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Prediction method of boundary layer transition in 3-D compressible flow

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1. Introduction

The requirement for a highly accurate method of predicting the 3-D compressible boundary layer transition location over a wing and body of a real aircraft is now increasing in relation to the worldwide trend of developing a so called "next generation" high speed civil transport (HSCT) or space plane with a large drag reduction.

Accurate prediction of transition location is crucial to the successful design of drag reduction techniques such as laminar-flow-control(LFC) and natural laminar flow(NLF). Direct numerical simulation of the transition process with the Navier-Stokes equation , the most rigorous treatment, is however still far beyond the capability of current computers, and is not practicable in the design phase. We must therefore rely on some approximate methods. The e^N method ,for example, is one of these and is a semi-empirical method in the sense that the factor N for the transition is determined only empirically about 9. But its less empiricism than other methods stimulates us to apply it to the three-dimensional boundary layer, although it is not so straightforward.

The numerical system for predicting the transition location of the boundary layers over supersonic wings described in this report consists of the calculations of the three-dimensional boundary layer , its linear stability characteristics and N factor. Each of the three calculations requires a lot of computing time and a rather complex interference procedure (interpolation or extrapolation of the data) between each calculation. Therefore to save computing time and make the resultant numerical system as simple as possible an efficient algorithm must be adopted: we have used the same computing algorithm for both the boundary layer and the linear stability calculation. The outline of the system will be described in this report.

2. e^N method

2.1 Governing equation

We examine if small disturbances (u,v,w,p,T,ρ) superimposed to 3-D compressible laminar boundary layer — the basic flow — would spatially grow or decay. The fundamental assumption in the analysis is that the basic flow is parallel, i.e. a function of only the y ordinate and the resultant governing equation for the disturbances derived from both the Navier-Stokes equation and the energy equation can be linearized because of the small disturbance assumption. The disturbances are written as

$$\begin{aligned}(u,v,w) &= \left[\tilde{u}(y),\tilde{v}(y),\tilde{w}(y)\right]\exp\left[i(\alpha x+\beta z-\omega t)\right]\\ p &= \tilde{p}(y)\exp\left[i(\alpha x+\beta z-\omega t)\right]\\ T &= \tilde{T}(y)\exp\left[i(\alpha x+\beta z-\omega t)\right]\\ \rho &= \tilde{\rho}(y)\exp\left[i(\alpha x+\beta z-\omega t)\right]\end{aligned} \qquad (1)$$

in which all variables are non-dimensionalized by some reference velocity and length. Then the disturbance equations are described in term of $(\tilde{u},\tilde{v},\tilde{w},\tilde{p},\tilde{T},\tilde{\rho})$, their y derivatives , wave numbers α and β, frequency ω and basic flow velocities $U(y),W(y)$. The boundary condition is

$$\begin{aligned}&\tilde{u}=\tilde{v}=\tilde{w}=\tilde{T}=0, &&\text{at } y=0\\ &\tilde{u},\tilde{v},\tilde{w},\tilde{T}\ \rightarrow\ 0 &&\text{as } y\rightarrow\infty\end{aligned} \qquad (2)$$

Following Mack[2], the disturbance equations are transformed into a system of first order ordinary differential equations

$$\frac{d\phi_i}{dy}=\sum_{j=1}^{8}a_{ij}\phi_j \qquad (i=1,2,\cdots 8) \qquad (3)$$

where

$$\begin{aligned}&\phi_1=\alpha\tilde{u}+\beta\tilde{w},\quad \phi_2=d\phi_1/dy,\quad \phi_3=\tilde{v},\quad \phi_4=\tilde{p},\\ &\phi_5=\tilde{T},\quad \phi_6=d\phi_5/dy,\quad \phi_7=\alpha\tilde{w}-\beta\tilde{u},\quad \phi_8=d\phi_7/dy,\end{aligned} \qquad (4)$$

and then the boundary condition equation (2) becomes

$$\begin{aligned}&\phi_1=\phi_3=\phi_5=\phi_7=0 &&\text{at } y=0\\ &\phi_1,\phi_3,\phi_5,\phi_7\ \rightarrow\ 0 &&\text{as } y\rightarrow\infty.\end{aligned} \qquad (5)$$

