A Variable Fidelity Response Surface Approach towards Integration of CFD and EFD

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Abstract

In this paper, a variable fidelity Kriging model approach, which is also referred to as cokriging, is proposed for efficient aerodynamic data modeling. In this approach, an accurate response surface is constructed by utilizing variable fidelity information. The variable fidelity information can be defined by different physical models, different accuracies of flow simulation as well as combination of experimental data and numerical data. The effectiveness of the developed variable fidelity Kriging model approach is discussed by using EFD/CFD aerodynamic data of a DLR-F6 configuration. The developed approach is promising for accurate aerodynamic data modeling by efficiently integrating EFD and CFD data.

Key words: Variable fidelity Kriging model, Aerodynamic data modeling, Response surface

Introduction

Recently, significances of experimental fluid dynamics (EFD) and computational fluid dynamics (CFD) are comparable in aerodynamic designs. Currently, CFD is not only a supplementary tool of EFD, but has intrinsic roles in aerodynamic design projects. In EFD approach, there are some restrictions in its measurement. For example, the difference of Reynolds number between wind tunnel testing and real flight testing, and the influence of wind tunnel wall/model support system are still essential problems to be taken into account in EFD approaches. Furthermore, the increase in lead time (mainly due to experimental model design/manufacture) is one of the major bottlenecks of EFD approach. In CFD approach, on the other hand, these issues can be eluded thanks to the availability of improved numerical algorithms and the growth of computer speed and memory. The CFD aerodynamic data is, however, considered to have less reliability than EFD data especially with fluid phenomena of large turbulence, transition and separation. The effect of numerical dissipation related to computational grid resolution is also significant for the accuracy of predicted aerodynamic performances. Since it may be difficult to resolve these problems individually in EFD and CFD fields, the integration of EFD and CFD data is promising to provide reliable aerodynamic data efficiently by utilizing the advantages of EFD/CFD approaches. In Japan Aerospace Exploration Agency (JAXA), a digital (CFD) / analog (EFD) hybrid wind tunnel system is being developed [1]. One of the major objectives of the development of this system is to comprehensively solve the issues mentioned above by effectively utilizing both EFD and CFD capabilities, resulting in the reduction of design time, cost, risk and the improvement of design data accuracy and reliability in the aircraft and aerospace vehicle development.

Response surface approaches have attracted increased attention recently in aerospace engineering since they offer substantial benefits for design optimization, aerodynamic database construction, and uncertainty quantification. The idea of a response surface approach is to replace expensive functional evaluations (i.e. costly EFD measurements or high-fidelity CFD simulations) with an analytical model which is constructed through selective sampling of the high-fidelity data. When a response surface model is constructed with given exact functional data, a designer can efficiently explore the approximated (high-dimensional) design space at very low computational cost. To realize an accurate/efficient exploration on the response surface, the construction of an accurate response surface is essential. The Kriging model [2-8], which was originally developed in the field of geological statistics, has often been found to perform well in other engineering fields and has thus gained popularity in aerospace engineering and design. This response surface model predicts the functional value by using stochastic processes, and has the flexibility to represent multimodal/nonlinear functions. One of the major approaches to enhance the accuracy of response surface models efficiently is to utilize the derivative information of the function [4,5,7]. Utilizing low-fidelity functional values as secondary information represents an alternative approach to improve the accuracy of response surface models [6-10]. This approach is referred to as cokriging method or variable fidelity (VF) approach [11]. These concepts of the response surface approaches are summarized in Fig.1. In the VF response surface approach, the trends of low-fidelity functional values as well as high-fidelity functional absolute values are simultaneously utilized to construct an accurate response surface. This approach is promising for efficient aerodynamic data modeling by integrating EFD and CFD data.

In this research, a variable fidelity Kriging model approach is utilized to produce accurate aerodynamic data by integrating EFD and CFD data. The EFD/CFD aerodynamic data of a DLR-F6 configuration, that are mutually managed in the digital/analog hybrid wind tunnel system of JAXA, are utilized in this study.

