KYUSHU UNIVERSITY

Lobe dynamicsにもとづくカオス的遷移軌道の設計

Design of Chaotic Transfers Based on Lobe Dynamics

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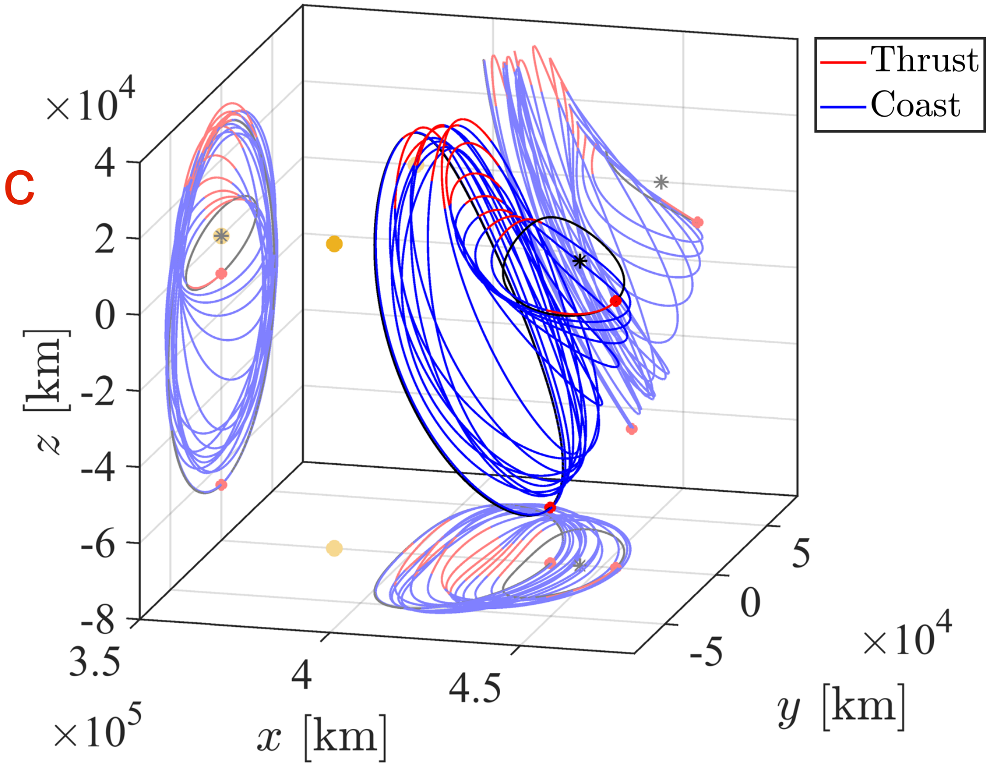
Background

Trajectory Design and Optimization

- Spacecraft dynamics: highly nonlinear and chaotic
- Desiging a fuel-optimal trajectory

- Leveraging the dynamical structure

1. Tube dynamics : Transported by the manifolds of libration point orbits
2. Lobe dynamics : Transported by the manifolds of resonant orbits



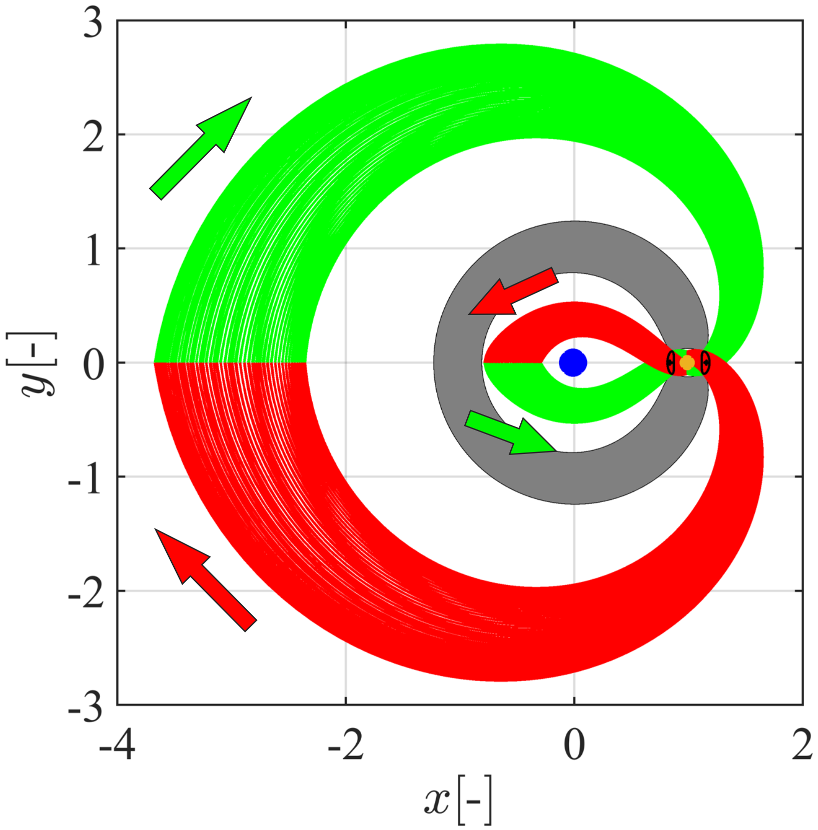
Example of the optimal transfer

Background

Tube dynamics vs. Lobe dynamics

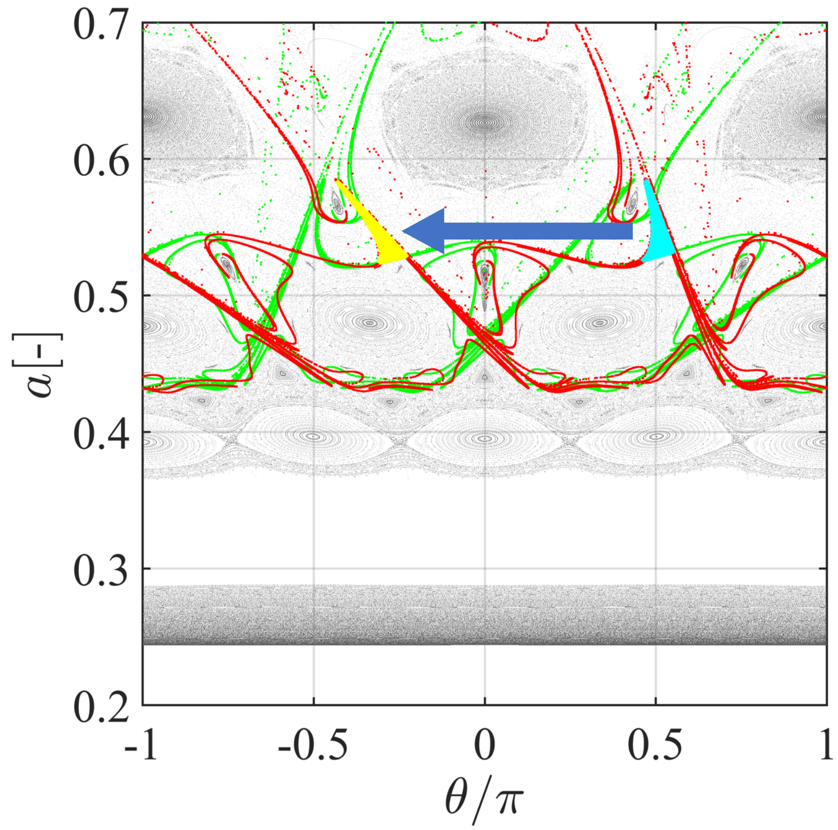
Tube dynamics

- Large transport structure
- Developed well in the literature



Lobe dynamics

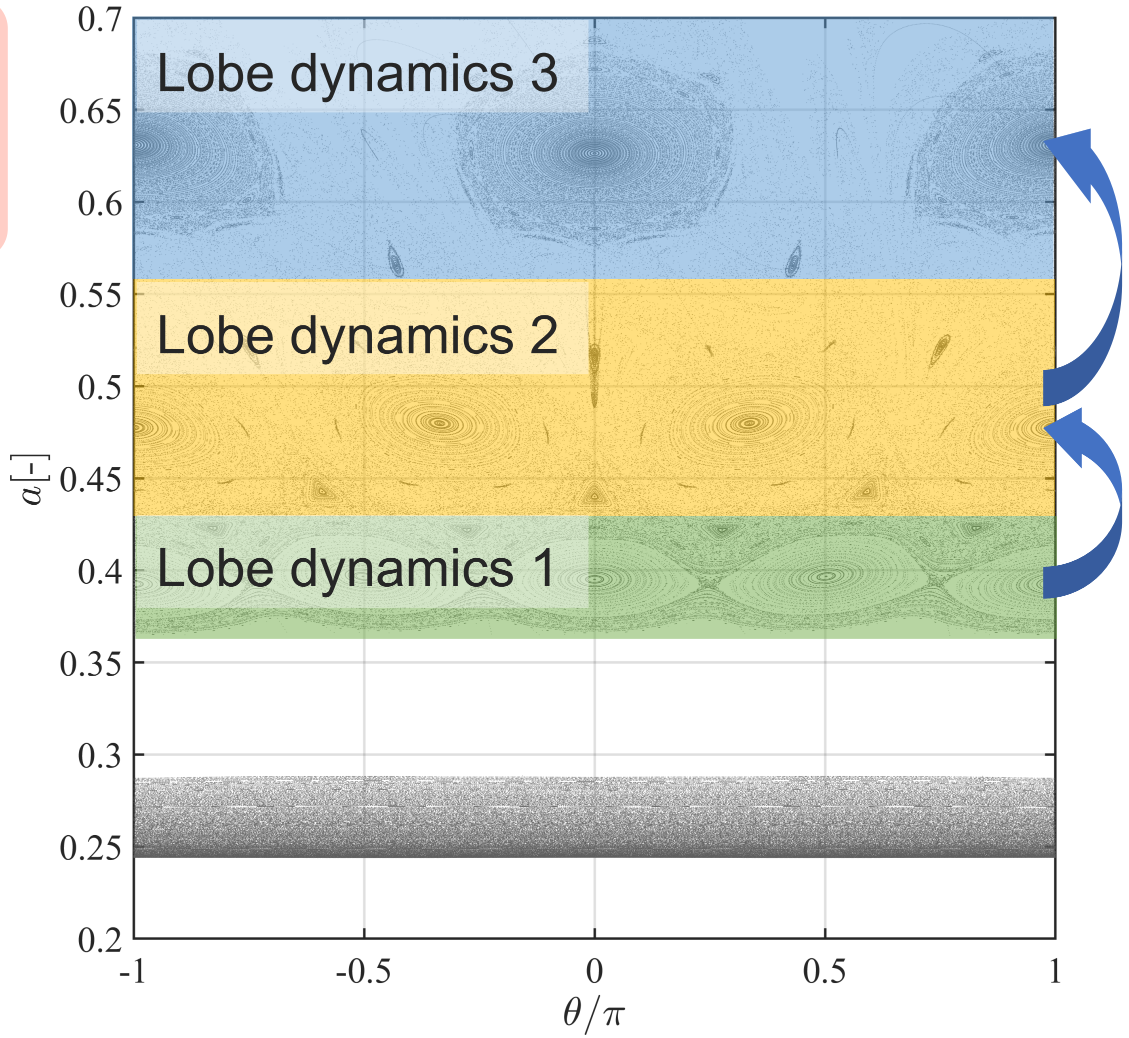
- Transport structure in chaotic sea
- Numerical difficulty in calculation



Research Purpose

Design low-energy chaotic transfer trajectories based on lobe dynamics

- Lobe dynamics reveals the structure of chaotic transport in the CR3BP
- Effectively combine lobe dynamics of various periodic orbits
- Construct low-energy transfer to deep space