We solve equation (3), following Malik[1], by the Euler-Maclaurin finite difference scheme:

$$\Psi^k-\Psi^{k-1}=\Delta_1\left(\frac{d\Psi^k}{dy}+\frac{d\Psi^{k-1}}{dy}\right)-\Delta_2\left(\frac{d^2\Psi^k}{dy^2}+\frac{d^2\Psi^{k-1}}{dy^2}\right)+o(h_k^5)\ , \qquad (6)$$

where $\Delta_1=h_k/2,\ \ \Delta_2=h_k^2/12$. To apply equation (6) to equation (3), we put

$$\Psi=\{\phi_i\}\ ,\quad d\Psi/dy=\{\sum_{j=1}^{8}a_{ij}\phi_j\}\ ,\quad d^2\Psi/dy^2=\{\sum_{j=1}^{8}b_{ij}\phi_j\} \qquad (7)$$

where

$$b_{ij} = da_{ij}/dy + \sum_{l=1}^{8} a_{il}a_{lj} \ .$$

Substitution of equation (7) into equation (6) yields

$$\phi_j^k - \Delta_1 \sum_{j=1}^{8} a_{ij}^k \phi_j^k + \Delta_2 \sum_{j=1}^{8} b_{ij}^k \phi_j^k - \{\phi_j^{k-1} - \Delta_1 \sum_{j=1}^{8} a_{ij}^{k-1}\phi_j^{k-1} + \Delta_2 \sum_{j=1}^{8} b_{ij}^{k-1}\phi_j^{k-1}\} = 0,$$

which is rewritten as

$$\sum_{j=1}^{8} P_{ij}^k \phi_j^k + \sum_{j=1}^{8} Q_{ij}^k \phi_j^{k-1} = 0 \quad , \qquad (8)$$

where

$$P_{ij}^k = \begin{cases} -\Delta_1 a_{ij}^k + \Delta_2 b_{ij}^k & (i \neq j) \\ 1 - \Delta_1 a_{ij}^k + \Delta_2 b_{ij}^k & (i=j) \end{cases}$$

$$Q_{ij}^k = \begin{cases} -\Delta_1 a_{ij}^{k-1} + \Delta_2 b_{ij}^{k-1} & (i \neq j) \\ 1 - \Delta_1 a_{ij}^{k-1} + \Delta_2 b_{ij}^{k-1} & (i=j). \end{cases}$$

2.2 Solution of eigenvalue problem and calculation of the N factor

As is well known , equation (8) and boundary condition (5) constitute an eigenvalue problem. To get its non-trivial solution we first drop the boundary condition ϕ_1=0 and instead impose ϕ_4=1 (this means that p is normalized);

$$\phi_3 = \phi_5 = \phi_7 = 0, \phi_4 = 1 \qquad \text{at } y=0 \ , \qquad (9a)$$

$$\phi_1, \phi_3, \phi_5, \phi_7 \rightarrow 0, \qquad \text{as } y \rightarrow \infty, \qquad (9b)$$

and then determine the eigenvalues to satisfy the dropped condition.

Numerical treatment of the boundary condition,equation (9b), is not easy in this form and so, following Mack[2) ,we replaced the condition by analytic solution of equation (3) which can be obtained by using the fact that as at y $\rightarrow\infty$ the characteristics of the basic flow are constant and y derivatives of flow variables are zero, a_{ij} in the right hand side of equation (3) becomes constant $\bar{a}_{ij}$.

Now equations (8), with boundary condition (9), are expressed in matrix-vector form

$$\Pi\delta = r \qquad (10)$$

where Π, δ and r are

$$\Pi = \begin{bmatrix} A_0 & C_0 & & & & & \\ B_1 & A_1 & C_1 & & & & \\ & \cdot & \cdot & \cdot & & & \\ & & B_i & A_i & C_i & & \\ & & \cdot & \cdot & \cdot & \cdot & \\ & & & B_{J-1} & A_{J-1} & C_{J-1} & \\ & & & & & & \end{bmatrix} \quad \delta = \begin{bmatrix} \delta_0 \\ \delta_1 \\ \\ \delta_i \\ \\ \delta_{J-1} \\ \delta_J \end{bmatrix} \quad r = \begin{bmatrix} r_0 \\ r_1 \\ \\ r_i \\ \\ r_{J-1} \\ r_J \end{bmatrix} \tag{11}$$

where

$$\delta_j = \begin{bmatrix} (\phi_1)_j \\ (\phi_2)_j \\ \\ \\ (\phi_8)_j \end{bmatrix} \qquad r_j = \begin{bmatrix} (r_1)_j \\ (r_2)_j \\ \\ \\ (r_8)_j \end{bmatrix}$$

and A_j, B_j, C_j are the 8×8 matrix (details are omitted here).