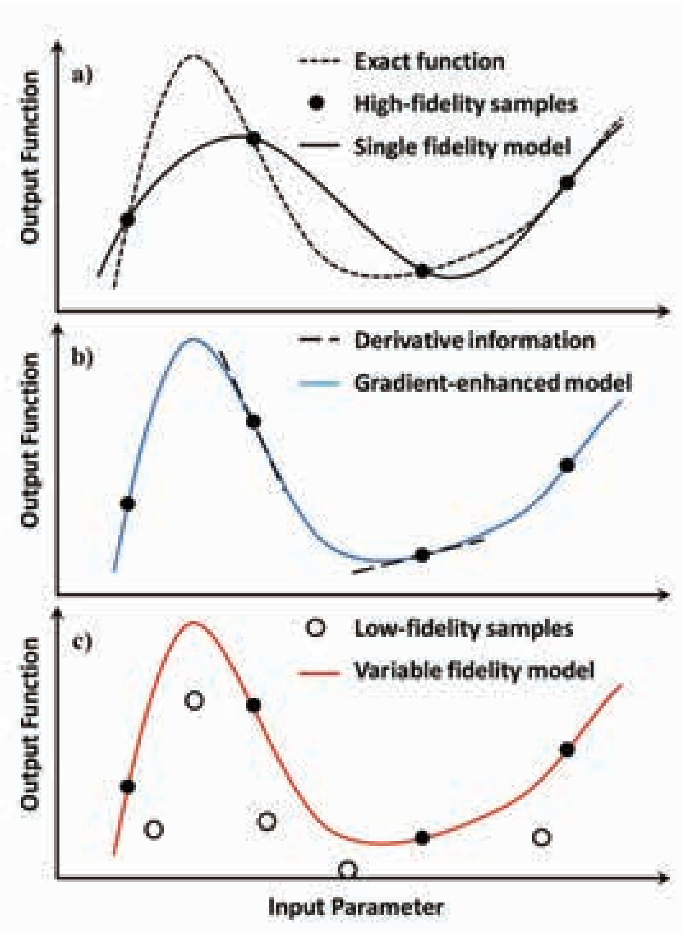

Fig.1 Concepts of Response Surface Approaches
a) Conventional Response Surface
b) Derivative-enhanced Response Surface
c) Variable Fidelity Response Surface

Variable Fidelity Kriging Model

In this section, our variable fidelity Kriging approach [7-8] is briefly introduced. In this approach, the high and low-fidelity functions are replaced by the following random functions:

$$\tilde{y}_{l(\mathbf{x})} = \beta_l + Z_{l(\mathbf{x})} \tag{1}$$

where l means the index of fidelity level. The first term β_l is a constant model and the second term Z_l represents a random process model with zero mean, variance σ_l^2 and the covariance of two locations $\mathbf{x}_1$ and $\mathbf{x}_2$ is given as follows:

$$\mathrm{cov}\left[Z_{l_1(\mathbf{x}_1)}, Z_{l_2(\mathbf{x}_2)}\right] = \sigma_{l_1}\sigma_{l_2} R_{l_1 l_2(\mathbf{x}_1,\mathbf{x}_2)} \tag{2}$$

where $R_{l_1 l_2}$ is a correlation function which is usually defined as a radial basis function based on a distance between the two locations. Then a linear combination of the high and low-fidelity information at given sample points is considered for the high-fidelity functional prediction as follows:

$$\hat{y}_{(\mathbf{x})} = \sum_{i=1}^{n_1} w_{1i} y_{1i} + \sum_{i=1}^{n_2} w_{2i} y_{2i} = \mathbf{w}_1^T \mathbf{y}_1 + \mathbf{w}_2^T \mathbf{y}_2 \tag{3}$$

where $\mathbf{y}_l$ and $\mathbf{w}_l$ are respectively the known function values on given sample points and their unknown weight coefficients, and n_l is the number of sample points at l-th fidelity level. The first fidelity level ($l = 1$) is considered as high-fidelity data in this study. The Kriging approach finds the best linear unbiased predictor which minimizes the mean square error (MSE):

$$MSE = E\left[\left(\hat{y}_{(\mathbf{x})} - \tilde{y}_{1(\mathbf{x})}\right)^2\right] \tag{4}$$

subject to the unbiasedness constraint of

$$E\left[\hat{y}_{(\mathbf{x})}\right] = E\left[\tilde{y}_{1(\mathbf{x})}\right] \tag{5}$$

The weight coefficients can be found by solving this constrained minimization problem with the Lagrange multiplier approach. Finally, the high-fidelity functional prediction is achieved by the following formula:

$$\hat{y}_{(\mathbf{x})} = \mathbf{L}^T \tilde{\beta} + \mathbf{r}_{(\mathbf{x})}^T \mathbf{R}^{-1}\left(\mathbf{Y} - \mathbf{F}\tilde{\beta}\right) \tag{6}$$

where

$$\begin{aligned} \tilde{\beta} &= \left(\mathbf{F}^T\mathbf{R}^{-1}\mathbf{F}\right)^{-1}\mathbf{F}^T\mathbf{R}^{-1}\mathbf{Y} \in \Re^{2\times 1} \\ \mathbf{Y} &= \left[\mathbf{y}_1^T, \mathbf{y}_2^T\,\sigma_1/\sigma_2\right]^T \in \Re^{(n_1+n_2)\times 1} \\ \mathbf{F} &= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \in \Re^{(n_1+n_2)\times 2} \\ \mathbf{L} &= \begin{bmatrix} 1 & 0 \end{bmatrix}^T \in \Re^{2\times 1} \end{aligned} \tag{7}$$

