

SPHERICAL COLLECTORS VERSUS BARE TETHERS FOR DRAG, THRUST, AND POWER GENERATION

Juan R. Sanmartin

Universidad Politecnica de Madrid, ph. 34-91-336 6302, fax 34-91-336 6303, jrs@faia.upm.es

Enrico Lorenzini

Harvard Smithsonian Center for Astrophysics, ph. 1-617-495 7211, fax 1-617-496 9323

Abstract

Performances of ED-tethers using either spherical collectors or bare tethers for drag, thrust, or power generation, are compared. The standard Parker-Murphy model of current to a full sphere, with neither space-charge nor plasma-motion effects considered, but modified to best fit TSS1R results, is used (the Lam, Al'pert/Gurevich space-charge limited model will be used elsewhere). In the analysis, the spherical collector is assumed to collect current well beyond its random-current value (thick-sheath). Both average current in the bare-tether and current to the sphere are normalized with the short-circuit current in the absence of applied power, allowing a comparison of performances for all three applications in terms of characteristic dimensionless numbers. The sphere is always substantially outperformed by the bare-tether if ohmic effects are weak, though its performance improves as such effects increase.

I. Introduction

A comparison of the performances of an electrodynamic (ED) tether using the tether itself, left bare, for electron (anodic) collection, and an ED tether using a large sphere as anodic device, as used in the TSS1 and TSS1R tether missions, involves a proper evaluation of the current-collection capabilities. This requires carrying the respective current laws to some common formulation, which is discussed in the next section. Independently, gauging performance is different for drag (or deorbit), power-generation, and thrust applications, and is carried out separately in Secs. III, IV, and V, respectively.

In this work we shall assume that current to the sphere follows a modified Parker-Murphy law, as claimed describing TSS1R results¹ (the well known space-charge-limited current law, also claimed as fitting those results,² will be considered elsewhere). That modified PM law is

$$I_{sph} = J_{th} \times 4\pi R_{sph}^2 \times a f_{PM}(\psi), \quad \psi = \frac{e\Phi_{sph}}{kT} \left(\frac{l_e}{R_{sph}} \right)^2, \quad (1a, b)$$

where R_{sph} and Φ_{sph} are the sphere radius and bias, and T is plasma temperature. Also, $a \approx 2.5$ is a fitting factor coming from TSS1R results, and J_{th} , f_{PM} and l_e are the thermal or random current density, the Parker-Murphy law function, and the electron thermal gyroradius, respectively,

$$J_{th} = eN_\infty \sqrt{\frac{kT}{2\pi m_e}}, \quad f_{PM} = \frac{1 + \sqrt{8\psi}}{2}, \quad l_e = \frac{\sqrt{m_e kT}}{eB}, \quad (2a-c)$$

with N_∞ being plasma density, and B a magnetic field. We shall assume that the sphere is an efficient collector in the sense that it is not so large that it just collects the thermal current. This requires ψ not to be small in (2b), the factor $\frac{1}{2}$ describing magnetic guiding of electrons.

We shall also assume that the bare-tether current follows the OML (orbital-motion-limited) collection law for a cylinder, whether a tape or a round wire. For an uniform bias Φ_{cyl} , this law is

$$I_{cyl}(OML) \approx J_{th} \times pL \times \sqrt{\frac{4}{\pi} \frac{e\Phi_{cyl}}{kT}} = eN_\infty \frac{pL}{\pi} \times \sqrt{\frac{2e\Phi_{cyl}}{m_e}}, \quad (3)$$

where p and L are cross-section perimeter and length of the cylinder. In the case of a bare tether, the current \bar{I}_{bt} involves some average of bias over the collecting length, which may be different for drag, thrust, and power generation applications.

