Attitude Control of Solar Sail-craft using Pseudo Equilibrium Point and its Application to Space Missions

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This study proposes an attitude control method of solar sail-craft. In spinning solar sail-craft, characteristic movements such as vortex motion (attitude drift motion) in which the spin axis tilts due to the influence of solar radiation pressure are observed. Therefore by inputting a bias input to create a pseudo equilibrium point , an arbitrary attitude is realized. Furthermore, stability based on pseudo equilibrium point is analyzed from eigenvalues of the system. By considering the pseudo equilibrium point and the control method using thermal radiation pressure of solar cells which is a new control method, the feasibility of application to the OKEANOS missions is clarified.

ソーラーセイルの擬似平衡点を用いた姿勢制御法とミッションへの応用

摘要

本研究では，擬似平衡点を用いたソーラーセイルの姿勢制御法を提案する．スピン型ソーラーセイルに姿勢運動はある平衡点の周りで姿勢ドリフト運動といった特徴的な運動をする．そこで平衡点を擬似的に作るバイアストルクを加えることで任意の姿勢を実現する．この擬似平衡点と新しい制御法である太陽電池の熱輻射圧を利用した制御を考えることで，特に OKEANOS ミッションを想定しフィージビリティを明らかにする．

Nomenclature

A= In-plane angle

δ= Out-of-plane angle

α_0= In-plane biased (pseudo) equilibrium point

δ_0= Out-of-plane biased (pseudo) equilibrium point

C_s, C_d, C_a= specular reflectivity, diffusive reflectivity, absorptivity

H= center of gravity offset

Ω= spin rate

ω_0= orbital angular velocity

T_x, T_y= solar radiation pressure

I_S, I_T= moment of inertia for the spin-axis, the orthogonal axis to the spin-axis

H= deflection angle

ξ= distortion angle

T_{cx}, T_{cy}= bias input

1. Introduction

Today, many interplanetary spacecrafts using solar radiation pressure are realized. As representative examples using solar radiation pressure for propulsion and attitude control, small solar power sail demonstrator IKAROS [1] and Asteroid explorer Hayabusa 2[2] can be mentioned. When the spacecraft has angular momentum, there are characteristic movements such as vortex motion[3] as figure.1. JAXA is developing solar power sail-craft OKEANOS that will launch on 2020s in figure 2. OKEANOS is spinning solar sail same as IKAROS. Thus occurring the vortex attitude motion is expected. Thus attitude stabilization is the most important subject.

Various attitude control of the spacecraft having angular momentum has been considered. In Hayabusa 2, attitude control was realized by one wheel control that changes angular momentum using only one reaction wheel [4]. Also, IKAROS demonstrated a method to control the attitude by changing the optical characteristics such as reflected control device [5]. Besides, the control method using thermal radiation pressure by turning ON/OFF of the solar cells is considered. Since solar cells are mounted for power generation, there is an advantage that there is no need to mount an extra control device if it can satisfy the power requirements of the spacecraft. In addition, since it is attitude control using solar radiation, it does not consume fuel and completely undisturbed control becomes possible, so that there is an advantage that high precision control requirements can also be satisfied.

Based on these control torque, this study propose attitude control method. In actual IKAROS operation, using vortex motion efficiently can be achieved. This this study also proposes attitude stabilization method taking advantage of vortex motion.

The objectives of this study is obtain the control law of solar sail. Especially this paper focuses on solar power sail-craft "OKEANOS". This paper first presents control law using biased (pseudo) equilibrium point, and then indicates the numerical analysis results by utilizing the proposed control method.

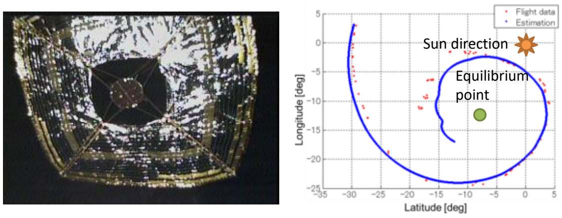

Figure 1. IKAROS actual picture taken by deployable camera (left), IAKROS flight data (right)

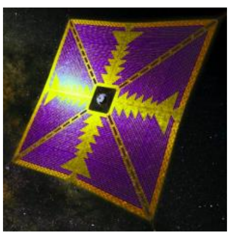

Figure 2. Solar power sail-craft "OKEANOS"

2. Definition of coordinate system

In this paper, projection plane is used to show visually the attitude motion of spinning type spacecraft, as illustrated in figure 3. Our interest is only in the relative motion of spin axis direction compared with sun direction. The plane consists of two direction such as In-plane angle and Out-of-plane angle. This paper define that α is In-plane angle and δ is Out-of-plane angle.

