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A method to predict meteor showers with application to Leonids

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Abstract: A method to predict meteor showers based on the analytical expressions for motions of meteoroids by Kozai (2002) is proposed and is applied to past Leonid meteor showers observed to test whether the method can really predict meteor showers or not. In fact by comparing the observed data with the computed ones the validity of the method is discussed. Predictions of future Leonid showers by 2034 are also given.
Key words: Leonid meteor showers; Motion of meteoroids; Predictions.

1. INTRODUCTION

In the previous paper Kozai (2002) derived analytical expressions for the motion of meteoroids by taking their ejection velocities from the parent comet and the solar radiation pressure without Poynting-Robertson effect into account. In fact the Poynting-Robertson effect is smaller than the main effect by a factor of 10,000, the ratio of the revolution speed and that of the light. Then the radiation pressure effect can be treated by reducing the heliocentric gravitational constant. Therefore, it is still a two-body problem.

In this paper it is assumed that the central stream of the meteoroids is that ejected along the velocity vector of the comet. For this case the inclination, the longitude of the ascending node, and the argument of perihelion as well as the true anomaly are the same as those of the comet. However, the semi-major axis and the eccentricity take different values from those of the comet. In fact they are derived analytically as functions of the ejection velocity, the correction of the heliocentric gravitational constant and the true anomaly at the ejection point. Then by Kepler's third law, the deviation of the mean motion is derived and by neglecting a small term it is found that it can be computed from the deviation of the semi-major axis only. And since the meteoroids are ejected from the comet, the deviation of the eccentricity can be computed from that of the semi-major axis. Scatterings of meteoroids around the central stream are estimated to be within 0.0006AU by assuming that the ejection velocity is of the order of 10m/s.

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Since any meteor shower occurs at one of the nodes of the orbital planes of the earth and the meteoroid, the orbital period of the meteoroid associated with any meteor shower can be estimated by assuming the ejection epoch. Then the value of the mean motion as well as the semi-major axis can be computed from the period. As the deviation of the semi-major axis is a function of a parameter depending on the ejection velocity, the correction of the gravitational constant and the mean anomaly at the ejection point, the orbit of the meteoroid can be computed with the value of the ejection velocity derived from the parameter by assuming that the meteoroid is ejected at its perihelion and by neglecting the solar radiation pressure effect. In fact several authors computed the orbit of the meteoroid by a similar way (Kondrat'eva et al 1997: McNaught & Asher 1999).

By using the ejection velocity thus derived the equation of motion is integrated by a numerical method. Then if the mutual distance between the earth and the meteoroid at one of the nodes of the two orbital planes is less than a certain value like 0.001AU, one can expect that a meteor shower occurs. And the maximum epoch is computed from the longitude of the node.

This method is applied to historical Leonid meteor showers and the results are compared with those by McNaught and Asher (1999). And by examining the observed data in the years, for which the computations are made, the validity of the method is discussed.

2. ANALYTICAL EXPRESSIONS

It is assumed that the parent comet of meteoroids moves on an elliptic orbit, for which the heliocentric radius, r, and the velocity, v, are expressed as,

$$r = a(1-e^2)/(1+e\cos f), \qquad v^2 = \mu(2/r - 1/a), \tag{1}$$

with the semi-major axis, a, the eccentricity, e, the true anomaly, f, and the heliocentric gravitational constant, μ.

Then it is assumed that the meteoroid is ejected from the comet with the velocity, dv, along the velocity vector of the comet. Then after formulating dr and dv, da and de are expressed under the initial condition, $dr = 0$ and df=0, as,

$$\begin{aligned} da/a &= [(1+e)/(1-e)][(1-(2/(1+e))\tan^2(f_0/2)](2dv/v_0 - d\mu/\mu), && (2) \\ de &= (1+e)[1-\tan^2(f_0/2)](2dv/v_0 - d\mu/\mu), && (3) \end{aligned}$$

where v_0 and f_0 are, respectively, the velocity and the true anomaly of the comet at the ejection point and $d\mu(< 0)$ is the correction to the heliocentric gravitational constant, μ, due to the solar radiation pressure effect. To derive these expressions, the quantity, $[(1-e)/(1+e)]^2\tan^2(f_0/2)$, is neglected with respect to 1.

Then the deviation of the mean motion, n, is derived by Kepler's third law, $n^2a^3 = \mu$, as,

$$dn/n = -(3/2)[(1+e)/(1-e)][1-(2/(1+e))\tan^2(f_0/2)](2dv/v_0 - d\mu/\mu) + (1/2)(d\mu/\mu), \tag{4}$$

where the last term can be neglected because it is smaller than the other term of $d\mu/\mu$, by the factor of $3(1+e)/(1-e)$, which is roughly 60 for the case of $e = 0.9$. Therefore, the following relation holds;

$$dn/n = -(3/2)(da/a). \tag{5}$$

Also by using the relation of da and de, the difference between the heliocentric radii of the meteoroid and the comet at $f = f$ is derived as,

$$dr/r = [\tan^2(f/2) - \tan^2(f_0/2)](2dv/v_0 - d\mu/\mu), \tag{6}$$

where the point of $f = f$ is assumed to be not far from the perihelion.