II. Normalized Currents

The double appearance of the radius R_{sph} in Eq.(1a) makes comparing performances difficult. It thus proves convenient to use Eq.(1b) itself to rearrange (1a) so that R_{sph} only appears once (through its ψ -dependence),

$$I_{sph} = J_{th} \times 4\pi l_e^2 \times a \frac{f_{PM}(\psi)}{\psi} \times \frac{e\Phi_{sph}}{kT}. \quad (4)$$

We note that the ratio $f_{PM}(\psi)/\psi$ in Eq. (4) decreases with ψ , keeping about unity over a wide $\psi > 1$ interval; it ranges from 1.91 at $\psi=1$ to 0.38 at $\psi=16$. TSS1R results reach as high as $\psi \sim 10$.^{1,2}

It is also convenient to normalize currents with the short-circuit value, $\sigma_c E_m A$, which is a bound to current in case of drag or power-generation applications; here σ_c , A and E_m are tether conductivity and cross-section area, and motional field, respectively. As we shall see, ohmic limitations are fundamental in the comparison between bare tethers and TSS1R-like spheres. We first introduce dimensionless values

$$i_{sph} = \frac{I_{sph}}{\sigma_c E_m A}, \quad \frac{\bar{I}_{bt}}{\sigma_c E_m A} \equiv \bar{i}_{bt}, \quad (5a, b)$$

and next introduce a characteristic length L^* that gauges bare-tether collection impedance against the ohmic impedance,³

$$\frac{4}{3} e N_\infty \frac{pL^*}{\pi} \sqrt{\frac{2eE_m L^*}{m_e}} \equiv \sigma_c E_m A. \quad (6)$$

For the relevant aluminum case one has

$$L^* = 1.47 \text{ km} \times \left(\frac{E_m}{150 \text{ V/km}} \right)^{1/3} \left(\frac{h}{0.1 \text{ mm}} \times \frac{3 \times 10^{11} \text{ m}^{-3}}{N_\infty} \right)^{2/3}, \quad (7)$$

where h is thickness for a thin tape and radius for a round wire.

Results for \bar{i}_{bt} versus L/L^* (and additional dimensionless numbers appropriate to the application considered) have been determined in the past and will be recalled in Secs. III-V. As regards current to a sphere, we directly obtain

$$i_{sph} = i^* \times \frac{\Phi_{sph}}{E_m L^*}, \quad i^* \equiv \frac{3\pi^{3/2} a}{2} \times \frac{f_{PM}(\psi)}{\psi} \frac{l_e}{p} \sqrt{\frac{eE_m l_e}{kT}} \sqrt{\frac{l_e}{L^*}}. \quad (8a, b)$$

For typical values

$$l_e \approx 2.5 \text{ cm}, \quad E_m \approx 150 \text{ V/km}, \quad kT = 0.15 \text{ eV},$$

and, say, $L^* = 1.5 \text{ km}$ in case of a thin tape, we have

$$i^* \approx 0.003 \frac{f(\psi)}{\psi} \times \frac{2l_e}{p}, \quad (9)$$

i^* thus turning out to be very small, typically about $10^{-2} - 10^{-3}$. Clearly, L^* would be always much larger for a round wire.

III. Deorbit missions

For the sphere, neglecting the small voltage drop at a hollow cathode that would establish electrical contact at the cathodic end, the total induced voltage would here equal the sphere bias plus the ohmic voltage drop,

$$E_m L = \Phi_{sph} + Z_t I_{sph} \quad (10)$$

where $Z_t = L/\sigma_c A$ is the tether resistance. One then finds $\Phi_{sph} = E_m L (1 - i_{sph})$, Eq. (8a) now becoming

$$i_{sph} = \frac{i^* \times L/L^*}{1 + i^* \times L/L^*}. \quad (11)$$

As regards the bare tether, \bar{i}_{bt} is a function of just the ratio L/L^* , which the analysis in Ref. 4 easily shows to be defined as

$$\bar{i}_{bt} = 1 - \frac{L^*}{L} \times \phi_A, \quad \int_0^{\phi_A} \frac{d\phi}{\sqrt{1 + \phi^{3/2} - \phi_A^{3/2}}} = \frac{L}{L^*}, \quad \text{for } L/L^* < 4, \quad (12a)$$