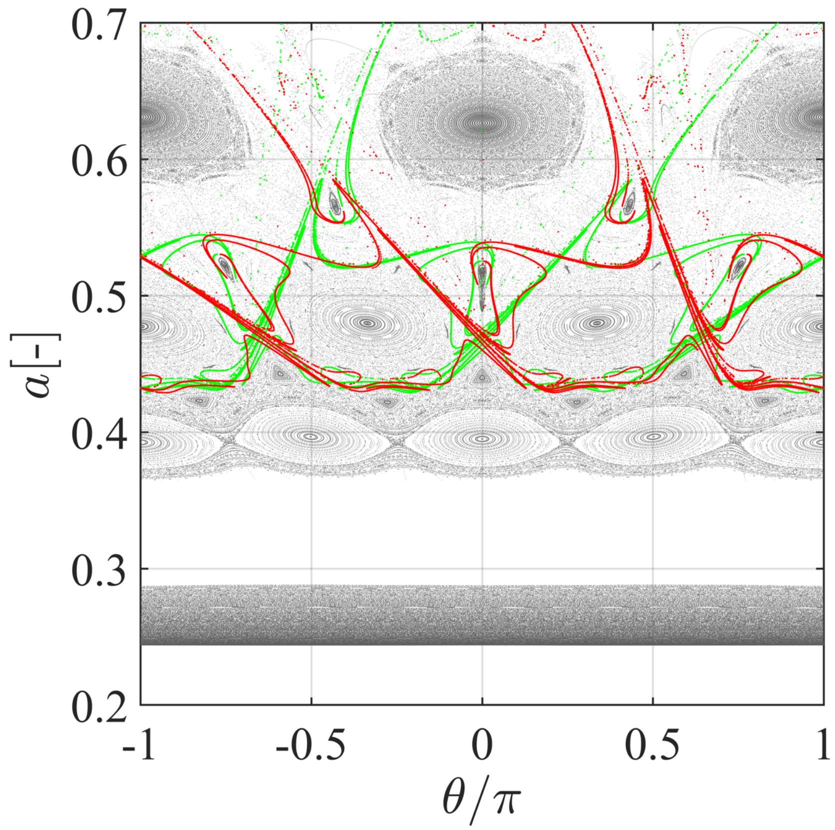


Methodology

Overview

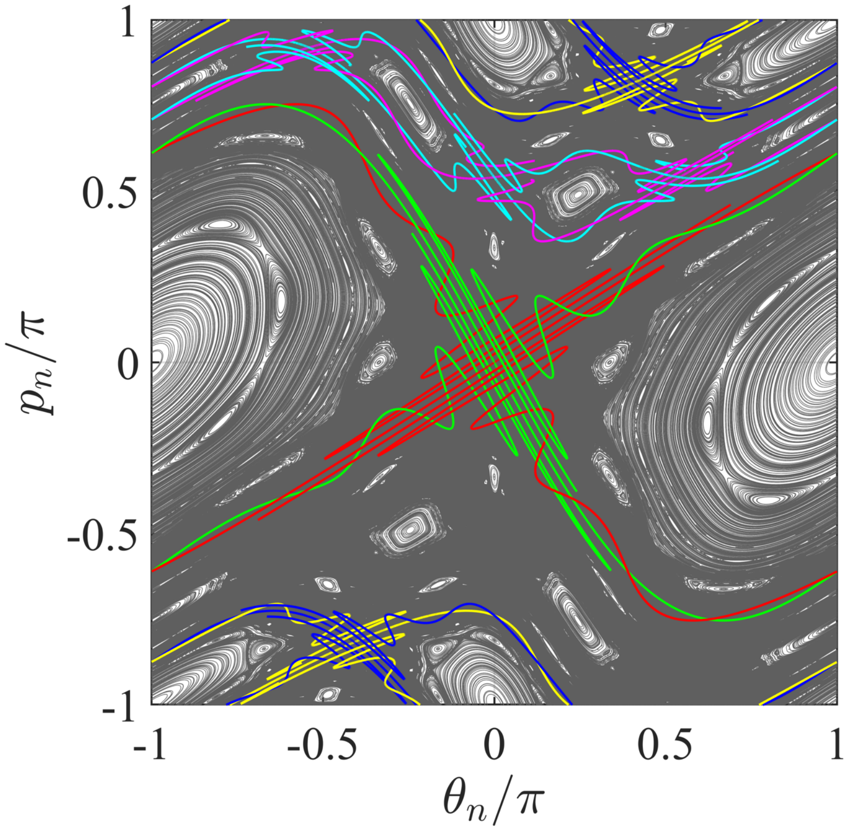
Poincaré map in CR3BP

- Chaos in the hamiltonian system
- Complex structure



Standard map

- Discrete system
- Simple example of chaos



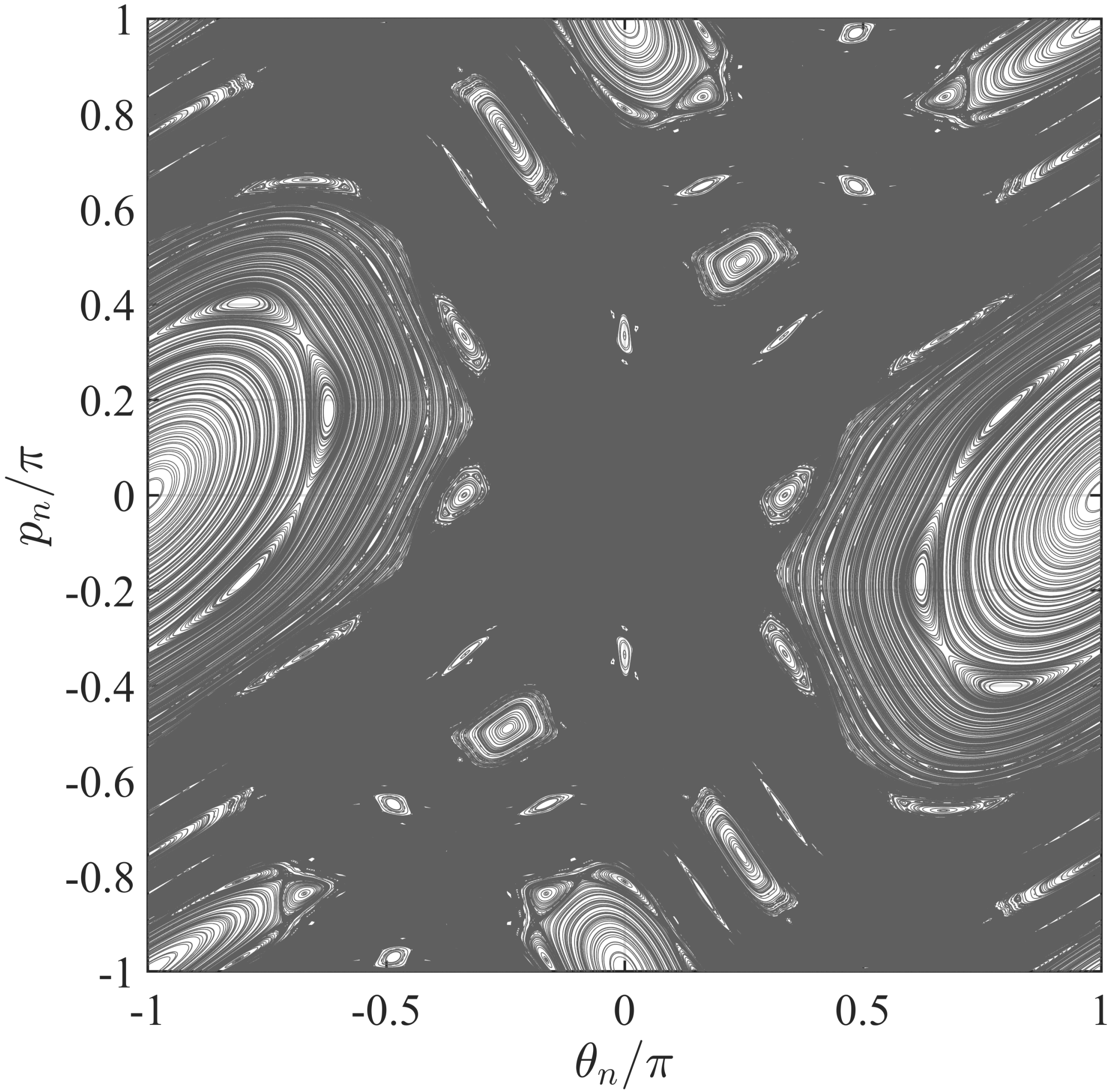
Standard Map

- A two-dimensional discrete map
- Basic model for chaotic dynamics

$$\begin{cases} \theta_{n+1} = \theta_n + p_{n+1} \\ p_{n+1} = p_n + K \sin \theta_n \end{cases} \quad (\text{mod } 2\pi)$$

Fixed point: $(\theta_n, \quad p_n) = (0, \quad 0)$ Unstable

$(\theta_n, \quad p_n) = (\pi, \quad 0)$ Stable



When $K = 1.2$

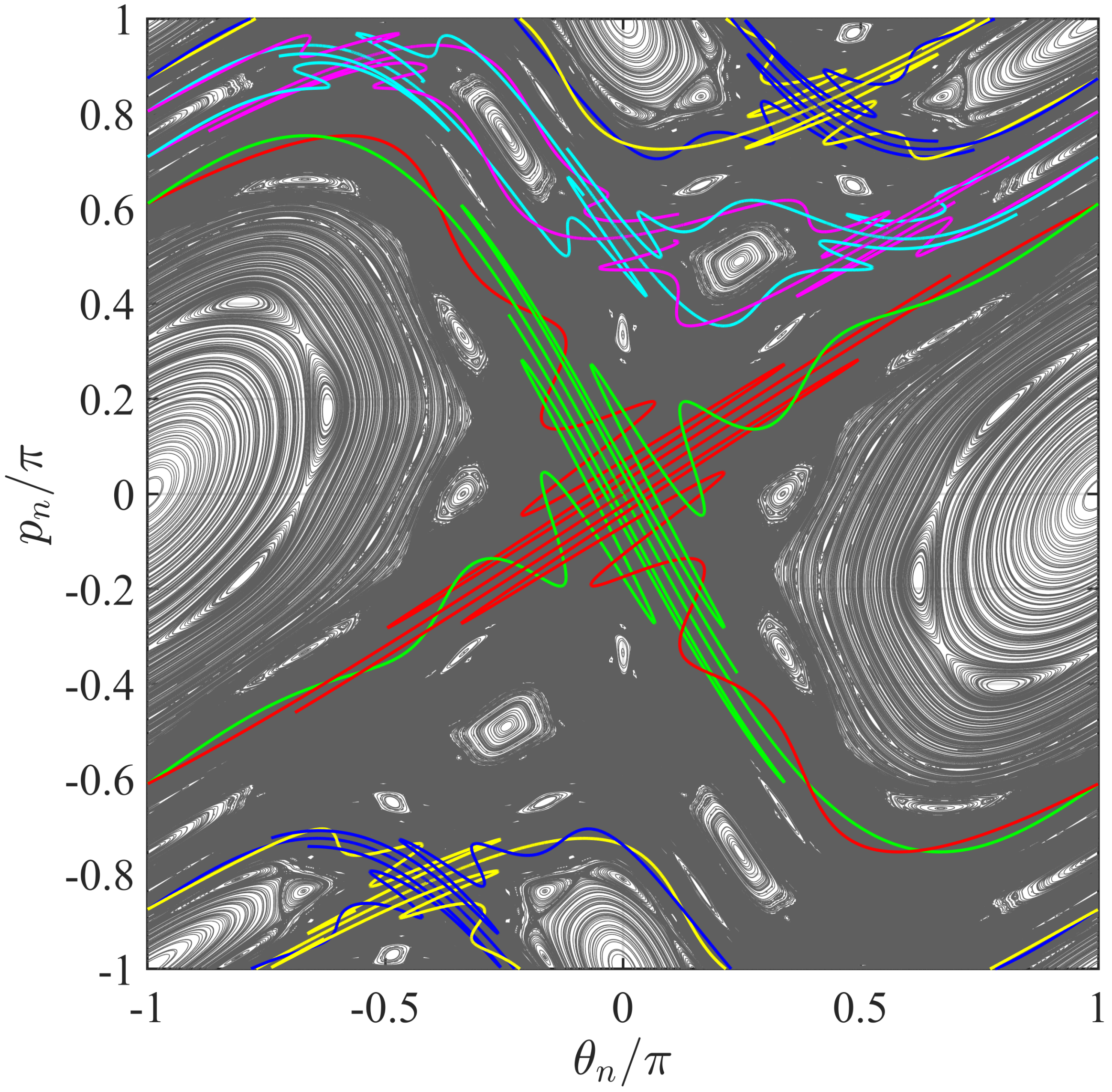
Standard Map

- Focus on three periodic orbits and their stable and unstable manifolds

Period 2
- Stable manifold (blue)
- Unstable manifold (yellow)

Period 3
- Stable manifold (cyan)
- Unstable manifold (magenta)

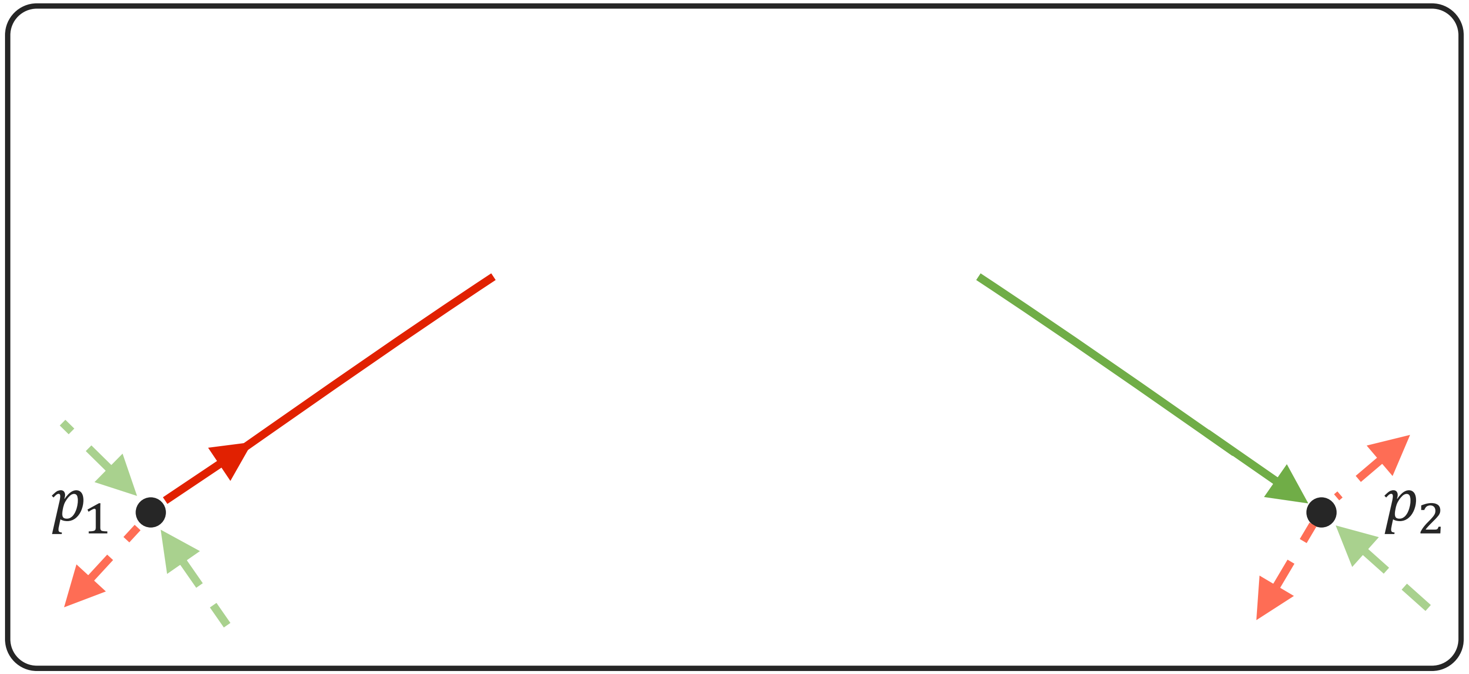
Fixed point
- Stable manifold (green)
- Unstable manifold (red)



When $K = 1.2$

Definition of lobe

- Lobe : Region bounded by the manifolds of resonant orbits
  on the Poincaré map



Poincaré map

p_i: Fixed point (resonant orbit)

: Stable manifold

: Unstable manifold

Definition of lobe

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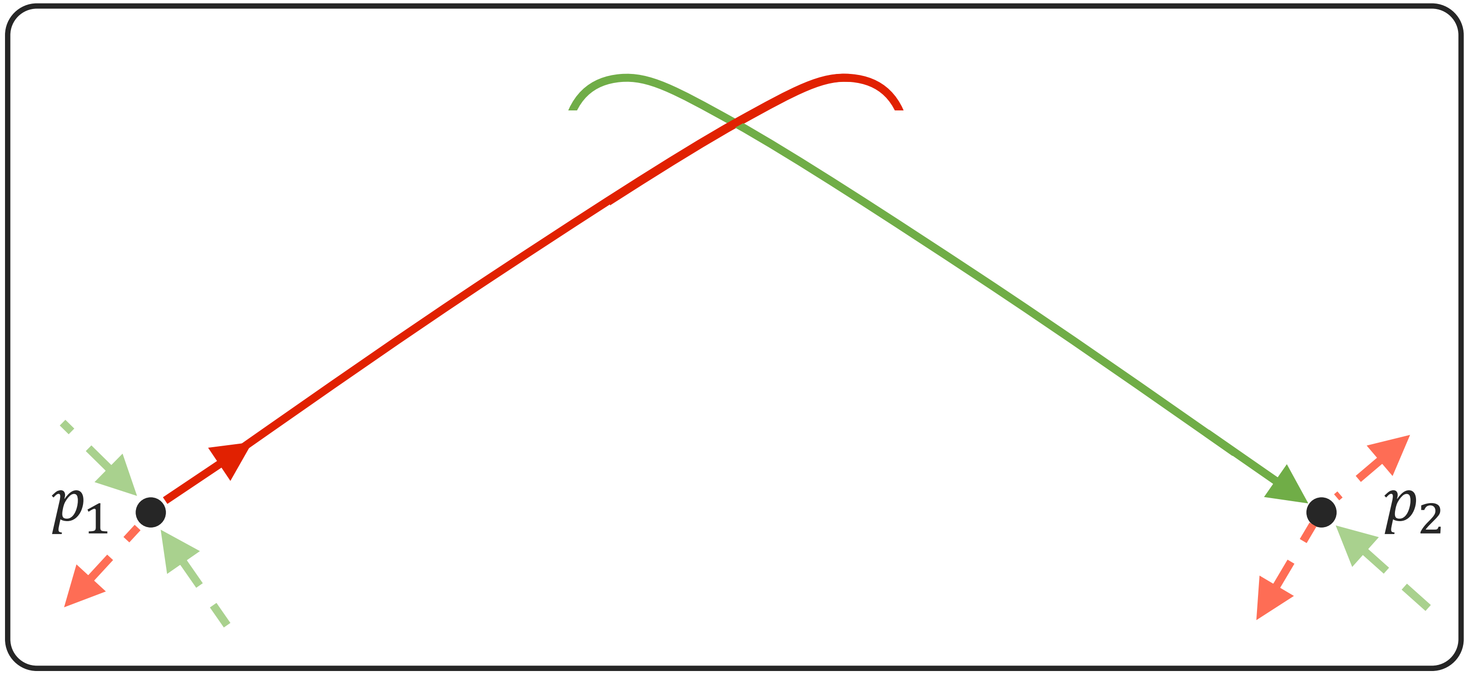

Poincaré map

p_i: Fixed point (resonant orbit)

