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Goodness-of-Fit Tests for the Type-I Extreme-Value and Two-Parameter Weibull Distributions with Unknown Parameters Estimated by Graphical Plotting Techniques —— Part 1 : Critical Values

Toshiyuki Shimokawa and Min Liao

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CHŌFU, TOKYO, JAPAN

Goodness-of-Fit Tests for the Type-I Extreme-Value and Two-Parameter Weibull Distributions with Unknown Parameters Estimated by Graphical Plotting Techniques —— Part 1: Critical Values*

Toshiyuki Shimokawa*1 and Min Liao*2

ABSTRACT

The objective of this study is to determine the critical values of the Cramer-von Mises(C-M) and Anderson-Darling(A-D) statistics for goodness-of-fit tests for the Type-I extreme-value and two-parameter Weibull distributions when the population parameters are estimated from a complete sample by graphical plotting techniques. Three kinds of graphical plotting technique, i.e., the median ranks, mean ranks, and symmetrical sample cumulative distribution (symmetrical ranks), are combined with the least-squares method on extreme-value and Weibull probability paper to estimate the parameters. Monte Carlo simulation is used to calculate the critical values of the C-M and A-D statistics, in which 1,000,000 sets of complete samples are generated ten times for each sample size of 3(1)20, 25(5)50, and 60(10)100. The critical values are discussed and tabulated for practical use.

Keywords : goodness-of-fit tests, critical values, Cramer-von Mises, Anderson-Darling, Type-I extreme-value distribution, Weibull distribution, graphical plotting techniques, Monte Carlo simulation

概　要

本研究の目的は、完全標本に対し確率紙を利用して母数を推定する場合を対象として、タイプI極値分布と２母数ワイブル分布に対する適合度検定を行うためのクレマー・フォンミーゼスおよびアンダーソン・ダーリング統計量に対する限界値を与えることである。極値確率紙とワイブル確率紙上で母数を推定するために、メジアンランク、平均ランク、および対称ランク（対称試料累積分布）の３種類プロット法を最小２乗法と組合わせる。標本の大きさとしては3(1)20、25(5)50、60(10)100を選び、クレマー・フォンミーゼスおよびアンダーソン・ダーリング統計量に対する限界値を、モンテカルロ・シミュレーションによりそれぞれ1,000,000組の標本を10回ずつ発生させることにより計算する。さらに、得られた限界値について議論し、実用的数表を与える。

1. INTRODUCTION

Visual and intuitive evaluation of observed data is very important and can be easily conducted in today's era of personal computer. First, several kinds of probability paper are displayed on a CRT screen and the data are plotted using graphical plotting techniques (GPTs). Then, the least-squares method (LSM) provides estimates of the population parameters for each distribution model and the best-fit line to the data is drawn. At the same time, the distribution form and the relative goodness-of-fit among candidate distribution models can be visually judged. Moreover, the quantitative goodness-of-fit can be tested on the basis of the estimated parameters. The estimated parameters give a one-sided lower tolerance limit for each distribution

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*1 Dr., Head of Fatigue Strength Laboratory, Structures Division, National Aerospace Laboratory (NAL).
Email : shimoka@nal.go.jp

*2 Dr., STA (Science and Technology Agency) Fellow at NAL (on leave from Northwestern Polytechnical University, Xi'an, P.R. China).

model. The design allowable should be given as the one-sided lower tolerance limit provided by the best-fit distribution model to avoid excessive under- or over-estimation. In this context, a goodness-of-fit test is a key procedure in selecting the distribution model that best fits the data.

In the past, the Chi-square and modified Kolmogorov-Smirnov (K-S) statistics, applicable for the cases of unknown population parameters, were used to test the goodness-of-fit of the observed data to the normality or a two-parameter Weibull distribution. Since the Chi-square test requires a large sample size, this test is not suitable for many practical cases. The modified K-S statistic is effective for small sample sizes. Shimokawa and Kececioglu[1] concluded that the median ranks, as a graphical plotting technique combined with the LSM, are recommended among three kinds of graphical plotting techniques, i.e., the median ranks, the mean ranks, and the symmetrical sample cumulative distribution (symmetrical ranks). Therefore, on the basis of the median ranks, Shimokawa[2] calculated the critical values of the modified K-S goodness-of-fit test for the Type-I extreme-value and two-parameter Weibull distributions using Monte Carlo simulation with 100,000 sets of complete samples for sample sizes, 3(1)20* , 25(5)50, and 60(10)100.

Over the next two decades, however, many authors[3)-10)] reported that the Cramer-von Mises (C-M) and Anderson-Darling (A-D) statistics are more powerful for goodness-of-fit tests than the K-S statistic in most cases. Furthermore, the K-S and Chi-square tests for normality have been replaced by the A-D test in MIL-HDBK-5E[11]. Although for the C-M and A-D goodness-of-fit tests for the Type-I extreme-value and two-parameter Weibull distributions with unknown parameters, there exist some critical values[3)-5)] and formulas[12] based on the maximum likelihood estimators (MLEs) for limited sample sizes or significance levels, they are still insufficient for practical use. Moreover, because the estimated parameters given by the GPTs are different from those given by the MLEs[3], the critical values of a statistic for goodness-of-fit tests by using the GPTs are different from those by using the MLEs. However, there are no critical values of the C-M and A-D statistics for goodness-of-fit tests for the GPTs.

