Analysis of swing-by trajectory in three-body problem using B-plane

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Abstract

In general, a swing-by trajectory is designed in a two-body problem in a preliminary design or a case of the trajectory of the spacecraft with high energy. In these cases, the swing-by trajectory is often designed by B-Plane with Patched conics method. However, when the spacecraft has low energy, the approximation of the two-body problem does not hold. Therefore, in this study, the B-Plane used in the two-body problem is extended to the three-body problem and analyze the influence of the secondary body on parameters of the B-Plane of the swing-by trajectories.

B-plane を用いた三体問題におけるスイングバイ軌道の解析

摘要

一般的に，スイングバイ軌道は初期段階での軌道もしくは高エネルギーを持つ宇宙機の軌道では二体問題で軌道設計される．この場合，スイングバイ軌道は Patched conics 法を用いた B-Plane によって軌道設計が行われることが多い．しかし，低エネルギーを持つ宇宙機の時．二体問題と近似することが出来なくなる．したがって，本研究では，二体問題で用いられる B-Plane を三体問題に拡張し，スイングバイ軌道のを用いて作られる B-Plane に対していくつかのパラメーターに注目して 2 天体目の影響を分析する．

1. Introduction

Swing-by orbit has been used in many missions and there are many studies on swing-by [1, 2]. As examples, Galileo, Voyager1 and Voyager2 utilized the swing-by trajectory. Currently, DESTINY+ under development by JAXA planes to use a multiple lunar swing-by in order to escape the Earth [3]. Swing-by can change the velocity and the energy of the spacecraft without thrusting. Therefore, it is possible to make the mission term longer and suppress the fuel consumption.

Dynamical models used in trajectory design can be generally divided into two types, a two-body problem and a multi-body problem. Although it is desirable to consider the influence of more celestial bodies, it takes a lot of effort to design the trajectory. For example, the case when the spacecraft has high orbital energy with respect to the Earth is shown in Fig. **1**. In this case, since the influence of the gravity other than the Earth is small, the dynamics of the spacecraft can be regarded as a the two body problem. The characteristic of designing a trajectory in a two-body problem is that it can be designed analytically. Therefore, the swing-by trajectory is often designed by using B-Plane [4]. B-Plane is a plane perpendicular to the speed of the spacecraft toward the swing-by target body, and the parameters on B-Plane has an analytical relationship with the swing-by trajectory. On the other hand, the case when the spacecraft has low orbital energy with respect to the Earth is shown in Fig.**2**. The approximation of the two-body problem with the Earth does not hold, therefore, the influence of other bodies than be Earth should be considered. In particular, trajectory design using the three-body problem considering the effects of two-celestial bodies is often used. The trajectory in the three-body problem can not be designed analytically.

In the low energy, the analytical solution by B-Plane may not be valid [5, 6]. Therefore, in this study, by designing the B-Plane using the dynamics of the three-body problem, the influence on the parameters constituting the B-Plane is investigated. Based on the obtained results, we compare the B-Planes of the two-body problem and the three-body problem, and conduct a physical study on the obtained B-plane.

This paper is organized as follows. Chapter 2 describes how to define B-Plane, and Chapter 3 shows simulation results. Finally, the conclusion is stated.

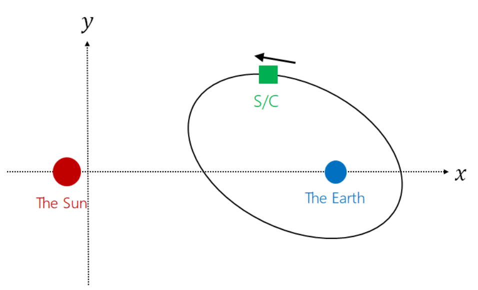

Fig. **1**: High Energy Orbit

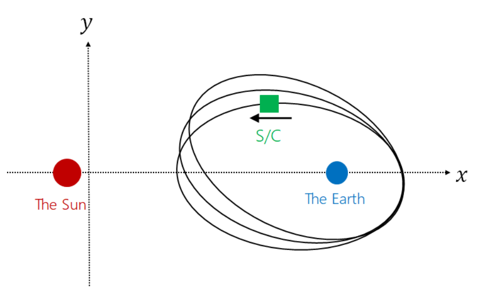

Fig. **2**: Low Energy Orbit

2. How to define and set B-Plane

2. 1 Equation of motion

The mass ration of the primary and secondary bodies are expressed as

$$\mu_{se} = \frac{\mu_e}{\mu_e + \mu_s} \tag{1}$$

μ_e and μ_s indicate the gravity constant of the Earth and the Sun, respectively. The equation of motion in two-body problem is expressed as

