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縮退化された制御変数及び多目的最適化による複数制御パターンを用いた多周回遷移軌道の設計

Design of Spiral Transfer Trajectory with Multiple Control Patterns using Reduced Control Variables and Multi-Objective Optimization

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Abstract（概要）

Many-revolution transfer trajectory (Spiral Trajectory) is an efficient method to transfer trajectories using low-thrust propulsion systems such as electric propulsion. Against the efficiency of the spiral trajectory, designing a trajectory while maintaining the freedom of low-thrust propulsion was problematic due to its long transfer time which increases the number of design variables. In order to overcome this problem, research on designing spiral trajectories with multi-objective optimization using methods that allow reducing design variables while maintaining freedom of thrust was proposed. Using this method and multi-objective optimization, spiral trajectories were successfully designed. Though because the trajectory was designed using only one thrust pattern throughout the transfer, the designed trajectories were not optimal. In this paper, we introduce a method to divide a spiral trajectory into multiple sections and designing a thrust pattern for each section. The results showed that this method partially improves the designed trajectories.

Nomenclature

V: Velocity
a: Semi-major axis
e: eccentricity
μ: Gravitational constant

1. Introduction

Currently in ISAS/JAXA, a deep space probe named DESTINY+ is under development in the budget of small-satellite program in JAXA[1]. The main mission of DESTINY+ is to perform a flyby observation of asteroid Phaethon which is an asteroid that produces dust particles that causes Gemini meteor showers. The DESTINY+ will also demonstrate advanced subsystem technologies such as, small bus-system, light weight solar array panels, innovative heat control units, and also a challenging trajectory design. The trajectory of DESTINY+ consists of a launch to a low altitude, highly elliptical orbit, then orbit raising by electric propulsion, departure with multiple lunar swing-by, and finally transfer to asteroid Phaethon. The specialty of DESTINY+'s trajectory is that it will be launched by JAXA's epsilon rocket, which is a small solid-fuel rocket, and then will leave Earth proximity using DESTINY+'s low thrust propulsion systems only. By demonstrating this kind of trajectory, it will enable a frequent and low-cost deep space exploration, which is a goal of JAXA.

2. Spiral Trajectory Design using TFCs and NSGA-II

2.1 Thrust Fourier Coefficients (TFC) Usually, designing an trajectory using low-thrust propulsion systems is very complicated due to the number of design variables. For example, when designing an orbit raising trajectory (usually called spiral trajectory from its shape) as in DESTINY+, the transfer time will be in order of years, and designing a control pattern during this time span with high degree of freedom in magnitude and direction of thrust, will consist of almost infinite number of control variables. To overcome this difficulty, Hudson and Scheeres introduced a method to design a control pattern of a low-thrust propulsion system during one revolution, using only 14 design variables called Thrust Fourier Coefficients (TFCs)[2]. In this method, thrust vector components of an spacecraft during one revolution are represented as Fourier series, and by averaging a set of variational equations of the spacecraft orbit over one revolution, secular equations can be obtained which gives variation of spacecraft orbit during one revolution using only 14 variables. Although this method was very effective in reducing design variables, it could not be applied to certain cases where singularity occurs with the Keplarian elements used in the method. Specifically, this method could not be applied to orbits that have eccentricity or inclination close to 0. To overcome this singularity problem, Ozawa modified the method using non-singular elements called equinoctial elements[3].

2.2 NSGA-II Multi-objective evolutionary algorithms (MOEAs) are very powerful tools to optimize an multiple objective problem with many design variables. Genetic algorithm is a type of MOEA which simulates the genetic manipulations such as selection, crossover, and mutation. By performing an optimization with a genetic algorithm, a group of solutions called a pareto front can be obtained, which contains the best solutions with different objective function values which allow performing trade-off between the objectives which plays a important role in designing spacecraft trajectories. NSGA-II is one of the representative genetic algorithms which implemented a non-dominated sorting technique to overcome difficulties of MOEAs developed in the past[4].

2.3 Spiral Trajectory Design By performing multi-objective optimization using NSGA-II with TFCs as design variables, Ozawa succeeded in designing spiral trajectories with two objective functions; time of flight and total delta V. Although the method of designing a spiral trajectory with NSGA-II and TFCs was successful, the solutions obtained by Ozawa consisted only of a single thrust pattern (thrust vector components) during the whole transfer. The problem of using a single thrust pattern over the whole transfer is that the optimal thrust pattern varies by the shape of the orbit. When using low-thrust propulsion systems, change in the spacecraft orbit during one revolution is very small, so in a short period of time, using a single thrust pattern would not be a problem from the view of optimality. But over a long period of time, the change in the shape of the spacecraft orbit cannot be neglected, so the thrust pattern must be redesigned according to the change in orbit shape. By designing a trajectory with multiple thrust profiles, we expect to obtain better results which means that the optimized pareto front will proceed toward the utopia point (the origin in a minimization problem).