In this paper, in the first part of this study, three kinds of graphical plotting techniques, namely the median ranks, mean ranks, and symmetrical ranks combined with the least-square methods, are used to estimate the population parameters of the Type-I extreme-value and two-parameter Weibull distributions. Monte Carlo simulation provides estimates of the parameters and calculates the critical values of the C-M and A-D goodness-of-fit test statistics, based on 1,000,000 sets of complete samples generated for each sample size of 3(1)20, 25(5)50, and 60(10)100. Furthermore, by performing this Monte Carlo simulation ten times, ten critical values are presented for each sample size and each significance level and the fluctuation of these critical values are evaluated. Finally, each critical value in this study is determined by the mean of the ten critical values and tabulated. In the second part of this study, the results of power investigation for the C-M and A-D statistics for goodness-of-fit tests will be presented in a separate paper[13], and compared with those of corresponding goodness-of-fit tests using the MLEs.

2. CONCEPTS OF THE CRAMER-VON MISES AND ANDERSON-DARLING GOODNESS-OF-FIT TESTS

Let $x_1<x_2<\ldots<x_n$ be order statistics of a sample size n from a population defined by a continuous distribution function $F(x)$. A goodness-of-fit test is to test the null hypothesis,

$$H_0 : F(x)=F_0(x;\theta), \tag{1}$$

where $F_0(x;\theta)$ is a specified family of a model that contains a set of parameters θ. The well-known C-M statistic for goodness-of-fit tests is based on the following equation[3],

$$W_n^2=n\int_{-\infty}^{+\infty}[F_n(x)-F_0(x;\theta)]^2dF_0(x;\theta), \tag{2}$$

where $F_n(x)$ is an empirical distribution function (EDF) for observations $x_i(i=1,2,\ldots,n)$ and a step function with a jump at each of the observed values of order statistics $x_1<x_2<\ldots<x_n$. Anderson and Darling[6),7)] improved the C-M statistic by introducing the weight function given by,

* 3(1)20 means sample sizes from 3 to 20 with interval 1.

$$A_n^2=n\int_{-\infty}^{+\infty}[F_n(x)-F_0(x;\theta)]^2\Psi(x;\theta)dF_0(x;\theta), \tag{3}$$

$$\Psi(x;\theta)=1/[F_0(x;\theta)\{1-F_0(x;\theta)\}], \tag{4}$$

where $\Psi(x;\theta)$ is the weight function and $n\Psi(x;\theta)$ is an inverse of the variance of $F_n(x)$. A_n^2 can detect discrepancies in both tail regions of $F_n(x)$ better than W_n^2 by giving greater weights in both regions[3),4),8),9)]. Usually, the following expressions are introduced for computational convenience[3)],

$$W_n^2=\sum_{i=1}^{n}[F_0(x_i;\theta)-\frac{i-0.5}{n}]^2+\frac{1}{12n}, \tag{5}$$

$$A_n^2=-\sum_{i=1}^{n}\frac{2i-1}{n}[\ln\{F_0(x_i;\theta)\}+\ln\{1-F_0(x_{n+1-i};\theta)\}]-n. \tag{6}$$

Because $F_0(x;\theta)$ depends on unknown parameters, for practical applications $F_0(x;\theta)$ must be modified as $F_0(x;\hat{\theta})$ by using estimates of the parameters. Then, the C-M and A-D statistics, W_n^2 and A_n^2, are changed to the modified C-M and A-D statistics, $\hat{W}_n^2$ and $\hat{A}_n^2$. Although a few special results are available for determining asymptotic distributions of $\hat{W}_n^2$ and $\hat{A}_n^2$, the theoretical distributions of $\hat{W}_n^2$ and $\hat{A}_n^2$ have not yet been obtained analytically[3)]. Generally, Monte Carlo simulation is used to obtain the critical values (or percentage points) of $\hat{W}_n^2$ and $\hat{A}_n^2$ [3)-5),8)-10)].

3. GRAPHICAL PLOTTING TECHNIQUES TO ESTIMATE POPULATION PARAMETERS

3. 1 Distribution Functions and Linear Relationships on Probability Paper

3. 1. 1 Type-I extreme-value distribution

The distribution function of a Type-I extreme-value distribution of the smallests, $F_E(y;a,b)$, is represented by

$$F_E(y;a,b)=1-\exp\{-\exp(\frac{y-a}{b})\}, \tag{7}$$

where a is the location parameter and b the scale parameter. The following linear equation is obtained by taking the logarithm of Equation (7).

$$y=a+b\ln[-\ln\{1-F_E(y;a,b)\}]. \tag{8}$$

This relationship is used for plotting the observed data on extreme-value probability paper.

Let a set of ordered observations be $y_i(i=1,2,...,n)$ and plotting positions be $p_i(i=1,2,...,n)$ given by a plotting method. If the data reasonably fits a Type-I extreme-value distribution, the plotted points (y_i, $\ln[-\ln\{1-p_i\}]$: $i=1,2,...,n$) should be distributed roughly on a straight line on this probability paper as indicated by Equation (8).

3. 1. 2 Two-parameter Weibull distribution

The distribution function of a two-parameter Weibull distribution, $F_W(x;\alpha,\beta)$, is represented by

$$F_W(x;\alpha,\beta)=1-\exp\{-(x/\alpha)^\beta\}, \qquad \alpha>0, \quad \beta>0, \tag{9}$$

where α is the scale parameter and β the shape parameter. Taking the logarithm of Equation (8), the following linear equation is obtained,

$$\ln x=\ln\alpha+\frac{1}{\beta}\ln[-\ln\{1-F_W(x;\alpha,\beta)\}]. \tag{10}$$

This relationship is used for plotting the observed data on Weibull probability paper.

Let $x_i(i=1,2,...,n)$ be a set of ordered observations. Let $y_i=\ln x_i$, $a=\ln\alpha$, and $b=1/\beta$, then Equation (10) agrees with

Equation (8). By these transformations, a two-parameter Weibull distribution is transformed into a Type-I extreme-value distribution. This means that the same statistical and probabilistic analysis methods can be used for both distribution models. Therefore, a Type-I extreme-value distribution is mainly discussed below.