$$\frac{d^2x}{dt^2} = -\frac{\mu_e}{R^3}x \tag{2}$$

$$\frac{d^2y}{dt^2} = -\frac{\mu_e}{R^3}y \tag{3}$$

$$\frac{d^2z}{dt^2} = -\frac{\mu_e}{R^3}z \tag{4}$$

where

$$R = \sqrt{x^2 + y^2 + z^2} \tag{5}$$

The equation of motion in three-body problem is expressed as

$$\frac{d^2x}{dt^2} = -\frac{\partial U}{\partial x} \tag{6}$$

$$\frac{d^2y}{dt^2} = -\frac{\partial U}{\partial y} \tag{7}$$

$$\frac{d^2z}{dt^2} = -\frac{\partial U}{\partial z} \tag{8}$$

where

$$\overline{U} = -\frac{1-\mu_{se}}{r_1} - \frac{1-\mu_{se}}{r_2} - \frac{1}{2}(1-\mu_{se})\,\mu_{se} \tag{9}$$

$$r_1 = \sqrt{(x+\mu_{se}\cos t)^2 + (y+\mu_{se}\sin t)^2 + Z^2} \tag{10}$$

$$r_2 = \sqrt{(x-(1-\mu_{se})\cos t)^2 + (y-(1-\mu_{se})\sin t)^2 + Z^2} \tag{11}$$

2. 2 Definition of each specification

In this study, the Sphere of Influence (SOI) is defined as the distance sufficiently away from the center of the gravity of the swing-by target body, and the distance is calculated by

$$r_{SOI} = r_{se}\left(\frac{\mu_e}{\mu_s}\right)^{\frac{2}{5}} \tag{12}$$

As shown in Fig. **3**, the velocity of the spacecraft at SOI assumes the infinite velocity. Here, the infinite velocity is made parallel to the x-axis.

2. 3 Definition of B-Plane

To generate the B-Plane, first find the position vector $\boldsymbol{B}$ from the center of the Earth when the infinite velocity crosses the B-Plane is calculated. Trajectories are propagated from the perigee to the SOI. At this time, the trajectory is a hyperbolic trajectory obtained analytically. When the asymptotic line of this hyperbolic orbit is parallel to the x-axis, which is a straight line connecting the Sun and the Earth, $\boldsymbol{B}$ is a vector whose direction is perpendicular to the asymptote from the center of the Earth. Figure **4** shows this series of flows. Next, when the position vector of the perigee $\boldsymbol{r}_p$ is rotated around the x axis, $\boldsymbol{B}$ rotates simultaneously. The surface formed by $\boldsymbol{B}$ is defined as B-Plane. In other words, when the velocity at infinity enters perpendicular to the B-Plane, the trajectory that reaches the perigee of the target is obtained. This is shown in Fig. **5**. The angle when the $\boldsymbol{r}_p$ is rotated around the x-axis is θ_x and 1 to 360 deg.

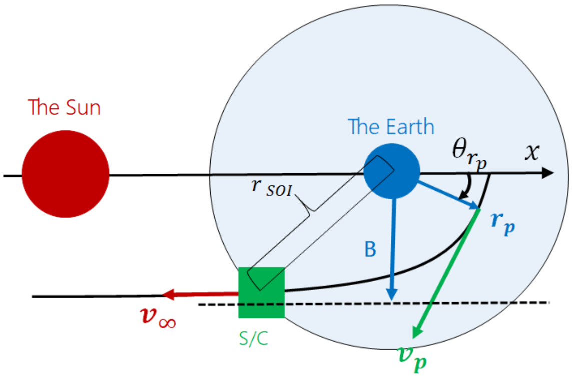

Fig. **3**: Explanation of values required for swing-by

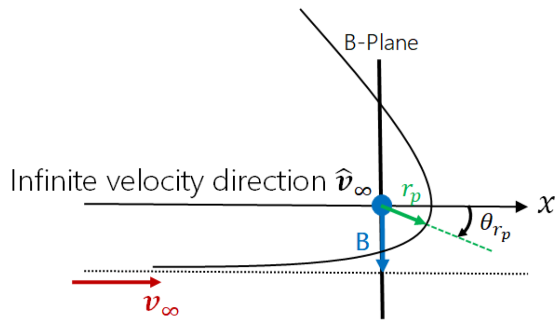

Fig. **4**: Definition of $\boldsymbol{B}$.

