Application of the Statistical Design Support System toward Vehicle Safety Design

by

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ABSTRACT

Recently, CAE technology has been widely applied to the crash safety design of vehicles, and made it possible for complicated non-linear problems such as a collision phenomenon to be simulated at high precision. However, the designers need not only to get the evaluation characteristics value, but also to improve it in a fixed development period. Therefore, an efficient design support system becomes indispensable to clear such critical requirements. The authors have proposed the Statistical Design Support System (SDSS). The effectivity analysis, which is the first step of the SDSS, combining the structural analysis and the design of experiments, is carried out to generate approximate evaluation functions called estimation expressions. And the correlation between the evaluation characteristic and design variables can be given in several simple estimation expressions. All the other steps such as optimization after that can be done based on the estimation expression. Therefore, the SDSS is one of the available optimal approaches for non-linear and dynamic phenomena such as vehicle collision. In this study, the authors applied the SDSS for the optimization of reinforced members considering occupant’s injuries. It is concluded that the SDSS is greatly useful for the realization of the multi-objective optimization. The reinforced members around the cabin frames are designed in order to secure the survival space in the case of the offset collision, and also designed considering occupant’s injury criteria in the case of full-lap collision. This design must be dealt as multi-objective optimization with considering the decrease of injury criteria due to full-lap collision and deformations due to offset collision. The authors applied the SDSS to such a design. It was found that the SDSS could satisfactorily be used for crashworthiness as a practical multi-objective optimization design tool, and that the design cycle could be significantly shortened using the SDSS design as well.

1. INTRODUCTION

Recently, CAE tools have made it possible for complicated nonlinear phenomena such as a collision to be simulated at high precision, and have contributed to reducing the development period required to finish a new design. Generally, existing CAE tools have been developed to focus on the simulation of the mechanical behavior of the prototype products, but they are difficult to use when learning how to improve the objective functions of the design. However, the engineering designers are being required not only to finish a new design in a strict restriction of the development period, but also to optimize the objective functions within this limited period. As a result, the efficient design support systems or optimum design systems become indispensable to clear such critical requirements, and nevertheless, the mathematic optimization techniques are of no service yet for the complicated collision. Furthermore, the optimum design of a complicated system product such as a vehicle, should be carried out not only for the weight and safety, but also for the production costs and so on. Therefore, an optimization design system, which can service the design of multi-objective and multi-disciplinary, needs to be developed. A Statistical Design Support System (SDSS) had been proposed to optimize the nonlinear design, and it was shown that this system could be applied for various design.

In this study, the authors applied the SDSS for optimization of reinforced members for the crash safety design of a vehicle. Furthermore, they concluded that SDSS is greatly hoping for the realization of the multi-objective optimization method which deals with more than one-purpose functions at the same time as in the

\begin{itemize}
  \item Sensitivity Analysis
  \item Reliability Evaluation
  \item Reanalysis
  \item Evaluation of Dispersion
  \item Effectiveness Analysis
  \item Optimization
  \item Robustness Analysis
\end{itemize}

Fig. 1 Overview of Statistical Design Support System

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2. MULTI-OBJECTIVE OPTIMUM DESIGN AND STATISTICAL DESIGN SUPPORT SYSTEM

2.1 STATISTICAL DESIGN SUPPORT SYSTEM

As shown in Fig.1, the SDSS is composed of seven functions; the effectiveness analysis, reanalysis, sensitivity analysis, evaluation of dispersion, robustness analysis, optimization analysis, and reliability evaluation. The basic concept of this system is the effectiveness analysis, where the effects of the design factors on the objective functions are evaluated quantitatively by combining the structural analyses and the design of experiments (orthogonal array designs). Based upon the analysis of variance for the results of the structural analysis the correlation between the objective functions and design variables can be given in some simple estimation expressions.

All of the subsequent treatments such as optimization are performed based on the estimation expressions. Because the structural analyses are separated from the other functions in this system, the optimization of the nonlinear phenomenon can be carried out within any commercial CAE environment. (Please refer to (1) and (2) for further details.)