The correlation matrix $\mathbf{R} \in \Re^{(n_1+n_2)\times(n_1+n_2)}$ expresses the correlations between all given sample points while the correlation vector $\mathbf{r} \in \Re^{(n_1+n_2)\times 1}$ expresses the correlations between all given sample points and a location $\mathbf{x}$. The matrix form of Eq.(6) as well as the definition of all matrices/vectors are very similar with that of the original Kriging formulation. The factor of σ_1/σ_2 is a special parameter required in this variable fidelity Kriging formulation. This factor has the role to take into account the influence of low-fidelity information. The MSE of Eq.(4) can be expressed as follows:

$$MSE = \sigma_1^2\left[1 - \mathbf{r}^T\mathbf{R}^{-1}\mathbf{r} + \varphi^T\left(\mathbf{F}^T\mathbf{R}^{-1}\mathbf{F}\right)^{-1}\varphi\right] \qquad \left(\varphi = \mathbf{F}^T\mathbf{R}^{-1}\mathbf{r} - \mathbf{L}\right) \tag{8}$$

As the conventional Kriging approach, hyper-parameters (which appear in the correlation function) as well as the factor of σ_1^2 are estimated by a likelihood maximization approach [2]. In this research, the additional factor of σ_1/σ_2 is also estimated by the likelihood maximization approach. The computational cost to construct a variable fidelity response surface model is primary dependent on the total number of sample points $n_1 + n_2$, since a huge number of calculations of $\mathbf{R}^{-1}$ and $|\mathbf{R}|$ are required with different sets of the hyper-parameters. Nevertheless, the computational cost is much smaller than that of a high-fidelity CFD computation with general numbers of sample points (< 500).

Validation using Analytic Function

In this section, the validity of the variable fidelity Kriging response surface approach is shown in an analytic functional problem. In this study, the following analytic function, which is similar to the Rastrigin function, is considered as the high-fidelity (exact) function:

$$F_{(\mathbf{x})} = 20 + \sum_{i=1}^{2}\left[x_i^2 - 10\cos(\pi x_i / 2)\right] \tag{9}$$

The shape of the exact function is shown in Fig.2. As its low-fidelity functions, the following functions are defined in this study:

$$\begin{aligned} f_{1(\mathbf{x})} &= F_{(\mathbf{x})} - 20 \\ f_{2(\mathbf{x})} &= \sum_{i=1}^{2}\left[x_i^2\right] \\ f_{3(\mathbf{x})} &= \sum_{i=1}^{2}\left[-10\cos(\pi x_i / 2)\right] \end{aligned} \tag{10}$$

By using 10 high-fidelity samples as well as 90 low-fidelity samples, single fidelity (SF) / VF response surface models were constructed. These sample points were generated by a Latin Hypercube Sampling (LHS) approach. The shapes of the approximated functions are shown in Fig.3. The accuracy of the approximated function was increased with the low-fidelity sample points of f_1 and f_3. By using the low-fidelity function of f_2 which is the quadratic part of the exact function, the approximated model only had the quadratic functional tendency. The accuracy of a response surface model is evaluated by the following Root Mean Square Error (RMSE):

$$RMSE = \frac{1}{M}\sqrt{\sum_{j=1}^{M}\left(\hat{y}_{(\mathbf{x}_j)} - F_{(\mathbf{x}_j)}\right)^2} \tag{11}$$

where the coordinates $\mathbf{x}_j$ define an equally spaced Cartesian mesh which covers the entire design space. In Fig.4, the RMSE values are compared between the SF and VF Kriging model approaches. In the VF approaches, the number of low-fidelity sample points is increased while the number of high-fidelity sample points is fixed to 10. With the low-fidelity functions of f_1 and f_3, the accuracies of the response surface models are improved with the increase in the number of low-fidelity sample points. Thus, the accuracy of VF response surface models can be increased with appropriate low-fidelity sample points. As understood from these results, the important aspect of the low-fidelity sample points is not the absolute values of the function, but the trends of the function. When the appropriate low-fidelity sample points can be obtained inexpensively, an accurate response surface can also be constructed efficiently.