The solution of equation (10) which can be obtained by the block-elimination method depends on α, β, ω, R i.e.

$$\delta = \delta(\alpha, \beta, \omega, R) \tag{12}$$

which in the spatial amplification theory (i.e. ω =real) means that δ depends on six scalar quantities $(\alpha_r, \alpha_i, \beta_r, \beta_i, \omega, R)$. The dropped boundary condition

$$\phi_1(\alpha, \beta, \omega, R) = 0 \qquad \text{at } y=0 \tag{13}$$

can determine only two eigenvalues. In the spatial amplification theory ω and R are given and thus a relationship between two wave numbers α and β is required. The relationship can be computed by making use of concepts based on group velocity using the saddle-point method (Gaster[3], Cebeci-Stewartson[4]).

According to this method, let us consider an oscillatory disturbance with period $2\pi/\omega$ generated at the origin so that the disturbance at large finite values of x,z may be assumed to be of the form

$$\widetilde{Q}(x, y, z, t) = e^{-i\omega t} \int_c Q(\alpha, \beta, \omega) e^{i(\alpha x + \beta z)} \, d\beta \tag{14}$$

where c is a contour in the complex plane of β extending to ∞ in either direction. Q is a determinate function whose properties are such that the integral converges. α in equation (14) is a function of β and ω found by solving equation (10). Now for any ray in the (x,z) plane and passing through the origin, we can write

$$z = x\tan\gamma \tag{15}$$

with γ denoting a constant, and look for the dominant contribution to Q along this ray as $x \to \infty$. This comes from the saddle point β^* of $\alpha x + \beta z$, regarded as a function of β, which occurs when

$$\left(\frac{\partial\alpha}{\partial\beta}\right)_{\omega R} x + z = 0 \quad . \tag{16}$$

Since x and z are real, the imaginary part of $\partial\alpha/\partial\beta$ must be zero to satisfy eqation (16). Combining equations (15) and (16) we can write the relation

$$\left(\frac{\partial\alpha}{\partial\beta}\right)_{\omega,R} = -\frac{z}{x} = -\tan\gamma \tag{17}$$

which gives another relationship between α and β and the wave orientaion and growth direction of the disturbance. Now we can solve equation (10) with equations (13) and (17). Before describing details of the solution procedure, we describe how to use the solution in the e^N method.

2.2.1 Neutral stability curve — Zarf [4)-8)]

The growth rate of spatially developing disturbance Γ is given from equation (17) as

$$\Gamma = \alpha_i - \beta_i \tan\gamma = \alpha_i - \beta_i(\partial\alpha/\partial\beta)_i \tag{18}$$

where $\Gamma < 0$ means amplified, $\Gamma > 0$ damped and $\Gamma = 0$ neutral. In order that $\Gamma = 0$ for any γ, the conditions

$$\alpha_i = \beta_i = 0 \quad \text{and} \quad \partial\alpha/\partial\beta = real \tag{19}$$

must be satisfied. The neutral stability curve on which the condition is satisfied is refered to as Zarf by Cebeci. In the e^N method the integration of the growth rate starts from the neutral point, so we must first determine the Zarf.

First fix the starting point of calculation (x,z) near the leading edge. The velocity and temperature distributions of the basic flow and Reynolds number are now given. Then equation (13) is written as

$$\phi_1(\alpha,\beta,\omega) = 0. \tag{20}$$

α_i, β_i being zero from equation (19), the relation contains three unknowns $(\alpha_r, \beta_r, \omega)$. The second equation in equation (19) and equation (20) can now determine α_r, β_r and ω as follows. When ω is constant in equation (20), it gives

$$d\phi_1 = \left(\frac{\partial\phi_1}{\partial a}\right)_\beta da + \left(\frac{\partial\phi_1}{\partial\beta}\right)_\alpha d\beta = 0 \tag{21}$$

Therefore $\partial\alpha/\partial\beta l$ in equation (19) is given by

$$\frac{\partial\alpha}{\partial\beta} = -\frac{\left(\partial\phi_1/\partial\beta\right)_\alpha}{\left(\partial\phi_1/\partial\alpha\right)_\beta} = e \quad . \tag{22}$$

