スーパーナイキスト周波数事象の解析

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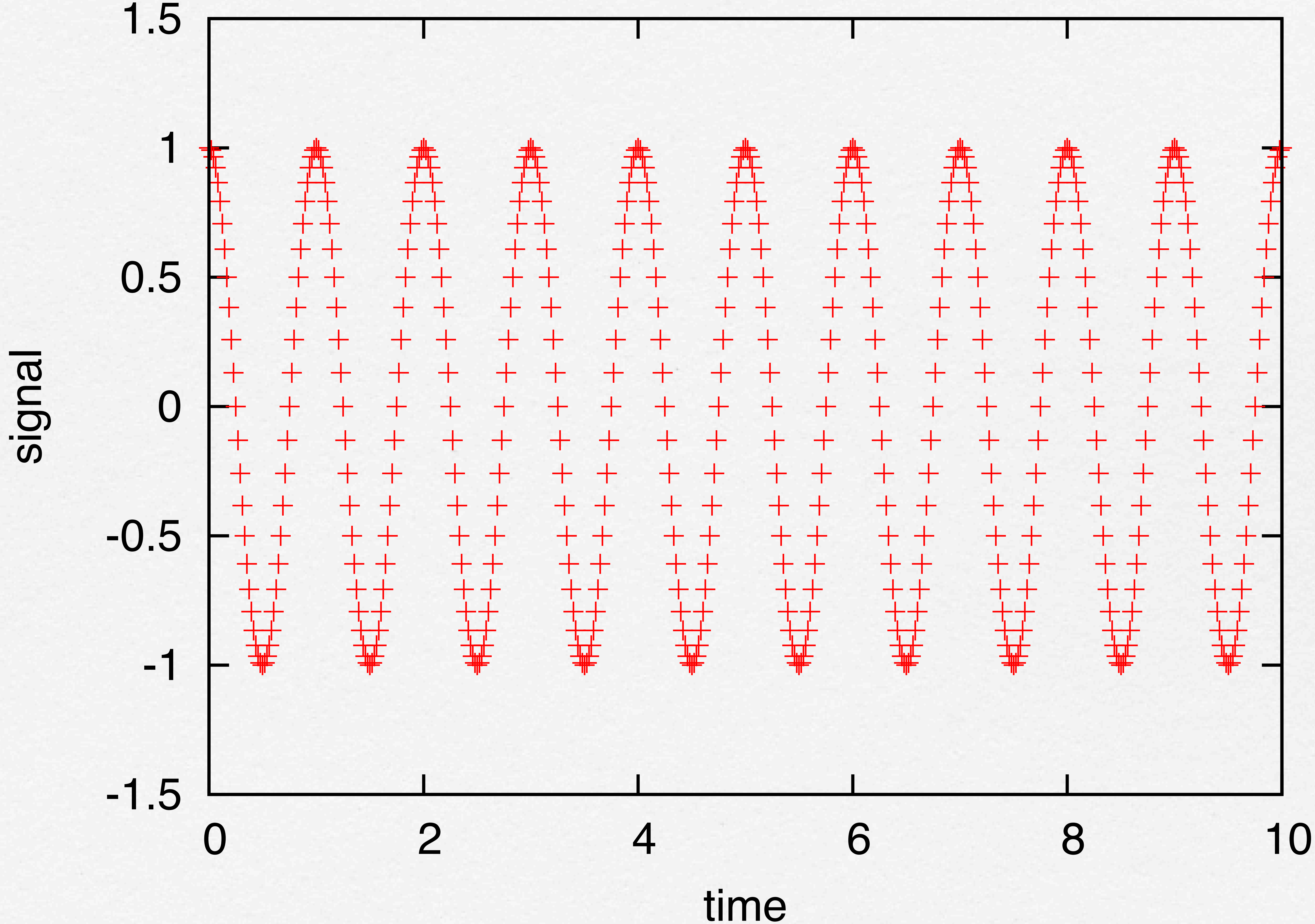
Question:

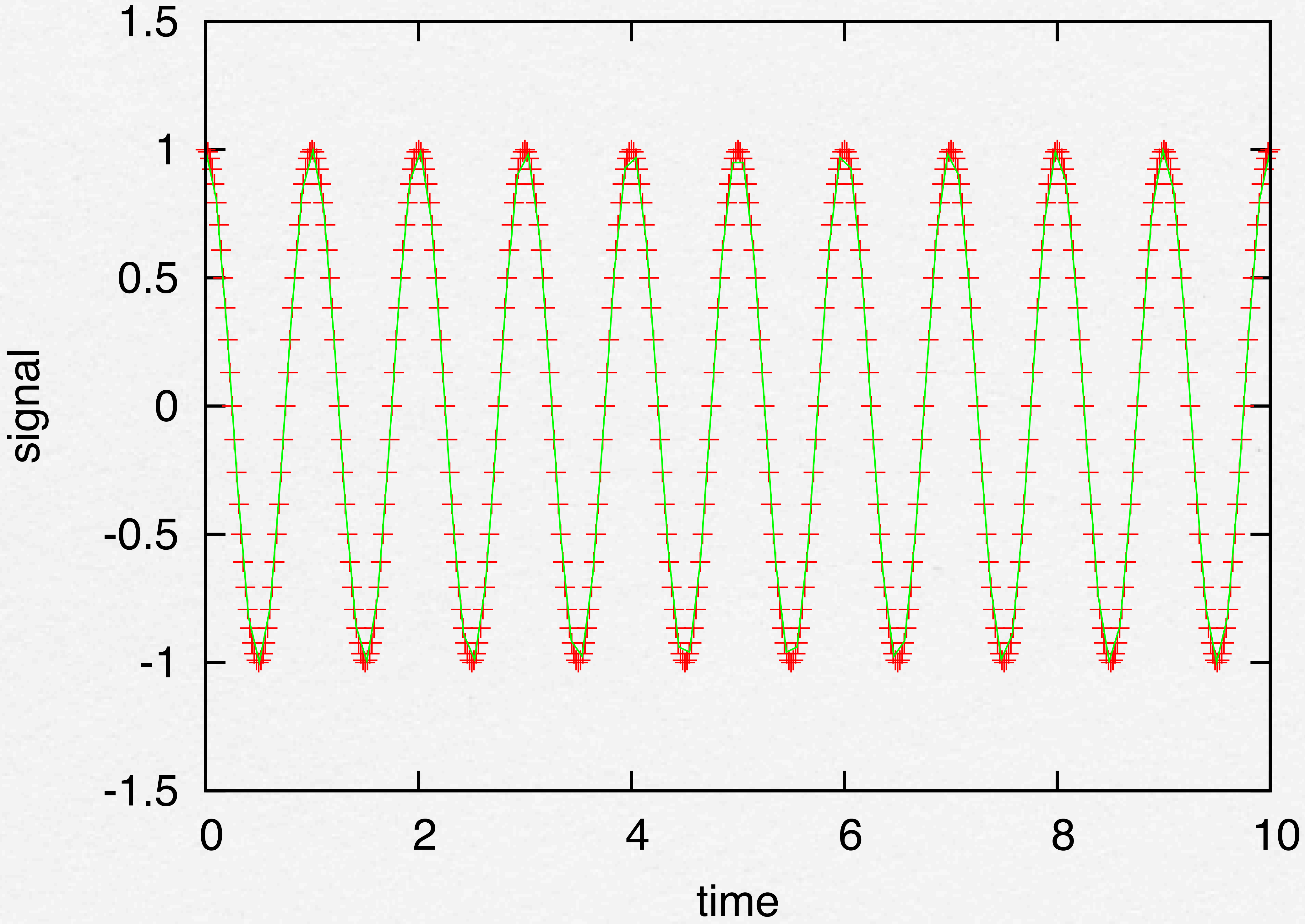
Are the *Kepler* LC (29.4-min sampling) data useless or still usefull for short period pulsators (roAp, delta Sct, sdB, WD) ?