2.2 MULTI-OBJECTIVE OPTIMUM DESIGN

Whether there was more than one objective function, only one was made an objective function from those, and the others of the objective functions have had to be handled as the constraints, until now. However, as an actual design, it is naturally thought to optimize more than one objective function at the same time. Such a situation is referred to a multi-objective optimization.

Generally, in the case of complex system design, the objective functions of plural to compete with each other exist, so the answer which makes all objective functions the smallest seldom exists with this multi-objective optimization. Therefore, one must do this at the cost of others as a next best thing to improve some certain objective any further. The answers which reduced the objective functions to the lowest levels are referred to as the pareto answers. Generally, the pareto answers are a set, and it is, therefore, necessary to choose one answer from these pareto answers by introducing a new measure criterion. In actual fact, there are many cases in which the criterion of the decision made by the designer is taken as this new measure.

In other words, due to how the information on the measure criterion of the decision made by the designer is adopted, several approaches become possible for the multi-objective optimization. The multi-objective optimization can be expressed in the following form:

Objective function: \( F = w_1f_1(x) + w_2f_2(x) + \cdots + w_nf_n(x) \) → minimum (2)

Where; \( w_1 + w_2 + \cdots + w_n = 1 \)

Constraints: \( g_1(x) \geq 0 \) \( (i = 1, 2, \cdots, k) \)
\( h_1(x) = 0 \) \( (j = 1, 2, \cdots, m) \)

A new objective function \( F \) is generated as shown in expression (2), where \( w_i \) is the weight to the \( i \)-th objective function \( f_i(x) \). Therefore, the multi-objective optimization (expression (1)) can be simplified as a single-objective optimization defined by the expression (2).

Multiple Grade Optimum Design Approach

There are \( l \) objective functions, \( F_1, F_2, \cdots, F_l \) and each of them has to be designed below a required level at least, and the required level vector and constraint vector of the multi-objective optimization are expressed as:

Required level vector \( F^r = (F_1^r, F_2^r, \cdots, F_l^r) \)

Constraint vector \( f^r = (f_1^r, f_2^r, \cdots, f_n^r) \)

When the order of the priority of each objective function is clear, the multi-objective optimization can be solved by the multiple grade optimum design approach with a 1-step analysis. If the order from high priority is \( F_1, F_2, \cdots, F_l \) at first, only \( F_1 \), the function with highest priority, is dealt with as the objective function, and the other functions \( F_2, \cdots, F_l \) are handled as the constraints. Then, the first step of the optimization analyses can be given as the following form;

Objective function: \( F_1(x) \) → minimum \( (F_1^r) \)

Constraints: \( F_1(x) \leq F_1^r \) \( (i = 2, 3, \cdots, l) \)
\( f_1(x) \leq f_1^r \) \( (i = 1, 2, \cdots, m) \) (3-1)

where \( F_1^r \) is the optimized result of \( F_1 \). In the second step, \( F_2 \) becomes the only one objective function, and the problem is expressed as;

Objective function: \( F_2(x) \) → minimum \( (F_2^r) \)

Constraints: \( F_2(x) \leq F_1^r + \Delta_1 \) \( (0 \leq \Delta_1 \leq F_1^r - F_1^r) \)
\( F_2(x) \leq F_2^r \) \( (i = 3, 4, \cdots, l) \)
\( f_2(x) \leq f_2^r \) \( (i = 1, 2, \cdots, m) \) (3-2)

where \( \Delta_1 \) is an allowable deviation for the objective function \( F_1 \). Similar increment analyses are then carried out for the other functions \( F_2, \cdots, F_l \), and the problem can be given as;

Objective function: \( F_p(x) \) → minimum \( (F_p^r) \)

Constraints: \( F_p(x) \leq F_p^r + \Delta_p \) \( (0 \leq \Delta_p \leq F_p^r - F_p^r) \)
\( F_p(x) \leq F_p^r \) \( (i = p+1, p+2, \cdots, l) \) (3-3)
\[ f(x) \leq f_i^* \quad (i=1,2,\ldots,m) \quad (3-3) \]

### 2.3 MULTI-OBJECTIVE OPTIMUM DESIGN IN STATISTICAL DESIGN SUPPORT SYSTEM

Since all optimum analyses are made with the estimation expressions in the SDSS, the multi-objective optimizations defined as expressions (2) and (3) can be carried out by SDSS easily.