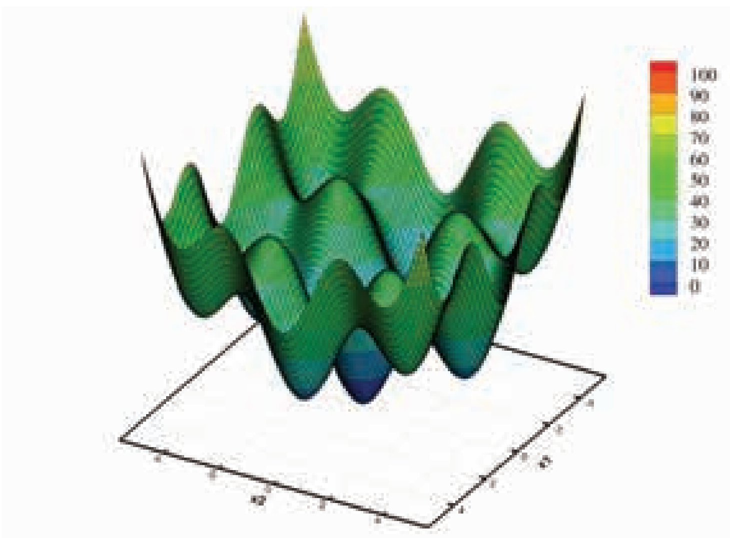

Fig.2 Exact Analytic Function

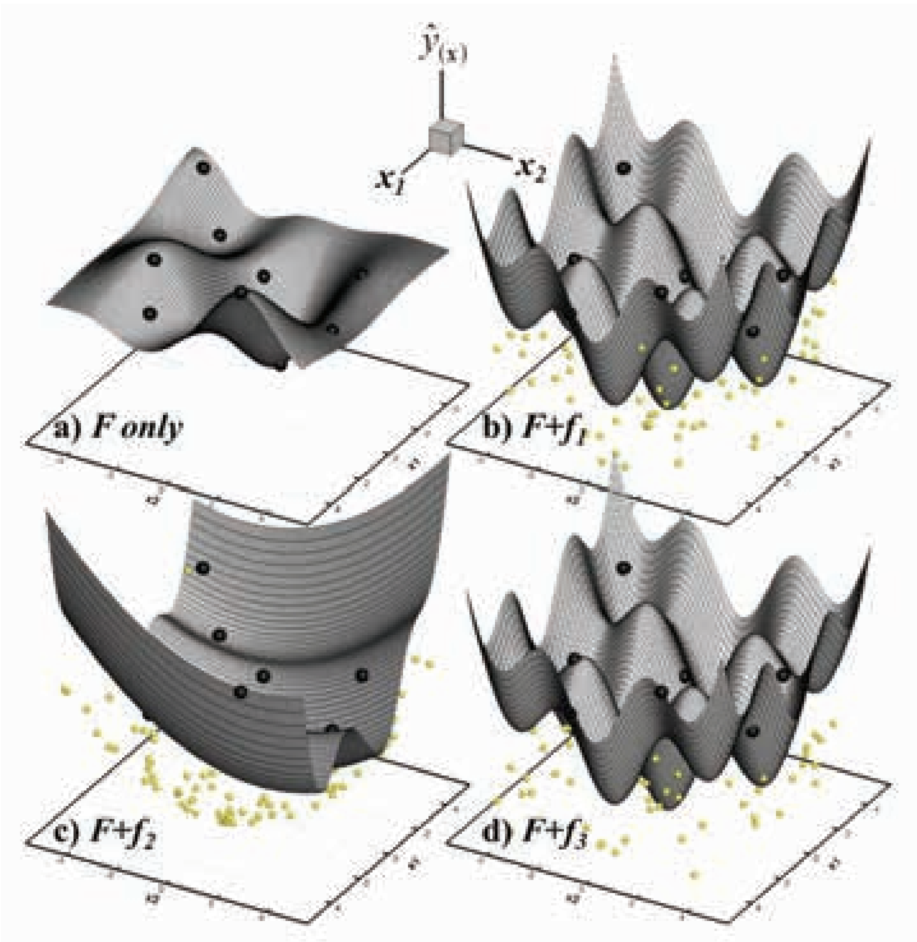

Fig.3 Estimated Functions by Response Surface Approaches,
Black Points: 10 High-Fidelity Samples
Yellow Points: 90 Low-Fidelity Samples