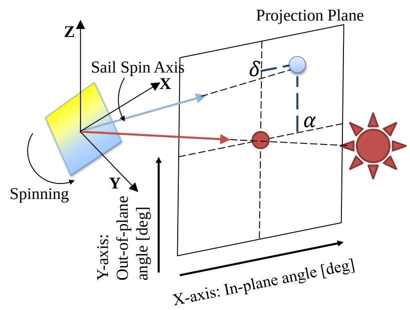

Figure 3. Projection plane and coordinate.

3. Generalized spinning sail model

From past research of IKAROS motion, generalized spinning sail model can be derived. The shape of spinning type solar sail is approximated to circular shape. Comparing with comparing normal spinning type solar sail, the deformation of membrane such as deflection and distortion angle considered in this sail model.

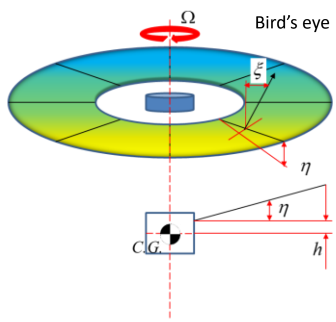

Figure 4. Spinning sail model.

By using this circular model, SRP torque can be derived as (1) to (4).

$$p_{n1} = P(2C_s), p_{n2} = P\left(B_f C_d + \kappa C_a\right),$$
$$p_s = P(C_a + C_d)\left(p = \frac{S_0}{c}\left(\frac{1}{AU}\right)^2\right) \quad (1)$$

$$\cos\beta = \cos\alpha \cos\delta \quad (2)$$

$$\begin{bmatrix} \frac{T_x^*}{\sin\beta} \\ \frac{T_y^*}{\sin\beta} \\ T_z^* \end{bmatrix} = \frac{\pi R^3}{3} \begin{bmatrix} -(2p_{n1}\cos\beta + p_{n2} + p_s\cos\beta) \\ -(2p_{n1}\cos\beta + p_{n2})\eta - 3p_s\left(\eta + \frac{h}{R}\right)\cos\beta \\ (2p_{n1}\cos^2\beta + 2p_{n2}\cos\beta - p_s\sin^2\beta)\xi \end{bmatrix} \quad (3)$$

$$\begin{bmatrix} T_x \\ T_y \\ T_z \end{bmatrix} = \begin{bmatrix} \sin\alpha & \cos\alpha\sin\delta & \\ -\cos\alpha\sin\delta & \sin\alpha & \\ & & 1 \end{bmatrix} \begin{bmatrix} \frac{T_x^*}{\sin\beta} \\ \frac{T_y^*}{\sin\beta} \\ T_z^* \end{bmatrix} \quad (4)$$

In this study, spin rate is constant for simplicity. Thus influence of T_z can be neglected. Considering this SRP torque, the equation of motion can be derive as (5).

$$\begin{aligned} &\ddot{\alpha}\cos\delta - 2(\dot{\alpha}-\omega_0)\dot{\delta}\sin\delta - \frac{I_S}{I_T}\Omega\dot{\delta} = -\frac{T_y}{I_T} \\ &\ddot{\delta} + (\dot{\alpha}-\omega_0)^2\sin\delta\cos\delta + \frac{I_S}{I_T}\Omega(\dot{\alpha}-\omega_0)\cos\delta = -\frac{T_x}{I_T} \end{aligned} \quad (5)$$

4. Equilibrium point

From Equations (4), α and δ are divided into two components as Equation (6).

$$\begin{aligned} \alpha &= \alpha_0 + d\alpha \\ \delta &= \delta_0 + d\delta \end{aligned} \quad (6)$$

The first terms of the Equations (6) show equilibrium point and the second term show circular motion. Furthermore the circular motion can be divided into precession and nutation. Therefore, the stabilization of system depends on the second term.