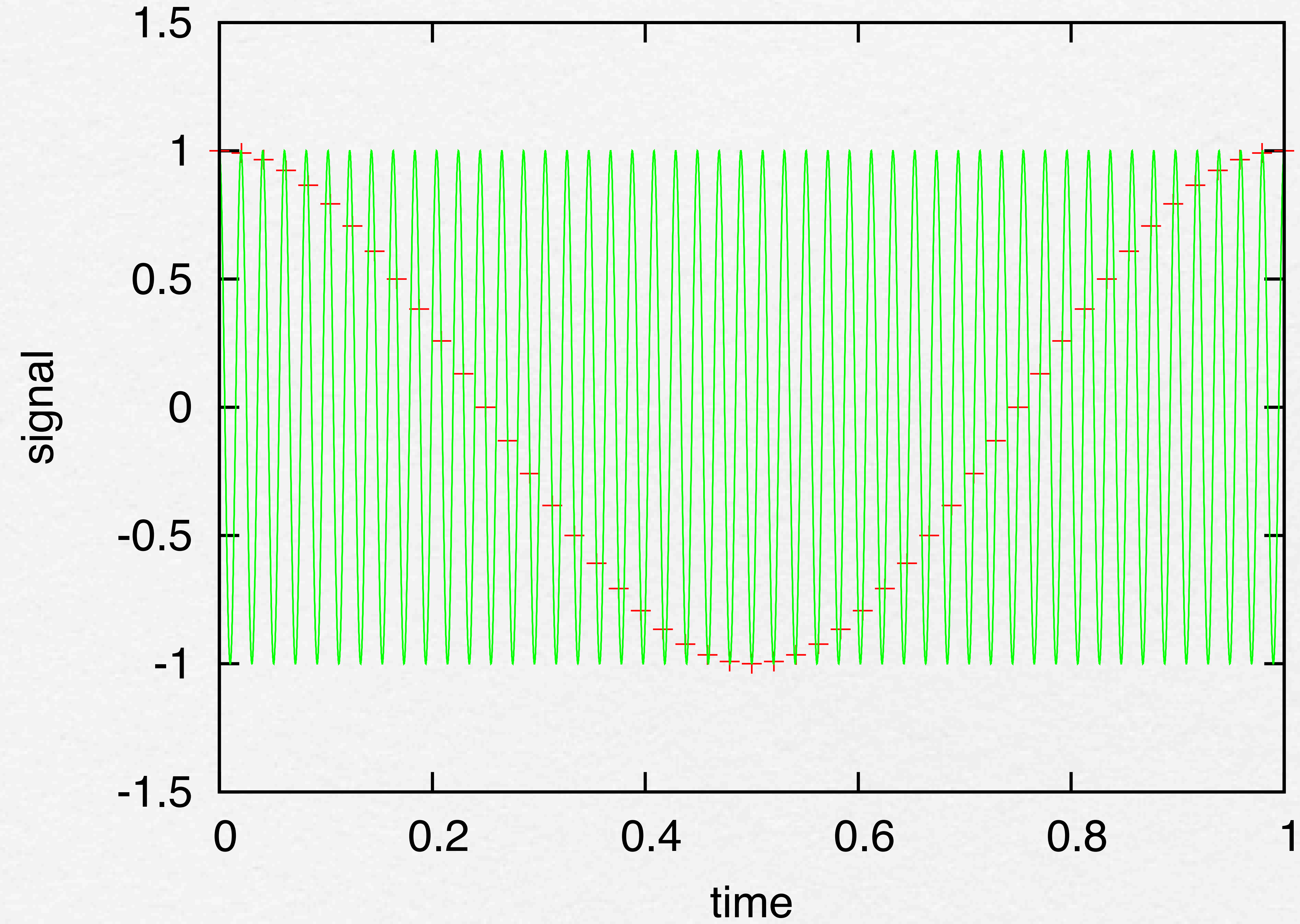
Nyquist frequency of the LC mode = 24.5 d^{-1}