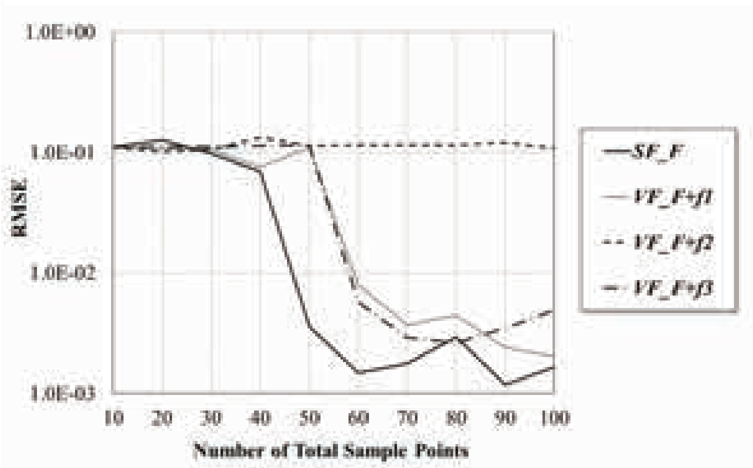

Fig.4 Comparison of Accuracies of Response Surfaces by RMSE Evaluations

Aerodynamic Data Modeling of DLR-F6 Configuration

The developed response surface approach is applied to the EFD/CFD aerodynamic data of a civil transport of DLR-F6 configuration [12]. The wind tunnel testing has been performed at JAXA 2m × 2m Transonic Wind Tunnel (JTWT) as shown in Fig.5. The CFD evaluation has been performed by a JAXA's in-house CFD solver of FaSTAR (FaST Aerodynamic Routine) [13] on JAXA Supercomputer System (JSS) as Fig.5. In this analysis, the Reynolds-Averaged Navier-Stokes (RANS) equations are solved with the Spalart-Allmaras turbulence model on a hexahedral grid [14]. The drag polar curves obtained by EFD/CFD analyses at Mach number (M_∞) of 0.75 and Reynolds number (Re) of 1.5×10^6 are compared in Fig.6. It can be confirmed that there is a certain level of difference between EFD and CFD data while the trends of drag polar curves are comparable between them except higher angles of attack (α). The EFD/CFD aerodynamic data were evaluated in the range of $0.6 \le M_\infty \le 0.85$, $0.8\times10^6 \le \mathrm{Re} \le 2.0\times10^6$ and $-5 \le \alpha \le 5$ degrees. In these evaluations, 58 EFD data as well as 44 CFD data have been obtained.

In Fig.7, conventional SF response surfaces of C_L and C_D that are constructed only by the EFD or CFD data are visualized. Spheres indicate (high-fidelity) sample points that are utilized to construct the response surfaces. It can be confirmed that the SF response surfaces constructed by EFD/CFD data have comparable tendencies. In this study, the accuracy of a response surface is evaluated by the following mean error (ME):

$$ME = \frac{1}{58}\sum_{j=1}^{58}\left|\hat{y}(\mathbf{x}_j) - y^{EFD}(\mathbf{x}_j)\right| \tag{12}$$

where j indicates the index of the EFD data. In Fig.8, the ME values of C_D are compared between the SF and VF Kriging model approaches. In the VF approaches, the number of high-fidelity (EFD) sample points is increased while the number of low-fidelity (CFD) sample points is fixed to 9. Various sets of the high/low-fidelity sample points are chosen by a LHS approach. The improvement in accuracy is observed by the VF approach with smaller numbers of the high-fidelity (EFD) sample points (<15). Although the accuracy of the VF model is not improved with larger numbers of the high-fidelity (EFD) sample points, this is due to the difference of functional tendencies between EFD/CFD data. The functional tendencies obtained from the 9 CFD sample points are no longer effective with larger numbers of EFD sample points. In Fig.9, the SF/VF response surfaces of C_L and C_D that are constructed only by 9 EFD and/or 9 CFD data are visualized. In Fig.10, estimated drag polar curves from the response surfaces of Fig.9 are indicated. It can be seen that the accuracies of the response surface models are increased by utilizing the VF Kriging approach. Despite only one EFD and two CFD sample points were set on (M_∞, Re) = (0.75, 1.5×10^6), the estimated drag polar curve by the VF approach showed a certain level of agreement with the EFD data.