3. APPLICATION TO LEONID METEOROIDS

In Table 1 the orbital elements of the comet 55P/Tempel-Tuttle are given. The elements in 1998 were determined by Nakano (1998) and those in the other years are computed by Nakano with two non-gravitational terms. In the last column the heliocentric radius of the comet, r_D, at its descending node is given. Note that the values of r_D are different from the heliocentric distance of the earth, r_E, at the descending node of the comet. In fact r_E is 0.989AU.

It is computed that v_0 near the perihelion is 41km/s and if dv=10m/s, dv/v_0 is 0.000244 and if the size of the meteoroid is 1mm, $d\mu/\mu$ is −0.00047. Of course, dv/v_0 is proportional to dv and $d\mu/\mu$ is reciprocally proportional to the size of the meteoroid expressed in mm.

The comet moves by 90° for 100 days near the perihelion, and, therefore, in 100 days the meteoroid moves by roughly $10 \times 86,400 \times 100$ m = 86,000 km =0.0006AU in any direction if the meteoroid has a velocity component of 10m/s in this direction. This is regarded also as the scattering around the central meteoroid stream near the perihelion. This value is not increased even after many revolutions if the motion is treated as that of the two-body problem. However, the scattering can be increased when any planetary perturbations are considered.

The equation (6) shows that dr at the descending node is very small as $f = 180° - \omega$ as well as f_0 at the ejection point do not take any large value. This equation also shows that the width of the main stream of meteoroids is narrow as $(2dv/v_0 - d\mu/\mu)$ takes a similar value for every particle in the stream arriving at the ascending node at almost same time.

Then numerical computations are made for the Leonid meteor showers between 1799 and 2034 and their results are shown in the next section.

Table 1: Orbital Elements for 55P/Tempel-Tuttle

T_0	q(AU)	e	ω	Ω	i	a(AU)	r_D(AU)
1600 July 21.217	0.98908	0.90445	167.412	229.018	161.836	10.35194	1.00050
1733 Oct. 01.396	0.96230	0.90799	170.324	232.222	162.952	10.45894	0.96886
1767 Feb. 23.997	0.97196	0.90627	171.000	232.908	162.824	10.36993	0.97768
1800 Mar. 02.914	0.97998	0.90481	170.901	232.959	162.864	10.29490	0.98587
1833 Jan. 02.925	0.98175	0.90474	170.825	233.113	162.699	10.30590	0.98775
1866 Jan. 11.627	0.97658	0.90605	170.905	233.248	162.699	10.39458	0.98245
1899 July 02.014	0.97261	0.90634	172.208	234.589	162.851	10.38458	0.97690
1932 July 12.723	0.97861	0.90508	172.686	235.056	162.707	10.30962	0.98241
1965 Apr. 30.009	0.98161	0.90447	172.564	235.115	162.706	10.27491	0.98555
1998 Feb. 28.098	0.97658	0.90553	172.499	235.258	162.486	10.33754	0.98057

4. HISTORICAL RECORDS AND PREDICTIONS

Since the epoch of any meteor shower is approximately known as it appears at the descending node of the meteoroid which enters the atmosphere of the earth, its revolution period, namely, the mean motion, can be estimated and from it da/a can be computed. As it is stated it is not necessary to estimate the radiation pressure effect by assuming the size of the meteoroid to investigate its motion. In fact dv/v_0 can be computed by putting $d\mu/\mu$ and f_0 to be zero. Then the equation of motion is numerically integrated including the attractions of all the planets and the three major minor planets by Schubart-Stumpff method with 0.015625 days as one time step. Such a short step must be taken as the meteoroid comes near the earth, where the speed is as high as 41km/s. Since the perturbations also affect the mean motion, several iterations are necessary by correcting the mean motion each time so that the stream comes to its descending node on time.

In Table 2 through 8 the results are given for the meteoroids, which come to its descending node with the distance to the earth within 0.001AU. The orbits ejected at the perihelion in 1600, 1733, 1767, 1800, 1833, 1866 and 1899 are computed and in the tables, each for the same ejection epoch, the ejection velocity, da/a, and the computed maximum epoch derived from the longitude of the descending node, $r_{\rm D} - r_{\rm E}$ in the unit of 0.001AU and the number of the meteors observed quoted mainly by Yeomans (1981) are tabulated. And when the data for the maximum epoch and/or $r_{\rm D} - r_{\rm E}$ derived by McNaught and Asher (1999) are available, they are given in parenthesis.