Observational data = Discrete function of time



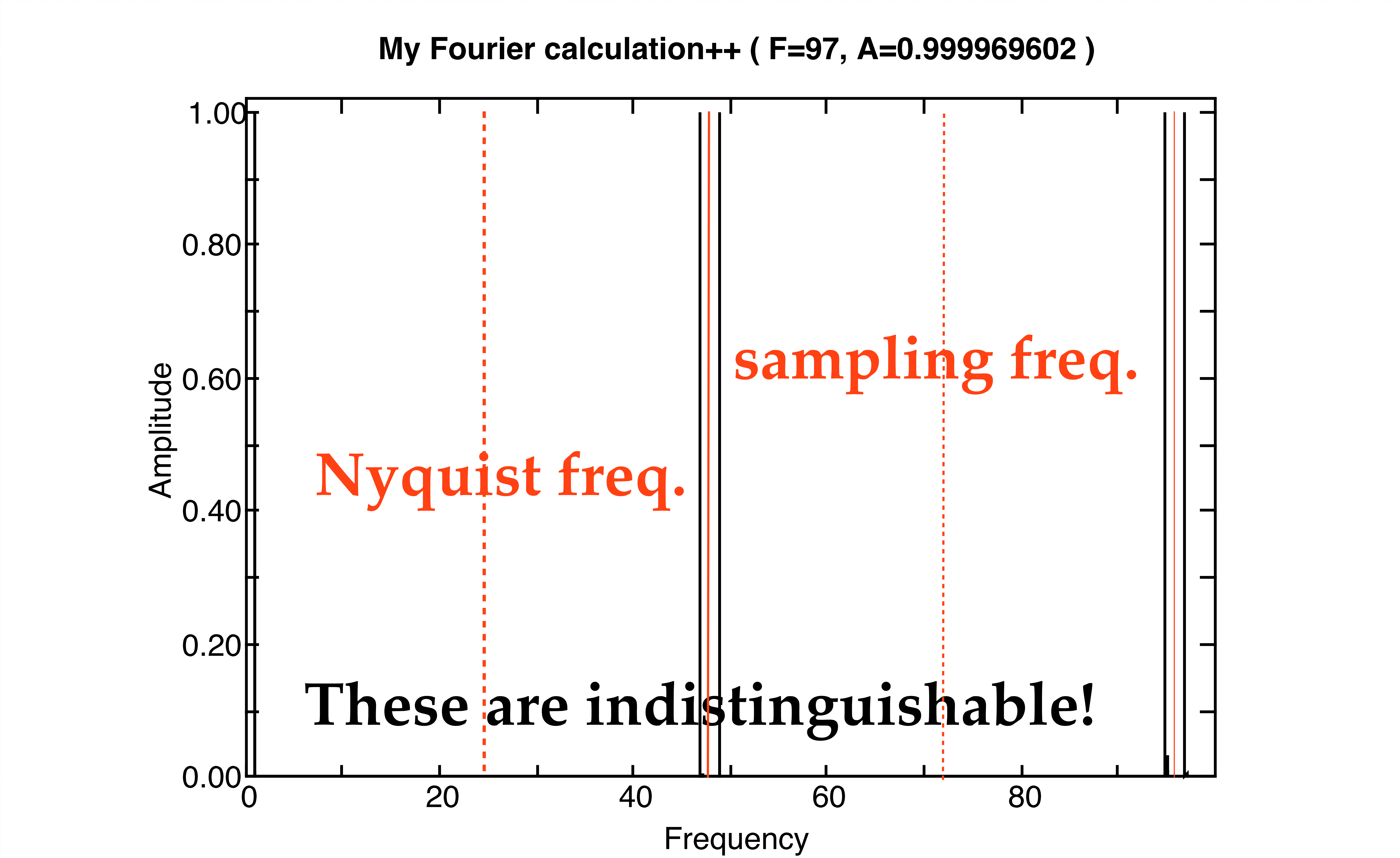
Sinusoidal fitting of discrete function





a multiply ambiguous fitting!

Fourier transform of discrete function



Fourier transform of continuous functions :

$$F(\omega) := \int_{-\infty}^{\infty} f(t) \exp(i\omega t)\, dt$$

Fourier transform of discrete functions :

$$f_{\rm obs}(t) := \sum_{n=-\infty}^{\infty} \int_{-\infty}^{\infty} f(t)\delta(t - t_n)\, dt$$

$$F_{\rm obs}(\omega) := \int_{-\infty}^{\infty} f_{\rm obs}(t) \exp(i\omega t)\, dt$$

Convolution of
the continuous function and the window spectrum

$$F_{\mathrm{obs}}(\omega) = (F * W)(\omega)$$

Window spectrum :

$$W(\omega) := \sum_{n=-\infty}^{\infty} \exp(i\omega t_n)$$
$$= \sum_{n=-\infty}^{\infty} \int_{-\infty}^{\infty} \exp(i\omega t)\delta(t - t_n)\, dt$$

Uniform cadence :

$$t_n := n\Delta t$$ sampling interval

$$\omega_{\mathrm{s}} := \frac{2\pi}{\Delta t}$$ sampling angular frequency

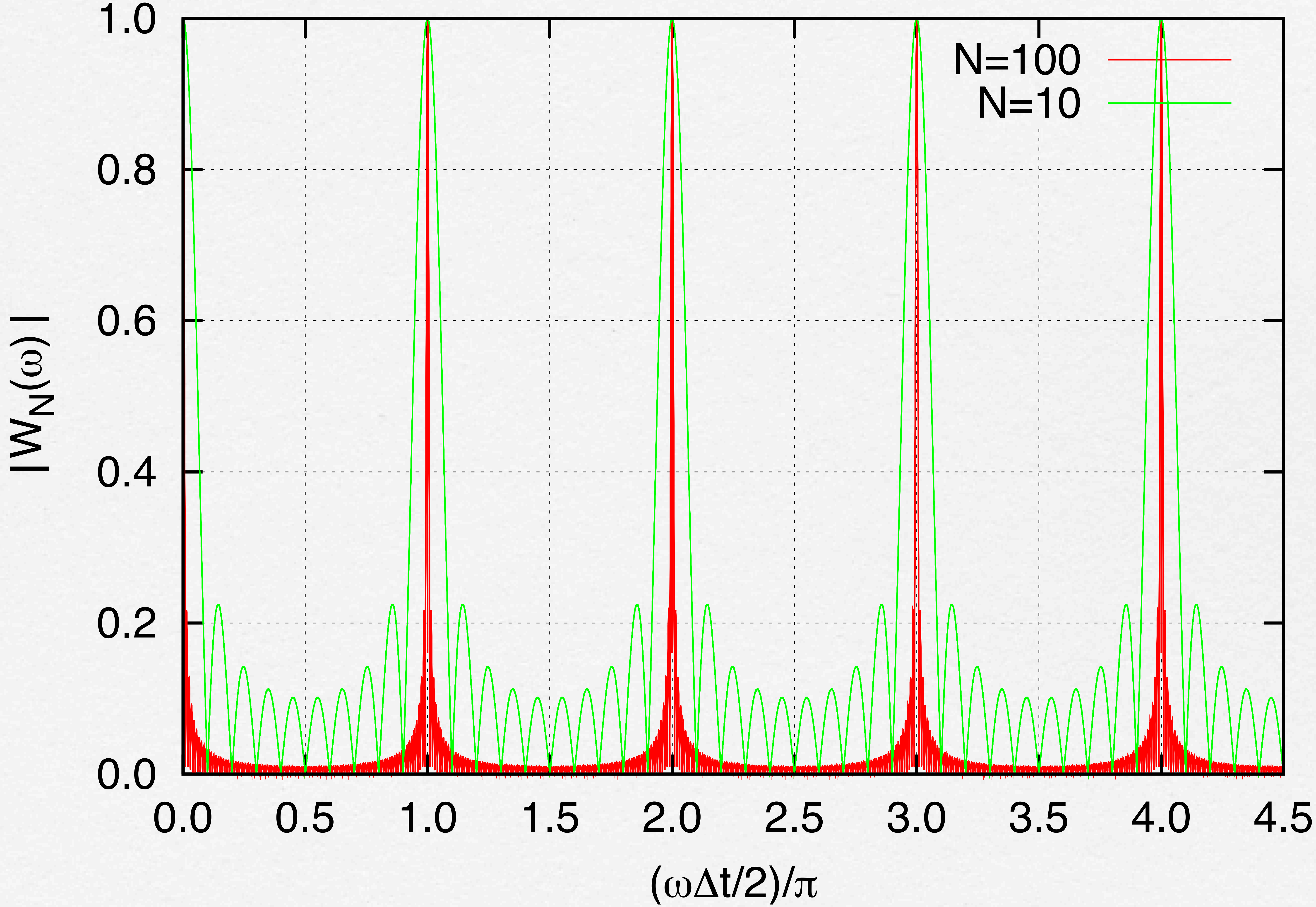
$$W(\omega) = \sum_{n=-\infty}^{\infty} \int_{-\infty}^{\infty} \exp(i\omega t)\delta(t - n\Delta t)\, dt$$