with $\bar{i}_{bt} \approx 0.3 \times (L/L^*)^{3/2}$ for L/L^* small; and

$$\bar{i}_{bt} = 1 - \frac{L^*}{L}, \quad \text{for } L/L^* > 4. \quad (12b)$$

To gauge performances we consider the ratio between the mass of the system dedicated to producing thrust, and the total impulse of the deorbiting mission, written as the product of drag-force F times the mission duration τ . That ratio, which should be as small as possible, is the inverse of the velocity of exhaust gases ($g_0 \times \text{specific impulse}$) in the case of chemical propulsion, where system mass is basically propellant mass. In the case of electrical propulsion one must also allow for the mass of a power supply. An ED-tether needs no power supply for deorbiting, and consumes very little expellant (mass) at the hollow cathode. Dedicated mass is then basically tether mass times some factor $\alpha_t \sim 2.5$ that accounts for tether-related hardware (*deployer / end ballast*).

Writing F as *average current* times LB , we have

$$\frac{\text{Dedicated mass}}{\text{Mission impulse}} \approx \frac{\alpha_t \rho AL}{F \tau} = \frac{\alpha_t \rho U_{orb}}{\sigma_c E_m^2 \bar{i} \tau}, \quad (13)$$

where \bar{i} is given by Eqs.(11) and (12a, b) for sphere and bare-tether, respectively. Since all other factors in (13) may have values common to both systems, performance is measured by the dimensionless average current, which should be as large as possible.

Both i_{sph} and \bar{i}_{bt} approach unity at large L/L^* , with ohmic effects limiting current collection in either case. This occurs for a bare tether at moderate L/L^* values, whereas for the sphere it occurs at much larger values. These are easier to reach with long, thin tapes at daytime. For $L = 20$ km, $N_\infty = 10^{12} \text{ m}^{-3}$, tape *thickness* $h = 0.1$ mm and *width* $= 25$ mm, and $\psi \sim 3$ (corresponding to a 4m sphere diameter), we find $i_{sph} \sim 0.12$ (note that $i^* \times L/L^*$ is independent of E_m). With $L^{*3/2} \propto N_\infty$, current to the sphere would drop by a factor of 10 at night, effectively switching off the drag F . For the sphere we would then have a normalized current

$$\bar{i}(\text{sphere}) = f_d \times i_d = f_d \times \frac{i_d^* \times L / L_d^*}{1 + i_d^* \times L / L_d^*} \quad (14)$$

where f_d is the *noneclipse* or *daytime* fraction, and i_d^* , L_d^* are i^* , L^* computed at day density. For a given (deorbiting) mission impulse, the bare-tape system appears then to be 10-20 times lighter than the system using a spherical anode.

IV. Power generation

For the sphere we would have

$$E_m L = \Phi_{sph} + (Z_t + Z_l) I_{sph}. \quad (15)$$

As with all generators, the impedance ratio Z_l / Z_t determines the efficiency η_g in taking energy (from the orbital motion) into useful energy at an electrical load of impedance Z_l in the tether circuit. Using

$$\eta_g = \frac{Z_l I_{sph}^2}{E_m L I_{sph}} = \frac{Z_l}{Z_t} i_{sph} \quad (16)$$

instead of Z_l / Z_t as free parameter in (15) leads to $\Phi_{sph} = E_m L (1 - i_{sph} - \eta_g)$, Eq. (8a) now becoming

$$i_{sph} = (1 - \eta_g) \times \frac{i^* \times L / L^*}{1 + i^* \times L / L^*}. \quad (17a)$$

As regards the bare tether, \bar{i}_{bt} is here a function of the ratio L/L^* and Z_L/Z_t , or alternatively η_g . From the analysis in Ref. 3 and assuming a large L/L^* ratio one finds

$$\bar{i}_{bt} \approx 1 - \eta_g - \frac{L^*}{L} \times F(\eta_g), \quad (17b)$$

$$(1 - \eta_g) \times F(\eta_g) \equiv \frac{4}{3} \eta_g^2 \int_0^1 \frac{d\eta}{(\eta^2 - \eta_g^2)^{2/3}} - (2\eta_g - 1) \times (1 - \eta_g^2)^{2/3}.$$