3. 2 Plotting Positions

This study uses the following three plotting methods as graphical plotting techniques. These methods are representative among many plotting methods and are generally used in graphical analysis. Let p_i be the plotting position for the i-th order of a sample of size n.

3. 2. 1 Median ranks[1),2),14)]

p_i = (Population probability corresponding to the median of the distribution of the i-th order statistic), namely,

$$0.5 = i\binom{n}{i}\int_0^{p_i} t^{i-1}(1-t)^{n-i}dt \quad (11)$$

3. 2. 2 Mean ranks[1),2),14)]

$$p_i = \frac{i}{n+1} \quad (12)$$

3. 2. 3 Symmetrical ranks[1),2),14)]

Plotting positions by the symmetrical sample cumulative distribution are simply called by the" symmetrical ranks "in this paper.

$$p_i = \frac{i-0.5}{n} \quad (13)$$

3. 3 Estimators by the Combination of the Least-Squares Method and a Graphical Plotting Method

Let c_i and T be

$$c_i = \ln[-\ln(1-p_i)], \quad (14)$$

$$T = \sum_{i=1}^{n}(y_i - \hat{a} - \hat{b}c_i)^2. \quad (15)$$

The least-squares method provides estimators of the parameters, $\hat{a}$ and $\hat{b}$, by minimizing T,

$$\hat{b} = \{\sum_{i=1}^{n}(y_i-\bar{y})(c_i-\bar{c})\}/\{\sum_{i=1}^{n}(c_i-\bar{c})^2\}, \quad (16)$$

$$\hat{a} = \bar{y} - \hat{b}\bar{c}, \quad (17)$$

where $\bar{c}$ and $\bar{y}$ are the averaged values, i.e., $\bar{c} = \sum_{i=1}^{n}c_i/n$ and $\bar{y} = \sum_{i=1}^{n}y_i/n$.

4. SIMULATION PROCEDURES

4. 1 A Sample Following a Type-I Extreme-Value Distribution

A sample of size n following a Type-I extreme-value distribution is generated by Monte Carlo simulation as follows. First n pseudo-uniform random numbers are generated by the composite generator proposed by Marsaglia and Bray[15)], which has been widely adopted in Monte Carlo simulation[15)]. Let descending ordered pseudo-random numbers be reliabilities, R_{si} (i = 1,2,...,n). Let s_i represent

$$s_i = \ln \ln(1/R_{si}), \tag{18}$$

and s denotes the averaged value, i.e., $\bar{s}=\sum_{i=1}^{n} s_i/n$. Monte Carlo simulation gives ascending ordered observations, $y_i (i=1,2,...,n)$, following a Type-I extreme-value distribution as

$$y_i = a + b \cdot s_i, \tag{19}$$

where a and b are the population parameters to be estimated. Meanwhile, the averaged value $\bar{y}$ of y_i is obtained by

$$\bar{y} = a + b \cdot \bar{s}. \tag{20}$$

4. 2 Discussion on the Estimators of the Population Parameters

Consider the estimators of the population parameters from a complete sample given by Monte Carlo simulation. Substituting Equations (19) and (20) into Equation (16), then

$$\hat{b} = \left\{\sum_{i=1}^{n} b \cdot (s_i - \bar{s})(c_i - \bar{c})\right\} \Big/ \left\{\sum_{i=1}^{n} (c_i - \bar{c})^2\right\} = B \cdot b, \tag{21}$$

where B is

$$B = \left\{\sum_{i=1}^{n} (s_i - \bar{s})(c_i - \bar{c})\right\} \Big/ \left\{\sum_{i=1}^{n} (c_i - \bar{c})^2\right\}. \tag{22}$$

Substituting Equations (20) and (21) into Equation (17), $\hat{a}$ is given as

$$\hat{a} = (a + b \cdot \bar{s}) - Bb\bar{c} = a + A \cdot b, \tag{23}$$

where A is

$$A = \bar{s} - B\bar{c}. \tag{24}$$

Equations (22) and (24) indicate that A and B are independent of the population parameters, a and b, but dependent on the plotting method used. This fact was also indicated by Shimokawa in 1990[2].

4. 3 The Value of the Estimated Distribution Function Corresponding to Each Observed Value Given by Monte Carlo Simulation

On the basis of Equations (21) and (23), the distribution function corresponding to each observed value is obtained as

$$F_E(y_i) = 1 - \exp\left[-\exp\left\{\frac{y_i - (a + bA)}{bB}\right\}\right]$$
$$= 1 - \exp[-\exp\{(s_i - A)/B\}]. \tag{25}$$

This equation indicates that $F_E(y_i)$ is independent of the population parameters a and b. Moreover, $F_E(y_i)$ corresponds to $F_0(x;\theta)$ in Equations (5) and (6). Therefore, W_n^2 and A_n^2 are not dependent on a and b, but dependent on the plotting method as shown by Equations (22) and (24).

Let $x_i = \exp y_i$, $\alpha = \exp \hat{a}$, and $\beta = 1/\hat{b}$, and substitute them into Equation (9). Equation (25) is thus derived and $F_E(y_i) = F_W(x_i)$ is confirmed. Therefore, the critical values of the C-M and A-D statistics for goodness-of-fit tests can be used for both Type-I extreme-value and two-parameter Weibull distributions.

4. 4 Calculation Procedures of Critical Values

A flow diagram for the calculation of critical values by Monte Carlo simulation is presented in Figure 1.

