not be thrown out completely, then the pressure at the input of the exhaust pipe will be the measure of the efficiency of the engine. As is shown in Fig. 21, the exhaust pipe, length l, is connected to the engine and the other end is opened to the air, then the simulated circuit will be as shown in Fig. 21(b). The pressure at the input of the exhaust pipe becomes high, when the length of the pipe is odd multiples of a quarter wave length.

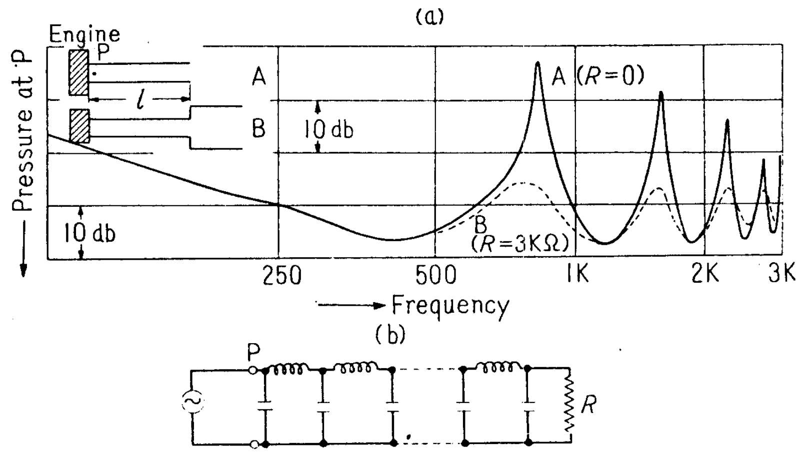

FIGURE 21. The exhaust pipe opened to the air.

The simulated circuit of the pipe is connected to the high impedance source which corresponds to the engine, and the other end is short circuited, which corresponds to the open air. But if we terminate with a resistance of some value (neglecting the end correction), it corresponds to the change of the cross section, namely for the characteristic impedance 10 kΩ of the exhaust pipe, the terminated resistance 2 kΩ, corresponds to the change of the cross section of the pipe which is 5 times that of the exhaust pipe. Experimental results showed that when the cross section of the terminated pipe is reduced to that of the exhaust pipe, the pressure at the input of the exhaust pipe becomes constant for the range of the frequencies under consideration. The terminated resistance of greater values than that of the characteristic impedance of the exhaust pipe corresponds to the smaller cross section of the terminated pipe as shown in Fig. 22. The pressure at the input of the exhaust pipe is illustrated in Fig. 22 too. As the resistance becomes greater, the fluctuation of the pressure becomes greater, but the frequency of the attenuation maximum corresponds to the minimum of that of Fig. 21(a). The former is considered to be the open pipe and the latter, the closed pipe respectively. The pressure at the input of the exhaust pipe depends on the length of the exhaust pipe and

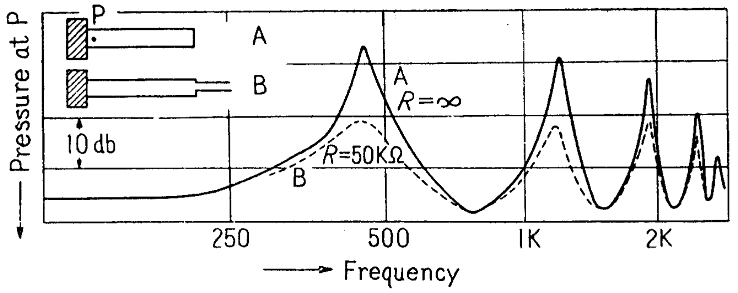

FIGURE 22. The exhaust pipe is closed or terminated by a smaller tube (the ratio of the cross section is 5).

the condition of the output of the pipe. When the cross section of the exhaust pipe is smaller than the following tube, the pressure at the input becomes high at the frequencies when the length of the exhaust pipe is odd multiples of a quarter wave length, on the contrary, it becomes high at the frequencies when the length becomes multiples of half wave length, for the larger cross section of the exhaust pipe.

Then it can be deduced that when the firing frequencies of the engine coincide to the resonance frequencies of the exhaust pipe, the efficiency of the engine must be reduced. Several experiments to remove the resonance in the exhaust pipe have been carried out by the simulator. The cavity, the cross section of which is ten times of the exhaust pipe, was replaced as one section of the exhaust pipe.

