A Modification of the e^N Method for Three-Dimensional Boundary Layers

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Transition prediction of three-dimensional boundary layer is numerically investigated by the use of a modified e^N method. Main idea is the corporation of a complex ray theory into the e^N method so as to uniquely determine a wavenumber which corresponds to a most amplified disturbance wave. From the calculation for a flow around a yawed circular cylinder with sweep angle 50°, it is found that the results agree well with a theoretical one obtained by N.Itoh (1986) excepting the N factor.

Key Words: 3-D Boundary Layer, Transition prediction, e^N method, Complex ray theory

1 Introduction

The prediction of laminar-turbulent transition is one of the most interesting subjects of aeronautical engineering. NASA developed a numerical prediction code with the e^N method called SALLY[1)] and it is available for relatively inexpensive calculation of the N factor. However the SALLY code and some other prediction codes[2)] have a crucial shortcoming that some of the unknown values can not be determined in principle. Thus, the user must arbitrarily determine the value of wavenumber or wave length of disturbance.

In order to overcome this unreasonable procedure, we developed a improved prediction code on the basis of a complex ray theory, which can determine a propagation path of a disturbance wave. From the comparison of the present result with a theoretical one, it is found that both results agree well, especially for wavenumbers.

2 Complex ray theory

This theory is proposed by N.Itoh[3)] on the basis of the extended kinematic wave theory, and its outline is given as follows.

Firstly, it is assumed that a disturbance is written in a wavy form

$$\boldsymbol{u}(\boldsymbol{X}) = \tilde{\boldsymbol{u}}(Z)\exp(i\Theta/\epsilon_0), \tag{1}$$

where Θ is a phase function, and the coordinates X, Y and Z denote the chord wise, the span wise and normal-to-the-wall directions, respectively. Then, complex chord wise wavenumber α, complex span wise wavenumber β and complex frequency ω are defined by

$$\frac{\partial\Theta}{\partial X} \equiv \frac{\alpha}{\Delta_1}, \qquad \frac{\partial\Theta}{\partial Y} \equiv \frac{\beta}{\Delta_1}, \qquad \frac{\partial\Theta}{\partial T} \equiv -\frac{\omega}{\Delta_2}. \tag{2}$$

Then, these quantities satisfy

$$\omega = \Omega(\alpha, \beta; X), \tag{3}$$

where $\epsilon_0, \Delta_1, \Delta_2$ are

$$\epsilon_0 = \sqrt{\nu l_s/U_\infty}/l_s, \Delta_1 = \delta/\delta_0, \Delta_2 = \Delta_1/(Q_E/Q_\infty), \tag{4}$$

and l_s is the surface length, and boundary layer thickness δ and δ_0 are

$$\delta \equiv \sqrt{\nu x/U_E}, \qquad \delta_0 \equiv \sqrt{\nu l_s/U_\infty}. \tag{5}$$

These quantities must satisfy the compatibility conditions

$$\frac{\partial}{\partial Y}(\frac{\alpha}{\Delta_1}) = \frac{\partial}{\partial X}(\frac{\beta}{\Delta_1}), \tag{6}$$

$$\frac{\partial}{\partial T}(\frac{\alpha}{\Delta_1}) = -\frac{\partial}{\partial X}(\frac{\omega}{\Delta_2}), \tag{7}$$

$$\frac{\partial}{\partial T}(\frac{\beta}{\Delta_1}) = -\frac{\partial}{\partial Y}(\frac{\omega}{\Delta_2}). \tag{8}$$

When we consider a wedge-shaped disturbance originating from a point source, imaginary part of ω and differentiation with respect to T are 0, namely

$$\omega_i = 0, \qquad \frac{\partial}{\partial T} = 0. \tag{9}$$

With these restrictions and the compatibility conditions (6)-(8), we can derive an equation

$$\frac{\partial}{\partial X}(\frac{\beta}{\Delta_1}) + \frac{\omega_\beta}{\omega_\alpha}\frac{\partial}{\partial Y}(\frac{\beta}{\Delta_1}) = 0. \tag{10}$$

This equation means that

" **β/Δ_1 does not change on $\frac{dY}{dX} = \frac{\omega_\beta}{\omega_\alpha} = C$** ".

Then imaginary part of C must satisfy a condition

$$Y_i = \int_{X_0}^{X_1} C_i \, dX = 0, \tag{11}$$

where X_0 and X_1 correspond to a source point of the disturbance and an observation location, respectively.

Consequently, from this realizable condition (11) and dispersion relation (3), the wavenumber of disturbance can be uniquely determined with the input of a frequency : although the growth rate of disturbance on the span wise direction remain as a parameter, this value is initially determined by physical demand.