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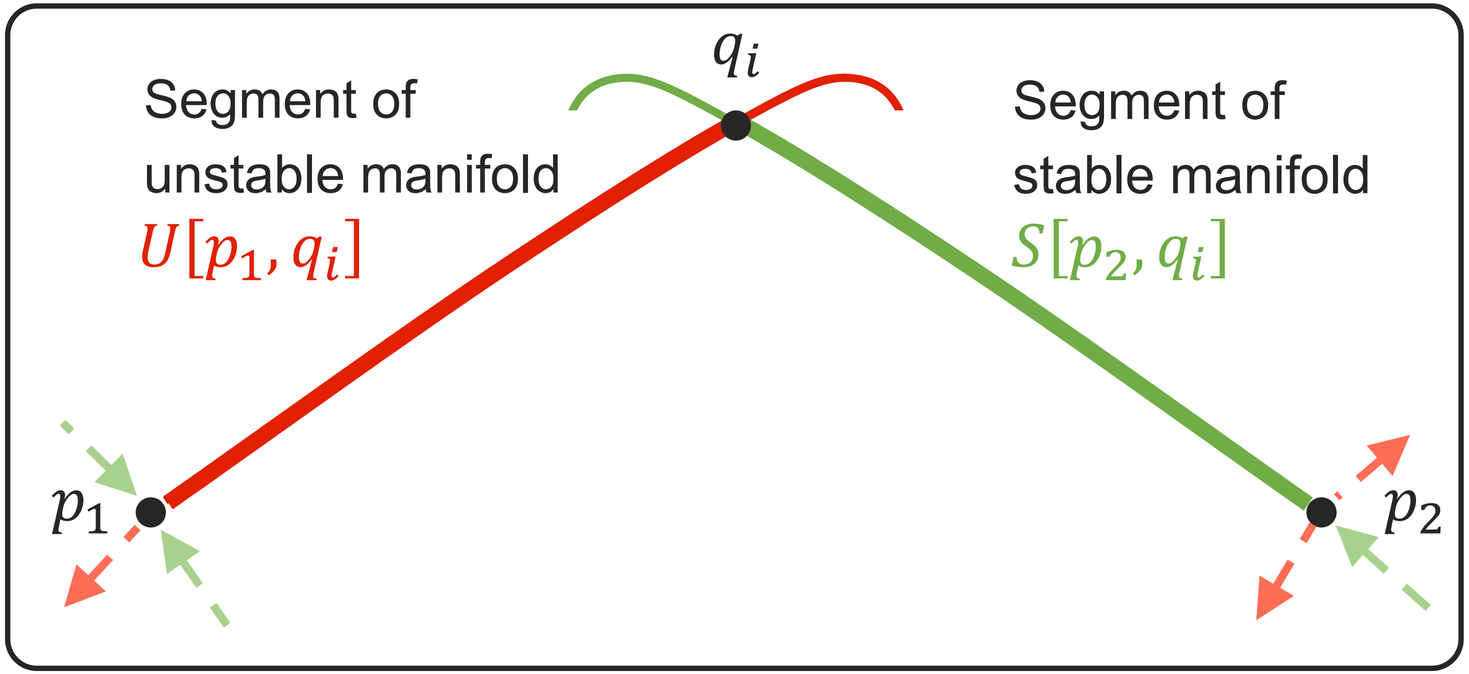
Definition of lobe

- Lobe : Region bounded by the manifolds of resonant orbits on the Poincaré map
- Primary intersection point : A heteroclinic point q_i if $U[p_1, q_i]$ and $S[p_2, q_i]$ intersects only in q_i



p_i: Fixed point (resonant orbit)

q_i: Primary intersection point

Poincaré map

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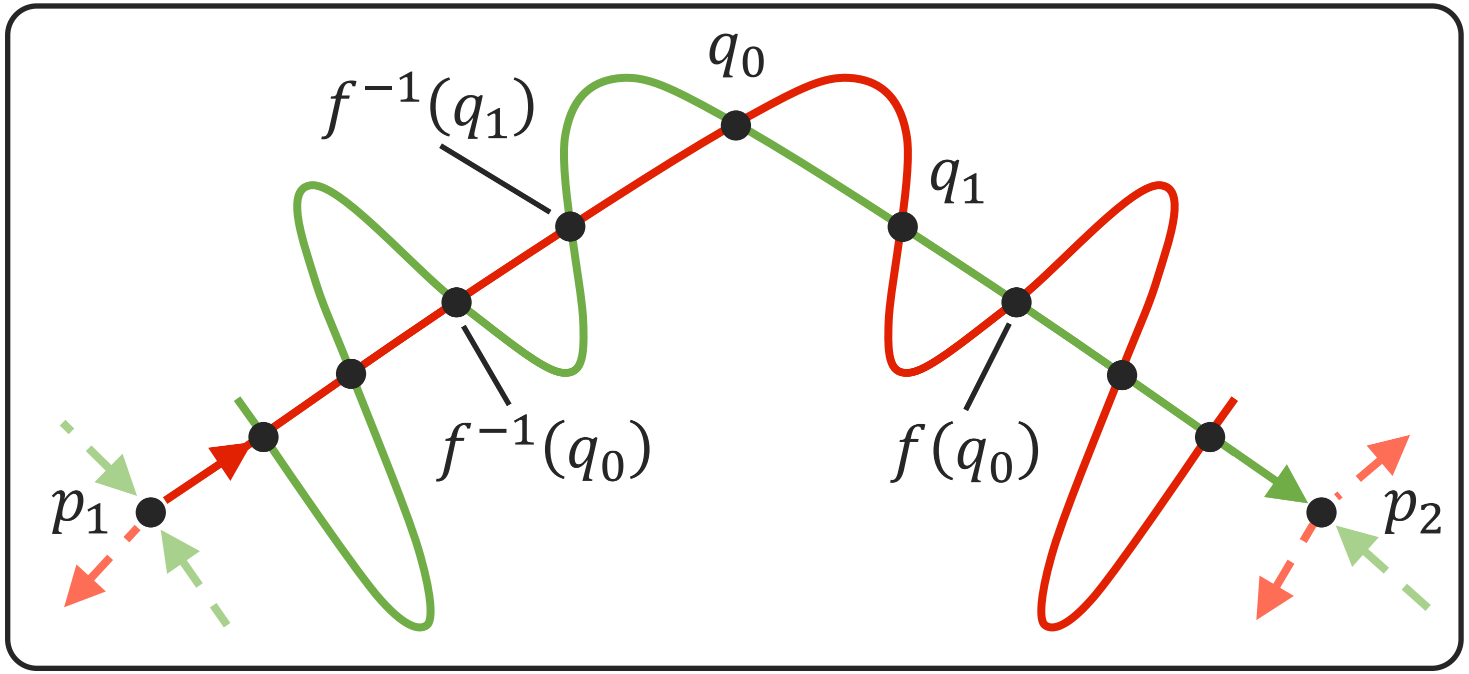

Poincaré map

p_i: Fixed point (resonant orbit)

q_i: Primary intersection point

f: Mapping function

Definition of lobe

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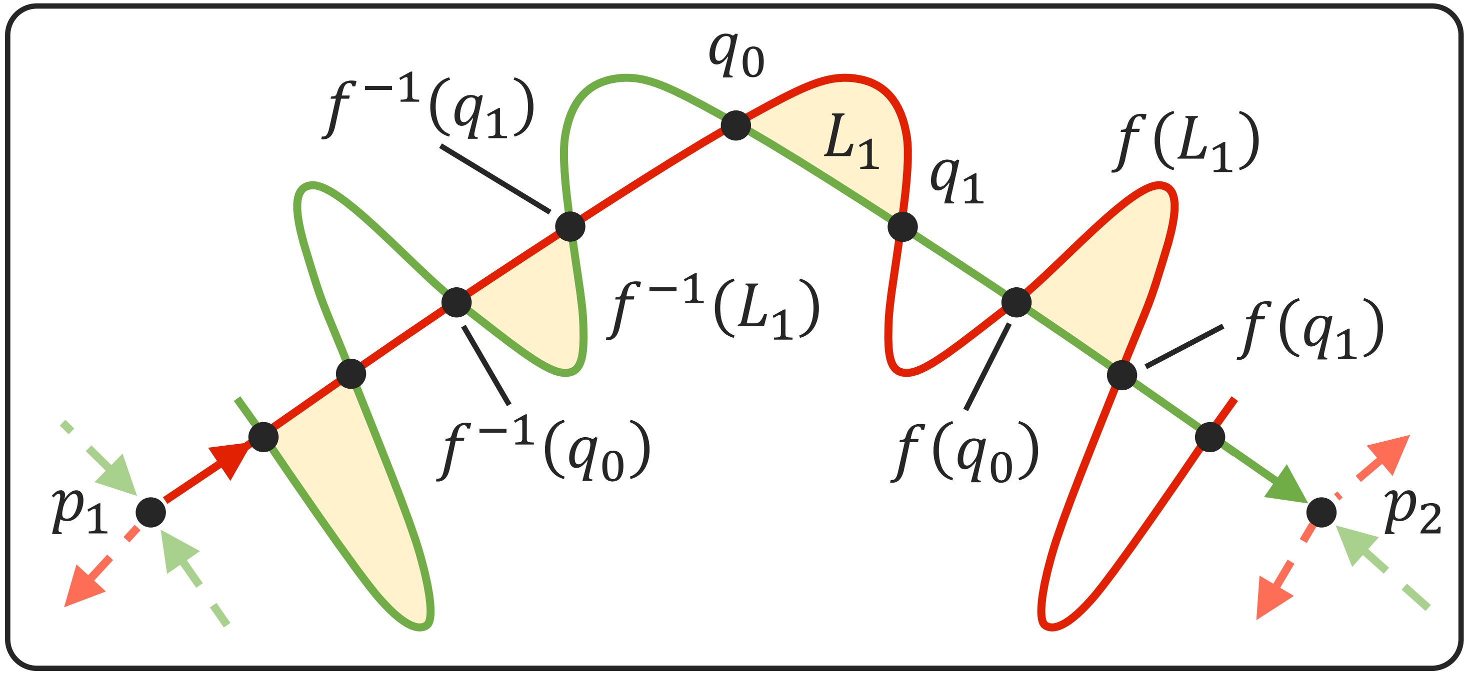

p_i: Fixed point (resonant orbit)

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L_i: Lobe

Poincaré map

Definition of lobe

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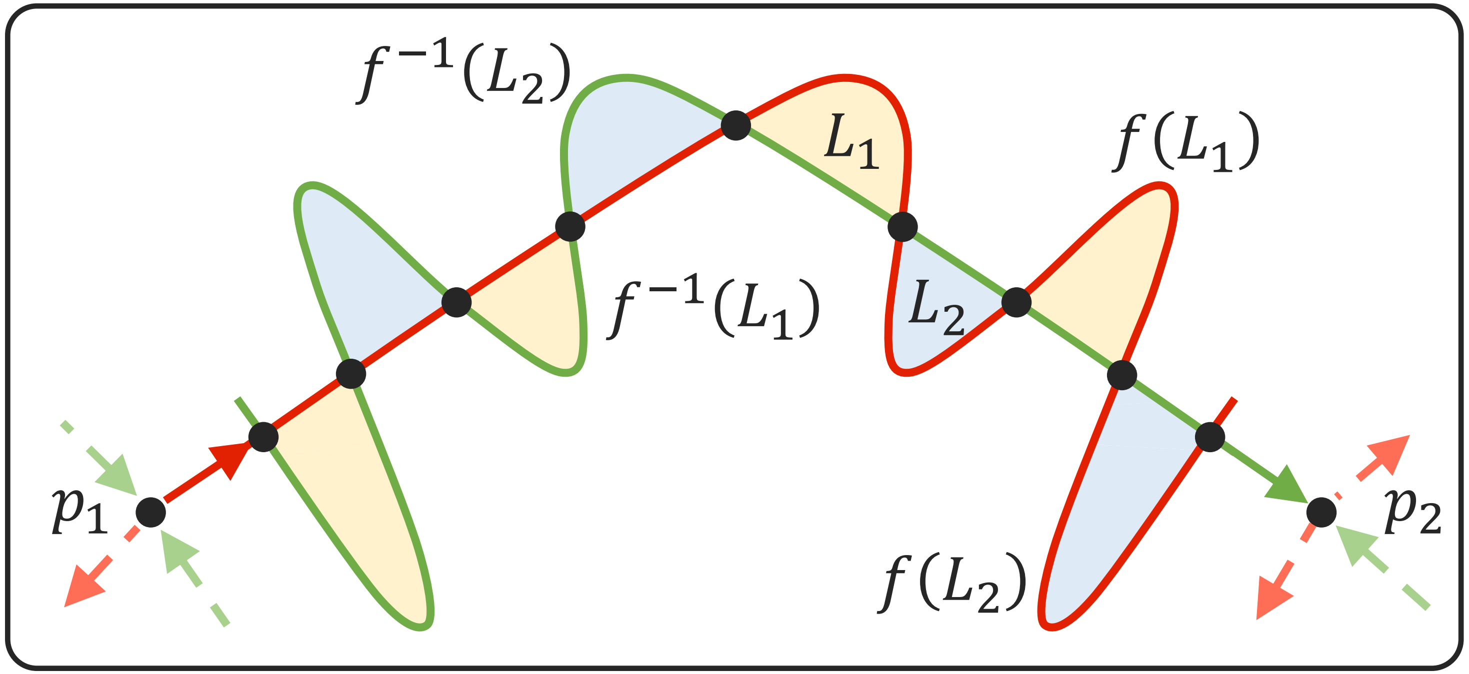


p_i: Fixed point (resonant orbit)

q_i: Primary intersection point

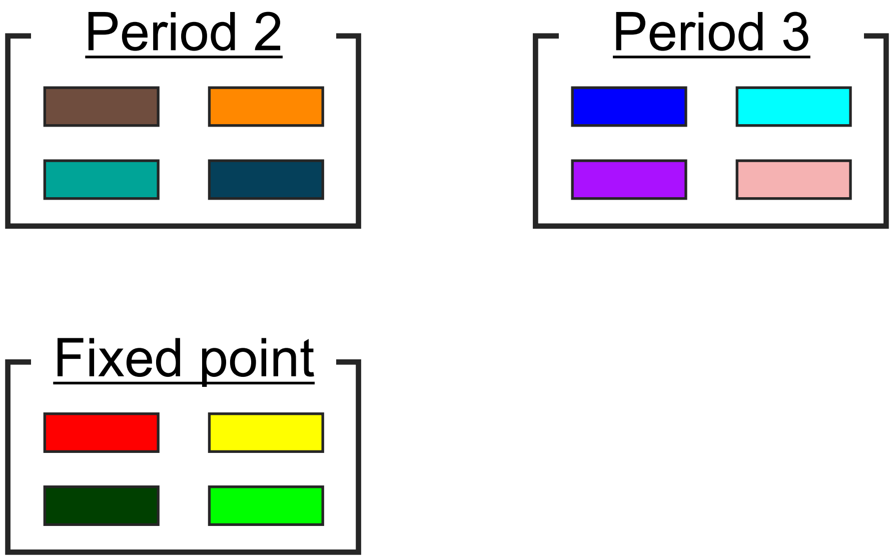
f: Mapping function

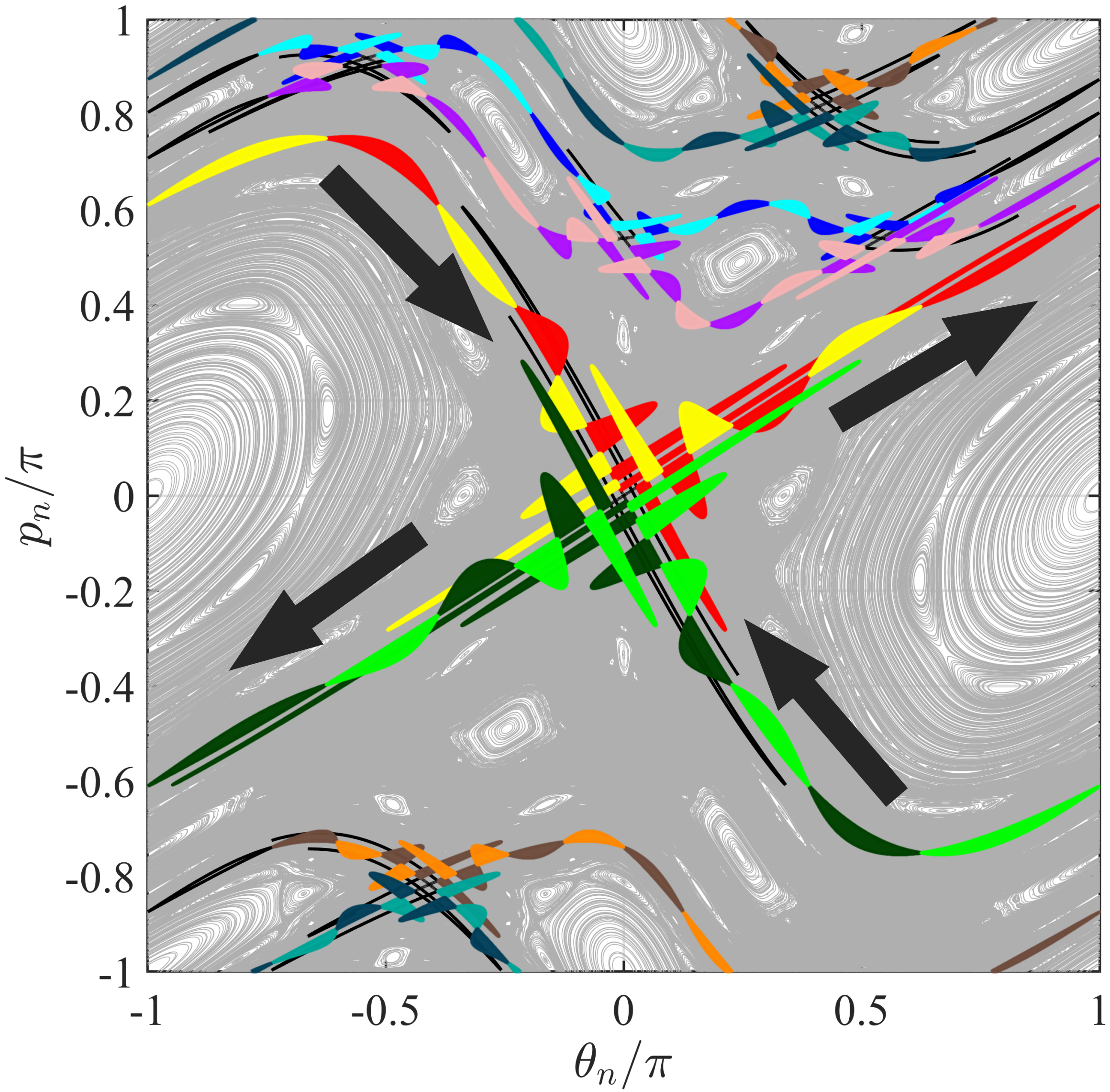
L_i: Lobe

Poincaré map

Lobe Dynamics

- Each periodic orbit has 4 lobe sequences
- Transfer occurs along the direction of unstable manifolds