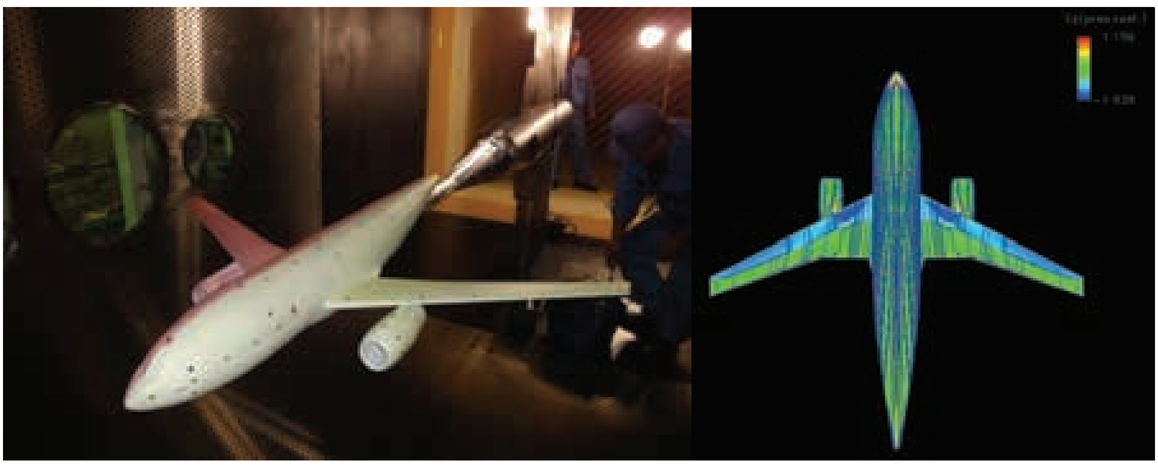

Fig.5 Aerodynamic Evaluations by EFD/CFD Approaches

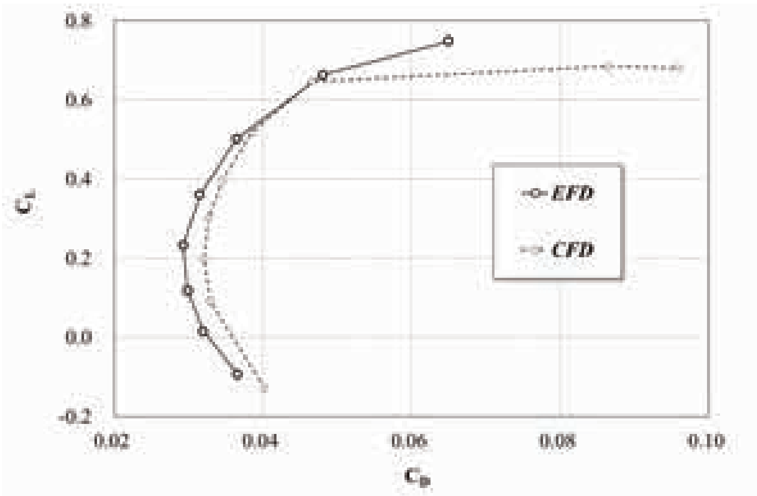

Fig.6 Comparison of Drag Polar Curves at M_∞ of 0.75 and Re of 1.5 × 10^6

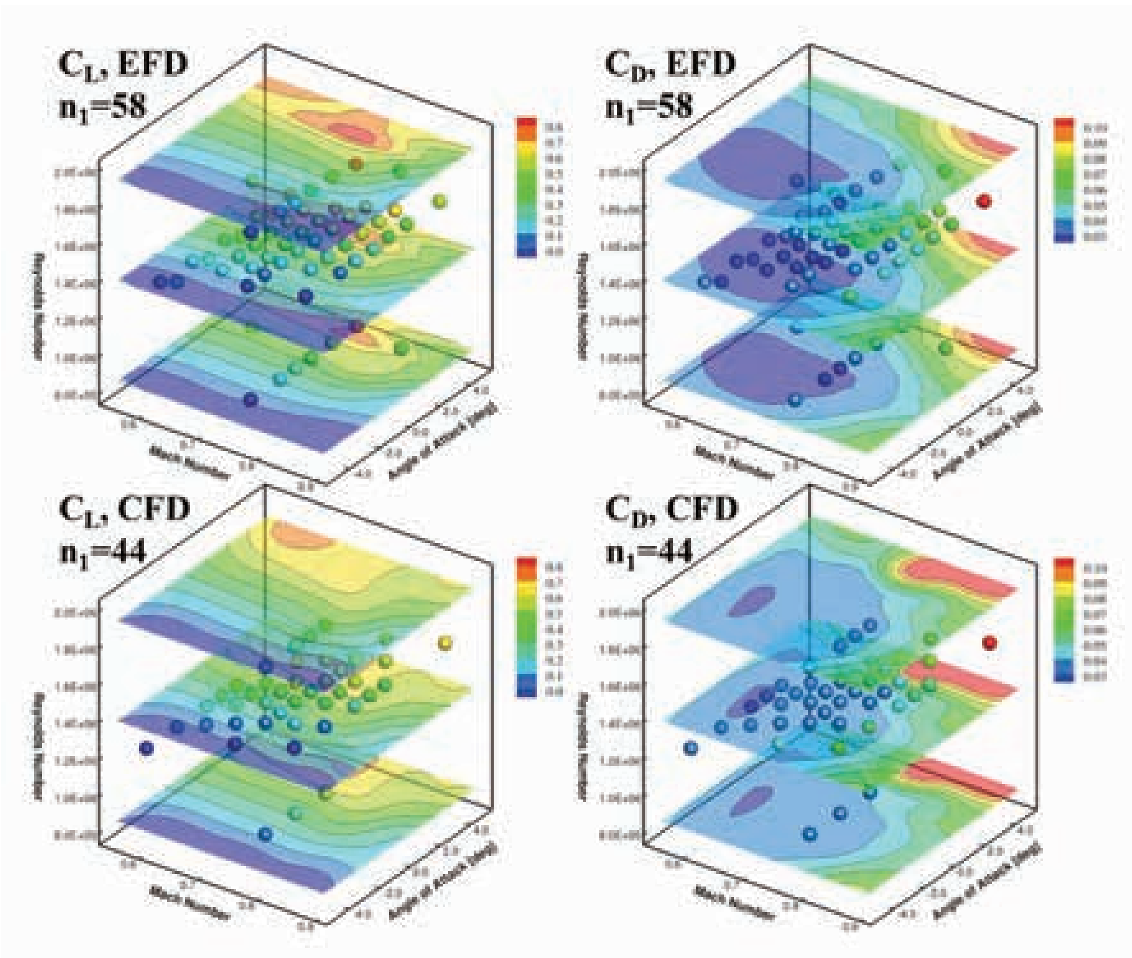

Fig.7 Single Fidelity Response Surfaces of C_L and C_D

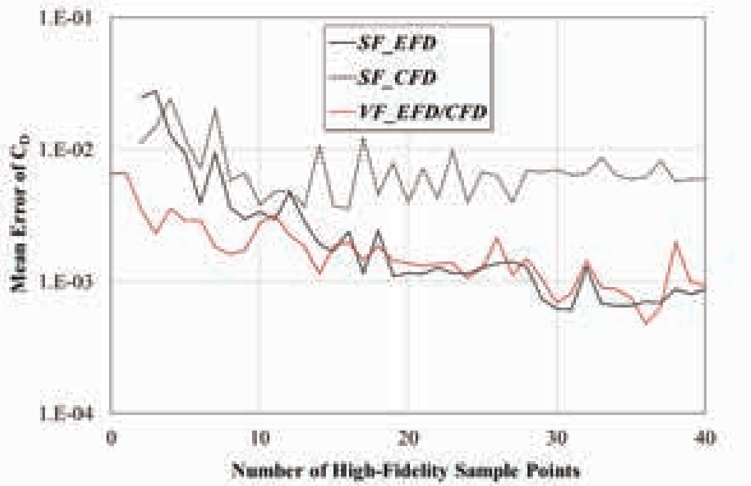

Fig.8 Comparison of Accuracies of Response Surfaces by ME Evaluations

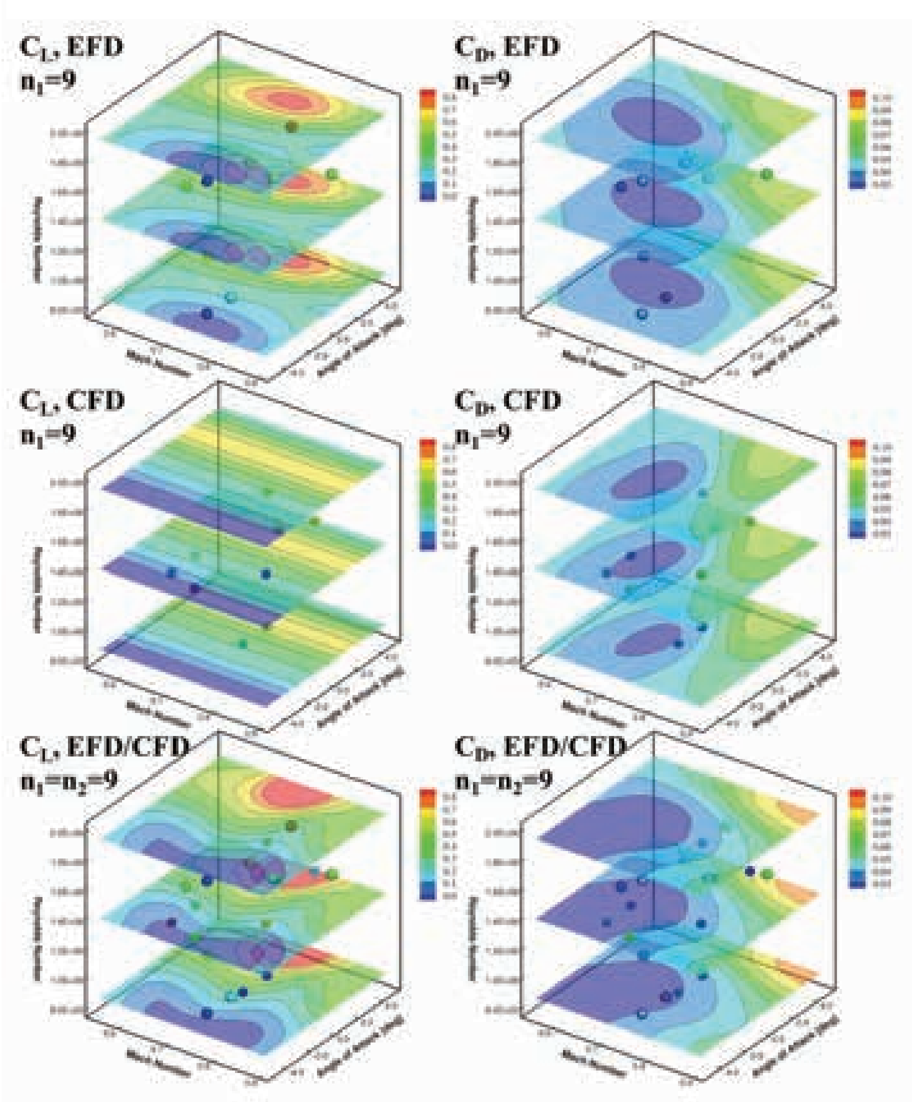

Fig.9 Single Fidelity / Variable Fidelity Response Surfaces of C_L and C_D
Upper: EFD-based SF, Middle: CFD-based SF, Lower: EFD/CFD-based VF Model
Spheres: EFD Samples, Cubes: CFD Samples

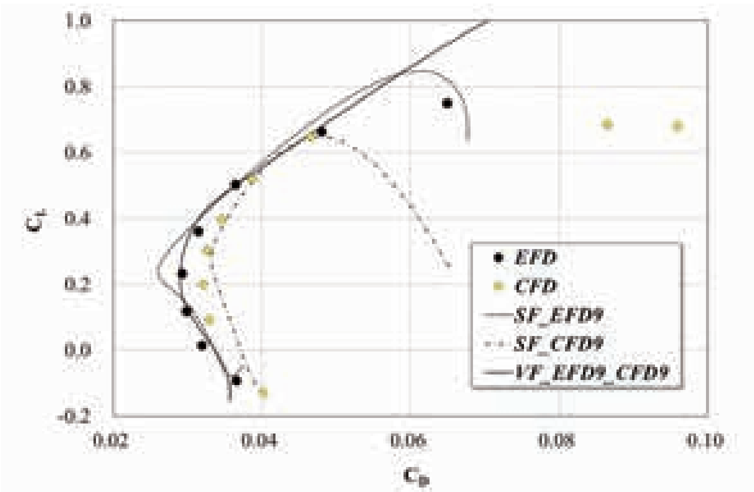

Fig.10 Estimated Drag Polar Curves at M_∞ of 0.75 and Re of 1.5 × 10^6

Summary & Future Directions