As we use an iteration method to get the solutions of α, β, ω, it is necessary to calculate the corrections $(\delta\alpha, \delta\beta, \delta\omega)$ to initial guesses of $(\alpha^v, \beta^v, \omega^v)$. If we expand equation (20) about $(\alpha^v, \beta^v, \omega^v)$ and neglect higher order terms, we get

$$\phi_{1r} + \frac{\partial\phi_{1r}}{\partial\alpha}\delta\alpha + \frac{\partial\phi_{1r}}{\partial\beta}\delta\beta + \frac{\partial\phi_{1r}}{\partial\omega}\delta\omega = 0 \tag{23a}$$

$$\phi_{1i} + \frac{\partial\phi_{1i}}{\partial\alpha}\delta\alpha + \frac{\partial\phi_{1i}}{\partial\beta}\delta\beta + \frac{\partial\phi_{1i}}{\partial\omega}\delta\omega = 0 \ . \tag{23b}$$

Similarly if we expand equation (22), we get

$$e + \frac{\partial e}{\partial\alpha}\delta\alpha + \frac{\partial e}{\partial\beta}\delta\beta + \frac{\partial e}{\partial\omega}\delta\omega = 0 \tag{24}$$

The imaginary part of α, β are zero and so is that of $d\alpha/d\beta$ or e. Therefore we must seek the values of α, β, ω which makes the imaginary part of equation (26) zero, i.e.

$$e_i + \frac{\partial e_i}{\partial\alpha}\delta\alpha + \frac{\partial e_i}{\partial\beta}\delta\beta + \frac{\partial e_i}{\partial\omega}\delta\omega = 0 \tag{25}$$

If we solve equation (23) and (25) for $\delta\alpha, \delta\beta, \delta\omega$, we get

$$\delta\alpha = \left[\phi_{1r}\left(\phi_{1i\omega}e_{i\beta} - \phi_{1i\beta}e_{i\omega}\right) + \phi_{1i}\left(\phi_{1r\beta}e_{i\omega} - e_{i\beta}\phi_{1r\omega}\right) - e_i\left(\phi_{1r\omega}\phi_{i\omega} - \phi_{1r\omega}\phi_{1i\beta}\right)\right]/\Delta \tag{26a}$$

$$\delta\beta = \left[\phi_{1r}\left(\phi_{1i\omega}e_i - \phi_{1i}e_{i\omega}\right) - \phi_{1i\alpha}\left(\phi_{1r\omega}e_i - e_i\phi_{1r\omega}\right) + e_{i\alpha}\left(\phi_{1r\omega}\phi_i - \phi_{1r}\phi_{1i\omega}\right)\right]/\Delta \tag{26b}$$

$$\delta\omega = \left[\phi_{1r\alpha}\left(\phi_{1i}e_{i\beta} - \phi_{1i\beta}e_i\right) - \phi_{1i\alpha}\left(\phi_{1r}e_{i\beta} - e_i\phi_{1r\beta}\right) + e_{i\alpha}\left(\phi_{1r}\phi_{1i\beta} - \phi_{1r\beta}\phi_{1i}\right)\right]/\Delta \tag{26c}$$

where

$$\Delta = e_{i\alpha}\left(\phi_{1r\beta}\phi_{1i\omega} - \phi_{1i\beta}\phi_{1r\omega}\right) - e_{i\beta}\left(\phi_{1r\alpha}\phi_{1i\omega} - \phi_{1i\alpha}\phi_{1r\omega}\right) - e_{i\omega}\left(\phi_{1r\alpha}\phi_{1i\beta} - \phi_{1i\alpha}\phi_{1r\beta}\right) \tag{27}$$

and suffix i and r means imaginary and real part, respectively and suffix α, β and ω means partial derivatives for each variable. The iteration will be continued until these correction terms $\delta\alpha, \delta\beta, \delta\omega$ becomes less than ε, a specified small quantity. When $\delta\alpha = \delta\beta = \delta\omega = 0$, equation (25) certainly gives e_i=0.