When $K = 1.2$

Problem settings

Selected Lobe Sequences

- Lobes are represented by their center of gravity for the preliminaly design

  $\times$: center of gravity

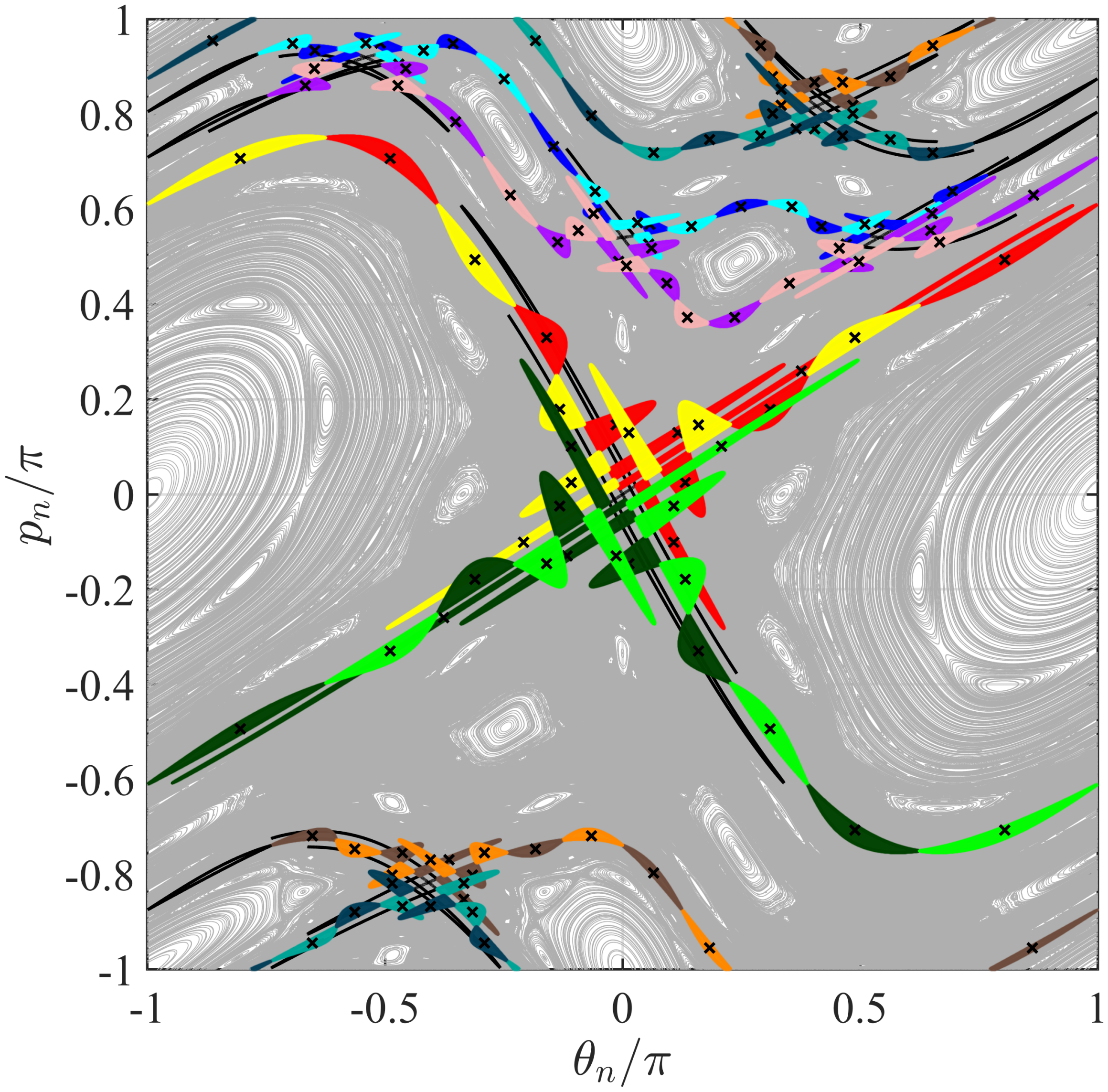
- For excluding explicit solutions, control input $\boldsymbol{u}$ should be small enough

$$\|\boldsymbol{u}\| \leq u^*$$

$\|\boldsymbol{u}\|$: the norm of $\boldsymbol{u}$

= the distance between lobes

$\boldsymbol{u}$: control input vector



When $K = 1.2$

Problem settings

Selected Lobe Sequences

- Large lobes are better for manuvers

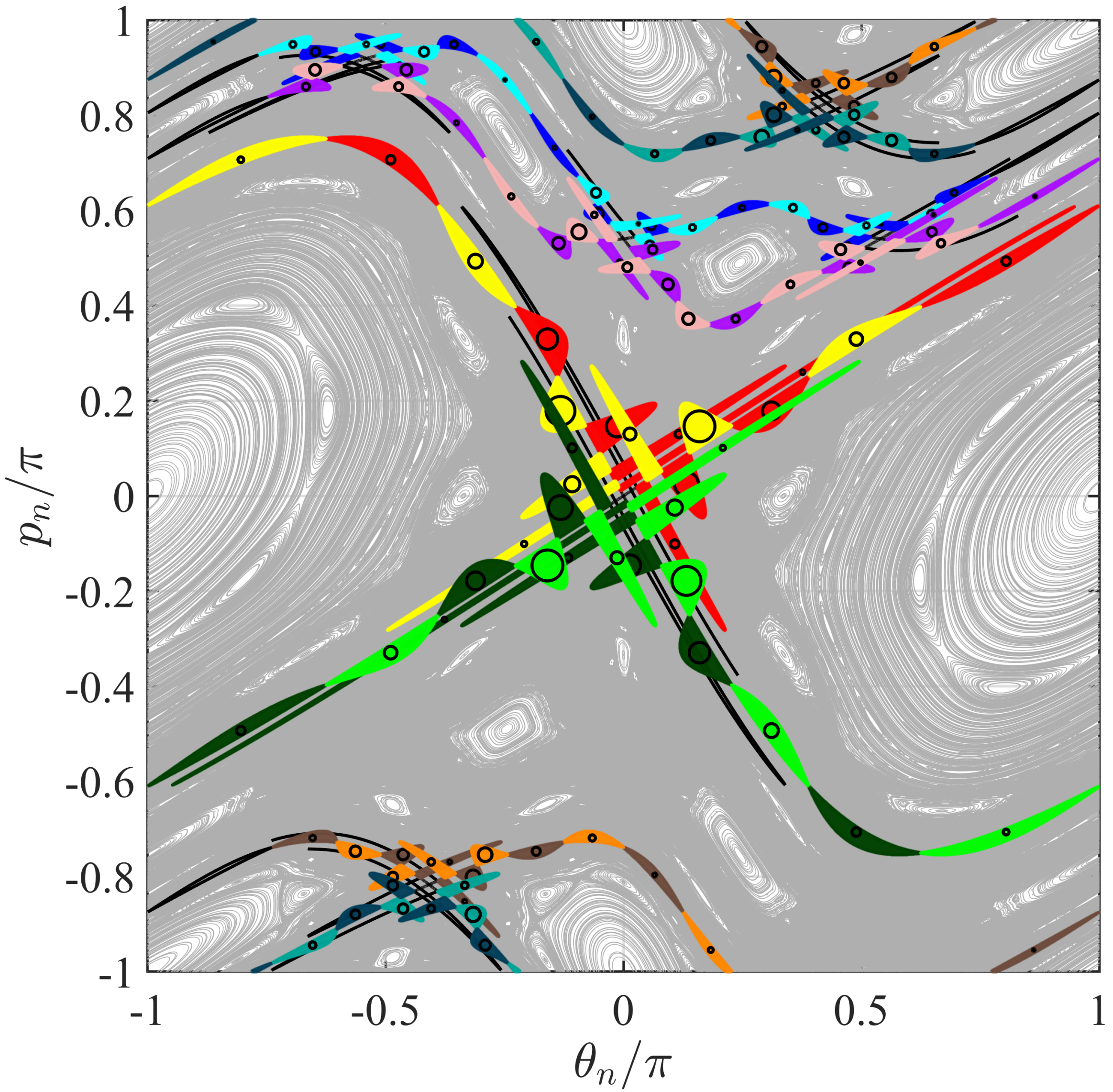
$$r \geq r^*$$

r : minimum distance from the center to the border

- In this case, $0.01 \leq r^* (\leq 0.03)$

Difficult to find lobes with $r^* < 0.01$

No good solutions when $r^* \geq 0.03$



When $K = 1.2$, $r^* = 0.01$

Problem settings

Selected Lobe Sequences

- Large lobes are better for manuvers

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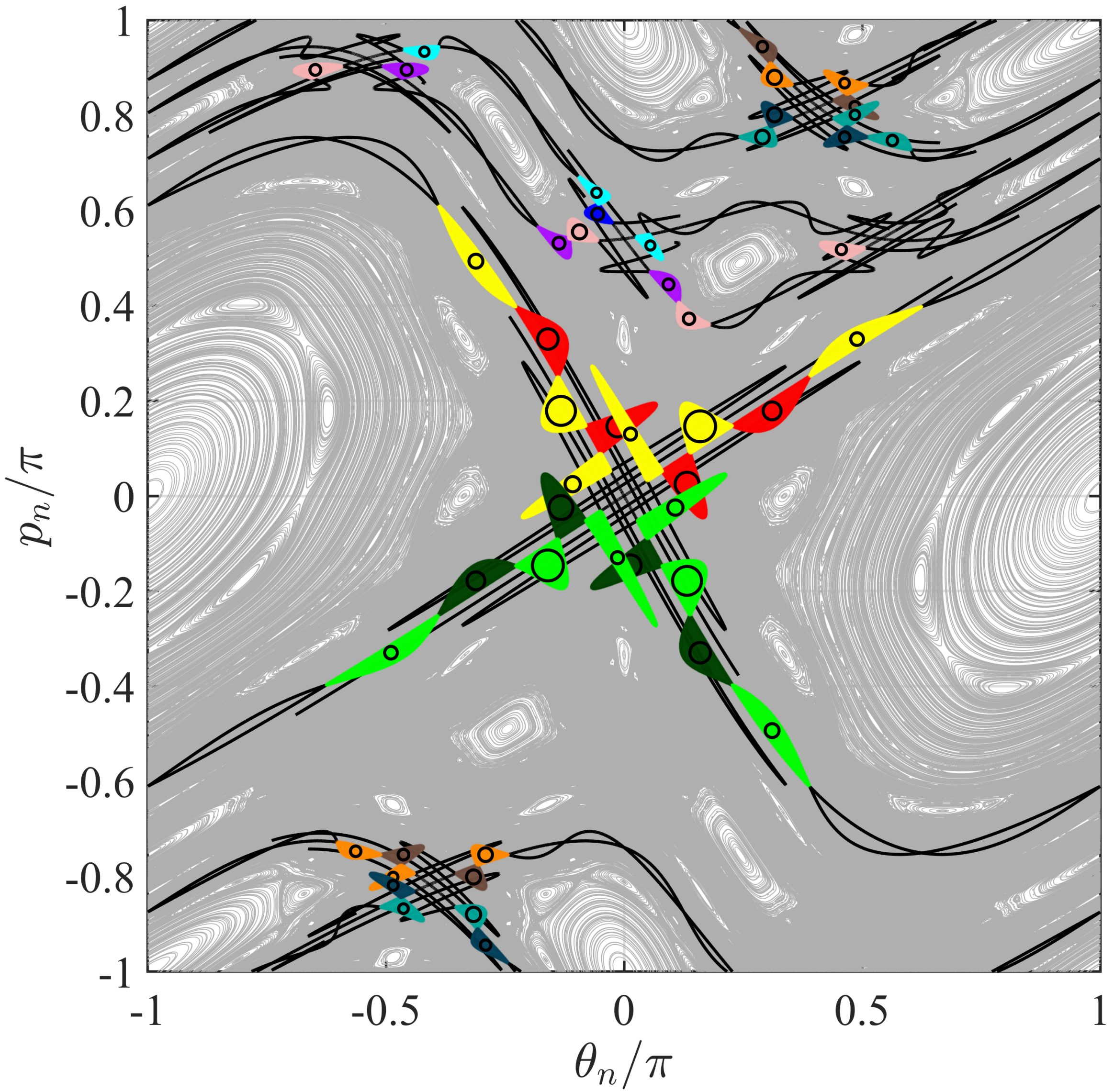