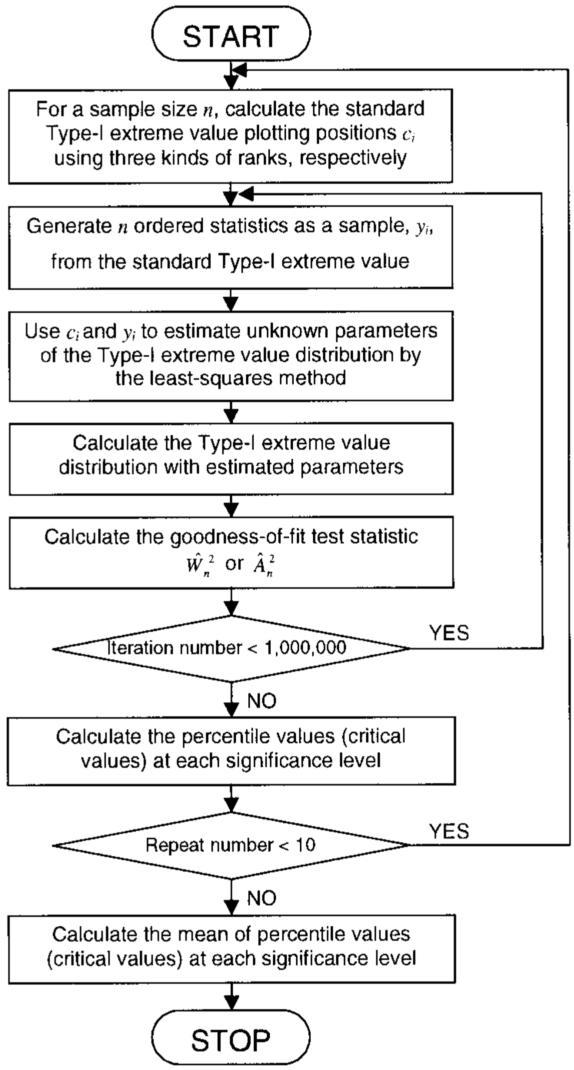

Figure 1 Flow diagram for the calculation of critical values.

5. CRITICAL VALUES FOR THE CRAMER-VON MISES AND ANDERSON-DARLING STATISTICS

5. 1 Sample Sizes of Observations and Percentage Points of the C-M and A-D Statistics

Sample sizes are selected as $n=3(1)20$, $25(5)50$, and $60(10)100$. For each n, Monte Carlo simulation provides the total sets of $M=1{,}000{,}000$ samples by resetting to the same seeds in the random number generator, then the values of W_n^2 and A_n^2 are calculated and decreasingly ordered, respectively. The ascending order is denoted by j. The cumulative probability is defined in this study as

$$P=(j-0.5)/M, \tag{26}$$

by using the concept of the symmetrical ranks. The critical values corresponding to $P\times 100$ percentage points (significance levels) of $W_n^2(P)$ and $A_n^2(P)$, are the values given by the order,

$$j=PM+0.5. \tag{27}$$

For example, $P=0.01$ gives $j=10000.5$, hence, the critical values of W_n^2 are given by

$$W_n^2(0.01)=\{W_n^2(j=10000)+W_n^2(j=10001)\}/2. \quad (28)$$

In this study, the entire calculation procedure by Monte Carlo simulation is repeated ten times, so ten critical values corresponding to $P\times 100$ percentage points for each n are calculated. At last, each critical value of W_n^2 or A_n^2 at each significance level for each n is obtained by computing the mean of the ten critical values. Therefore, 1,000,000×10 samples in total are generated for each sample size.

5. 2 Calculated Results and Discussion

The critical values of W_n^2 and A_n^2 for the three kinds of graphical plotting techniques are listed in Tables 1 to 6. Figures 2 to 7 represent the relationships between the critical values, W_n^2 and A_n^2, and sample size n at different significance levels γ. These tables and figures present the following characteristics.

(1)The critical values of W_n^2 and A_n^2 increase monotonically as n becomes larger and γ lower, except for that of A_n^2 for the symmetrical ranks and $\gamma=0.01$.

(2)All the critical values of W_n^2 and A_n^2 increase quickly for n less than 10. There is only a small change of the critical values of W_n^2 and A_n^2 for n larger than 40.

To select a proper goodness-of-fit test statistic, the power to distinguish the difference between the applied distribution models is very important. The detailed power comparisons of the C-M and A-D statistics using three graphical plotting methods will be presented in the second part of this study[13].

Table 1 Critical values of the Cramer-von Misese, W_n^2, for the median ranks.

Sample Size	Significance Level γ						
n	0.01	0.025	0.05	0.1	0.15	0.2	0.25
3	0.0935	0.0890	0.0833	0.0776	0.0723	0.0673	0.0628
4	0.1429	0.1249	0.1083	0.0889	0.0777	0.0699	0.0639
5	0.1598	0.1338	0.1140	0.0947	0.0835	0.0753	0.0687
6	0.1686	0.1412	0.1203	0.0988	0.0862	0.0774	0.0707
7	0.1755	0.1464	0.1240	0.1018	0.0888	0.0795	0.0724
8	0.1809	0.1503	0.1272	0.1040	0.0905	0.0810	0.0737
9	0.1851	0.1536	0.1297	0.1059	0.0920	0.0823	0.0747
10	0.1888	0.1563	0.1318	0.1074	0.0933	0.0833	0.0756
11	0.1918	0.1586	0.1335	0.1086	0.0943	0.0842	0.0764
12	0.1945	0.1606	0.1350	0.1098	0.0953	0.0850	0.0770
13	0.1967	0.1622	0.1363	0.1108	0.0960	0.0856	0.0776
14	0.1987	0.1638	0.1376	0.1117	0.0967	0.0862	0.0781
15	0.2005	0.1652	0.1387	0.1125	0.0974	0.0868	0.0786
16	0.2021	0.1664	0.1396	0.1132	0.0980	0.0873	0.0790
17	0.2038	0.1675	0.1405	0.1138	0.0985	0.0877	0.0794
18	0.2049	0.1686	0.1414	0.1145	0.0990	0.0881	0.0797
19	0.2063	0.1694	0.1420	0.1149	0.0994	0.0885	0.0801
20	0.2074	0.1704	0.1428	0.1155	0.0998	0.0888	0.0804
25	0.2123	0.1740	0.1454	0.1176	0.1015	0.0903	0.0816
30	0.2159	0.1767	0.1476	0.1191	0.1028	0.0913	0.0825
35	0.2188	0.1787	0.1493	0.1203	0.1037	0.0921	0.0832
40	0.2209	0.1805	0.1505	0.1213	0.1045	0.0928	0.0838
45	0.2230	0.1819	0.1516	0.1221	0.1052	0.0934	0.0843
50	0.2245	0.1831	0.1525	0.1228	0.1058	0.0939	0.0847
60	0.2271	0.1850	0.1540	0.1238	0.1066	0.0946	0.0853
70	0.2292	0.1866	0.1552	0.1248	0.1074	0.0952	0.0859
80	0.2305	0.1878	0.1562	0.1255	0.1080	0.0957	0.0863
90	0.2317	0.1887	0.1569	0.1260	0.1084	0.0961	0.0867
100	0.2331	0.1896	0.1576	0.1266	0.1089	0.0965	0.0869