In this research, a variable fidelity Kriging response surface approach has been utilized for aerodynamic data modeling by integrating EFD/CFD data. In this approach, a response surface model can be constructed by the absolute functional values of high-fidelity (EFD) sample points as well as the functional trends of low-fidelity (CFD) sample points. The EFD/CFD aerodynamic data of a DLR-F6 configuration, that are managed in the digital/analog hybrid wind tunnel system of JAXA, are utilized in this study. The variable fidelity approach provided better aerodynamic data modeling than the single fidelity conventional response surface approach with smaller numbers of EFD sample points. This result indicates the validity of the variable fidelity Kriging model approach for the fusion of EFD/CFD data.

Once an experimental model has been manufactured, it is not difficult to make massive aerodynamic database with respect to flow conditions (M_∞ , Re , α etc) as long as wind tunnel facilities are available. In this context, the fusion of EFD/CFD data within the M_∞ - Re - α parameter space, which was examined in this paper, may not be interested by aerodynamic designers. One of the promising directions for the practical application of the variable fidelity Kriging approach is the fusion of EFD/CFD data between various model configurations. Although the manufacturing of various experimental models is difficult in terms of cost effectiveness, it is relatively easy in CFD by applying computational grid deformation techniques. By integrating EFD/CFD data between various configurations, efficient/reliable aerodynamic design (optimization) can be achieved with the variable fidelity Kriging response surface approach.

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