When $K = 1.2$, $r^* = 0.03$

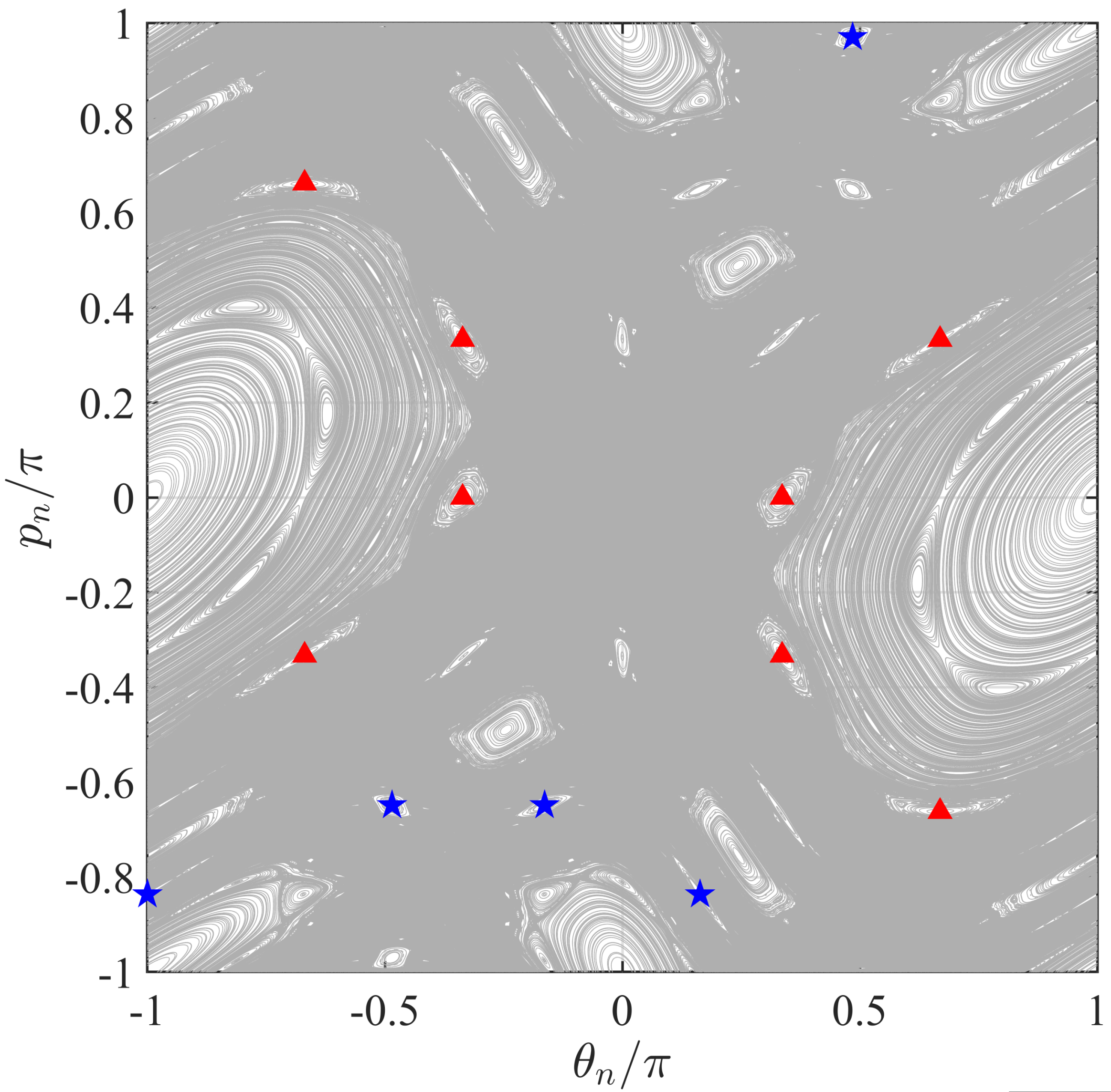
Problem settings

Transfer Problem

- Design transfers between stable periodic orbits using lobe sequences

▲ : Departure orbit (period 8)

★ : Arrival orbit (period 5)



When $K = 1.2$

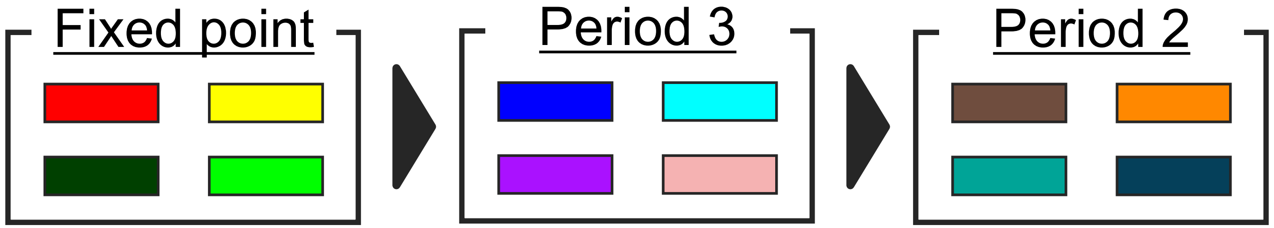
Transfer Problem

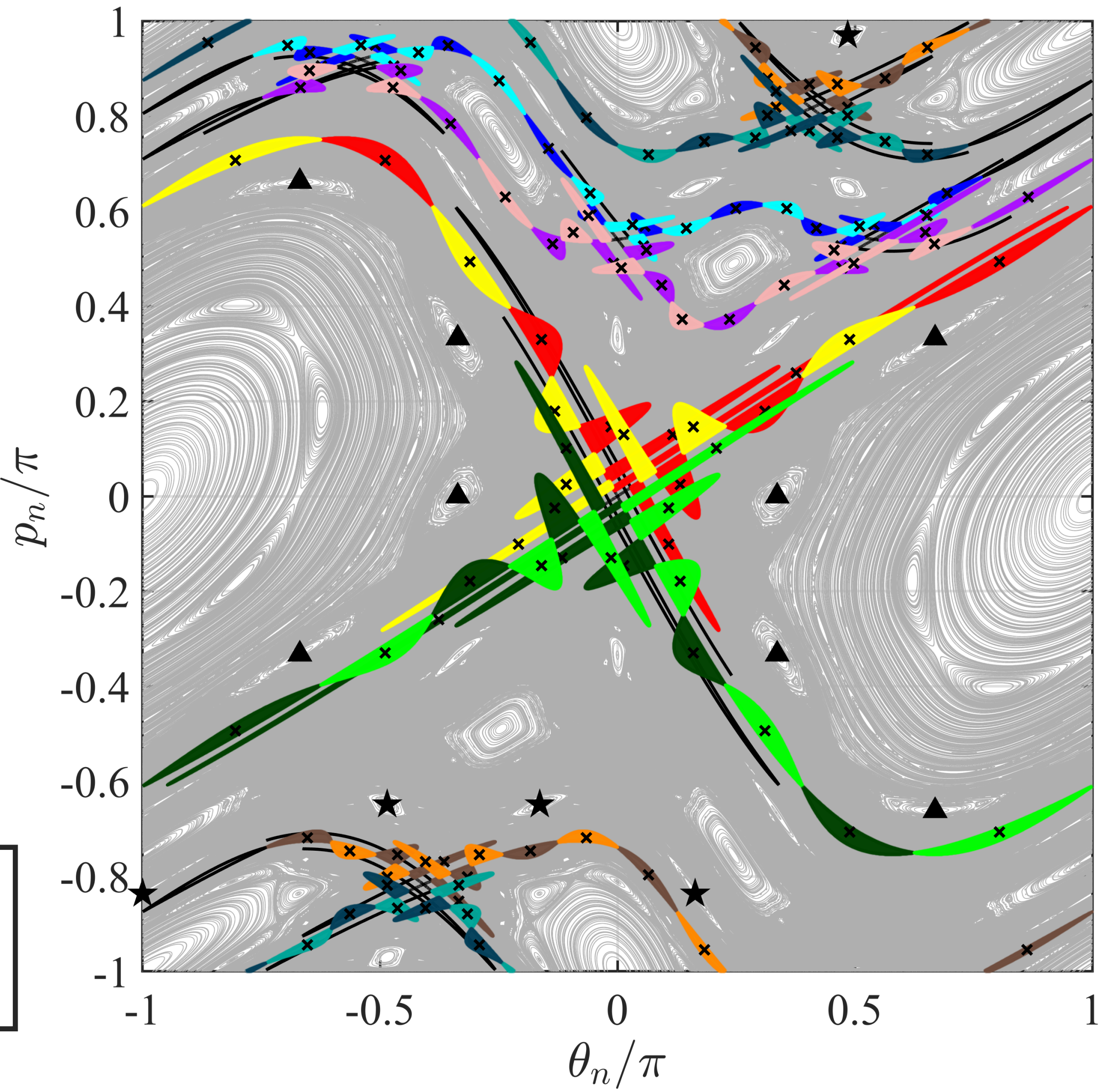
- Design transfers between stable periodic orbits using lobe sequences

▲ : Departure orbit (period 8)

★ : Arrival orbit (period 5)

- The order of lobe sequences is designed





When $K = 1.2$

Problem settings

Formulation

- Problem:

minimize $J = \sum \|\boldsymbol{u}_k\|$

subject to $\|\boldsymbol{u}_k\| \le u^*$

$r \ge r^*$

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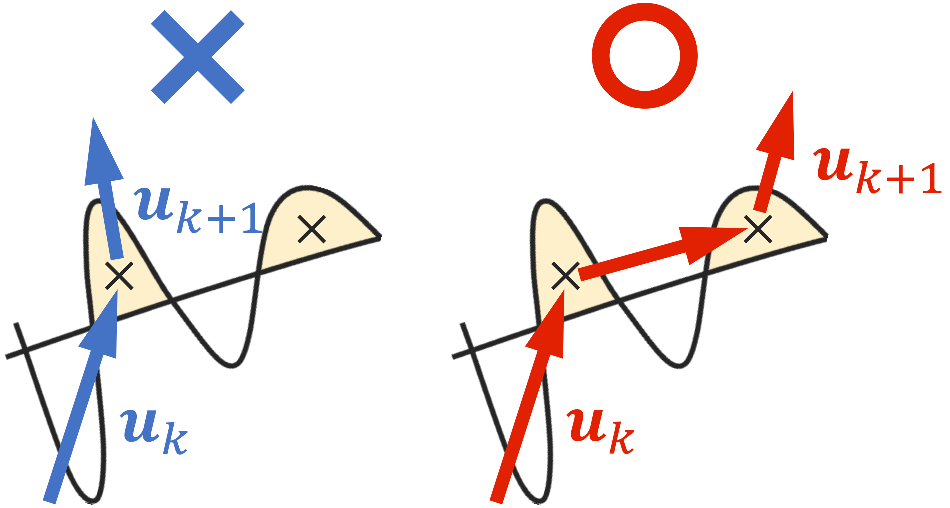
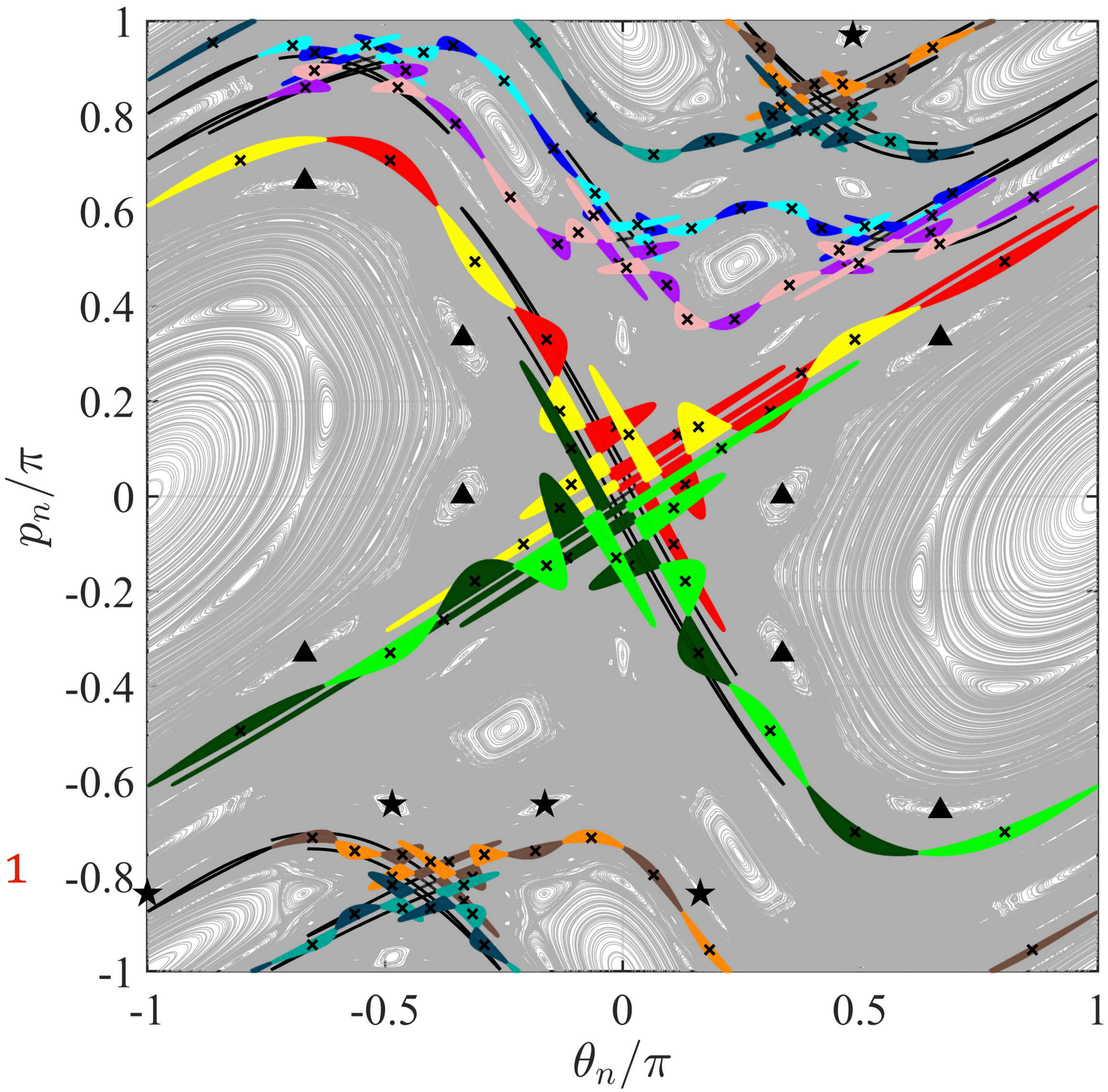
Use lobe sequences properly





When $K = 1.2$, $r^* = 0.01$

Formulation

- Problem:

minimize	$J = \sum \lVert \boldsymbol{u}_k \rVert$
subject to	$\lVert \boldsymbol{u}_k \rVert \leq u^*$
	$r \geq r^*$

Graph analysis

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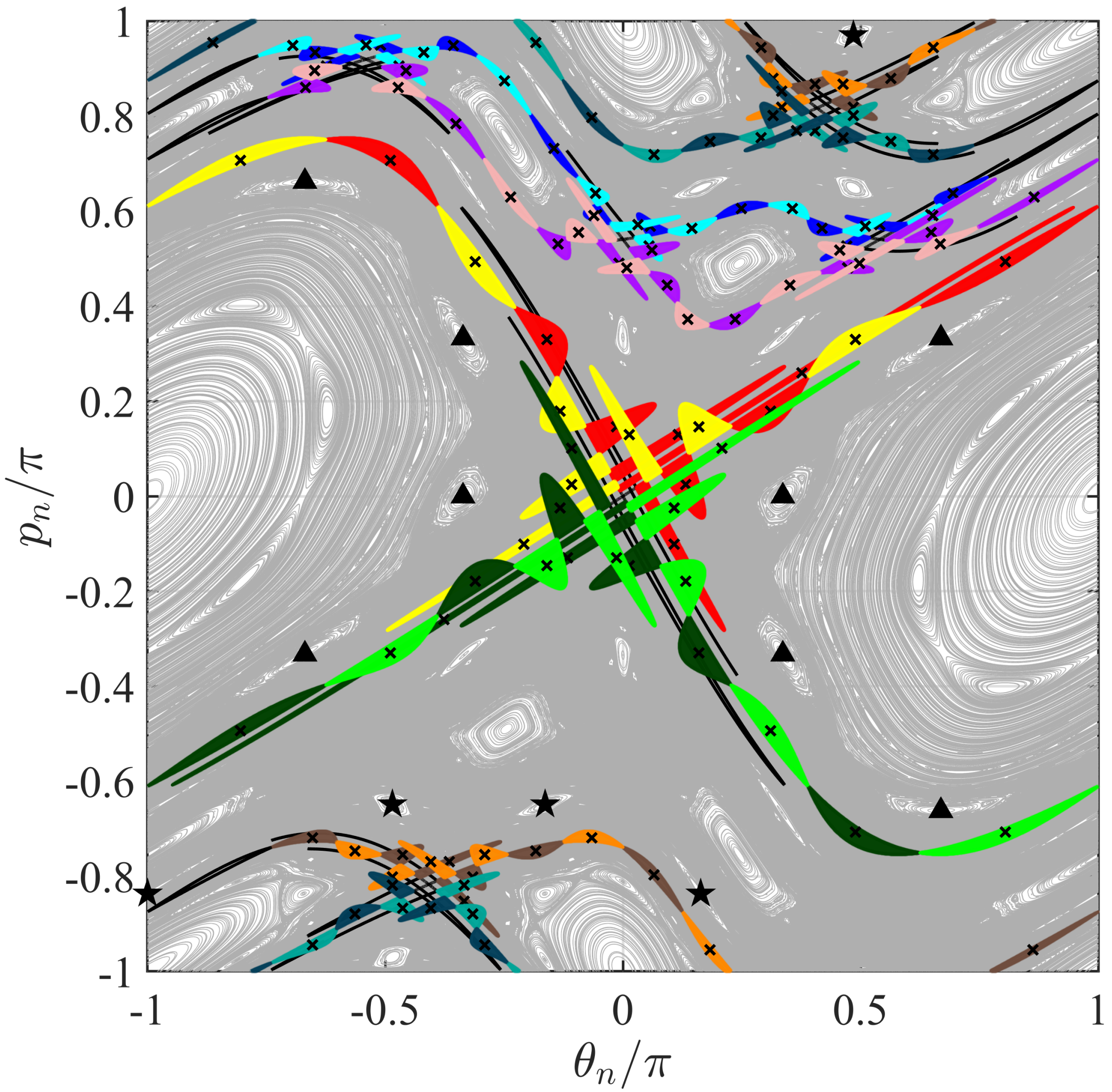
Use lobe sequences properly