Table 2 Critical values of the Cramer-von Misese, W_n^2, for the mean ranks, $P_i=i/(n+1)$.

Sample Size	Significance Level γ						
n	0.01	0.025	0.05	0.1	0.15	0.2	0.25
3	0.0841	0.0827	0.0805	0.0762	0.0722	0.0685	0.0651
4	0.1314	0.1179	0.1046	0.0885	0.0782	0.0713	0.0660
5	0.1551	0.1320	0.1128	0.0942	0.0840	0.0766	0.0707
6	0.1663	0.1402	0.1204	0.1000	0.0879	0.0793	0.0728
7	0.1754	0.1473	0.1255	0.1036	0.0909	0.0819	0.0749
8	0.1826	0.1526	0.1296	0.1065	0.0931	0.0837	0.0764
9	0.1885	0.1569	0.1329	0.1089	0.0950	0.0852	0.0776
10	0.1933	0.1605	0.1357	0.1109	0.0966	0.0865	0.0787
11	0.1974	0.1636	0.1380	0.1126	0.0979	0.0875	0.0795
12	0.2009	0.1662	0.1400	0.1140	0.0990	0.0885	0.0803
13	0.2038	0.1683	0.1417	0.1153	0.1000	0.0893	0.0810
14	0.2065	0.1704	0.1433	0.1164	0.1008	0.0900	0.0816
15	0.2087	0.1722	0.1447	0.1174	0.1017	0.0906	0.0821
16	0.2109	0.1737	0.1458	0.1183	0.1024	0.0912	0.0826
17	0.2129	0.1752	0.1469	0.1190	0.1030	0.0917	0.0830
18	0.2145	0.1765	0.1479	0.1198	0.1035	0.0922	0.0834
19	0.2160	0.1776	0.1488	0.1204	0.1040	0.0926	0.0838
20	0.2175	0.1788	0.1496	0.1210	0.1045	0.0930	0.0841
25	0.2235	0.1830	0.1529	0.1234	0.1064	0.0945	0.0854
30	0.2275	0.1862	0.1552	0.1251	0.1077	0.0956	0.0863
35	0.2306	0.1885	0.1571	0.1263	0.1087	0.0964	0.0870
40	0.2332	0.1903	0.1585	0.1274	0.1095	0.0971	0.0875
45	0.2354	0.1918	0.1596	0.1281	0.1102	0.0976	0.0880
50	0.2370	0.1930	0.1605	0.1288	0.1107	0.0981	0.0884
60	0.2396	0.1949	0.1618	0.1298	0.1115	0.0987	0.0889
70	0.2418	0.1965	0.1630	0.1306	0.1121	0.0993	0.0894
80	0.2431	0.1976	0.1639	0.1313	0.1127	0.0997	0.0897
90	0.2442	0.1983	0.1646	0.1317	0.1130	0.1000	0.0900
100	0.2455	0.1991	0.1652	0.1321	0.1133	0.1003	0.0902

Table 3 Critical values of the Cramer-von Misese, W_n^2, for the symmetrical ranks, $P_i = (i - 0.5)/n$.