The derivatives $\phi_{1\alpha}, \phi_{1\beta}$ and e_α, e_β in equations (22),(23) and (24) are obtained as follows. From equation (22)

$$e = -\frac{\phi_{1\beta}}{\phi_{1\alpha}}$$

we can get the partial derivatives of e as follows:

$$e_\alpha = -\frac{\phi_{1\beta\alpha}}{\phi_{1\alpha}} + \frac{\phi_{1\beta}}{\phi_{1\alpha}^2}\phi_{1\alpha\alpha} \tag{28a}$$

$$e_\beta = -\frac{\phi_{1\beta\beta}}{\phi_{1\alpha}} + \frac{\phi_{1\beta}}{\phi_{1\alpha}^2}\phi_{1\alpha\beta} \tag{28b}$$

$$e_\omega = -\frac{\phi_{1\beta\omega}}{\phi_{1\alpha}} + \frac{\phi_{1\beta}}{\phi_{1\alpha}^2}\phi_{1\alpha\omega} \tag{28c}$$

The α, β and ω derivatives of ϕ_1 in equation (28) are calculated as follows: for example in the case of $\phi_{1\alpha\beta}$, if we first differentiate equation (10) with α and then with β, we get

$$\Pi\frac{\partial^2\delta}{\partial\alpha\partial\beta} = -\frac{\partial\Pi}{\partial\beta}\frac{\partial\delta}{\partial\alpha} - \frac{\partial\Pi}{\partial\alpha}\frac{\partial\delta}{\partial\beta} - \frac{\partial^2\Pi}{\partial\alpha\partial\beta}\delta \tag{29}$$

As the coefficient matrix Π of this equation is the same as that of equation (10), we can use the solution algorithm of equation (10) when the right hand side of equation (29) is known. The derivatives of Π can immediately be obtained from equation (11) and $\partial\delta/\partial\alpha, \partial\delta/\partial\beta$ can be obtained by solving the following equations

$$\Pi\frac{\partial\delta}{\partial\alpha} = -\frac{\partial\Pi}{\partial\alpha}\delta, \quad \text{and} \quad \Pi\frac{\partial\delta}{\partial\beta} = -\frac{\partial\Pi}{\partial\beta}\delta \tag{30}$$

In the same way the $\alpha\alpha, \beta\beta, \alpha\omega, \beta\omega$ derivatives can be obtained.

We now summarize the first step of eigenvalue calculation on Zarf :

(I) solve equation (10) for some initial guess of α, β, ω .

(II) if the value of ϕ_1 at y=0 does not satisfy equation (20), the correction terms $\delta\alpha, \delta\beta, \delta\omega$ are calculated from equation (26).

(III) if $\delta\alpha, \delta\beta, \delta\omega < \varepsilon$ the iteration is stopped, and if not, return to step (I) and repeat the same procedure. When the iteration finishes, e is the real value which gives the direction of wave propagation.

(IV) repeat the calculation of the eigenvalues for various (x,z) positions.

In practical calculation some care is required because, although in this method a good initial guess for the eigenvalues is required to give a converged value, it is rather difficult to give such a value. Thus we start the calculation from the basic flow whose eigenvalue is already known, for example the flat plate flow, and then vary the flow gradually to the target flow with gradual change of eigenvalues.

2.2.2 computation of eigenvalues for general point

For the basic flow at a general point on a supersonic wing surface $\alpha_r, \alpha_i, \beta_r$ and β_i in equation (13) and (17) are solved for a given frequency ω and Reynolds number R by Newton's method. The solution procedure is almost the same as that for Zarf except that the corrections are obtained in the following manner. First expanding equation (13) about some initial guess of $\alpha_r, \alpha_i, \beta_r, \beta_i$ we get

$$\phi_{1r} + \frac{\partial \phi_{1r}}{\partial \alpha_r}\delta\alpha_r + \frac{\partial \phi_{1r}}{\partial \alpha_i}\delta\alpha_i + \frac{\partial \phi_{1r}}{\partial \beta_r}\delta\beta_r + \frac{\partial \phi_{1r}}{\partial \beta_i}\delta\beta_i = 0$$

$$\phi_{1i} + \frac{\partial \phi_{1i}}{\partial \alpha_r}\delta\alpha_r + \frac{\partial \phi_{1i}}{\partial \alpha_i}\delta\alpha_i + \frac{\partial \phi_{1i}}{\partial \beta_r}\delta\beta_r + \frac{\partial \phi_{1i}}{\partial \beta_i}\delta\beta_i = 0$$

and then from equation (17) and (24),

$$e_r + \frac{\partial e_r}{\partial \alpha_r}\delta\alpha_r + \frac{\partial e_r}{\partial \alpha_i}\delta\alpha_i + \frac{\partial e_r}{\partial \beta_r}\delta\beta_r + \frac{\partial e_r}{\partial \beta_i}\delta\beta_i = -z/x = \gamma$$

$$e_i + \frac{\partial e_i}{\partial \alpha_r}\delta\alpha_r + \frac{\partial e_i}{\partial \alpha_i}\delta\alpha_i + \frac{\partial e_i}{\partial \beta_r}\delta\beta_r + \frac{\partial e_i}{\partial \beta_i}\delta\beta_i = 0$$