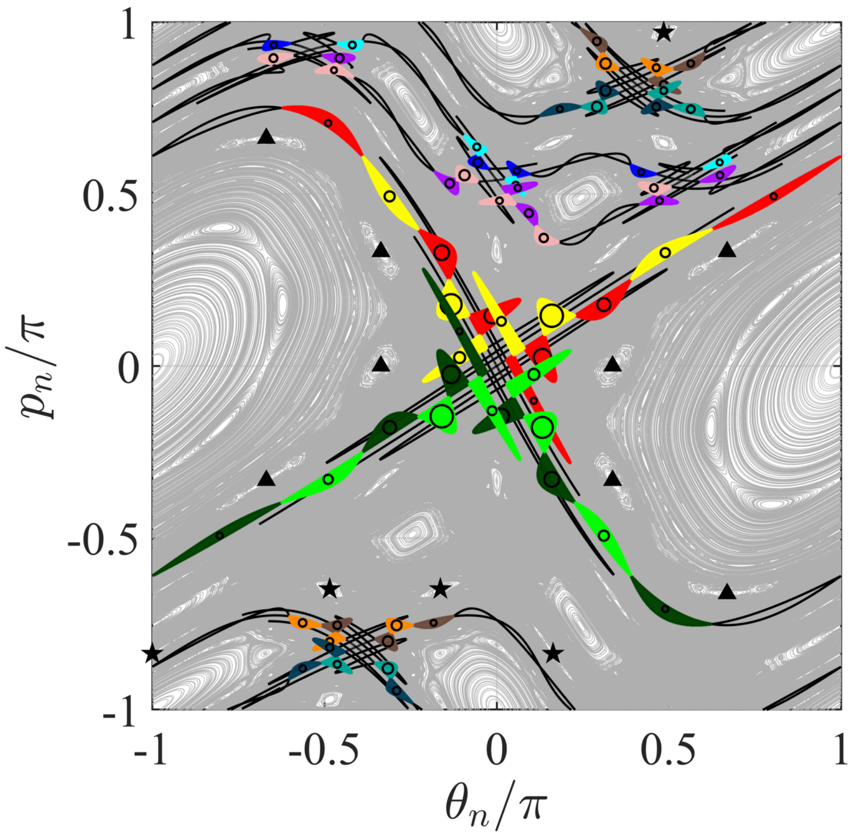
- Pick up low-cost paths from the graph and find the optimal transfer