In this study, the weighting method and multiple grade optimum design approach were adopted into the SDSS to optimize the reinforced members of a vehicle. In the SDSS, because all of the objective functions and the constraints are given in simple polynomial expressions and the repetition calculation can be carried out very quickly, the non-efficiency of the multiple grade optimum design approach can be covered easily. If no result exists for the optimizations defined as expressions (2) or (3), the recalculation can be made easily, after loosening the constraints. The Sequential Quadratic Programming is used in the SDSS to conduct the optimization calculation.

### 3. OPTIMIZATION IN THE CRASH SAFETY DESIGN OF VEHICLES

#### 3.1 ANALYTICAL MODEL OF OFFSET CRASH

Here, the behavior of an offset collision was studied, the thickness of 11 reinforced members was chosen as the design variables, and the weight and deformations of the front and cabin were evaluated to be the objective functions.

The model of the full-scale collision introduced to the public by the United States Road Traffic Safety Office (NHTSA) was partly improved and used as shown in Fig. 2. The initial speed of the vehicle is 56 km/h, before it crashes into the deformable barrier. The total element number is about 37,000, and the nodal number is about 51,200. The dynamic simulation software PAM-CRASH on the market was used for the structural analyses. The computing time of each run was about 40 hours, when SGI D-200 was used as the hardware.

#### 3.2 EFFECTIVITY ANALYSIS

As shown in Fig. 3, the thickness of 11 reinforced members around the front and the cabin was chosen as the design variables which significantly lessen the extent of the crew's injuries upon collision. An 18-run analysis was carried out to study the effectivity of the 11 design variables on the objective functions, where each run was defined by an orthogonal array L_{18}(2^2 \times 3^2)^10. The level values of all design factors were set as table 1, where the level values of each design variable were normalized by their upper limit value respectively. The initial levels of all variables were the third levels. The intrusion of Toe-Board (Y_{Toe-Board}) and the deformation of the A-C pillar space (Y_{A-C-pillar}) shown in Fig. 4 were evaluated as the objective functions, besides the weight.

![Fig.2 FEM model of offset crash](image)

![Fig.3 Analytical model and design variables](image)

![Fig.4 Analytical results of offset model](image)

<table>
<thead>
<tr>
<th>Table 1: Levels of design variables</th>
</tr>
</thead>
<tbody>
<tr>
<td>All Factor</td>
</tr>
<tr>
<td>0.5</td>
</tr>
</tbody>
</table>

Based upon 18 results of \( Y_{Toe-Board} \), \( Y_{A-C-pillar} \) and the weight, the analysis of variance was conducted to generate the estimation expressions for these objective functions. Here, the estimation expressions for \( Y_{Toe-Board} \), \( Y_{A-C-pillar} \), and the weight were given by Chebyshev's orthogonal functions, as shown as equations (4), (5) and (6).
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3.3 MULTI-OBJECTIVE OPTIMIZATION

Problem Description

The multi-objective optimization discussed in this study was given in the following form;

Objective function : \( \{ Y_{\text{weight}}, Y_{\text{Toc-Board}}, Y_{\text{A-C pillar}} \} \)

Constraints : \( 0.5 \leq X_i \leq 1 \quad (i=1,2,\cdots,6) \)

The weighting method and multiple grade optimum design approach were applied in SDSS for the above optimization. Here, only the results of the multiple grade optimum design approach were discussed.