It is noted that the maximum epoch is computed from the longitude of the descending node of the meteoroid of the central stream, which does not arrive at the atmosphere of the earth but approaches it with the distance of $r_{\rm D} - r_{\rm E}$. Since the earth moves by 1° (0.017AU) per day, the maximum epoch can be off by 0.06 days (1.5 hours), if the position of the descending node is off by 0.001AU. Namely errors of this order seem to be possible.

Table 2: Meteoroid ejected on 1600/07/21.217.

dv(m/s)	da/a	maximum epoch	$r_{\rm D} - r_{\rm E}$ (0.0001AU)	numbers
−0.298	−0.0002856	1799/11/12.266(.339)	−0.5	storm
		1866/11/13.746	−4.5	2,000
−0.277	−0.0002657	1799/11/12.275(.339)	−1.5	storm
		1832/11/12.891(.091)	4.2	20,000
+0.004	+0.0000038	1832/11/13.027(.091)	−5.2	20,000

In Table 2 results for meteor streams ejected during the perihelion passage in 1600 are given. They produced remarkable me0.298 teor showers in 1799, 1832 and 1866, which were observed in Europe and U.S.A.

Table 3 is for meteoroid streams ejected in 1733 and coming near the earth in 1801, 1832, 1833, 1866, 1901, 1931, 1963, 1965 and 2000. There are records describing that remarkable meteor showers occurred in some of those years. However, no such record has been found for 1801, 1931 and 1963. There are two streams with dv=6.253 and 6.254 m/s, very close to each other. Although both of them approached the earth in 1866, one of them did not in 1931 and 1963. Therefore, it is reasonable to conclude that only a few meteoroids with a critical

Table 3: Meteoroid ejected on 1733/01/01.396.

dv(m/s)	da/a	maximum epoch	$r_D - r_E$ (0.0001AU)	numbers
02.986	0.00286312	1832/11/13.122(.174)	4.7	20,000
		1833/11/13.374(.455)	4.4	50,000
		1965/11/17.801	−8.5	5,000
06.253	0.0059962	1866/11/13.994(.994)	3.6(3.6)	2,000
		1931/11/17.741	−6.3	no record
		1963/11/17.988	−0.9	no record
06.254	0.0059969	1866/11/13.993(.046)	3.6(3.6)	2,000
06.383	0.0061206	1866/11/13.995(.046)	5.1	2,000
		2000/11/18.163(.156)	−8.3	5,000
16.658	0.0159737	1801/11/13.106(.211)	−8.4	no record
		1901/11/15.685	1.1	144,000
18.638	0.0178726	1801/11/13.134(.211)	−5.7	no record

Table 4: Meteoroid ejected on 1767/02/23.997.

dv(m/s)	da/a	maximum epoch	$r_D - r_E$ (0.0001AU)	numbers
02.925	0.0027808	1832/11/13.114(.176)	−3.5	20,000
08.496	0.0080776	1866/11/13.982(.022)	−1.9	2,000
		2001/11/18.414(.417)	4.7	600
		2034/11/18.944	4.0	
31.786	0.0302225	1869/11/13.965(.016)	6.2(5.3)	no record

Table 5: Meteoroid ejected on 1800/03/02.914.

dv(m/s)	da/a	maximum epoch	$r_D - r_E$ (0.0001AU)	numbers
02.941	0.0028319	1832/11/13.115(.175)	−9.1	20,000
		1833/11/13.367(.435)	−8.8	50,000
17.683	0.0170293	1833/11/13.370(.435)	2.7(2.9)	50,000

condition became meteors in 1931 and 1963.

Table 4 is for meteoroid streams ejected in 1767. Except for 1869 and 2034 there are records of remarkable meteor showers. The value of dv like 31,788 m/s is larger than the real ejection velocity due to the radiation pressure effect. Therefore, only dark meteors are expected to appear as meteoroids in this stream are estimated to be small.

Table 5 is for those ejected in 1800 and it is known that there were very remarkable showers observed in Europe and North America in 1832 and 1833.

In Table 6 results for only one stream ejected in 1833 are given. It is noted that in 2014 this

Table 6: Meteoroid ejected on 1833/01/02.925.

dv(m/s)	da/a	maximum epoch	$r_D - r_E$ (0.0001AU)	numbers
36.853	0.0352818	1867/11/14.342(.392)	2.0(2.1)	5,000
		2014/11/17.070	0.7	

Table 7: Meteoroid ejected on 1866/01/11.627.

dv(m/s)	da/a	maximum epoch	$r_D - r_E$ (0.0001AU)	numbers
11.678	0.0112340	2000/11/18.323(.327)	−7.4	5,000
14.399	0.0138517	2001/11/18.759(.763)	−1.8	5,000
		2034/11/19.298(.222)	2.1	
17.341	0.0166819	2002/11/19.436(.442)	1.1	

Table 8: Meteoroid ejected on 1899/07/02.014.

dv(m/s)	da/a	maximum epoch	$r_D - r_E$ (0.0001AU)	numbers
13.925	0.0135837	1966/11/17.470(.495)	−2.7	150,000
		1999/11/18.085(.089)	7.1	1,500
16.708	0.0162984	1966/11/17.479(.495)	1.8(1.4)	150,000
		1990/11/17.688	0.9	60

stream will approach the earth because of very rapid change of the revolution period due to the action of the earth when it was near the earth in 1867. Therefore, very remarkable showers seem not to be expected as meteoroids under critical situations only will arrive near the earth.