Sample Size n	Significance Level γ 0.01	0.025	0.05	0.1	0.15	0.2	0.25
3	0.1133	0.1079	0.0993	0.0869	0.0802	0.0741	0.0683
4	0.1615	0.1377	0.1175	0.0972	0.0852	0.0764	0.0697
5	0.1718	0.1448	0.1243	0.1030	0.0903	0.0810	0.0737
6	0.1816	0.1519	0.1290	0.1061	0.0927	0.0831	0.0756
7	0.1871	0.1562	0.1325	0.1086	0.0946	0.0846	0.0769
8	0.1918	0.1595	0.1350	0.1105	0.0961	0.0859	0.0780
9	0.1952	0.1622	0.1371	0.1120	0.0973	0.0869	0.0788
10	0.1983	0.1644	0.1388	0.1132	0.0982	0.0877	0.0795
11	0.2009	0.1663	0.1401	0.1142	0.0990	0.0884	0.0801
12	0.2031	0.1679	0.1414	0.1151	0.0998	0.0890	0.0806
13	0.2049	0.1693	0.1424	0.1158	0.1004	0.0895	0.0811
14	0.2065	0.1705	0.1435	0.1166	0.1010	0.0900	0.0815
15	0.2082	0.1717	0.1444	0.1172	0.1015	0.0904	0.0819
16	0.2095	0.1727	0.1451	0.1178	0.1019	0.0908	0.0822
17	0.2109	0.1737	0.1459	0.1183	0.1024	0.0911	0.0825
18	0.2119	0.1745	0.1465	0.1188	0.1028	0.0915	0.0828
19	0.2128	0.1752	0.1470	0.1192	0.1031	0.0917	0.0830
20	0.2138	0.1760	0.1476	0.1196	0.1034	0.0920	0.0832
25	0.2179	0.1789	0.1499	0.1212	0.1047	0.0931	0.0842
30	0.2210	0.1812	0.1515	0.1224	0.1057	0.0939	0.0849
35	0.2232	0.1827	0.1528	0.1234	0.1064	0.0945	0.0854
40	0.2250	0.1842	0.1539	0.1242	0.1071	0.0951	0.0859
45	0.2267	0.1853	0.1547	0.1248	0.1076	0.0955	0.0862
50	0.2280	0.1863	0.1555	0.1254	0.1080	0.0959	0.0865
60	0.2301	0.1879	0.1567	0.1262	0.1087	0.0964	0.0870
70	0.2319	0.1892	0.1577	0.1269	0.1093	0.0969	0.0874
80	0.2329	0.1900	0.1584	0.1275	0.1097	0.0973	0.0877
90	0.2341	0.1909	0.1590	0.1279	0.1101	0.0976	0.0880
100	0.2350	0.1917	0.1596	0.1283	0.1104	0.0979	0.0882

Table 4 Critical values of the Andeson-Darling, A_n^2, for the median ranks.

Sample Size n	Significance Level γ 0.01	0.025	0.05	0.1	0.15	0.2	0.25
3	0.527	0.501	0.460	0.426	0.400	0.376	0.354
4	0.927	0.731	0.612	0.504	0.445	0.405	0.374
5	1.076	0.825	0.682	0.557	0.489	0.442	0.406
6	1.146	0.895	0.733	0.592	0.517	0.466	0.427
7	1.192	0.932	0.766	0.619	0.539	0.484	0.443
8	1.218	0.960	0.791	0.639	0.556	0.498	0.455
9	1.237	0.981	0.811	0.655	0.569	0.510	0.465
10	1.253	0.998	0.826	0.668	0.581	0.520	0.474
11	1.264	1.011	0.839	0.679	0.590	0.528	0.481
12	1.276	1.024	0.850	0.688	0.598	0.536	0.488
13	1.284	1.034	0.860	0.696	0.605	0.541	0.493
14	1.293	1.043	0.869	0.704	0.611	0.547	0.498
15	1.304	1.053	0.877	0.710	0.617	0.552	0.503
16	1.310	1.061	0.884	0.716	0.622	0.556	0.506
17	1.318	1.067	0.890	0.721	0.626	0.560	0.510
18	1.324	1.074	0.895	0.726	0.631	0.564	0.513
19	1.331	1.080	0.901	0.730	0.634	0.567	0.516
20	1.337	1.086	0.906	0.735	0.638	0.571	0.519
25	1.366	1.111	0.927	0.752	0.652	0.583	0.531
30	1.388	1.130	0.944	0.764	0.663	0.593	0.539
35	1.406	1.145	0.957	0.775	0.672	0.600	0.545
40	1.422	1.159	0.968	0.783	0.679	0.606	0.551
45	1.436	1.170	0.977	0.790	0.685	0.611	0.555
50	1.448	1.180	0.985	0.797	0.690	0.616	0.559
60	1.468	1.196	0.997	0.806	0.698	0.623	0.565
70	1.485	1.210	1.008	0.814	0.704	0.628	0.570
80	1.497	1.220	1.016	0.821	0.710	0.633	0.574
90	1.508	1.228	1.023	0.826	0.714	0.637	0.577
100	1.521	1.237	1.030	0.830	0.718	0.640	0.580

Table 5 Critical values of the Andeson-Darling, A_n^2, for the mean ranks, $P_i = i/(n+1)$.

Sample Size n	Significance Level γ 0.01	0.025	0.05	0.1	0.15	0.2	0.25
3	0.454	0.447	0.437	0.418	0.400	0.384	0.369
4	0.695	0.619	0.554	0.480	0.437	0.407	0.384
5	0.846	0.711	0.614	0.523	0.474	0.438	0.411
6	0.931	0.776	0.666	0.561	0.501	0.460	0.428
7	0.993	0.824	0.704	0.589	0.524	0.479	0.444
8	1.038	0.861	0.734	0.612	0.542	0.494	0.457
9	1.073	0.892	0.759	0.631	0.558	0.507	0.468
10	1.104	0.917	0.780	0.647	0.571	0.518	0.477
11	1.130	0.938	0.798	0.661	0.582	0.527	0.485
12	1.153	0.957	0.814	0.673	0.592	0.536	0.492
13	1.172	0.974	0.827	0.683	0.601	0.543	0.498
14	1.191	0.990	0.840	0.693	0.609	0.549	0.504
15	1.207	1.003	0.851	0.702	0.616	0.556	0.509
16	1.222	1.015	0.861	0.710	0.622	0.561	0.514
17	1.236	1.027	0.870	0.716	0.628	0.566	0.518
18	1.249	1.038	0.879	0.723	0.633	0.570	0.522
19	1.262	1.047	0.886	0.729	0.638	0.574	0.525
20	1.273	1.056	0.894	0.734	0.643	0.578	0.529
25	1.321	1.094	0.924	0.757	0.661	0.594	0.542
30	1.357	1.123	0.947	0.774	0.675	0.605	0.552
35	1.387	1.145	0.965	0.787	0.686	0.614	0.559
40	1.412	1.165	0.980	0.798	0.694	0.621	0.566
45	1.432	1.180	0.992	0.807	0.701	0.628	0.571
50	1.450	1.193	1.002	0.815	0.707	0.633	0.575
60	1.479	1.215	1.018	0.826	0.717	0.640	0.582
70	1.503	1.233	1.032	0.836	0.724	0.647	0.587
80	1.520	1.246	1.042	0.843	0.730	0.652	0.591
90	1.535	1.257	1.050	0.849	0.735	0.656	0.595
100	1.550	1.267	1.058	0.855	0.739	0.659	0.598

Table 6 Critical values of the Andeson-Darling, A_n^2, for the symmetrical ranks, $P_i = (i - 0.5)/n$.