Results of Multiple Grade Optimum Design Approach

As for the order of priority of the objective functions, No. 1 was the weight, and No. 2 was the deformation of the A-C pillar. The intrusion of the Toc-Board was made the third. The required level for each objective function was; \( Y_{\text{weight}}=31.63 \text{kg}, \quad Y_{\text{A-C pillar}}=0.0350 \text{m}, \quad Y_{\text{Toc-Board}}=0.10 \text{m} \), respectively. Therefore, the first step of this analysis was given in the following form;

The first step

Objective function : \( Y_{\text{weight}} \)

Constraints : \( Y_{\text{A-C pillar}} \leq 0.0350, \quad Y_{\text{Toc-Board}} \leq 0.10, \quad 0.5 \leq X_i \leq 1 \quad (i=1,2,\cdots,6) \)

The optimization calculation was carried out by SQP, and the optimized results \( Y^* \) are shown in Table 2. For the next step, the allowable deviation \( \Delta \) was defined as \( \Delta = |Y^*-Y^'|/2 \). And then the second step was given as;

The second step

Objective function : \( Y_{\text{A-C pillar}} \) \quad minimum

Constraints : \( Y_{\text{Toc-Board}} \leq 0.10, \quad Y_{\text{weight}} \leq 29.57+1.03, \quad 0.5 \leq X_i \leq 1 \quad (i=1,2,\cdots,6) \)

Finally, the third step is given as;

The third step

Objective function : \( Y_{\text{Toc-Board}} \) \quad minimum

Constraints : \( Y_{\text{weight}} \leq 29.57+1.03, \quad Y_{\text{A-C pillar}} \leq 0.0310+0.0021, \quad 0.5 \leq X_i \leq 1 \quad (i=1,2,\cdots,6) \)

As a result, the answers given from the third step are the final answers of the multi-objective optimization.

\[
Y_{\text{weight}} = 3.315 + 9.626 X_1 + 4.481 X_1^2 + 7.500 X_2 + 2.241 X_2^2 \\
+ 12.26 X_3 + 2.241 X_3^2 + 4.212 X_4 + 4.483 X_4^2 - 0.949 X_5 + 4.483 X_5^2 \\
+ 11.97 X_6 + 2.242 X_6^2 \quad [\text{kg}] \quad (6)
\]

\[
Y_{\text{Toc-Board}} = |\Delta|_{\text{Toc-Board}} = 0.2436 + 0.00883 X_1 - 0.0259 X_1^2 \\
- 0.0964 X_1^2 - 0.0512 X_1 X_2 - 0.0613 X_2 - 0.0882 X_3 + 0.0399 X_4^2 \\
- 0.0198 X_5 + 0.0363 X_6 - 0.0265 X_6^2 \quad [\text{m}] \quad (4)
\]

\[
Y_{\text{A-C pillar}} = |\Delta|_{\text{A-C pillar}} = 0.207 - 0.0866 X_1 + 0.0415 X_1^2 \\
- 0.0199 X_1 + 0.0725 X_2 + 0.0243 X_2^2 + 0.0284 X_3 + 0.0276 X_4^2 \\
- 0.162 X_5 + 0.0968 X_6 - 0.0372 X_6 + 0.019 X_6^2 \quad [\text{m}] \quad (5)
\]

Table 2 Optimal solution

<table>
<thead>
<tr>
<th></th>
<th>Weight</th>
<th>A-C pillar</th>
<th>Toc-Board</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial</td>
<td>46.04kg</td>
<td>0.0226m</td>
<td>0.0720m</td>
</tr>
<tr>
<td>First step</td>
<td>29.58kg</td>
<td>0.0350m</td>
<td>0.0997m</td>
</tr>
<tr>
<td>Second step</td>
<td>30.60kg</td>
<td>0.0310m</td>
<td>0.0927m</td>
</tr>
<tr>
<td>Third step</td>
<td>30.60kg</td>
<td>0.0330m</td>
<td>0.0870m</td>
</tr>
</tbody>
</table>

Table 3 Optimized design variables

<table>
<thead>
<tr>
<th></th>
<th>X_1</th>
<th>X_2</th>
<th>X_3</th>
<th>X_4</th>
<th>X_5</th>
<th>X_6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
</tr>
<tr>
<td>Optimum</td>
<td>0.50</td>
<td>0.80</td>
<td>0.96</td>
<td>0.97</td>
<td>0.70</td>
<td>0.50</td>
</tr>
</tbody>
</table>

Fig.5 Optimization step vs normalized objective function
Table 4 Reanalysis solution