This paper proposes the biased (pseudo) equilibrium point as control method. Spinning motion converges or diverges to equilibrium point. Taking advantage of this vortex motion, slight change of equilibrium point lead to stabilization of the motion. Only constant input achieves this proposed method.

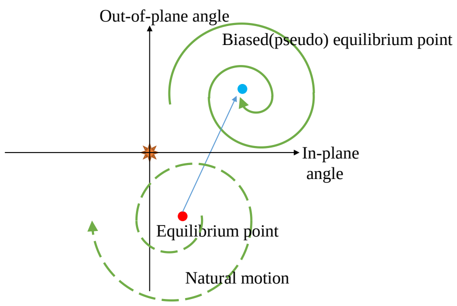

Figure 5. Biased equilibrium point.

To achieve this proposed method, the equation of motion including bias input is derived as equations (7).

$$\begin{aligned} &\ddot{\alpha}\cos\delta - 2(\dot{\alpha}-\omega_0)\dot{\delta}\sin\delta - \frac{I_S}{I_T}\Omega\dot{\delta} = -\frac{T_y}{I_T} + \frac{T_{cy}}{I_T} \\ &\ddot{\delta} + (\dot{\alpha}-\omega_0)^2\sin\delta\cos\delta + \frac{I_S}{I_T}\Omega(\dot{\alpha}-\omega_0)\cos\delta = -\frac{T_x}{I_T} + \frac{T_{cx}}{I_T} \end{aligned} \quad (7)$$

Now if the variable is defined as $\alpha = \alpha_0, \delta = \delta_0, \dot{\alpha} = \dot{\delta} = \ddot{\alpha} = \ddot{\delta} = 0$, bias torque can be derived as equation (8).

$$\begin{aligned} T_{cx} &= I_T\omega_0^2\sin\delta_0\cos\delta_0 - I_S\Omega\omega_0\cos\delta_0 + T_x \\ T_{cy} &= T_y \end{aligned} \quad (8)$$

If this bias is continuously applied as the control torque, the motion becomes stable around the biased equilibrium point. As the source of this bias torque, reflectivity control device that is used IKAROS mission, and thermal radiation pressure of solar cells are considered. Especially thermal radiation pressure is new idea of attitude control. The heat is released from the surface of solar cells. Reaction of the thermal radiation applies the force on the surface. By using the thermal radiation pressure, the thermal radiation torque acts on the spacecraft and this torque can be used for control input. Specifically, the control is realized by changing the ON and OFF states synchronized with spacecraft's spinning.

5. Linearization and eigenvalue analysis

Equations (5) can be linearized around equilibrium point by using first-order approximation of Taylor expansion. The equation of motion can be expressed in state matrix as Equation (9).

The stability of system can be obtained from eigenvalue. This state matrix consists of 4 rows and 4 columns. Therefore the eigenvalue number is 4. There are 3 patterns of eigenvalues such as 4 conjugate complex numbers as Equation (18) or 2 conjugate complex numbers and 2 real numbers as Equation (19) or 4 real numbers as Equation (20).

$$\begin{bmatrix} d\ddot{\alpha} \\ d\ddot{\delta} \\ d\dot{\alpha} \\ d\dot{\delta} \end{bmatrix} = \begin{bmatrix} 0 & \frac{\tilde{\Omega} - 2\omega_0 \sin\delta_0}{\cos\delta_0} & \frac{\pi R^3}{3I_T \cos\delta_0}\left[-\{(2p_{n1} + p_s)A + p_{n2}C\}\xi + \{(2p_{n1} + 3p_s)E + p_{n2}G\}\eta + \frac{3h}{R} p_s E \right] & \frac{\pi R^3}{3I_T \cos\delta_0}\left[-\{(2p_{n1} + p_s)B + p_{n2}D\}\xi + \{(2p_{n1} + 3p_s)F + p_{n2}H\}\eta + \frac{3h}{R} p_s F \right] \\ -\tilde{\Omega}\cos\delta_0 + 2\omega_0 \sin\delta_0 \cos\delta_0 & 0 & \frac{\pi R^3}{3I_T}\left[\{(2p_{n1} + p_s)E + p_{n2}G\}\xi + \{(2p_{n1} + 3p_s)A + p_{n2}C\}\eta + \frac{3h}{R} p_s A \right] & \frac{\pi R^3}{3I_T}\left[\{(2p_{n1} + p_s)F + p_{n2}H\}\xi + \{(2p_{n1} + 3p_s)B + p_{n2}D\}\eta + \frac{3h}{R} p_s B \right] - {\omega_0}^2 (\cos^2\delta_0 - \sin^2\delta_0) - \tilde{\Omega}\omega_0 \sin\delta_0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} d\dot{\alpha} \\ d\dot{\delta} \\ d\alpha \\ d\delta \end{bmatrix}$$