Table 7 is for meteoroid streams ejected in 1866 and includes the predictions in 2002 and 2034 as well as showers observed in 2000 and 2001.

Table 8 is meteoroid streams ejected in 1899 and includes the very remarkable shower observed at Kitt Peak Observatory, U.S.A. in 1966 and those observed in east Asia in 2001. The results for 1990 are recorded in Japan as the stream arrived in this year as the revolution period was changed due to the earth when it approached it in 1966. The number of meteors observed was not so large, however, it is noted that this number was much higher than those in nearby years.

Table 9 summarizes the results in Tables 2-8 according to the years of meteor showers, for which the computations are made. However, the data of the 4th line in Table 2 and of the 1st line in Table 5 for 1832, and that of 8th line in Table 3 for 1867 are not reproduced to save space in Table 9. There the numbers of revolutions after the ejection, the computed maximum epochs, $r_D - r_E$ in the unit of 0.0001AU, the observed hourly rate (HR) of meteors and the places, where they were observed are tabulated. In the last column the references for the observed data are given.

Table 9: Observed data.

Year	Rev	Comp. epoch day/hour	$r_D - r_E$ (0.001AU)	HR	Place	ref.
1799	6	12/06.4,6,5	−0.5,−1.5	30,000	Atlantic, America	a)
1801	2	13/02.5,05.1	−8.4,−5.7	no record		
1832	7,3,2	13/00.1,02.9,02.7	−5.2,4.7,−3.5	20,000	E.and W.Europe	a)
1833	1	13/08.9	+2.7	100,000	N.America	a)
1866	3,4,5	13/23.6,23.8,17.9	−1.9,3.6,−4.5	6,000	U.K, America	a)
1867	1	14/08.2	+2.0	2,184	N.America	a)
1869	3	13/23.2	+8.2	80	UK	a)
1901	5	15/16.4	+1.1	144,000	UK	a)
1931	6	17/17.6	−6.3	30−90	UK	a)
1963	7	17/23.0	−0.9	no record		
1965	7	17/19.2	−8.5	400	USSR	a)
1966	2	17/11.3,11.5	−2.7,1.8	140,000	Kitt Peak,U.S.A.	a)
1990	3	17/16.5	+0.9	60	Japan	b)
1999	3	18/02.1	+7.1	1,500	Europe	c)
2000	8,4	18/03.9,07.8	−8.3,−7.4	5,000	Europe	d)
2001	7,4	18/09.9,18.2	+4.7,−1.8	600,5,000	USA,Japan	e)

References. a) Kazimircak-Polonskaya et al. (1968), b) *Circulars of Japan Meteor Soc.*, (1990). c)*IAU Circulars* 7311 (1999), d) *ibid* 7522 (2000), e) *ibid* 7755 (2001).

5. DISCUSSIONS

In the reference a) there is no record, in which meteor shower with much more than HR 1,000 was observed except those listed in Table 9. Still there is no record for meteor shower in 1801, and in 1869, 1965 and 1990 not very many meteors were observed according to the records. In those years including 1801 the values of $r_D - r_E$ are larger than 0.0005AU. Already probable reasons why not so many meteors were observed in 1931, 1963 and 1990 were described, As earlier computations showed, the error for the computed maximum epoch which are regarded also as the possible duration time of the shower are estimated to be 1.5 hours. Still in 1901 the predicted maximum epoch, 16.4 hours UT, is far from that observed in England, where the showers were observed by morning (ref. a)). And according to the predicted maximum epoch showers would have been visible in Japan, where no such records were found. Therefore, it may be conjectured that the meteor stream observed in England in 1901 had a large velocity component perpendicular to the orbital plane of the comet when it was ejected.

Therefore, it cannot be concluded that the method proposed here can predict meteor showers with 100% probability. And if the value of $|r_D - r_E|$ is larger than 0.0005, it is probable that any very remarkable shower will not appear. Note that there is a case such as that in 1901 as it is described. It is also true that even if the value of $|r_D - r_E|$ is larger than 0.001, meteor shower with HR 1,000 or so may appear and that the method proposed here cannot predict how many meteors will be visible in each year.

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