<table>
<thead>
<tr>
<th></th>
<th>Weight</th>
<th>A-C pillar</th>
<th>Toe-Board</th>
</tr>
</thead>
<tbody>
<tr>
<td>Optimum</td>
<td>30.60</td>
<td>0.0330</td>
<td>0.0870</td>
</tr>
<tr>
<td>Reanalysis</td>
<td>30.40</td>
<td>0.0348</td>
<td>0.0764</td>
</tr>
</tbody>
</table>

3.4 MULTI-OBJECTIVE OPTIMUM DESIGN FOR CABIN DEFORMATION (OFFSET) AND ACCELERATION (FULL LAP)

Since the design variables, which were discussed in the optimization of cabin deformation, also offset the behavior of collision acceleration, it is considered that the optimization of the acceleration should also be examined in addition to the cabin deformation. Here, the approach to conduct the simultaneous optimization (multi-objective optimization) for the cabin deformation and the acceleration was discussed using the above offset model and a full-lap collision model.

3.5 ANALYTICAL MODEL OF FULL-LAP CRASH

The model of the full-scale collision introduced to the public by the National Highway Traffic Safety Agency (NHTSA) was partly improved and used as shown in Fig.6. The initial speed of the vehicle is 56 km/h, the same as that of offset collision simulation. The total element number is about 22,700, and the nodal number is about 24,000. The computing time of each run was about 20 hours, when SGI D-200 was used as the hardware.

3.6 EFFECTIVITY ANALYSIS

Here, \( X_1 \sim X_9 \) as shown in Fig.3, and Table 3, were chosen as the design variables, too. An 18-run analysis was carried out to study the effectivity of the 11 design variables on the objective functions, where values of the variables for each run were defined by an orthogonal array \( L_{18} \). As shown in Fig.7, the integrated value \( Y_{\text{Acceleration}} \) of the acceleration history under the floor was chosen as the objective function, and the intrusion of the Toe-Board \( Y_{\text{Toe-Board}} \) and the deformation of the A-C pillar \( Y_{A-C \text{ Pillar}} \) were also similarly evaluated as the objective functions, besides the weight.

Based upon 18 results of \( Y_{\text{Acceleration}} \), the analysis of variance was done to generate the estimation expression (response surface) between the objective functions \( Y_{\text{Acceleration}} \) and design variables. \( X_1, X_2 \sim X_9 \) as,

\[
Y_{\text{Acceleration}} = -13.71 - 15.4 X_1 + 6.67 X_2^2 + 19.43 X_3 - 13.73 X_4^2 + 25.067 X_5 - 18.13 X_6 X_1 + 31.93 X_6 X_7 X_5 + 20.533 X_6 X_8 - 3.133 X_6 + 17.53 X_8 - 8.533 X_9^2
\]

By using the characteristic value of this acceleration \( Y_{\text{Acceleration}} \) with \( Y_{\text{Toe-Board}}, Y_{A-C \text{ Pillar}}, \) and \( Y_{\text{weight}} \), the multi-objective optimization was carried out.

3.7 MULTI-OBJECTIVE OPTIMIZATION

As for the order of priority of the objective functions, No. 1 was the weight, and No. 2 was the deformation of the A-C pillar. No. 3 was the acceleration, and the intrusion of the Toe-Board was made the fourth. The required level for each objective function was; \( Y_{\text{weight}} = 31.63 \text{kg}, Y_{A-C \text{ Pillar}} = 0.0350 \text{m}, Y_{\text{Acceleration}} = 12, Y_{\text{Toe-Board}} = 0.10 \text{m}, \) respectively. Therefore, the first step of the multi-objective optimization analysis was given in the following form;

The first step

Objective function : \( Y_{\text{weight}} \rightarrow \text{minimum} \)

Constraints : \( Y_{A-C \text{ Pillar}} \leq 0.0350, Y_{\text{Toe-Board}} \leq 0.10, \)

\[
Y_{\text{Acceleration}} \leq 12.0
\]

\[
0.5 \leq X_i \leq 1 \quad (i=1,2,\cdots,6)
\]