(9)

$$A = -2\sin\alpha_0 \sin\delta_0 \cos\alpha_0 \cos\delta_o \tag{10}$$

$$B = \cos^2\alpha_0 \cos^2\delta_0 - \sin^2\delta_0 \cos^2\alpha_0 \tag{11}$$

$$C = -\sin\delta_0 \sin\alpha_0 \tag{12}$$

$$D = \cos\delta_0 \cos\alpha_0 \tag{13}$$

$$E = -\sin^2\alpha_0 \cos\delta_0 + \cos^2\alpha_0 \cos\delta_0 \tag{14}$$

$$F = -\sin\alpha_0 \sin\delta_0 \cos\alpha_0 \tag{15}$$

$$G = \cos\alpha_0 \tag{16}$$

$$H = 0 \tag{17}$$

$$\lambda = \begin{cases} a_1 \pm b_1 i \\ c_1 \pm d_1 i \end{cases} \qquad (18)$$

$$\lambda = \begin{cases} a_2 \pm b_2 i \\ c_2 \\ d_2 \end{cases} \qquad (19)$$

$$\lambda = \begin{cases} a_3 \\ b_3 \\ c_3 \\ d_3 \end{cases} \qquad (20)$$

In many cases, eigenvalue has Equation (18) pattern. This eigenvalues express two circular motions such as nutation and precession. If the components number can be defined as $b_1<d_1$, a_1 means precession divergence speed and c_1 means nutation divergence speed. The motions are assumed to 4 patterns from combinations of the real parts. The state of motions can be expressed as figure 6.

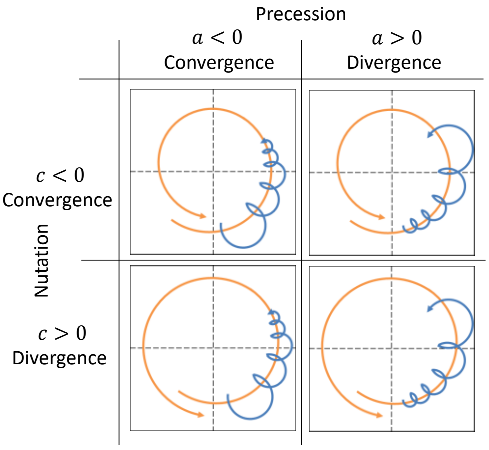

Figure 6. Combinations of real numbers.

6. Simulations

This section prove that the state motion can be assumed from eigenvalues, and the slight change of equilibrium point lead to stabilization of the motion.

With reference to the OKEANOS configuration, this simulation uses a circular approximation model. The circular model has same area of OKEANOS. The parameters is as follows table (1).

Table 1. Parameters

parameter	value
$I_{xx}, I_{yy}(I_T)$[kg・m^2]	48109
$I_{zz}(I_S)$ [kg・m^2]	64696
Area [m^2]	1576
Orbit [AU]	1
C_s (specular reflectivity)	0.192
C_d (diffuse reflectivity)	0.160
C_a (absorptivity)	0.618
Spin rate [rpm]	0.1
C.G offset [m]	0.1
Calculation time [day]	40
Initial attitude error [deg]	2
deflection[deg]	1
torsion[deg]	0.4

This study takes a biased equilibrium point as a variable. The results of numerically eigenvalue analysis is shown in figure (7). The color bar means the value of each eigenvalue.