We can now solve these four equations for the corrections $\delta\alpha_r, \delta\alpha_i, \delta\beta_r, \delta\beta_i$ to the initial guess values.

At any downstream position x the initial guess may be given by the values of the previous calculation step $x - \Delta x$.

2.2.3 Prediction of the transition point

In three-dimensional flow, the N factor is calculated from

$$N = -\int_{x_o}^{x} \Gamma \, dx \tag{31}$$

where

$$\Gamma = \alpha_i - \beta_i \tau . \tag{32}$$

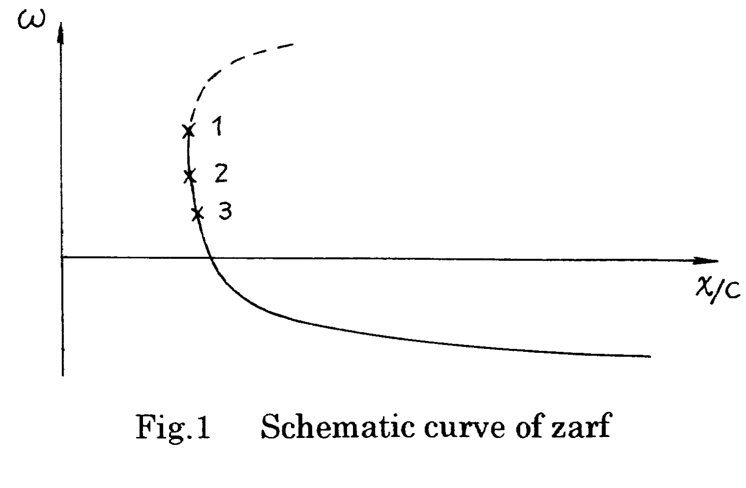

Fig.1 Schematic curve of zarf

The detailed procedure of the transition point is as follows:

① First calculate the neutral stability curve (zarf) for some x positions (figure 1)

② Start the calculation from, for example, position 1 in figure 1, x_1/c . At the next x/c step solve the eigenvalue problem with the frequency $\omega = \omega_1$ at x_1/c and the relation

$$\partial\alpha/\partial\beta = \left(\partial\alpha/\partial\beta\right)_{x=x_1} = -\tau^{(1)}$$

to get α and β which give Γ from equation (32). Repeat the eigenvalue problem for various $\tau^{(1)}$ and find $\tau = \tau_{min}$ for which $\Gamma = \Gamma_{min}$. Then proceed to the next x/c step.

③ The values of $\Gamma = \Gamma_{min}$ which corresponds to $\tau = \tau_{min}$ at each x/c step give the N factor for the frequency ω_1 from equation (31).

④ Repeat steps ② and ③ for other frequencies $\omega_2, \omega_3, \cdots$. (we do not consider a negative ω because its growth rate is smaller than that of a positive ω .)

⑤ Finally we can obtain a family of N factor curves as is shown in figure 2. If we choose nine as the value of N as a criterion of the transition, the transition point is given by $(x/c)_{tr}$ in the figure.