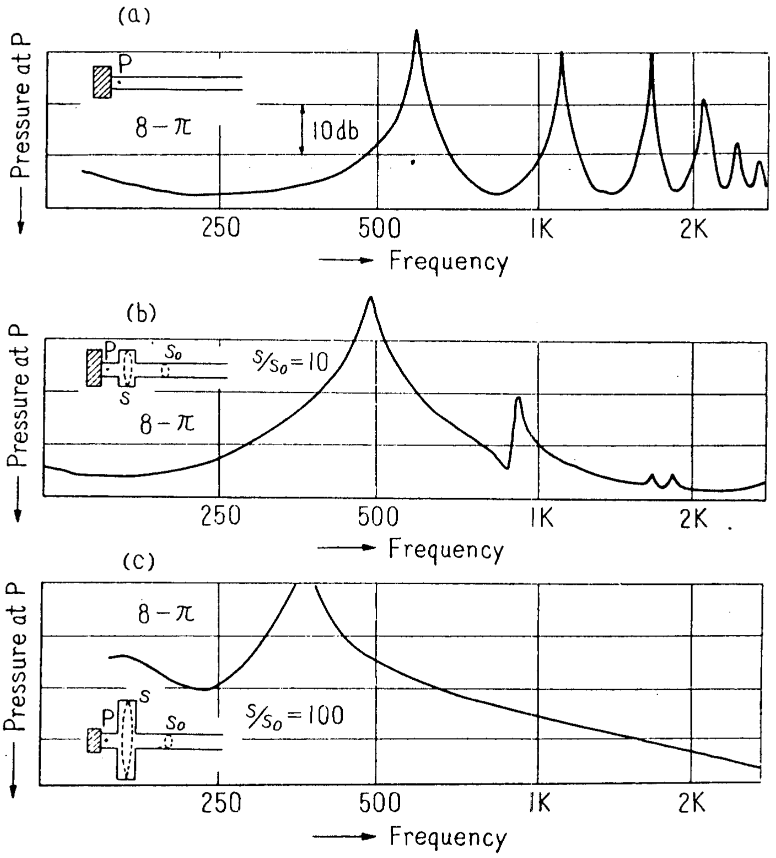

FIGURE 23. The effects of the cavity connected in series with the exhaust pipe (prechamber).

As shown in Fig. 23, the position of the cavity affects the frequency response, but we can not obtain the required characteristics. The cavity of large dimension such as the ratio of the cross section is 100 times of that of the exhaust pipe shows the desirable response as shown in Fig. 23(c). It will not be suitable for practical use, and the resonator of proper resonance frequency was attached to the second section of the exhaust pipe. Some improvement was observed at a low frequency. The acoustic model and the equivalent circuit are illustrated in Fig. 24.

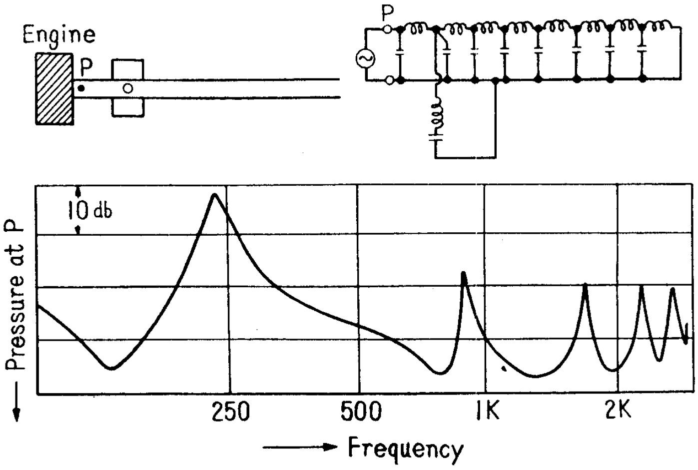

FIGURE 24. The effects of the resonator at the exhaust pipe.

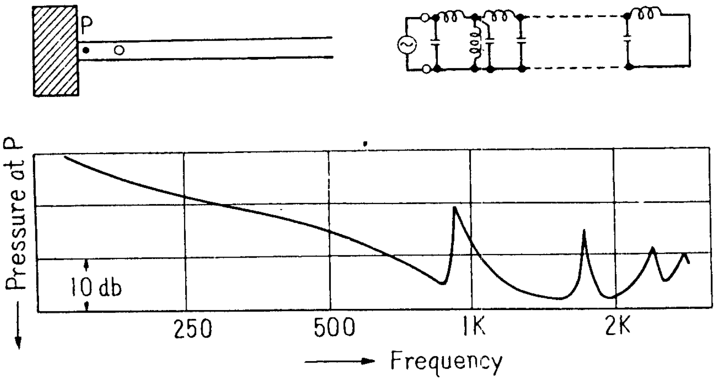

FIGURE 25. The effects of holes at the input of the exhaust pipe.

Fig. 25 shows the reduction of the pressure at the input of the exhaust pipe which has holes at the second section of the pipe. When the holes are opened to the exterior air, the pressure at the input is greatly reduced especially at low frequencies. In the actual design the holes must be closed by the cavity and the absorbing material should be packed in it.

7. CONCLUSION

The acoustic filters have been studied by the equivalent electric simulator. The attenuation characteristics of the several elements which have been obtained by the acoustic measurement could be simulated completely by the *L-C-R* circuits. Without constructing any acoustic models we can get the transmission characteristics of various filters very easily. At the acoustic elements, the change of the dimensions or the position of the holes means the construction of another model, but by the simulator, we can get the equivalent circuit of the different dimensions only changing the values of L, C, R. And we can get the voltage (which corresponds to the pressure at the acoustic elements) very easily at any points of the electric circuit. We expect that the simulator will be used conveniently for the study of the transmission characteristics of the acoustic systems.

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December 18, 1959

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