3. Spiral Trajectory Design with multiple control patterns

3.1 Method In this research, we divided the spiral trajectory into desired number of sections by choosing an index element and switching the sections according to the change of the index. We applied this method to an apogee raising transfer trajectory and chose apogee as the index element. To compare results of a single and multiple thrust profile, we optimized an trajectory with one section, and 8 sections with the following settings.

3.2 Problem Setting

Object:

$$\text{minimize } f(x), \tag{3.1}$$

$$f = [\text{time of flight},\ \Delta V], \tag{3.2}$$

$$x = [\text{TFCs, max time of flight}] \tag{3.3}$$

Initial Condition:

Table 1 initial condition

Semi-major axis, a [km]	*42240*
Eccentricity, e [-]	0.1
Inclination, i [deg]	10
Right ascension of ascending node, Ω [deg]	20
Argument of periapsis, ω, [deg]	90
Mean Anomaly, M [deg]	0

Goal:

$$\text{Goal apogee} = 130{,}000 \text{ [km]} \tag{3.4}$$

$$\text{Initial apogee} = 46{,}464 \text{ [km]} \tag{3.5}$$

$$\Delta \text{Apogee for each section (8 section)} = 10{,}422 \text{ [km]} \tag{3.6}$$

3.3 Result Below are the optimization results of the trajectory designed with 1 section, and 8 sections. From the result, we can observe that the pareto front did not proceed completely with 8 sections compared to 1 section as expected. Although we can see an area in the bottom right of the pareto front where results improved by designing a trajectory with multiple thrust patterns.

In order to verify that having multiple sections improved the results, we looked into the values of the optimized TFCs. To begin, apogee altitude can be expressed using eccentricity and semi-major axis as below.

$$\text{Apogee} = a(1+e) \tag{3.7}$$

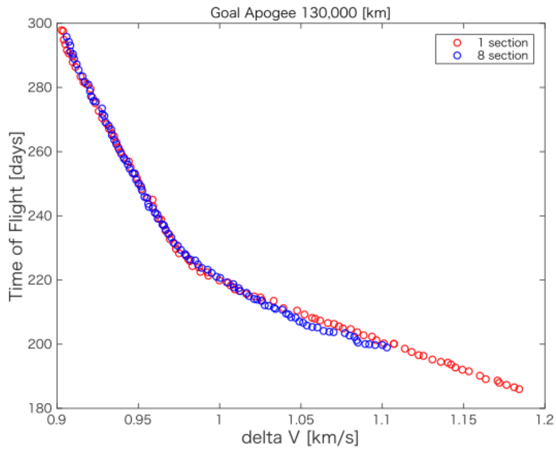

Figure 1 optimization results

Next, an averaged variational equation for semi-major axis with TFCs can be expressed as below[2)].

$$\left\langle\frac{da}{dt}\right\rangle = 2\sqrt{\frac{a^3}{\mu}}\left[\frac{1}{2}e\beta_1^R + \sqrt{1-e^2}\alpha_0^S\right] \tag{3.8}$$

From equation (3.7), in order for the value of apogee to increase, the semi-major axis and eccentricity needs to be increased. Because eccentricity can be changed only between 0~1 for the trajectory to be an orbit, we will focus on the change of the semi-major axis. From equation (3.8), for the semi-major axis to increase efficiently, relationships below need to be met in the beginning of the transfer and the end of the transfer.

Table 2 Relationships of variable value

	Beginning	*End*
eccentricity	Small	Large
TFC β_1^R	Small	Large
TFC α_0^S	Large	Small

To check if the optimized results with multiple sections follow the relationships in Table 2, we chose two populations from the pareto front that have the same amount of ΔV. The amount of ΔV was 1.084[km/s], and time of flight was 203.7 [days] and 201.2 [days] for 1 section, and 8 sections respectively. We also checked the TFCs of the population that had the minimum amount of time of flight at 198.9 [days]. Below are the values of the TFCs.

Table 3 TFC values

		Beginning	*End*
1 section	TFC β_1^R	0.378	-
	TFC α_0^S	0.723	-
8 sections	TFC β_1^R	0.358	0.433
	TFC α_0^S	0.848	0.578
8 sections (min ToF)	TFC β_1^R	0.370	0.771
	TFC α_0^S	0.849	0.488

From Table 3, we can observe that the results of TFCs with 8 sections followed the relationships that need to be met based on orbital dynamics. Also, from the minimum time of flight results, we can observe that the TFCs strongly follow the relationships that need to be met.

4. Conclusion

In this paper we presented an method to design an spiral trajectory with multiple thrust patterns, to obtain better trajectories for a space probe. The results showed that dividing a spiral trajectory into multiple sections and optimizing a thrust pattern for each section improved the pareto front partially. Also, by observing the optimized design variable values, the results followed the relationships that were expected based on orbital dynamics. For future work, the goal apogee altitude needs to be increased to match the trajectory of DESTINY+, in which a better solution is expected for the reason of longer transfer time, and also, changing the index element from apogee altitude might lead to better solutions.

References

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