Sample Size n	Significance Level γ 0.01	0.025	0.05	0.1	0.15	0.2	0.25
3	0.725	0.683	0.619	0.510	0.456	0.422	0.391
4	1.319	1.025	0.778	0.604	0.522	0.468	0.429
5	1.488	1.090	0.865	0.675	0.574	0.509	0.461
6	1.541	1.159	0.910	0.708	0.605	0.536	0.485
7	1.565	1.182	0.939	0.731	0.624	0.553	0.501
8	1.571	1.196	0.956	0.748	0.639	0.566	0.512
9	1.571	1.206	0.967	0.760	0.650	0.576	0.521
10	1.568	1.211	0.976	0.769	0.659	0.584	0.528
11	1.562	1.214	0.982	0.776	0.665	0.590	0.534
12	1.562	1.218	0.988	0.782	0.671	0.596	0.539
13	1.556	1.220	0.992	0.787	0.675	0.600	0.543
14	1.556	1.223	0.997	0.791	0.680	0.604	0.546
15	1.557	1.226	1.000	0.795	0.683	0.607	0.550
16	1.554	1.228	1.004	0.799	0.686	0.610	0.552
17	1.553	1.229	1.006	0.802	0.689	0.613	0.555
18	1.553	1.231	1.009	0.805	0.692	0.615	0.557
19	1.554	1.232	1.011	0.807	0.694	0.617	0.559
20	1.552	1.234	1.014	0.809	0.697	0.619	0.561
25	1.556	1.243	1.023	0.819	0.705	0.627	0.568
30	1.559	1.250	1.031	0.825	0.711	0.633	0.573
35	1.563	1.255	1.037	0.831	0.717	0.638	0.577
40	1.566	1.260	1.042	0.836	0.720	0.641	0.581
45	1.571	1.266	1.047	0.840	0.724	0.644	0.584
50	1.575	1.270	1.051	0.843	0.727	0.647	0.586
60	1.581	1.277	1.057	0.848	0.731	0.651	0.589
70	1.588	1.283	1.063	0.853	0.735	0.654	0.592
80	1.591	1.287	1.067	0.856	0.738	0.657	0.595
90	1.596	1.291	1.070	0.859	0.741	0.659	0.597
100	1.602	1.296	1.074	0.862	0.743	0.661	0.598

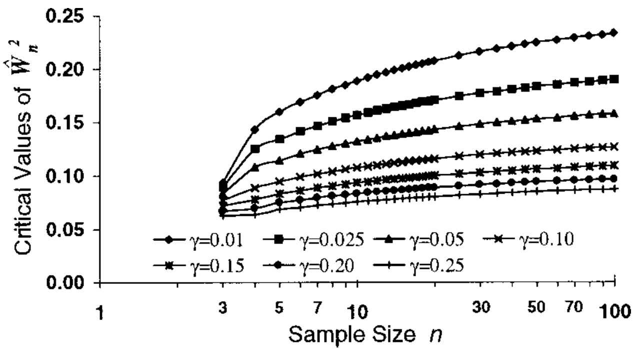

Figure 2 Critical values of the Cramer-von Mises, W_n^2, for the median ranks.

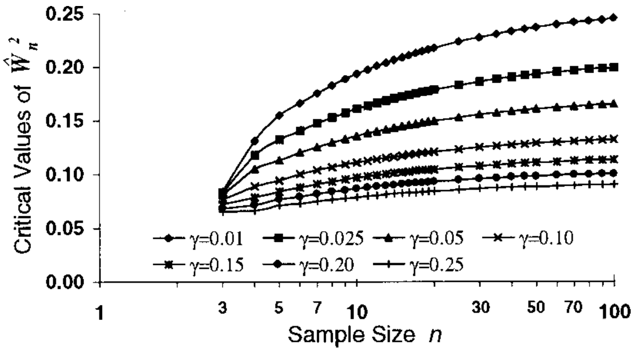

Figure 3 Critical values of the Cramer-von Mises, W_n^2, for the mean ranks. $P_i = i/(n+1)$.

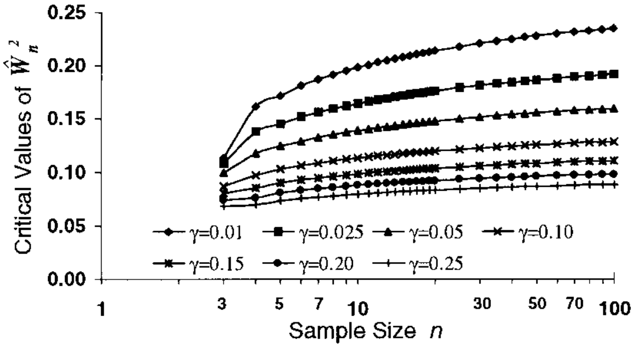

Figure 4 Critical values of the Cramer-von Mises, W_n^2, for the symmetrical ranks. $P_i = (i-0.5)/n$.