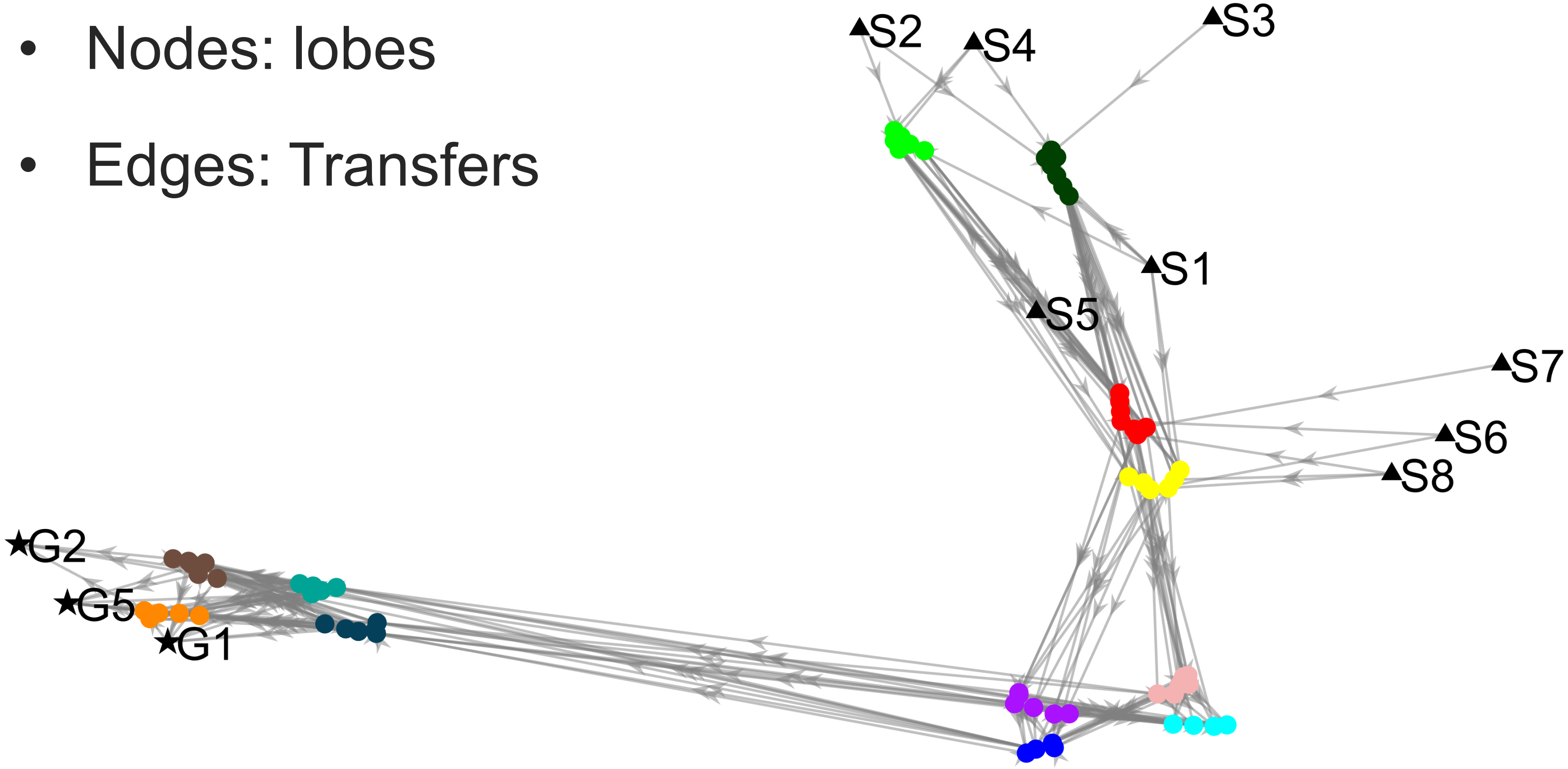
When $K = 1.2$, $r^* = 0.01$

Graph Analysis

- Example: $u^* = 0.88, \ r^* = 0.025$



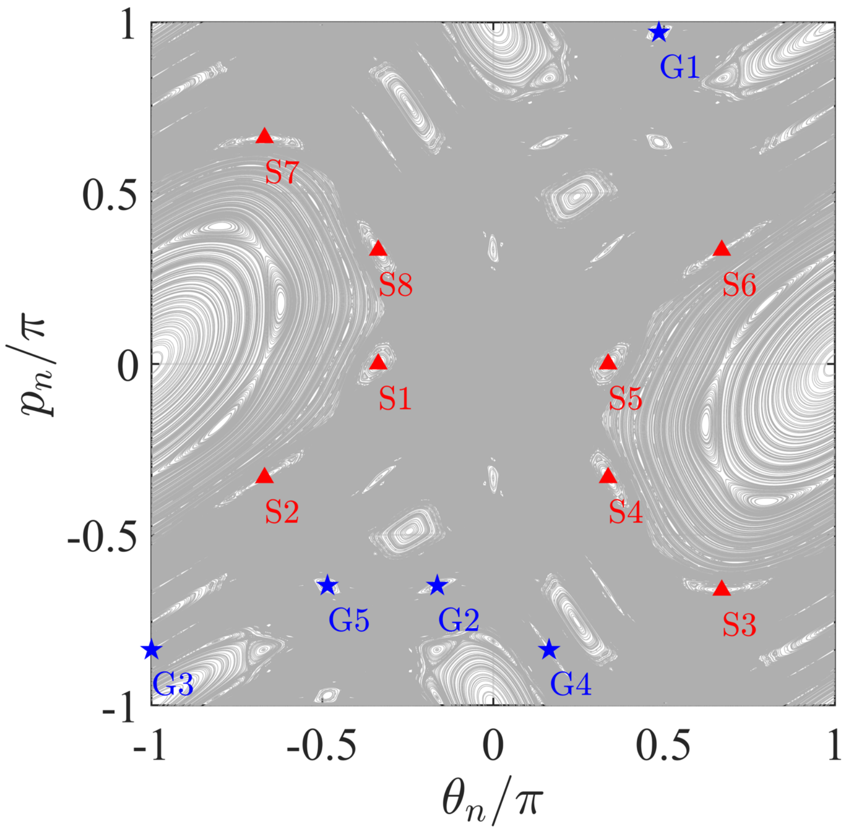
- Nodes: lobes
- Edges: Transfers



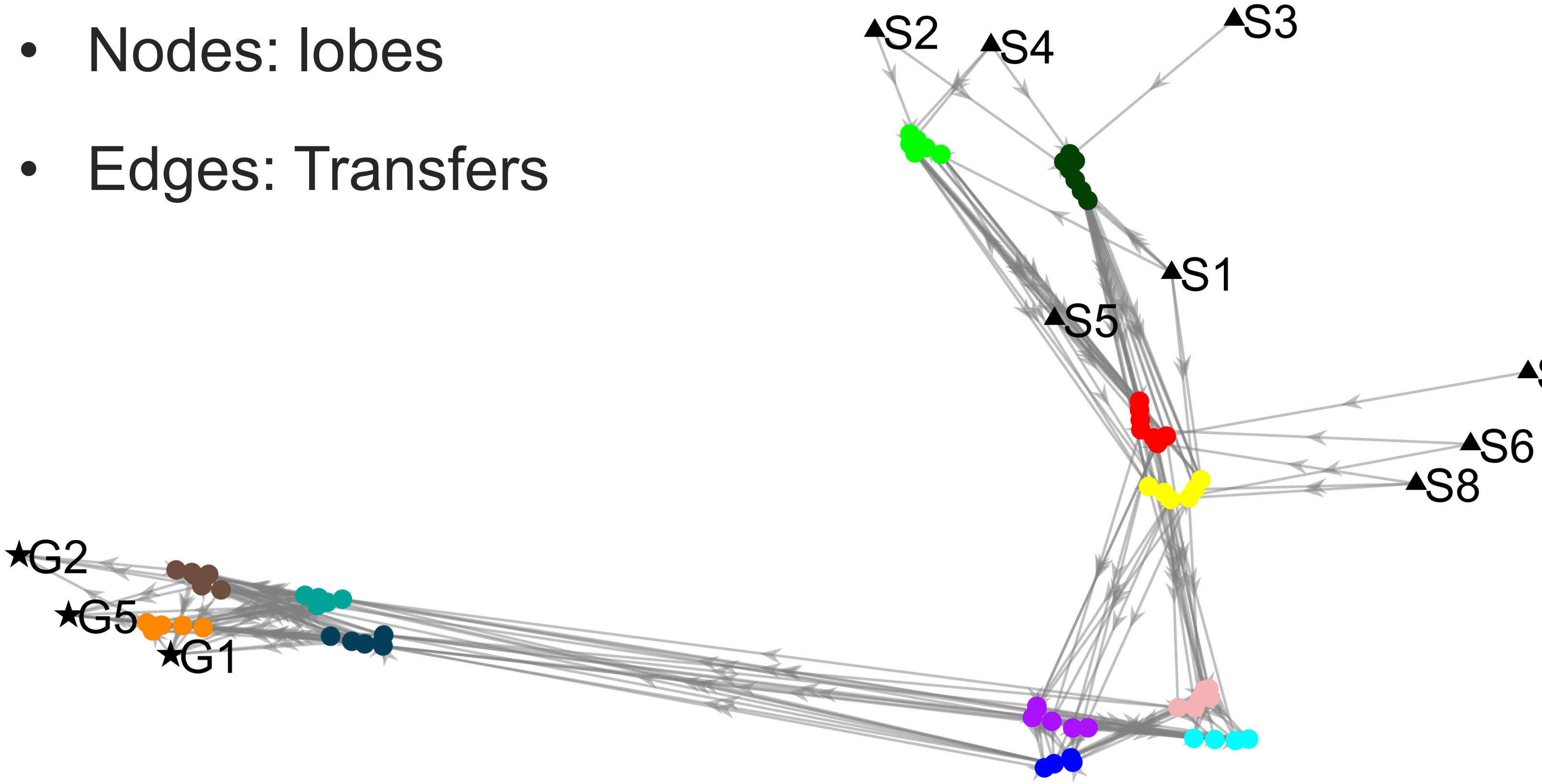
Results

Graph Analysis

- Example: $u^* = 0.88$, $r^* = 0.025$



- Nodes: lobes
- Edges: Transfers



Results

Graph Analysis

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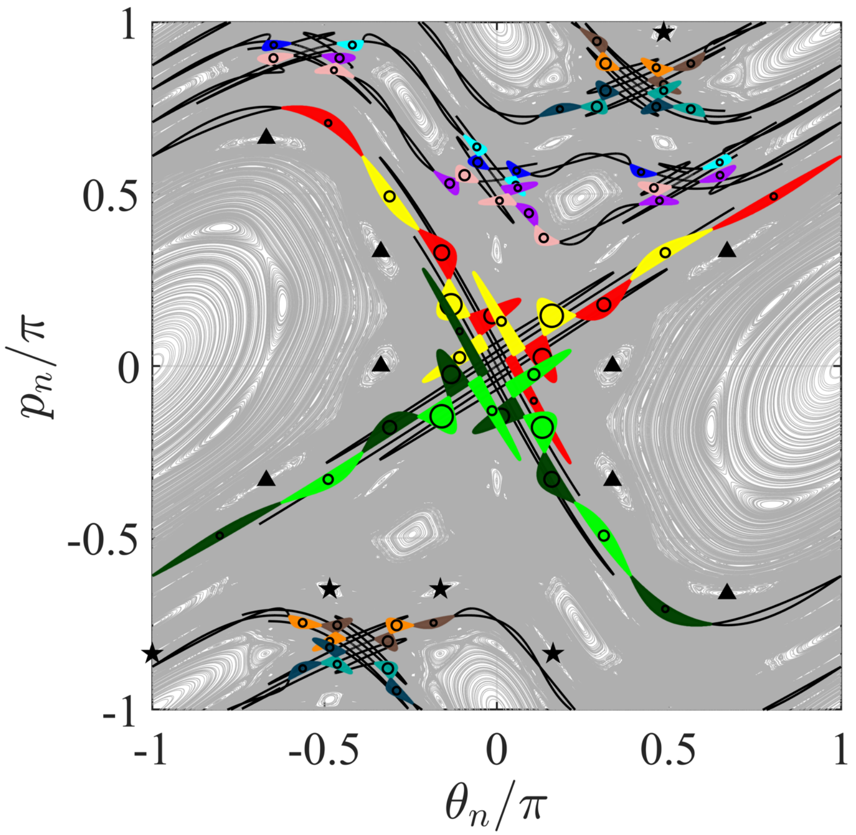
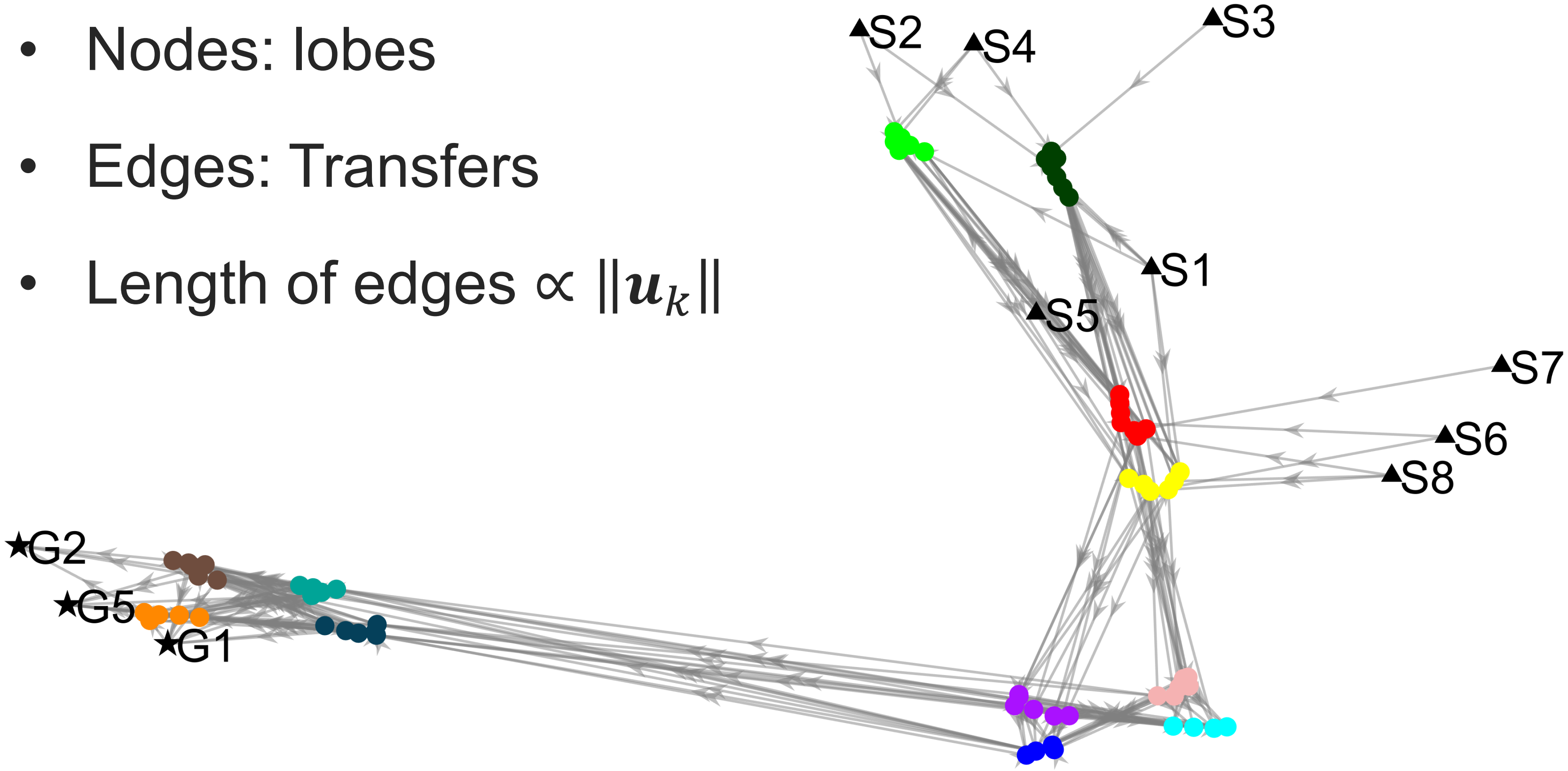
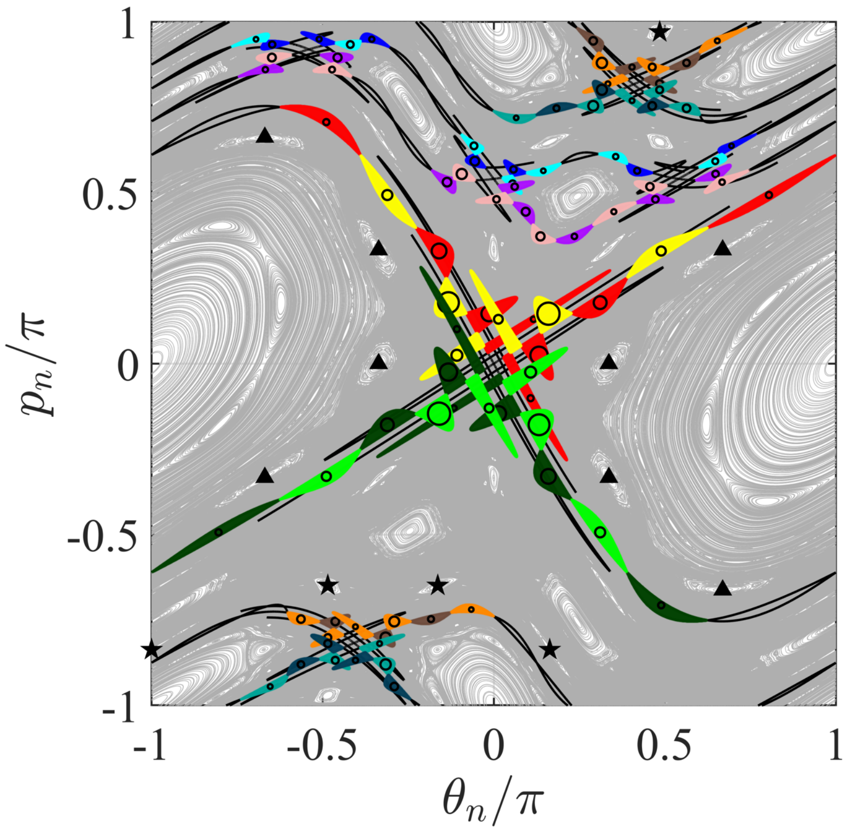


- Nodes: lobes
- Edges: Transfers
- Length of edges $\propto \|\boldsymbol{u}_k\|$

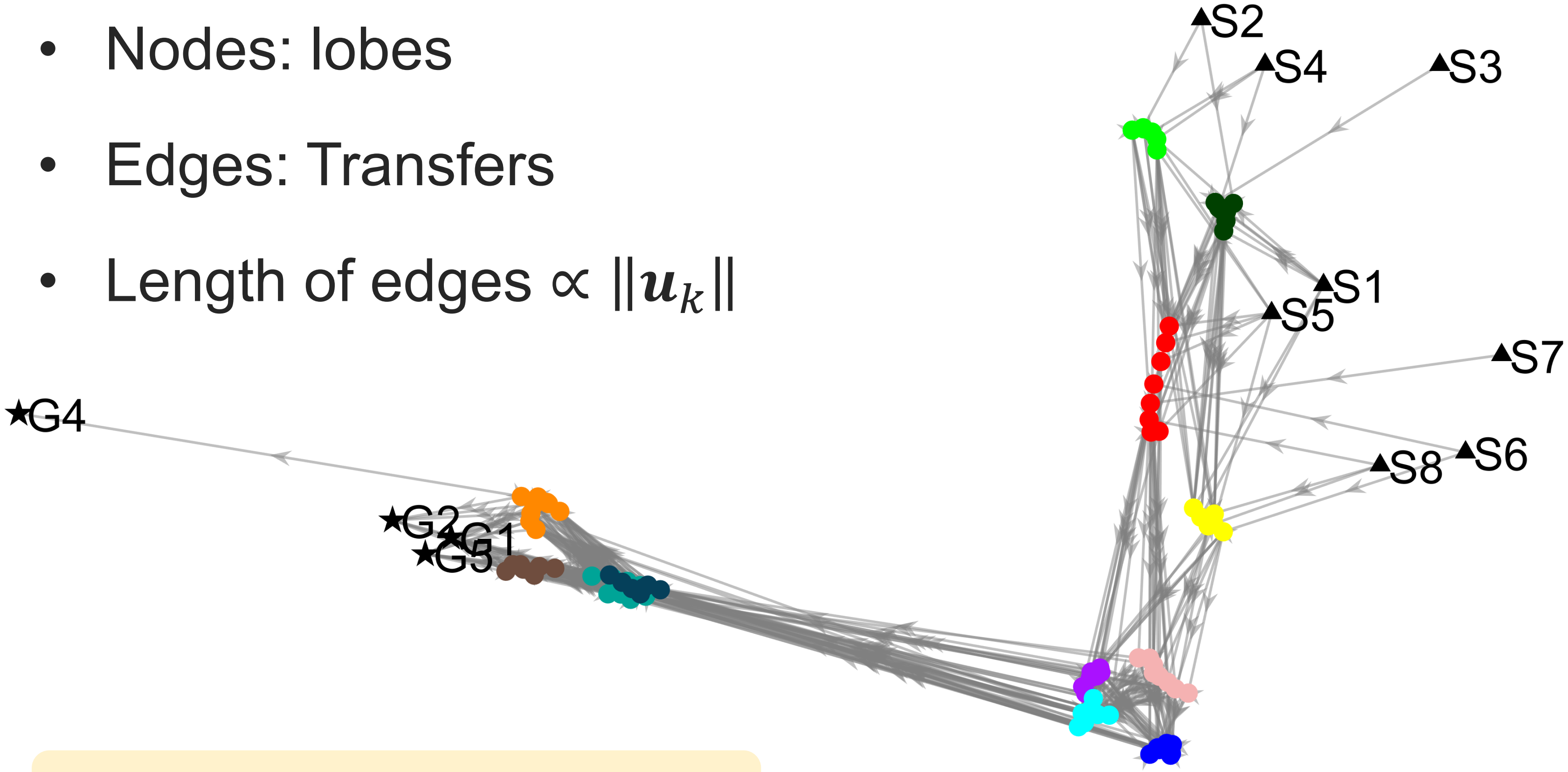


Graph Analysis

- Example: $u^* = 0.88$, $r^* = 0.02$



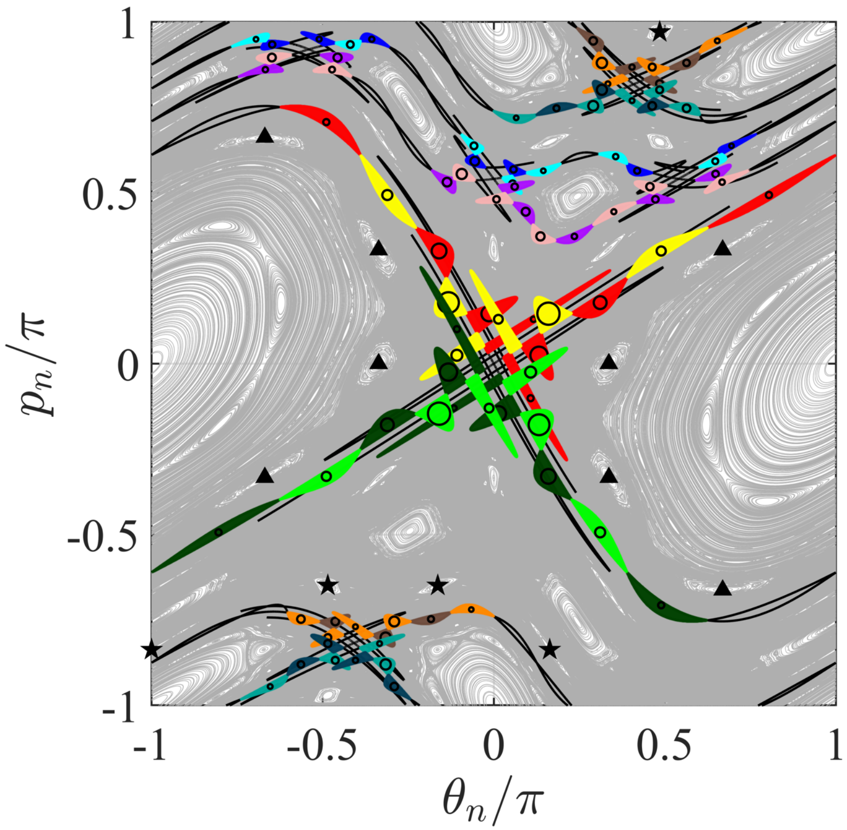
- Nodes: lobes
- Edges: Transfers
- Length of edges $\propto \|\boldsymbol{u}_k\|$