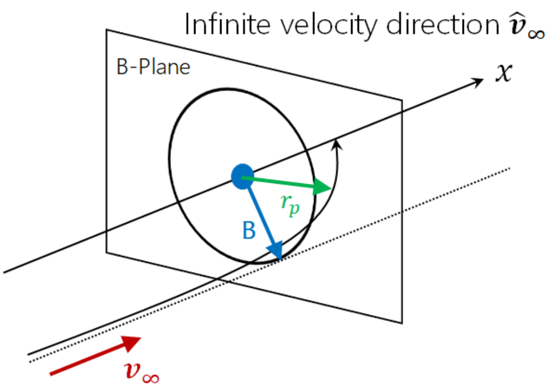

Fig. **5**: Definition of B-plane.

Specifically, to find $\boldsymbol{B}$, we make a vector $\boldsymbol{r}_{final}$ using the viewpoint of $\boldsymbol{v}_\infty$ from the center of the Earth. Find the angle $\cos\theta_s$ between $\boldsymbol{r}_{final}$ and $\boldsymbol{v}_\infty$ by taking the inner product.

$$\cos\theta_s = \frac{\boldsymbol{v}_\infty \cdot \boldsymbol{r}_{final}}{|\boldsymbol{v}_\infty||\boldsymbol{r}_{final}|} \tag{13}$$

Find B.

$$\boldsymbol{B} = r_{final\_point} - b_{start\_point} \tag{14}$$

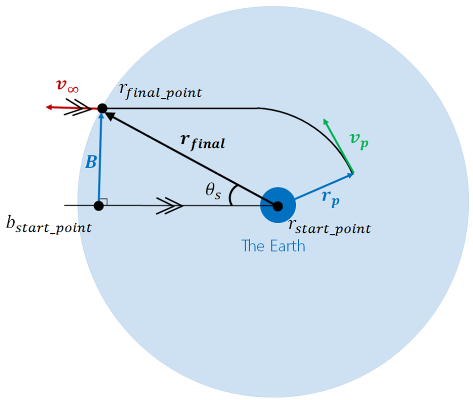

Fig. **6**: Calculation of B

3. Simulation Results

3. 1 Simulation for two-body problem

The parameters used in the simulation conditions for the two-body problem are shown in Table **1**.

Table **1**: Simulation condition values for two-body problems

Earth's gravity constant μ_e[m^3/s^2]	:	3.986×10^5
Sun's gravity constant μ_s[m^3/s^2]	:	1.327×10^{11}
Earth's revolution radiusr_{se} [km]	:	1.496×10^8
Earth revolution cycle T_e [s]	:	3.156×10^7
r_{SOI} [km]	:	9.247×10^5
Peripheral distance r_p [km]	:	1.000×10^4
Peripheral speed v_p [km/s]	:	8.929
The angle of the near point vector θ_{r_p} [deg]	:	1.192×10^{-3}

The plane spanned by the velocity vector v_∞ and $\boldsymbol{B}$ at infinity is defined as the orbital plane. As shown in Fig. **7**, let θ_{deep} be the angle between $\boldsymbol{B}$ and $\boldsymbol{v_p}$ in the orbital plane. Also, as shown in Fig. **8**, let θ_{cross} be the angle between $\boldsymbol{B}$ and $\boldsymbol{v_p}$ around the x axis. The simulation results using $\boldsymbol{B}$ and $\boldsymbol{r}_p$ are

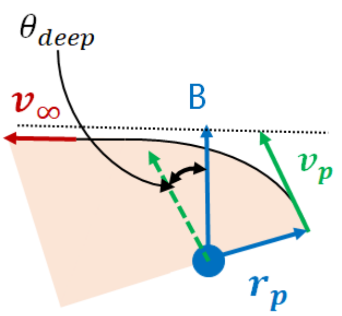

Fig. **7**: Definition of find θ_{deep}

shown in Fig. **11** and Fig. **12**.θ_{deep} was constant at -5.969deg and θ_{cross} was almost constant at 0.00deg, where, the radius of the circle indicates the size of $\boldsymbol{B}$, and the color bar indicates

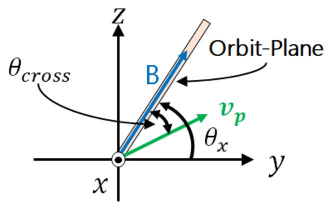

Fig. **8**: Definition of find θ_{cross}

the size of each angle.