The optimization calculation was carried out by SQP, and the optimized results \( Y^* \) are shown in Table 5. For the next step, the allowable deviation \( \Delta \) was defined as \( \Delta_{\text{weight}} = Y_i^{\text{weight}} Y_i^{\text{weight}} \times 0.3 \). And then the second step was given as;

The second step

Objective function : \( Y_{A-C \text{ Pillar}} \rightarrow \text{minimum} \)
Constraints: 

\[ Y_{\text{Acceleration}} \leq 12.0, \quad Y_{\text{Toe-Board}} \leq 0.10, \]
\[ Y_{\text{Weight}} \leq 30.58 + 0.32, \]
\[ 0.5 \leq X_i \leq 1 \quad (i=1,2,\ldots,6) \]

Similarly, the next step, the allowable deviation \( \Delta \) was defined as 

\[ \Delta_{A-C \text{ pillar}} = |Y_{\text{A-C pillar}} - Y'_{\text{A-C pillar}}| \times 0.5. \]

And then the third step was given as:

The third step

Objective function: \( Y_{\text{Acceleration}} \rightarrow \text{minimum} \)

Constraints:

\[ Y_{\text{Toe-Board}} \leq 0.10, \quad Y_{\text{Weight}} \leq 30.58 + 0.32, \]
\[ Y_{\text{A-C pillar}} \leq 0.0333 + 0.0009, \]
\[ 0.5 \leq X_i \leq 1 \quad (i=1,2,\ldots,6) \]

For the last step, the allowable deviation \( \Delta \) was defined as 

\[ \Delta_{\text{acceleration}} = |Y_{\text{acceleration}} - Y'_{\text{acceleration}}| \times 0.3. \]

And then the fourth step was given as:

The fourth step

Objective function: \( Y_{\text{Toe-Board}} \rightarrow \text{minimum} \)

Constraints:

\[ Y_{\text{acceleration}} \leq 11.81 + 0.06, \]
\[ Y_{\text{Weight}} \leq 30.58 + 0.32, \]
\[ Y_{\text{A-C pillar}} \leq 0.0333 + 0.0009, \]
\[ 0.5 \leq X_i \leq 1 \quad (i=1,2,\ldots,6) \]

The solution obtained by the fourth step is the final multi-objective optimization solution, and as shown in Fig. 8, the objective functions were optimized by that each step. In the event that, every objective function was normalized by its required level value. That is to say, the required level is being satisfied, if it is over the line of 1. The finally optimized solution shows that all objective functions satisfy similarly required level.

The re-analysis was carried out to verify the optimum solution.

<table>
<thead>
<tr>
<th>Table 5 Optimal solution</th>
</tr>
</thead>
<tbody>
<tr>
<td>Weight</td>
</tr>
<tr>
<td>Initial</td>
</tr>
<tr>
<td>First step</td>
</tr>
<tr>
<td>Second step</td>
</tr>
<tr>
<td>Third step</td>
</tr>
<tr>
<td>Forth step</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Table 6 Optimal parameter</th>
</tr>
</thead>
<tbody>
<tr>
<td>( X_i )</td>
</tr>
<tr>
<td>Initial</td>
</tr>
<tr>
<td>Optimum</td>
</tr>
</tbody>
</table>

The error of weight for the optimum solution was 1% less than the result of the re-analysis. The error of the acceleration was 5.6%, and the error of the Toe-Board was over 30%. Though the intrusion of the Toe-Board is in the constraints, it is necessary to quantitatively analyze for the error. In the future, it will become important that the optimum solution be analyzed taking such an error into consideration.

Since using the estimation expressions for the objective functions were used, there was only a short calculation time required on the optimum calculation.

4. CONCLUSIONS

The multi-objective optimization approaches were proposed in the Statistical Design Support System (SDSS). It was shown that both the weighting method and multiple grade optimum design approach could be adopted into the SDSS. The proposed approaches were applied for the multi-objective optimization design of reinforced members for the crash safety of a vehicle. It was shown that for the weight, crash deformation and collision acceleration could be carried out very easily by the proposed approach. It may be concluded that this system can be used as a practical design tool for the multi-objective optimization design of complex and nonlinear systems.

REFERENCES