$$= \sum_{n=-\infty}^{\infty} \delta(\omega - n\omega_{\mathrm{s}})$$

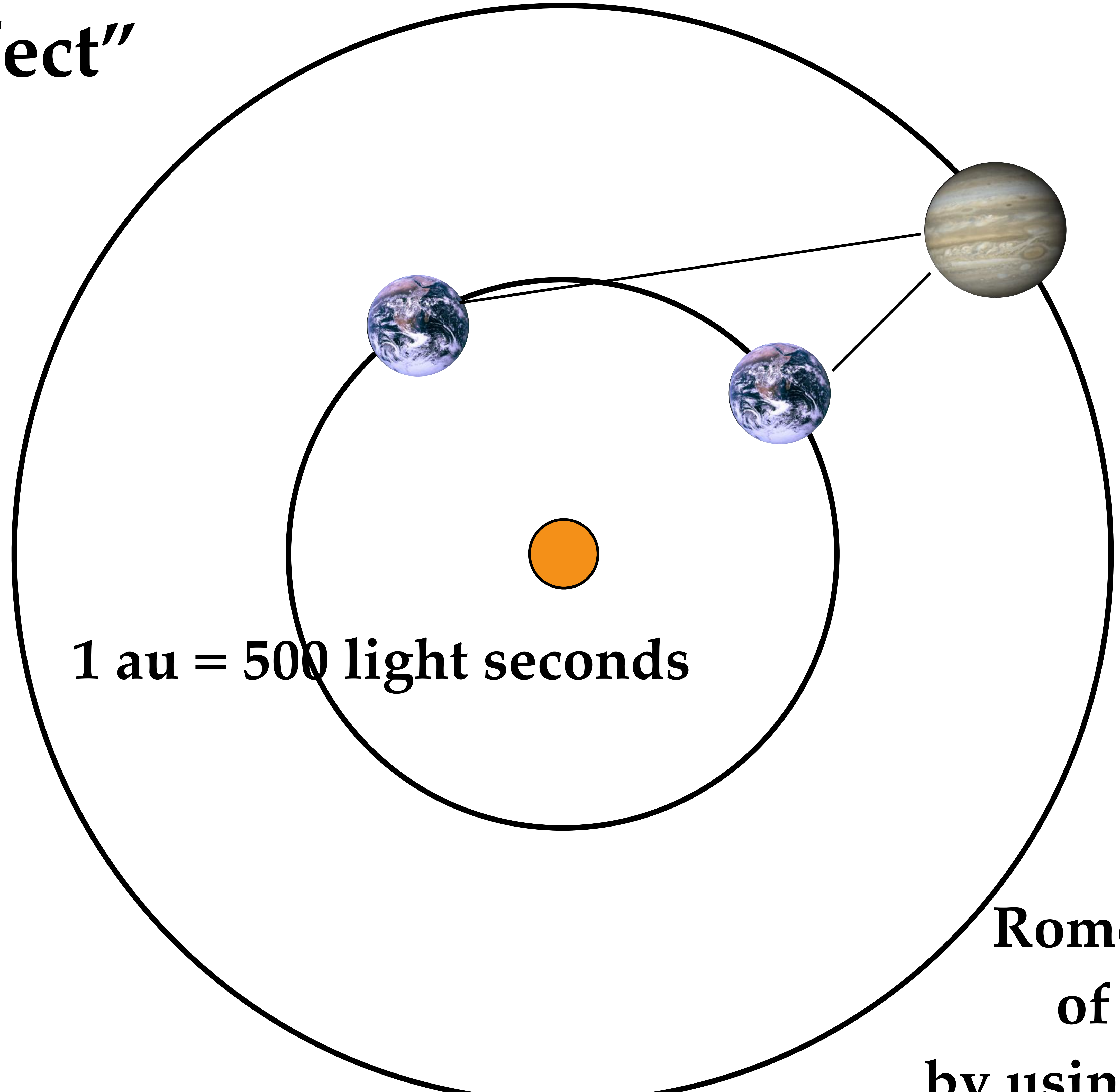
$$F_{\mathrm{obs}}(\omega) = \sum_{n=-\infty}^{\infty} F(\omega - n\omega_{\mathrm{s}})$$

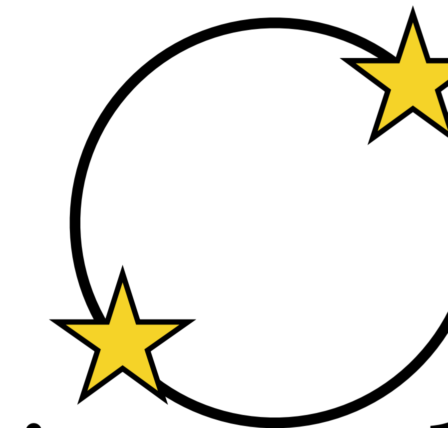
Window spectrum in the case of uniform cadence with Δt

$$|W_N(\omega)| = (N+1)^{-1} \left| \frac{\sin\{(N+1)\omega\Delta t/2\}}{\sin(\omega\Delta t)/2} \right|$$



“light-time effect”





Binary pulsar
(cf. Taylor & Hulse)

Romer’s measurement of the light speed by using Io’s eclipse timing

Kepler data:

1. taken with regular time interval according to the clock on-board

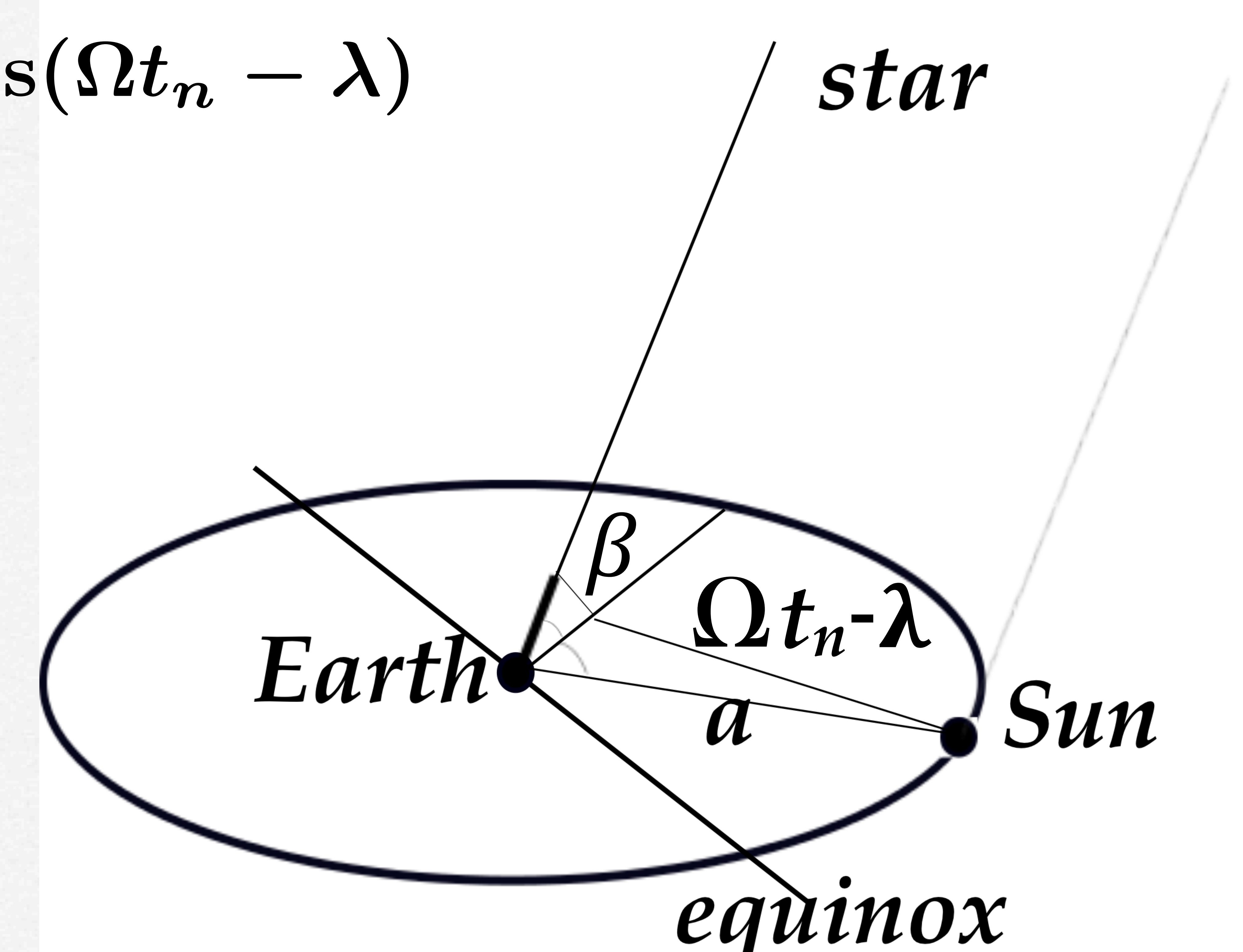
$$t_n = t_0 + n\Delta t$$

2. the time stamps converted to the barycentre arrival times

$$t_{\odot,n} = t_n + \tau \cos(\Omega t_n - \lambda)$$

$$\tau = (a/c)\cos\beta$$

"Romer delay"



Modulated sampling :

$$t_n = n\Delta t + \tau \sin \Omega t$$

$$\exp\{i\omega(n\Delta t + \tau \sin \Omega t)\} \ ?$$

Bessel function : $\exp(ix \sin\theta) = \sum_{k=-\infty}^{\infty} J_k(x) \exp(ik\theta)$

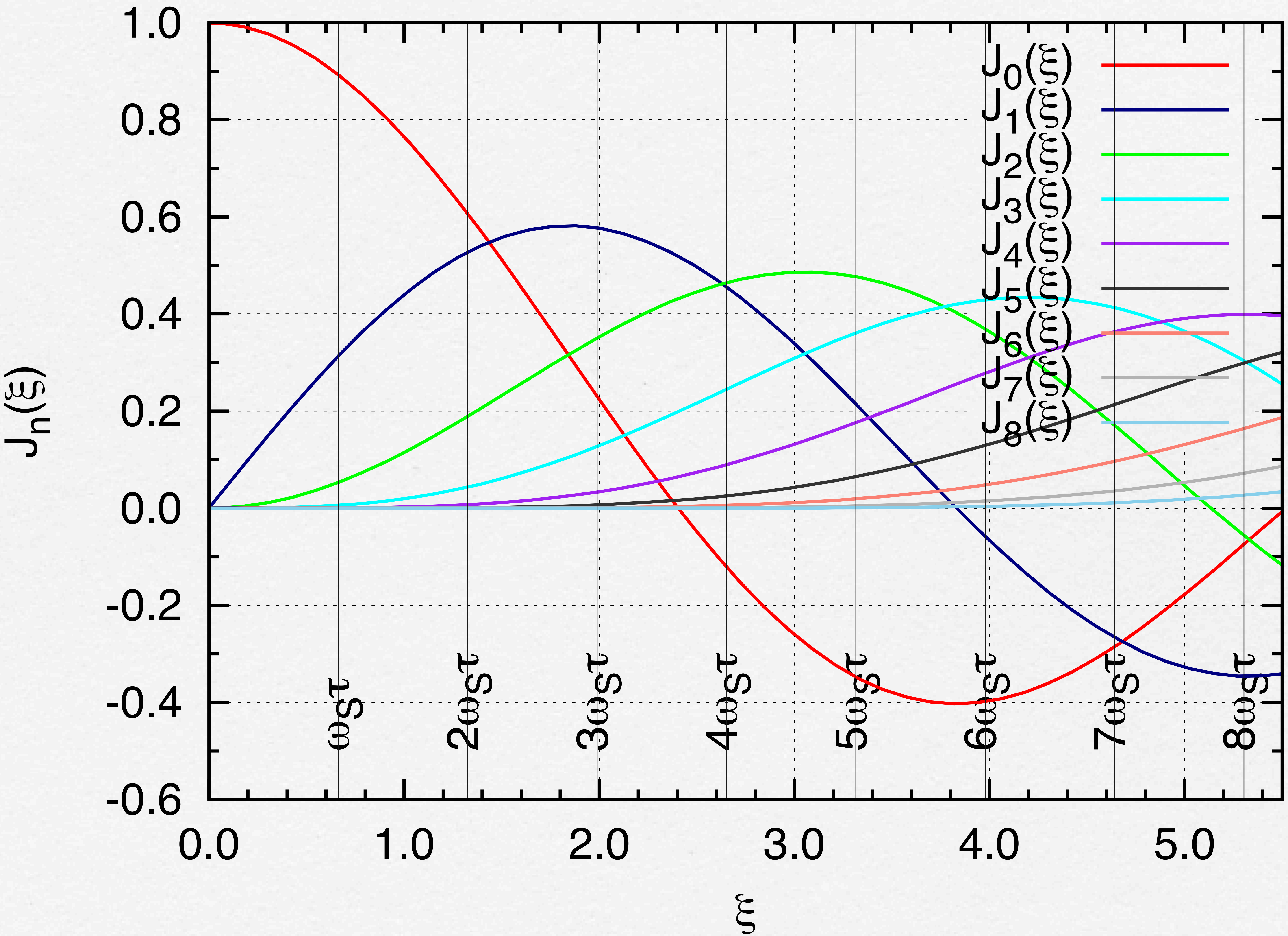
$$W(\omega) = \sum_{n=-\infty}^{\infty} \sum_{k=-\infty}^{\infty} J_k(\omega\tau)\delta(\omega - n\omega_{\mathrm{s}} - k\Omega)$$