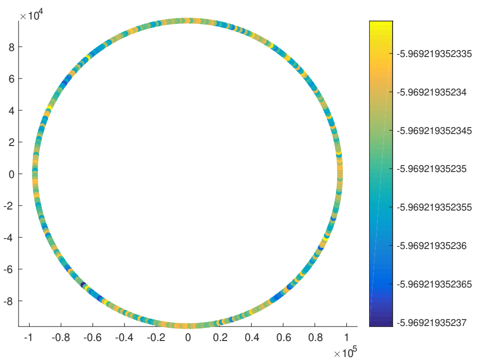

Fig. **9**: Angle θ_{deep} between $\boldsymbol{B}$ and $\boldsymbol{v_p}$ in the orbital plane

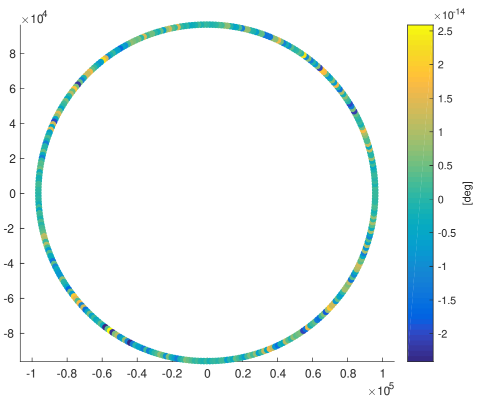

Fig. **10**: The angle θ_{cross} between $\boldsymbol{B}$ and $\boldsymbol{v_p}$ around the x axis

3. 2 Simulation for three-body problem

Table **2** shows the parameters used in the simulation conditions for the three-body problem. This time, in the three-body problem of the sun-earth system, the Earth's revolution cycle is 2π and the earth's revolution radius is 1.

Table **2**: Simulation condition values for three-body problems

Non-dimensional gravity constant μ_{se}[−]	:	3.003×10^{-6}
Earth's revolution radiusr_{se} [-]	:	1.000
Earth revolution cycle T_e [-]	:	2π
r_{SOI} [-]	:	6.181×10^{-3}
Peripheral distance r_p [-]	:	6.6845×10^{-5}
Peripheral speed v_p [-]	:	0.2998
The angle of the near point vector θ_{r_p} [deg]	:	1.192×10^{-3}

The vector $\boldsymbol{B}$ and B-Plane in the same way as in the two-body simulation. Based on this $\boldsymbol{B}$, the values of θ_{deep} and θ_{cross} are found. However, the radius of the circle indicates

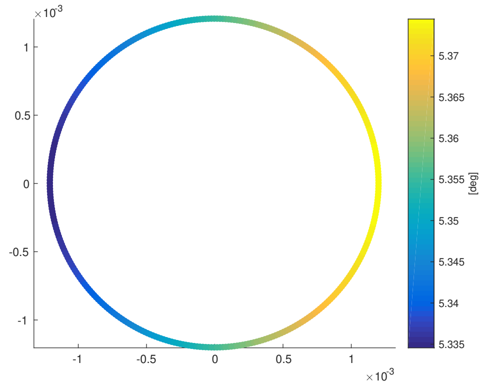

Fig. **11**: Angle θ_{deep} between $\boldsymbol{B}$ and $\boldsymbol{v_p}$ in the orbital plane

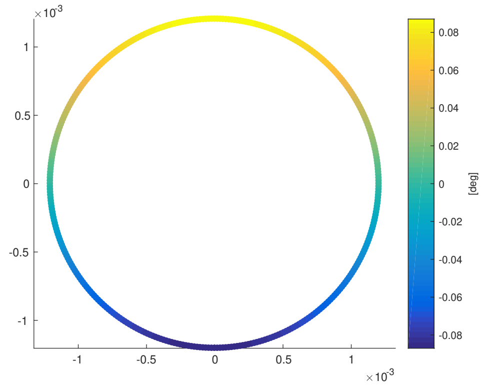

Fig. **12**: The angle θ_{cross} between $\boldsymbol{B}$ and $\boldsymbol{v_p}$ around the x axis

the size of $\boldsymbol{B}$, and the color bar indicates the size of each angle.

It was found that θ_{deep} takes the maximum value when θ_x = 180deg, and takes the minimum value when θ_x = 360deg. It was also found that θ_{cross} takes the maximum value when θ_x = 91deg, and takes the minimum value when θ_x = 269deg.

4. Conclusion

In this study, we investigated the effect of the B-Plane used in the trajectory design for the two-body problem on the parameters constituting the B-Plane when the three-body problem was designed with the dynamics. It was confirmed that the B-Plane changed under the influence of the Sun. Future work includes the analysis of swing-by trajectories with different energies to give more physical insights into trajectory design.

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