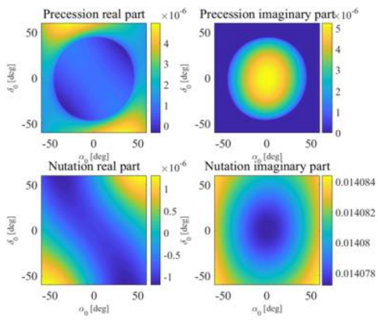

Fig.7 Eigenvalue.

The equilibrium point of natural motion that does not include bias torque is (-0.36,-2.09) from numerically analysis. From the contour information, natural motion precession real part eigenvalue is light blue area that is positive value. If the equilibrium point bring the dark blue area of precession real part, the eigenvalue order gradually changes to smaller value. It means that the divergence speed of is also slowly. The state of motion is shown in figure 8.

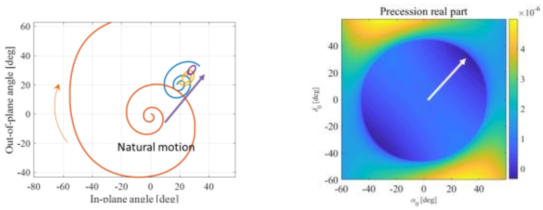

Fig.8. Result of numerically analysis (left), Eigenvalue (right).

Left figure 8 is the result of numerically solving nonlinear equation of Equation (6). From the information of nutation real part contour figure, the nutation divergence speed gradually becomes high, but the rate is very small. Thus the divergence cannot be seen visually. Since the results is in agreement with the contour information, the motion can be estimated from the information.

Focusing on the dark blue area of precession real part in figure 7, the negative value exists. In other words, the precession characteristic changes to convergence. The equation of motion can be derived in figure 9.

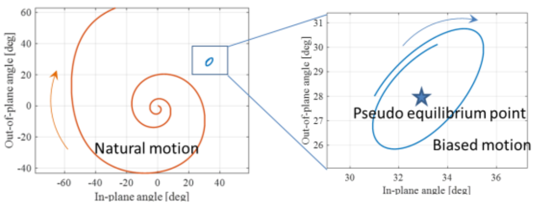

Fig.9. Stabilization of motion.

In this area like dark blue, the stabilization such as convergence of precession can be obtained. This characteristic depends on deflection and distortion angle. Thus additional analysis is derived for different value of deflection and distortion angle. Results of analysis are summarized in this table 2. In this table, this study defines stability as having a better equilibrium point than natural motion that does not include bias input. If the natural motion is stable, the table of stabilization is blank.

If the distortion angle is defined as none (0), the natural motion is neutral stable as circular motion. The stabilizations are improved comparing natural motion. This stabilization method is shown to be useful.

Table.2. Stabilization of motion for difference value

Deflection	Distortion	Overall stabilization of natural motion	Overall stabilization of biased motion
Positive	Positive	Divergence	Improving
	Negative	Divergence	Improving
	None	Divergence	Improving
Negative	Positive	Convergence	-
	Negative	Convergence	-
	None	Convergence	-
None	Positive	Circular	Improving
	Negative	Circular	Improving
	None	Circular	Improving

7. Conclusion

The objective of this study was to stabilize spinning sail motion. Primary, this paper presents the mechanism of spinning type spacecraft and spinning sail. To explain the mechanism and dynamics of spinning sail motion, the motion can be divided into equilibrium point and circular motion. Since this stabilization depends on this two components, changing the equilibrium point lead to stabilization of the motion. This paper proposes the method using this equilibrium point. The paper called this point biased (pseudo) equilibrium point.

To analysis the stabilization, eigenvalue analysis is taken in this study. By linearizing equation of motion, eigenvalue can be derived numerically. From the eigenvalue at biased equilibrium point, the motion can be estimated. Focusing the precession real part, the negative value exists. Thus the whole motion converges around biased equilibrium point. But this characteristic of the motion depends on values of deflection and distortion angle. Summarizing the result of eigenvalue analysis, it can be seen that the biased equilibrium point that stabilize more than natural motion in many cases. This stabilization method is shown to be useful.

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