At an ED-tether, power is generated at the expense of orbital energy; a tether can thus serve as a primary power source only for very short times. It turns out, however, that for longer than 1 - 2 weeks, with solar power not available, power generation by a combination *ED-tether / rocket*, with the tether providing electrical power and the chemical rocket providing thrust to compensate the magnetic drag on the tether, proves more efficient as regards fuel consumption than the alternative power source, which would be a fuel cell for direct generation. Note that the magnetic power $\dot{m}_{rk} v_{rk} U_{orb}$ (magnetic drag F being assumed equal to rocket thrust $\dot{m}_{rk} v_{rk}$ to keep the orbit stationary) is greater than the rocket output power $\dot{m}_{rk} v_{rk}^2 / 2$. Here \dot{m}_{rk} and v_{rk} are propellant mass-flow-rate and velocity at the rocket exhaust; for LOX-LH₂ (*specific impulse* ~ 460 s), we have $v_{rk} \sim 4.5$ Km/s $< 2U_{orb} \sim 15$ km/s. The decrease in rocket energy of motion due to the fuel-mass loss makes for the excess over the rocket output.

Performance is gauged by the ratio between the mass of the system dedicated to producing power, and the total energy generated, written as the product of mission duration τ times the electrical power generated, $W_g = \eta_g \times FU_{orb}$. System mass is basically made of rocket propellant mass and tether related mass,

$$\frac{\text{Dedicated mass}}{\text{Energy generated}} \approx \frac{\dot{m}_{rk} \tau + \alpha_t \rho AL}{\eta_g FU_{orb} \tau}. \quad (18)$$

Using $\dot{m}_{rk} v_{rk} = \bar{I}LB$ we find

$$\frac{\text{Dedicated mass}}{\text{Energy generated}} = \frac{1}{\eta_g U_{orb} v_{rk}} + \frac{\alpha_t \rho}{\eta_g \sigma_c E_m^2 \bar{i} \tau} \propto \frac{1}{\eta_g} \left[1 + \frac{\tilde{\tau} / \tau}{\bar{i}(\eta_g)} \right], \quad (18')$$

$$\tilde{\tau} \equiv \alpha_t \rho U_{orb} v_{rk} / \sigma_c E_m^2 \sim 0.5 \text{ weeks} \quad (19)$$

for aluminum and $E_m \sim 150$ V/km.

At the large L/L^* values favoring the spherical collector we just set $\bar{i}_{bt} \approx 1 - \eta_g$ in (16b) for the bare tether. For the sphere, with drag again effectively switched off at night, we set $\bar{i}(\text{sphere}) \approx (1 - \eta_g) f_d i_d$, with i_d as given by Eq. (14). For a given mission duration τ , the ratio in (18') has a minimum at

$$\frac{2\eta_g - 1}{(1 - \eta_g)^2} = \frac{\tau}{\tilde{\tau}} \quad \text{for the bare tether,} \quad (20a)$$

$$= \frac{f_d i_d \times \tau}{\tilde{\tau}} \quad \text{for the sphere.} \quad (20b)$$

One finds $\eta_g \approx 0.5$ at short times; $\eta_g \approx 0.59$ at $\tau = \tilde{\tau}$ (or $\tilde{\tau} / f_d i_d$); $\eta_g \approx 1 - \sqrt{\tilde{\tau} / \tau}$ at long times. For the case considered in Sec.III, having $1/f_d i_d$ about 15, the system with the spherical collector is heavier than the bare-tether system by a factor of 10 for mission duration $\tau \approx \tilde{\tau}$, and a factor of 4.5 for $\tau \approx \tilde{\tau} / f_d i_d$, or about two

months. Both systems would have similar masses for durations much longer than two months, where system mass is basically propellant mass for the rocket.

V. Thrusting missions

For the sphere we would now have

$$W_s = \varepsilon_s \times I_{sph} = (E_m L + \Phi_{sph} + I_{sph} Z_t) \times I_{sph}, \quad (21a)$$

where W_s and ε_s are the supply power and voltage, with I_{sph} and Φ_{sph} related by Eq. (8a).