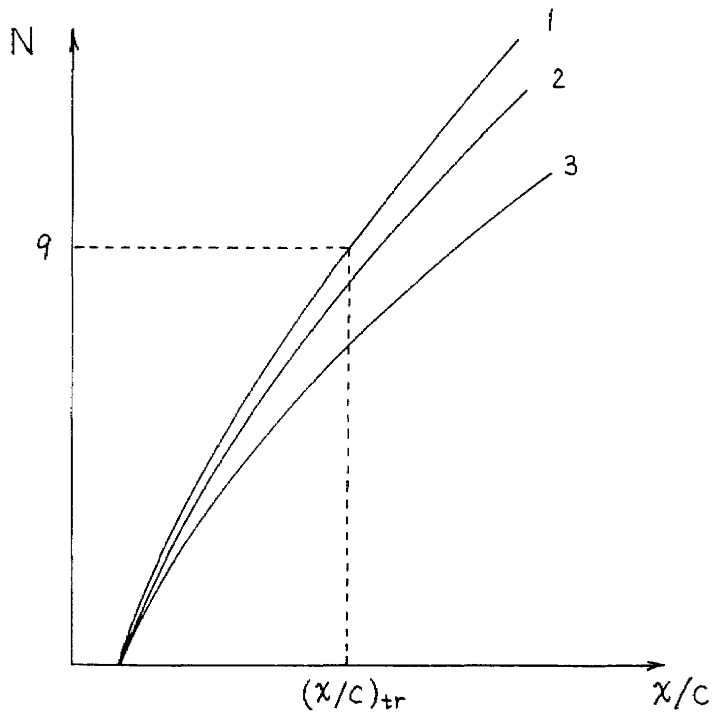

Fig.2 prediction of transition point by e^N method

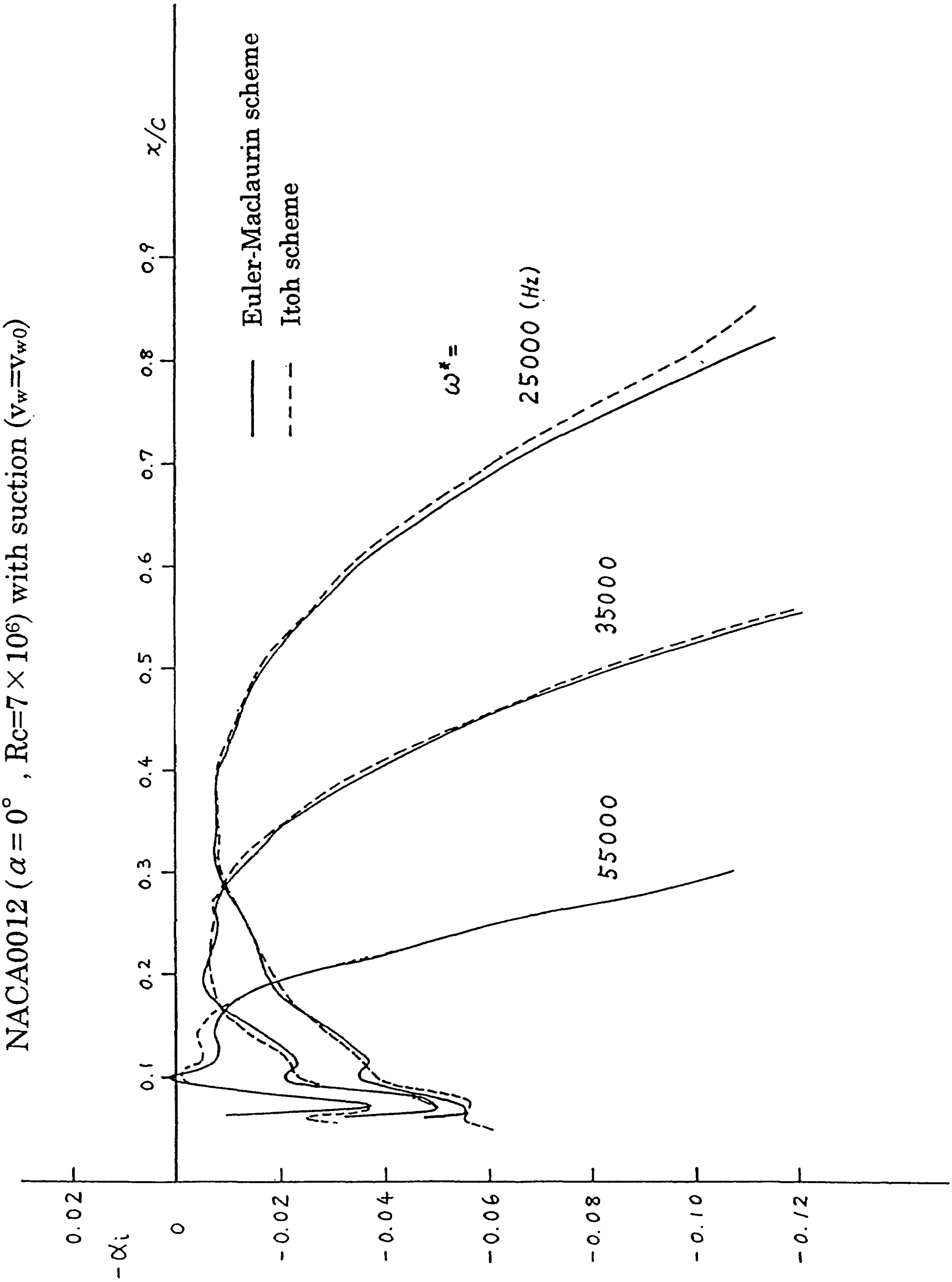

Fig.3 Accuracy of the Euler-Maclaurin scheme

2.3 Result

We have already finished the mathematical formulation of the linear stability calculation and now are developing a program code. Final results will be published in the near future. However as an incompressible version of the method is now available, we have executed a spatial linear stability calculation for the incompressible laminar boundary layer on the NACA0012 airfoil with suction ($\alpha = 0^\circ$, $R=7\times10^6$) to check the accuracy of the Euler-Maclaurin scheme. The result in figure 3 shows a comparison of amplification rate $-\alpha_i$ between the Itoh (ref.9) and E-M schemes. The agreement is good and the E-M scheme has a good degree of accuracy.

3. Boundary layer calculation

As was stated in the introduction, we have used the same algorithm as that of the linear stability theory stated in §2 for the solution of the boundary layer equation. Thus only the outline of the solution procedure will be described. Following Iyer[10] the 3-D compressible boundary layer equation written in the body-fitted non-orthogonal curvilinear coordinate system is transformed into a system of partially differential equation:

continuity equation;

$$W_\varsigma = A_1 F_\xi + A_2 F + A_3 G_\eta + A_4 G \tag{33}$$

ξ - momentum equation;

$$(lL - WF)_\varsigma = B_1 (F^2)_\xi + B_2 (FG)_\eta + B_3 F^2 + B_4 FG + B_5 G^2 + B_6 \theta \tag{34}$$

η - momentum equation;

$$(lM - WG)_\varsigma = C_1 (FG)_\xi + C_2 (G^2)_\eta + C_3 F^2 + C_4 FG + C_5 G^2 + C_6 \theta \tag{35}$$

energy equation;

$$(l/P_r - WH)_\varsigma = D_1 (FH)_\xi + D_2 (GH)_\eta + D_3 FH + D_4 GH + D_5 \tag{36}$$

where $F = u/u_e, G = v/v_r$ and

$$F_\varsigma = L \tag{37}$$

$$G_\varsigma = M \tag{38}$$

$$H_\varsigma = I \tag{39}$$

(for the nomenclature see original paper). Equations (33)~(39) can be written in the vector form as