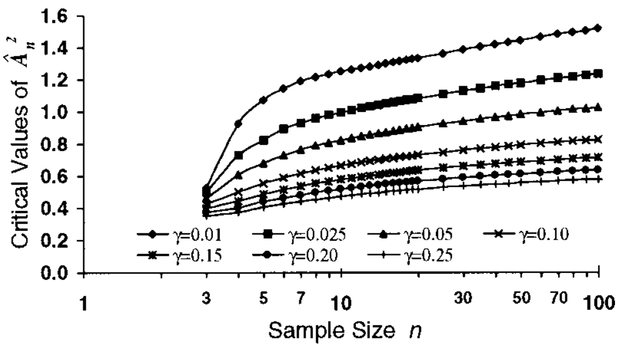

Figure 5 Critical values of the Anderson-Darling, A_n^2, for the median ranks.

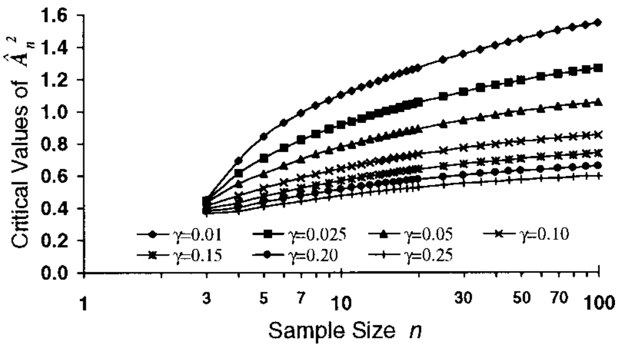

Figure 6 Critical values of the Anderson-Darling, A_n^2, for the mean ranks. $P_i = i/(n+1)$.

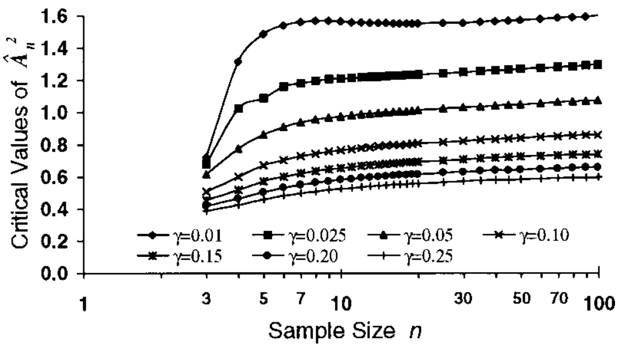

Figure 7 Critical values of the Anderson-Darling, A_n^2, for the symmetrical ranks. $P_i = (i-0.5)/n$.

Table 7 Standard deviations of Anderson-Darling critical values for the symmetrical ranks (sample size $n=10$).

Repcat No.	Sample Size n	Significance Level γ						
		0.01	0.025	0.05	0.1	0.15	0.2	0.25
1	10	1.5736	1.2152	0.9763	0.7686	0.6580	0.5832	0.5275
2	10	1.5718	1.2109	0.9761	0.7699	0.6590	0.5844	0.5285
3	10	1.5685	1.2126	0.9760	0.7693	0.6592	0.5846	0.5286
4	10	1.5670	1.2076	0.9767	0.7691	0.6590	0.5845	0.5281
5	10	1.5700	1.2116	0.9772	0.7692	0.6590	0.5844	0.5284
6	10	1.5652	1.2112	0.9776	0.7707	0.6600	0.5852	0.5288
7	10	1.5655	1.2086	0.9758	0.7680	0.6589	0.5842	0.5282
8	10	1.5637	1.2076	0.9744	0.7680	0.6580	0.5837	0.5278
9	10	1.5665	1.2121	0.9770	0.7683	0.6583	0.5836	0.5278
10	10	1.5643	1.2082	0.9756	0.7690	0.6592	0.5840	0.5282
Mean	/	1.5676	1.2106	0.9763	0.7690	0.6589	0.5842	0.5282
Standard Deviation	/	0.0033	0.0025	0.0009	0.0008	0.0006	0.0006	0.0004

5. 3 Fluctuation of Critical Values by Monte Carlo Simulation

The fluctuation of critical values of the A-D statistic for the symmetrical ranks is investigated in this section, because the A-D statistic coupled with the symmetrical ranks is found to have the highest power among all test statistics in this study. This will be presented in the second part of this study[13]. For sample size $n=10$, Table 7 presents ten critical values for each of seven significance levels that are calculated by performing the Monte Carlo simulation ten times. Then, the mean and standard deviation of critical values at each significance level are calculated from the ten critical values at the significance level. Table 7 shows that the standard deviations of critical values are very small, for instance, the standard deviation of critical value at $\gamma=0.05$ is 9.0×10^{-4}. The following characteristics are summarized.

(1)The standard deviations indicated that the fluctuation of critical values by Monte Carlo simulation is very small.

(2)Each critical value in this study determined by the mean of ten critical values is more precise than any one of the ten critical values, because the error of critical value by Monte Carlo simulation is proportional to the factor; $1/\sqrt{M_R}$[16], where M_R is the repeated times of the Monte Carlo simulation.

6. CONCLUDING REMARKS

This study presented the practical tables of critical values of the Cramer-von Mises and Anderson-Darling statistics for goodness-of-fit tests for the Type-I extreme-value and two-parameter Weibull distributions using Monte Carlo simulation. It is shown that the obtained tables could be used for both distribution models. The underlying conditions in calculating the critical values were as follows: (1) a complete sample is chosen; (2) the parameter estimates are provided by the combination of a graphical plotting technique and the least-squares method on probability paper; (3) three graphical plotting techniques are used, i.e., the median ranks, mean ranks, and symmetrical ranks; (4) $1{,}000{,}000\times10$ samples in total are generated for each sample size; (5) sample sizes are $n=3(1)20$, 25(5)50, and 60(10)100 ; and (6) significance levels are $\gamma=0.01$, 0.025, 0.05, 0.10, 0.15, 0.20, and 0.25.

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