Then the total amplification N between X_0 and X_1 is defined as

$$N = -S \int_{X_0}^{X_1} [\alpha_i + \beta_i C_r] \, dX, \quad (S : \text{const.}), \qquad (12)$$

which represents the value of $\log(A_1/A_0)$.

3 Results and discussion

We examined the instability of a 3-D boundary layer around a infinite yawed circular cylinder with the sweep angle 50° as shown in Fig.1. Reynolds number, which is defined with a uniform flow velocity and the chord length, is 0.5×10^6. In the linear stability analysis, the Orr-Sommerfeld equation is used and the velocity profiles of the boundary layer are obtained from a similarity solution[4)].

Figure 2 shows the variation of the wavenumber and the N factor with frequency (rigid lines : present results, broken lines : theoretical ones[3)]). In this case, β_i and ω_i are 0. This condition means that the disturbance waves can not amplify in time and in the span wise direction. A quite large difference of the N factor between the theoretical and the present result is found in this figure, although the wave numbers show good agreement. This difference probably originated from the difference in the stability equations used. It is interested that the peak value of N appears at non-zero frequency. This means that a stationary wave is not dominant in this case.

The variation of N factor with the change of the observation points X_1 from 0.15 to 0.40 is shown in Fig.3. The N factor of each disturbance increases as the move of X_1 toward the downstream. This means that the disturbances are always unstable at least in this observation range. Furthermore, at each observation locations, the value of frequency corresponding to the most amplified disturbance is always about $\omega_r \simeq 0.8$. This fact shows that the disturbance wave being dominant at the upstream location is always dominant at each downstream location.

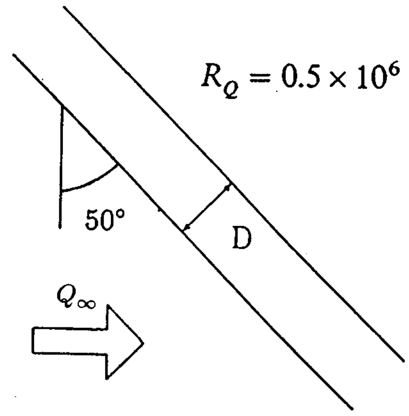

Fig.1 The flow field.

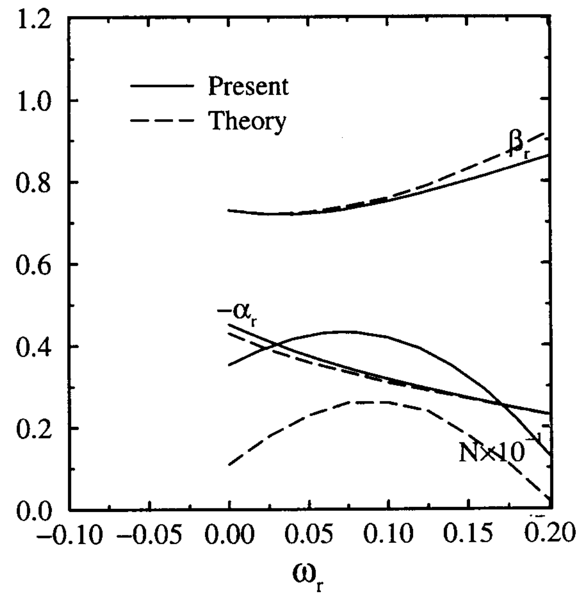

Fig.2 The variations of wavenumbers and N factor with frequency ($\beta_i = \omega_i = 0.0$).

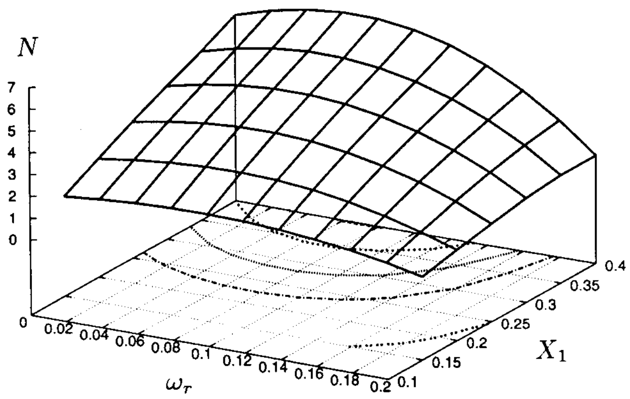

Fig.3 The variation of N factor with frequency at several observation points ($\beta_i = \omega_i = 0.0$).

References

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