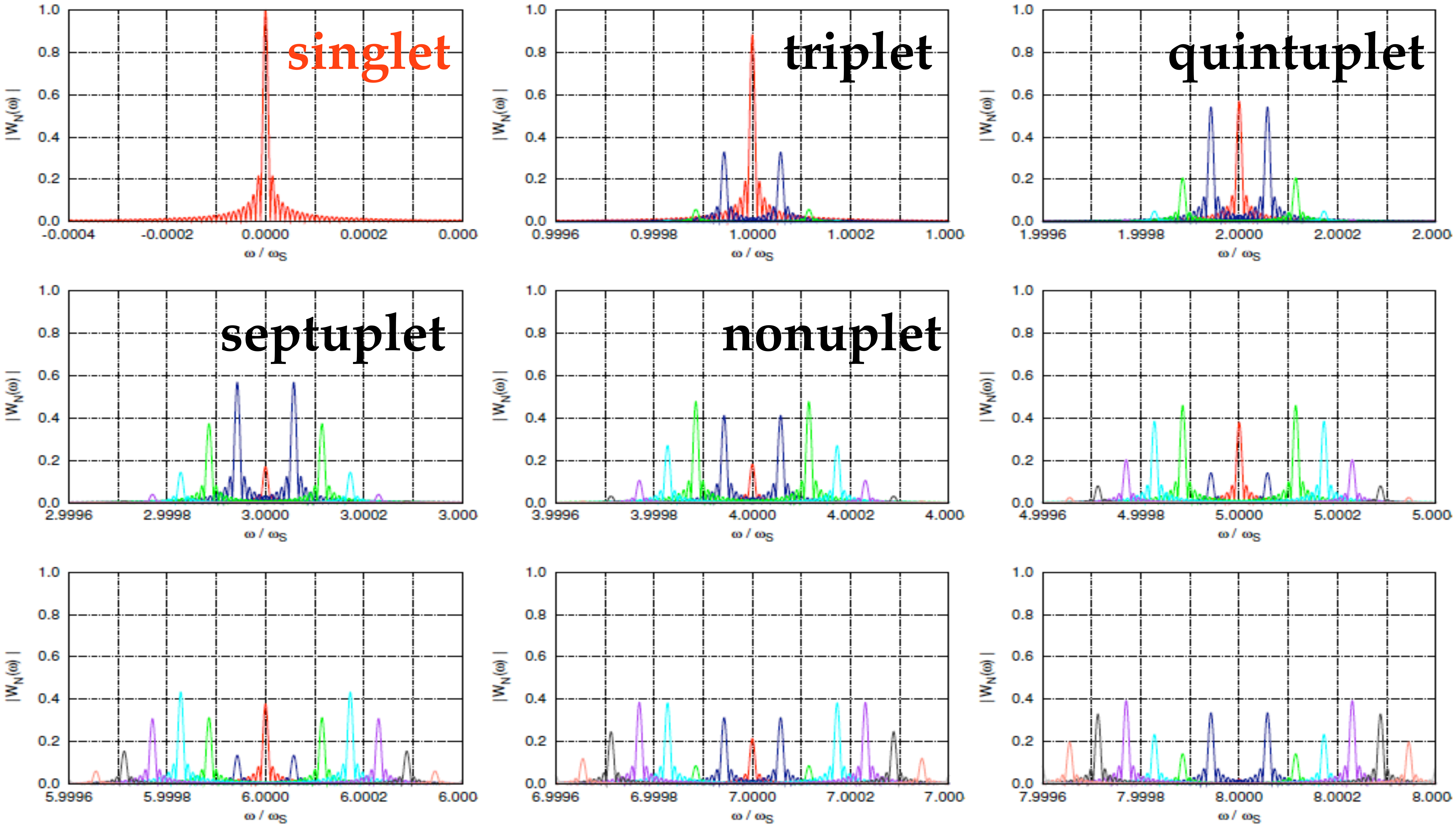
$$F_{\mathrm{obs}}(\omega) = \sum_{n=-\infty}^{\infty} \sum_{k=-\infty}^{\infty} J_k(\omega\tau)F(\omega - n\omega_{\mathrm{s}} - k\Omega)$$

Window spectrum for the *Kepler* LC mode

$$|W_N(\omega)| = (N+1)^{-1} \sum_{k=-\infty}^{\infty} \left| J_k(\omega\tau) \frac{\sin\{(N+1)(\omega + k\Omega)\Delta t/2\}}{\sin\{(\omega + k\Omega)\Delta t/2\}} \right|$$

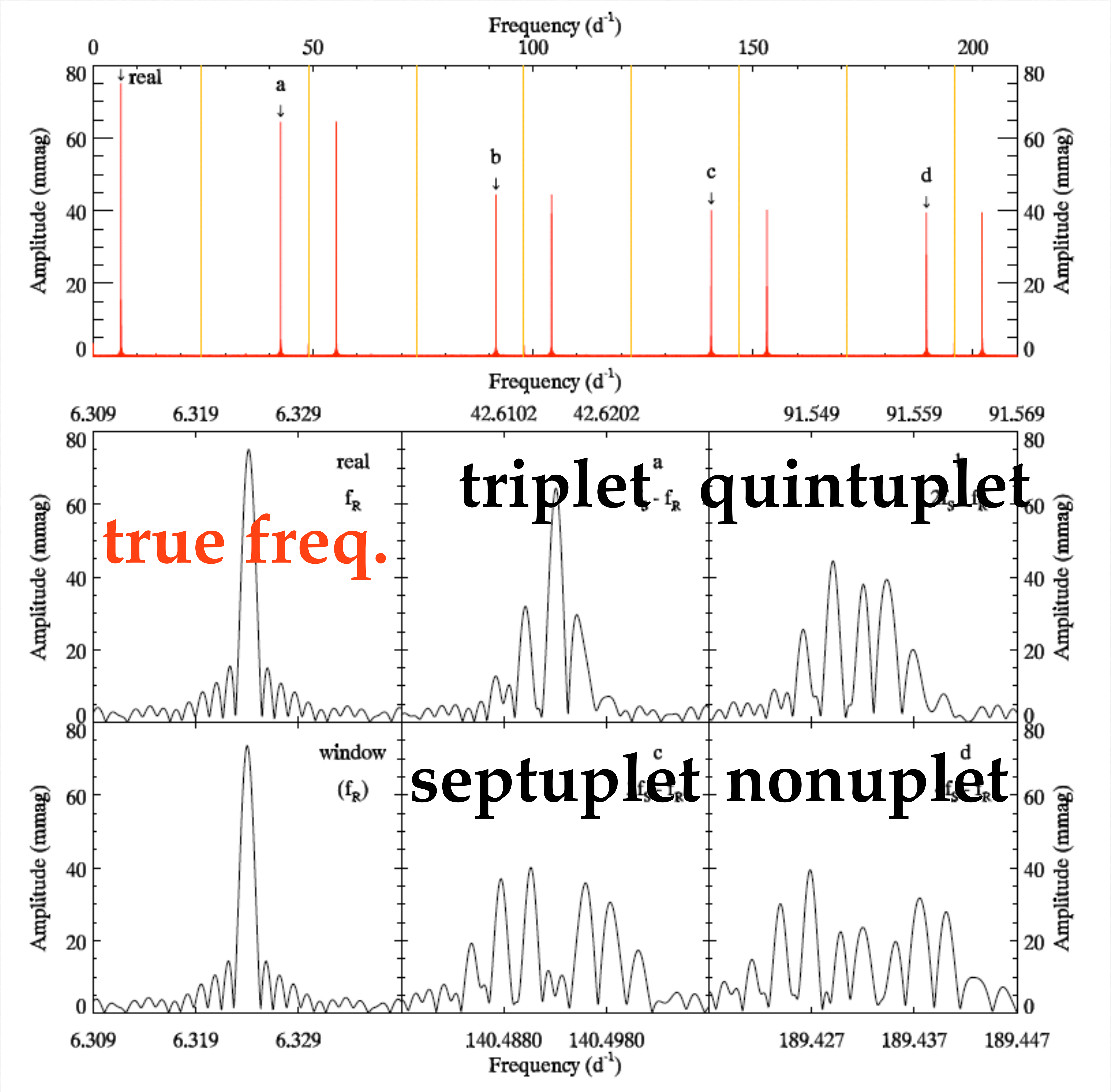
Long Cadence (30-min sampling)