$$Q_\varsigma = R \tag{40}$$

where

$$Q = (Q_i) = (W, lL - WF, lM - WG, l_p I - WH, F, G, H) \tag{41}$$

Equation (40) has the identical form as equation (10) and thus the same solution algorithm can be used. In real calculation, equation (40) is further transformed into the equation of the solution vector S_i

$$S_\zeta = R^* \tag{42}$$

where

$$S = (S_i) = (W, F, G, H, L, M, T) \tag{43}$$

and then is solved for S_i. Of course there are some different points between the system of equation (10) and of equation (40) or (42). The first difference is that the latter is a nonlinear system. The second is that the right hand side of equations (33)~(36) i.e. (42) contain ξ and η derivatives and thus the former term is approximated by a finite difference scheme with second order accuracy and the latter by a second order or zigzag scheme respectively, depending on the sign of the cross flow velocity component. The third is that the boundary condition is different, it is homogeneous and non-homogeneous in the stability and boundary layer calculations respectively. However it is only an apparent difference, because the stability calculation also uses a non-homogeneous boundary condition as a technique for getting a nontrivial solution. In this way the system of equation (42) can now be solved with the same algorithm as equation (10) except that as the former is nonlinear about the solution vector it must be solved iteratively by using Newton's method, the calculation in each iteration being, of course, the same.

4. Conclusion

In this report we propose an efficient numerical calculation method of the transition location of the boundary layer over a supersonic wing which consists of the boundary layer, its linear stability and N factor calculations. We have used the same solution algorithm for both boundary layer and linear stability calculations to save computing time and make the calculation system as simple as possible.

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