In thrusting, a bare tether is more efficient if some upper segment is insulated.³ The analysis in Ref.5 shows that at low L^*/L , the lower segment left bare should be a small fraction of the full tether length.⁵ Then the required power supply is just

$$W_s = \varepsilon_s \times \bar{I}_{bt} = (E_m L + \bar{I}_{bt} Z_t) \times \bar{I}_{bt}, \quad (21b)$$

corresponding to negligible impedance in bare-tether collection.

To gauge performances we consider the ratio between the mass of the system dedicated to producing thrust, and the total impulse of the deorbiting mission,

$$\frac{\text{Dedicated mass}}{\text{Mission impulse}} = \frac{\beta W_s + \alpha_t \rho A L}{F \tau} \quad (22)$$

where β is the inverse specific power of the supply. Typically, β is a few tens of kg/kW, where a dedicated solar-power is required, as in the case of a 'space-tug'; and less than 10 kg/kW, where a solar-power system can be taken for free, as in the case of reboost of the International Space Station.

For the bare tether one then finds

$$\frac{\text{Dedicated mass}}{\text{Mission impulse}} = \frac{\beta U_{orb}}{\tau} \left(1 + \bar{i}_{bt} + \frac{1}{\tilde{E}_m^2 \bar{i}_{bt}} \right). \quad (23a)$$

The expression in the parenthesis is minimum for W_s such that $\bar{i}_{bt} = 1/\tilde{E}_m$, yielding a minimum *mass-to-impulse* ratio

$$\frac{\beta U_{orb}}{\tau} \left(1 + \frac{2}{\tilde{E}_m} \right).$$

Here, \tilde{E}_m is a normalized value of the motional field, $E_m / \sqrt{\alpha_t \rho / \beta \sigma_c}$, which is typically near unity.

For the tether with a sphere, taking Φ_{sph} from Eq. (8a), one finds

$$\frac{\text{Dedicated mass}}{\text{Mission impulse}} \approx \frac{\beta U_{orb}}{f_d \tau} \left[1 + \frac{i(sph)}{i_d} + \frac{1}{\tilde{E}_m^2 i(sph)} \right]. \quad (23b)$$

A minimum now occurs at $i(sph) = \sqrt{i_d} / \tilde{E}_m$ yielding a *mass-to-impulse* ratio

$$\frac{\beta U_{orb}}{f_d \tau} \left(1 + \frac{2}{\tilde{E}_m \sqrt{i_d}} \right).$$

For a given mission impulse in the case considered in Sec.III, the system with the spherical collector would be heavier than the bare-tether system by a factor of about 4.

VI. Conclusions

We have shown that, in the absence of ohmic effects, bare tethers substantially outperform tethers using a sphere as passive anodic collector. For wires with not unreasonably thin cross sections, ohmic effects are typically small. On the other hand, ohmic effects on (long) thin tapes, may be strong. The greater current-collecting capability of a bare tape makes it more vulnerable to ohmic limitations. As a result, spherical-collector performances do approach performances by bare tether. Systems with tapes 20 km long and as thin as 0.1 mm, left bare, are lighter than insulated-tape systems with collecting spheres, by factors of about 10-15 in deorbiting; about 6-8 in power generation; and about 4 in thrusting.

References

- 1 D.C. Thompson, C. Bonifazi, B.E. Gilchrist, S.D. Williams, W.J. Raitt, J.P. Lebreton, W.J. Burke, N.H. Stone and K.H. Wright, Jr., *Geophys. Res. Lett.* **25**, 413-416 (1998).
- 2 G. Vannaroni, M. Dobrowolny, J.P. Lebreton, E. Melchioni, F. De Venuto, C.C. Harvey, L. Iess, U. Guidoni, C. Bonifazi and F. Mariani, *Geophys. Res. Lett.* **25**, 749-752 (1998).
- 3 J.R. Sanmartin, M. Martinez-Sanchez, and E. Ahedo, *J. Prop. Power* **9**, 353-360 (1993).
- 4 E. Ahedo and J.R. Sanmartin, *J. Space. Rockets* **39**, 198-205 (2002).
- 5 J.R. Sanmartin, R.D. Estes, E.C. Lorenzini and S.A. Elaskar, *The Efficiency of Electrodynamic Tether Thrusters*, submitted to *J. Space. Rockets*.