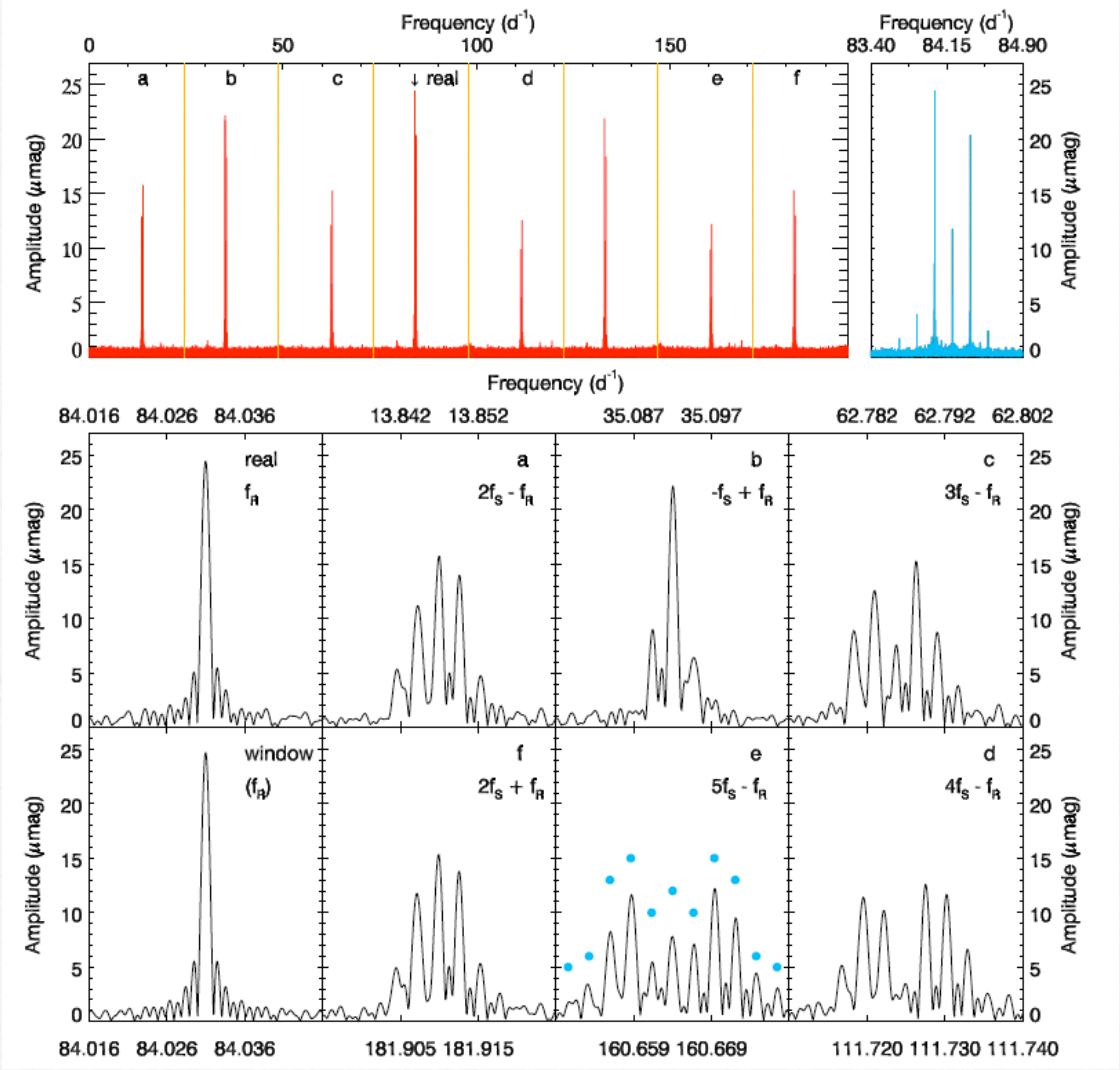


singlet
triplet
quintuplet
septuplet
nonuplet
|W_N(ω)|
ω / ω_S

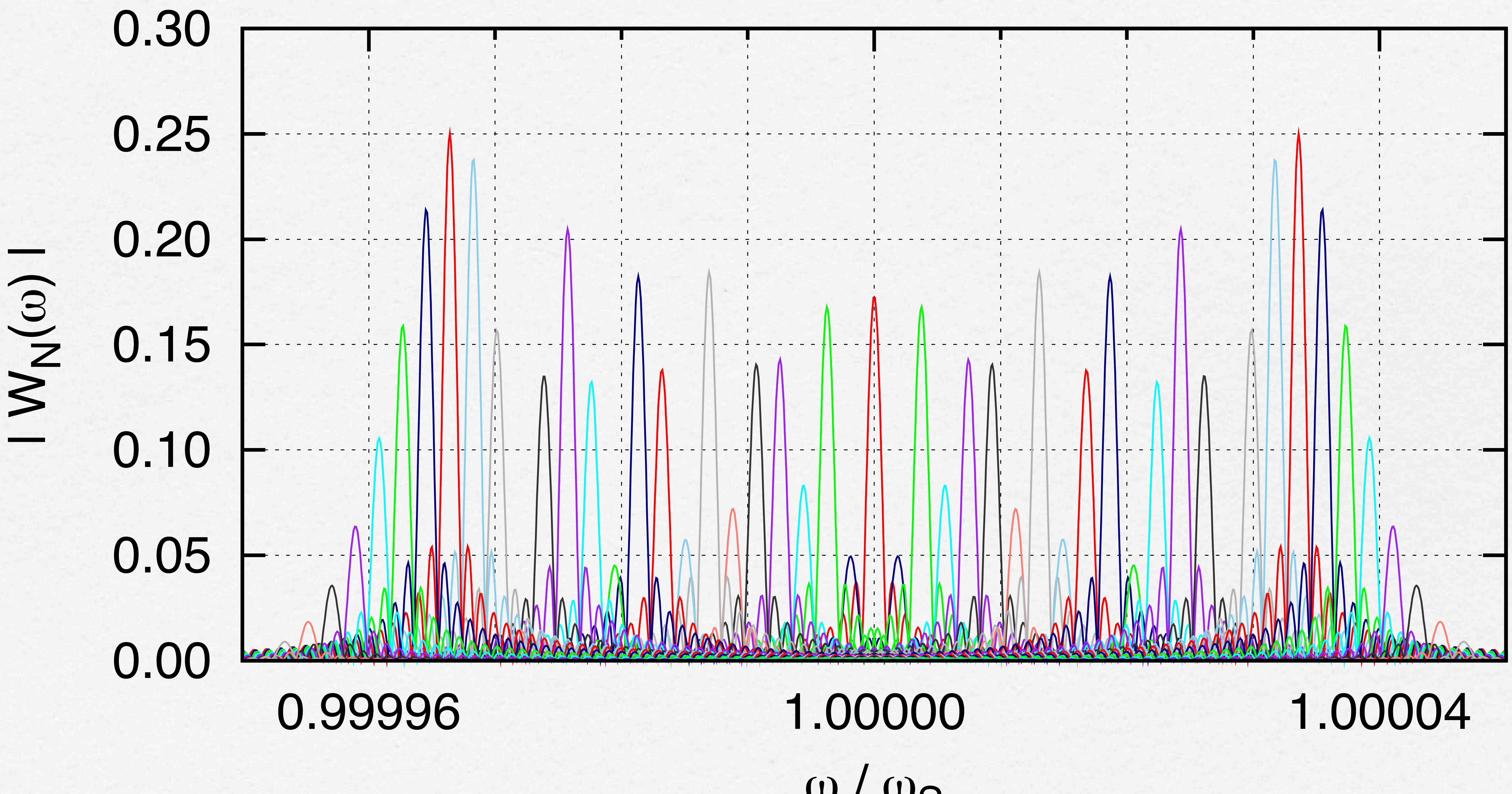
Example: KIC 6861400 (δSct star)



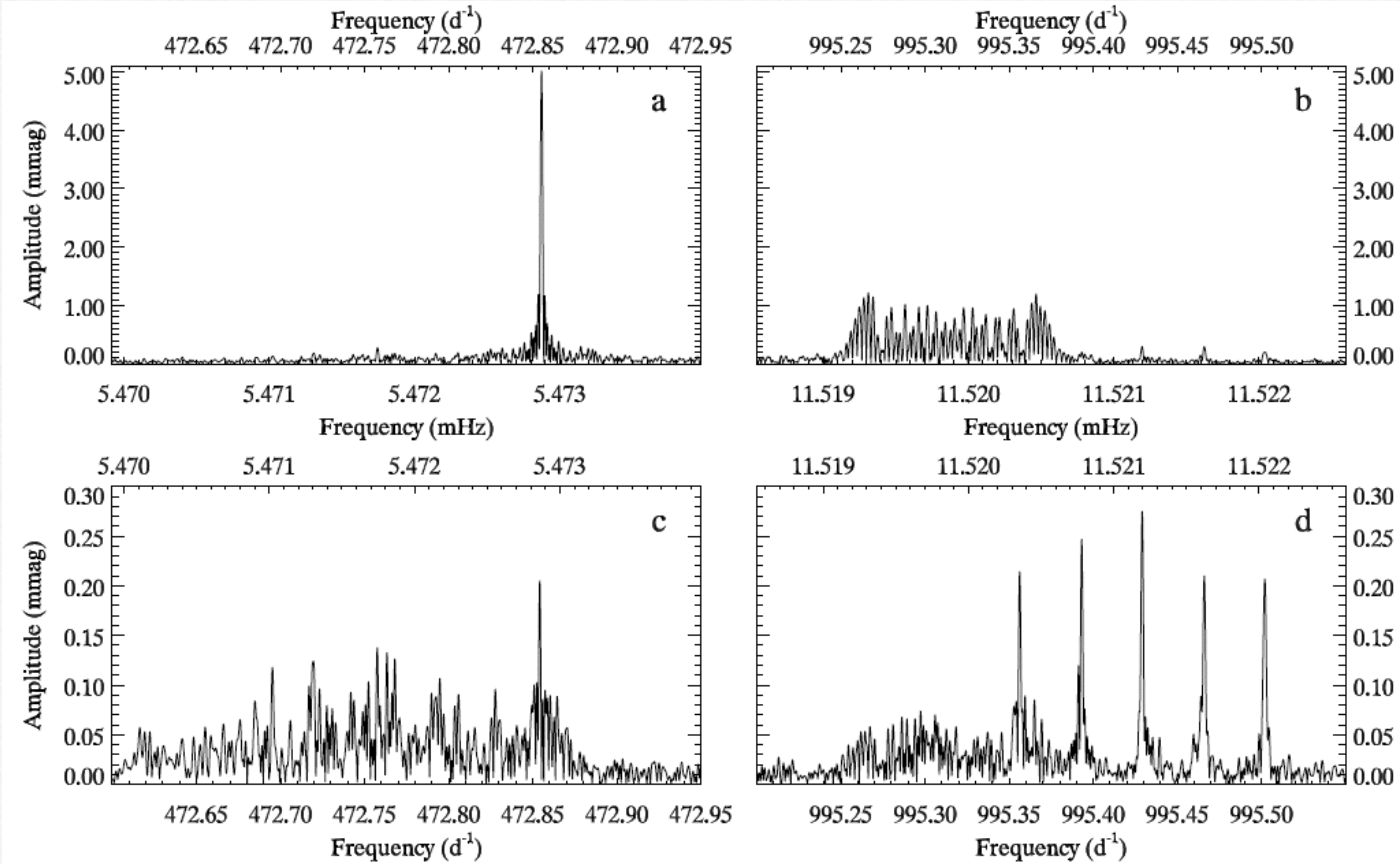
Beyond the Nyquist frequency: KIC 10195926 (roAp star)



Short Cadence (1-min sampling) window spectrum near the sampling frequency



KIC 10139564 (sdB star)



結論

- サンプリングに周波数変調を導入すれば、ナイキスト周波数に関わらずに、唯一的に事象を解析可能。
- スーパーナイキスト事象も問題なく解析可能。
- *Kepler* データに応用、成果大。
- 時間についてのフーリエ変換（周波数解析）のみならず、空間についてのフーリエ解析（波長解析）についても同様。