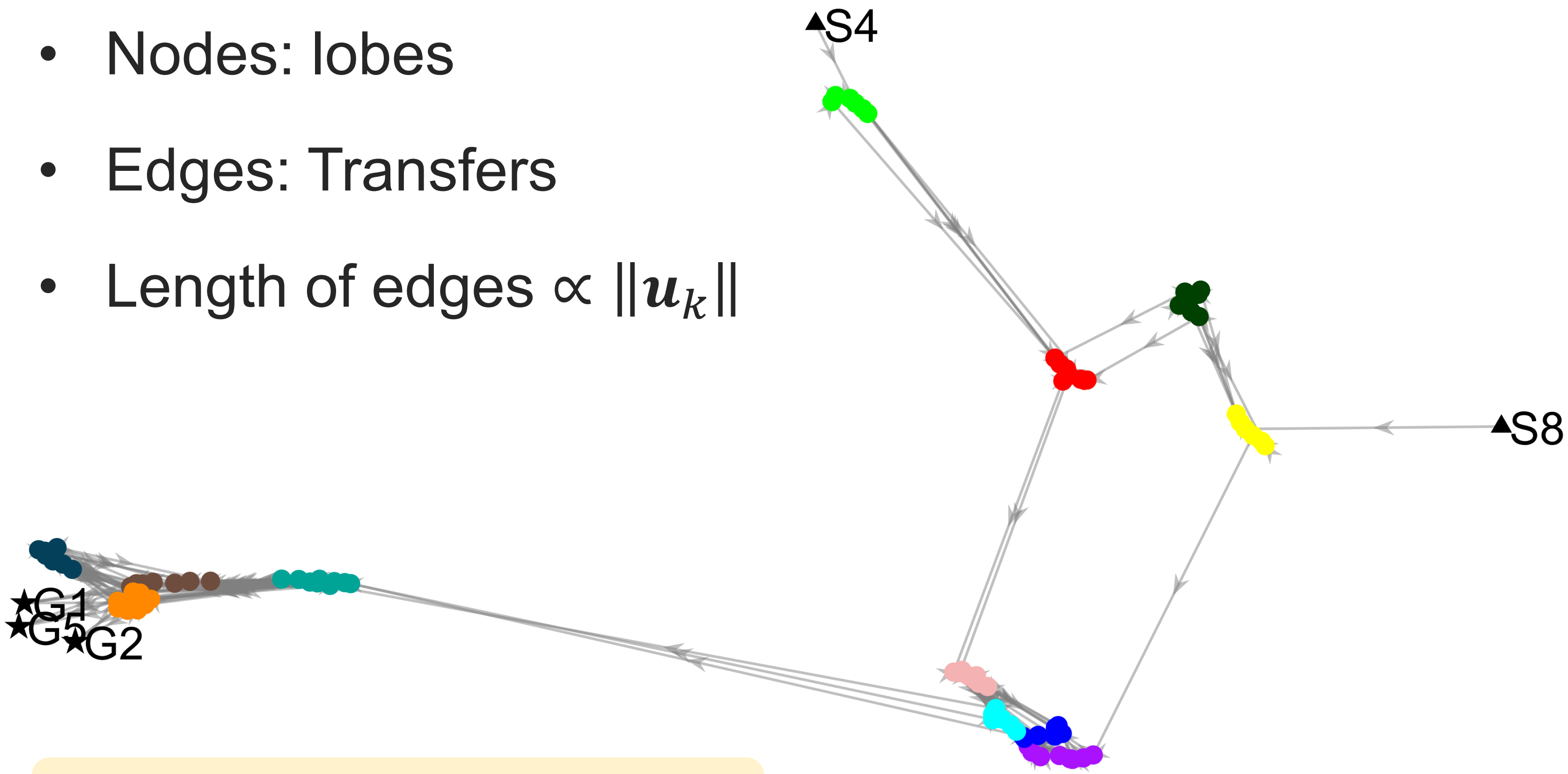
Small r^* → More edges

Graph Analysis

- Example: $u^* = 0.52$, $r^* = 0.02$



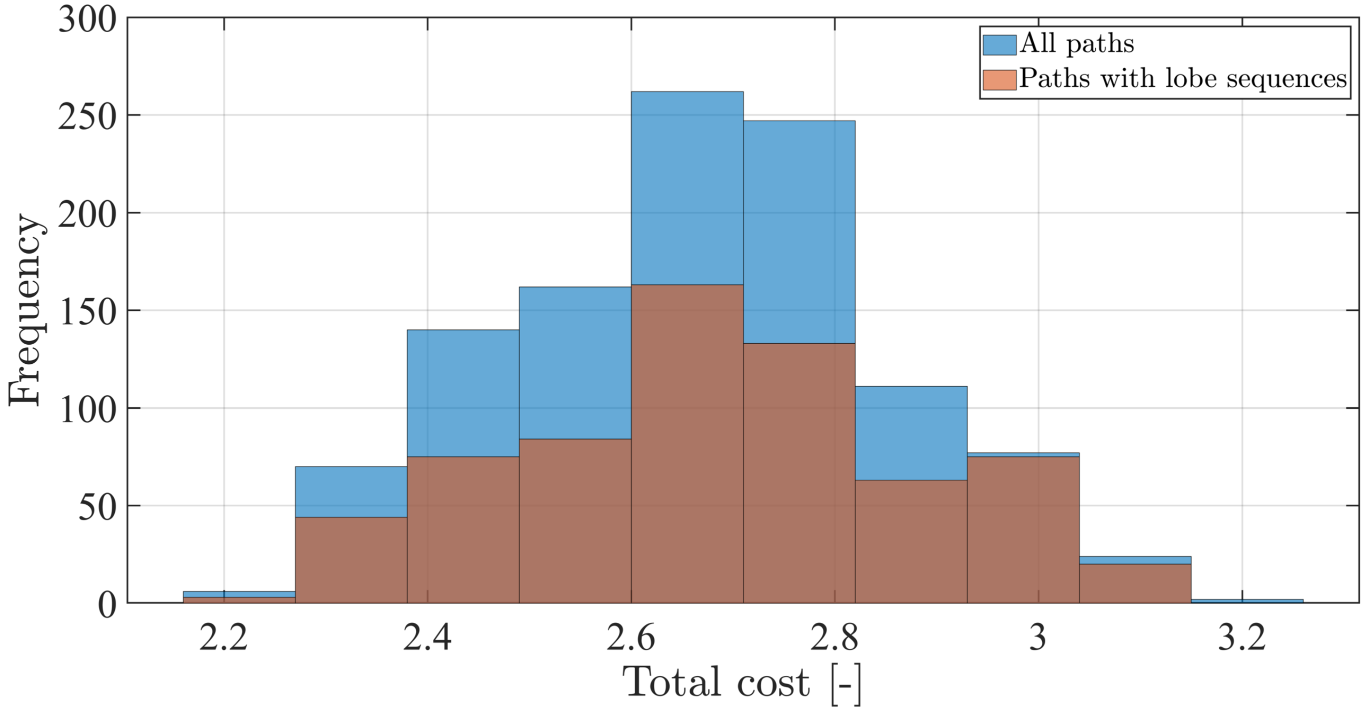
- Nodes: lobes
- Edges: Transfers
- Length of edges $\propto \|\boldsymbol{u}_k\|$

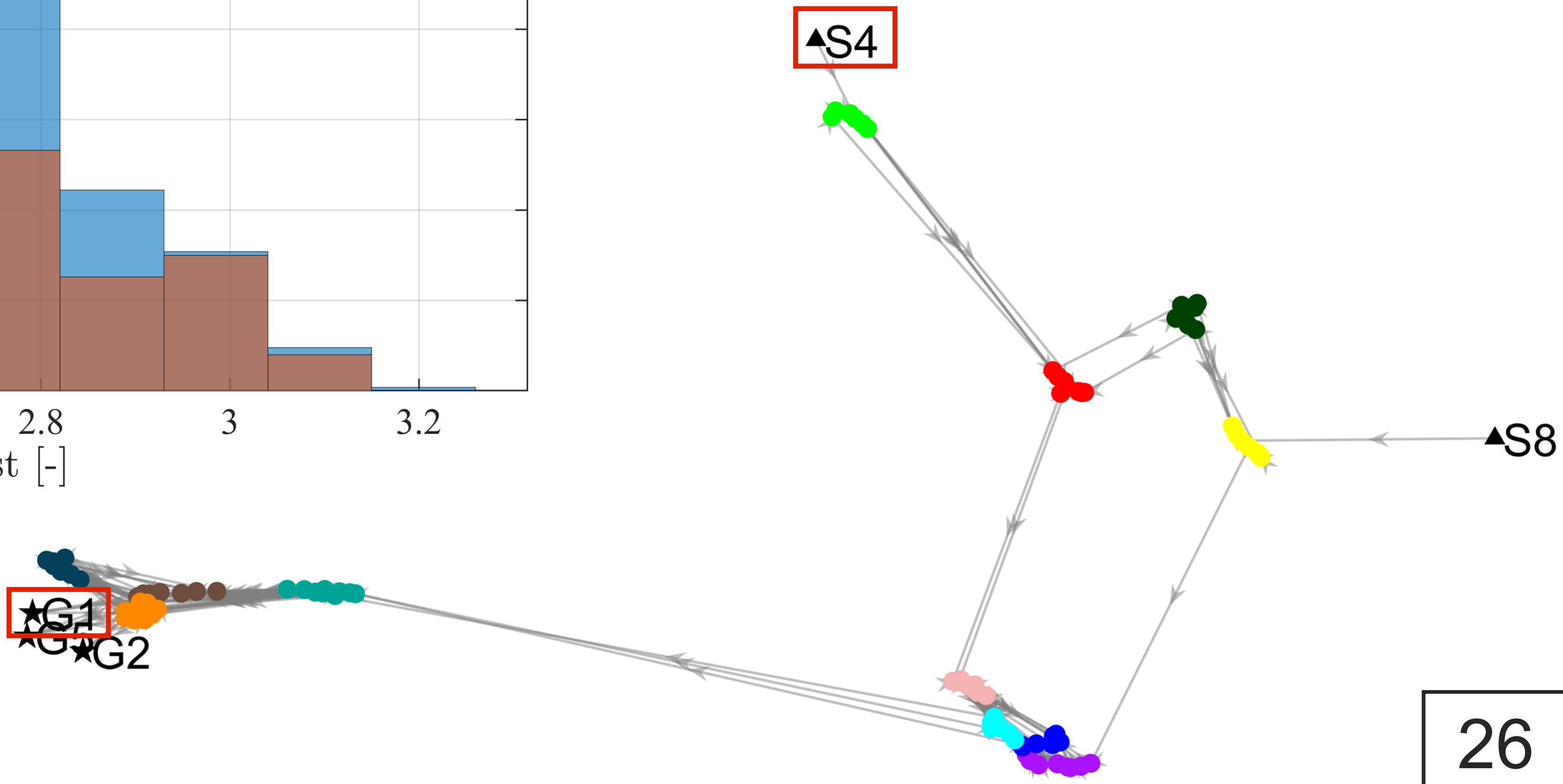


Small u^* → Less edges

Graph Analysis

- Example: $u^* = 0.52, r^* = 0.02$





Graph Analysis

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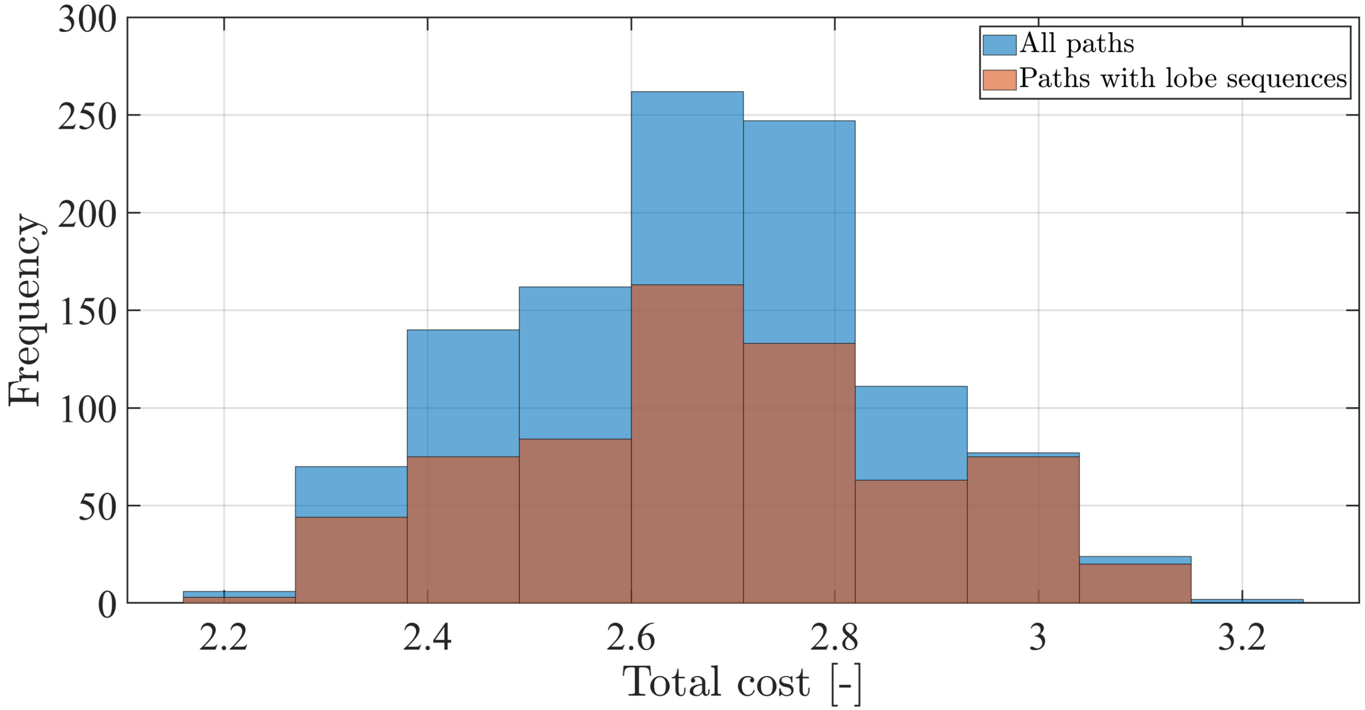

$$\begin{aligned} &\text{minimize} && J = \sum \|\boldsymbol{u}_k\| \\ &\text{subject to} && \|\boldsymbol{u}_k\| \leq u^* \\ & && r \geq r^* \end{aligned}$$

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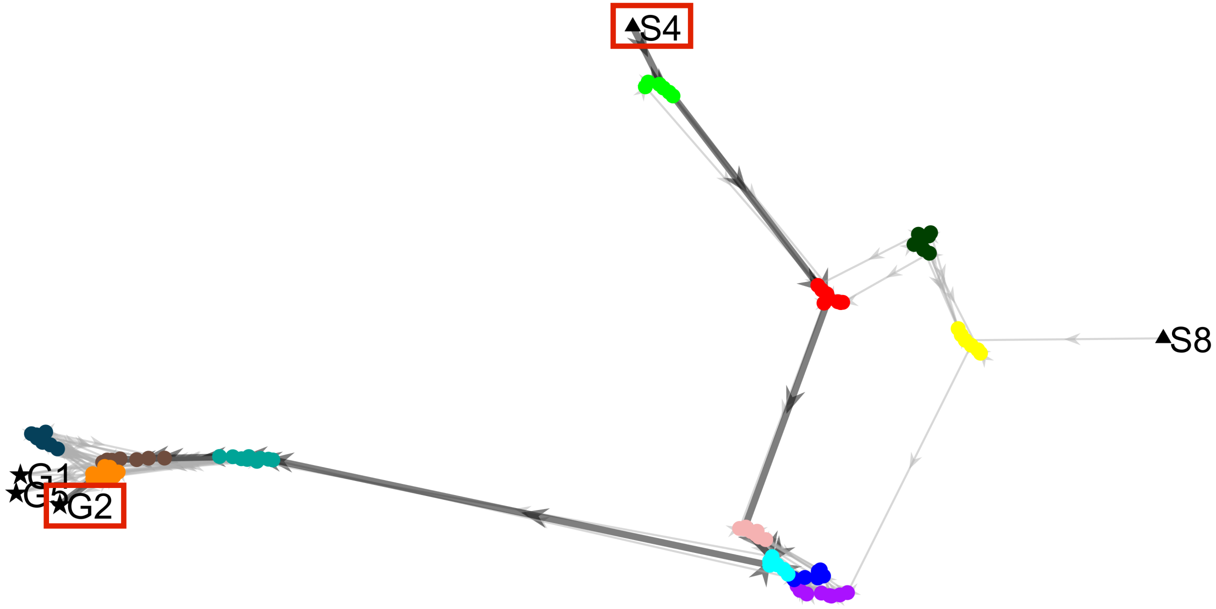
Use lobe sequences properly

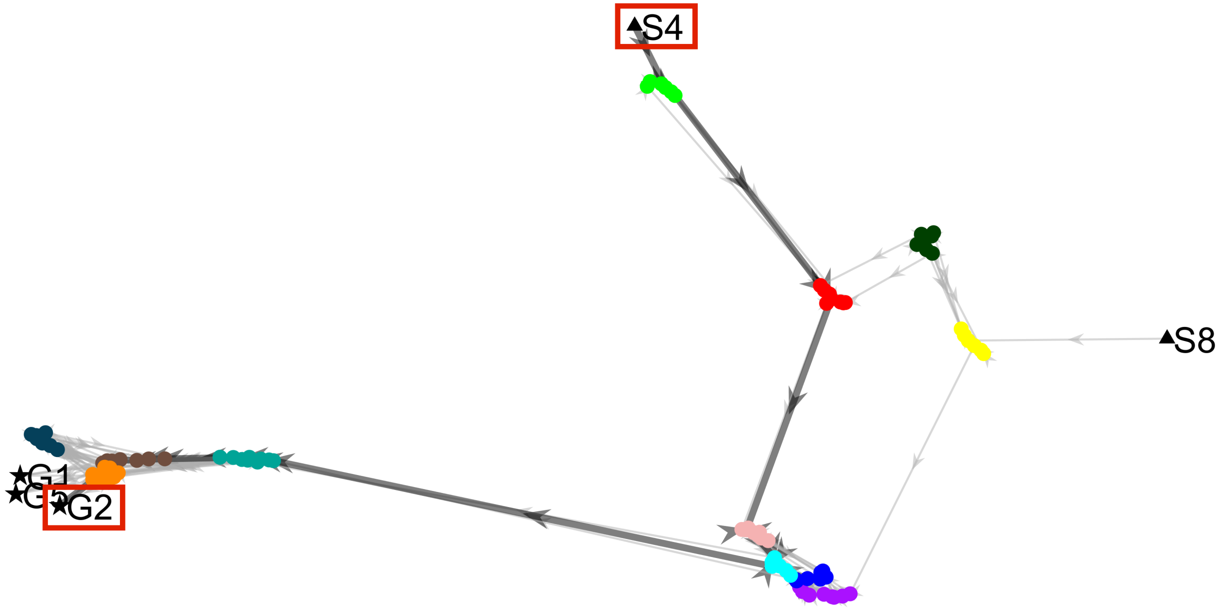
→ Paths with lobe sequences

Graph Analysis

- Example: $u^* = 0.52$, $r^* = 0.02$



Optimal transfer ($J = 2.1673$)

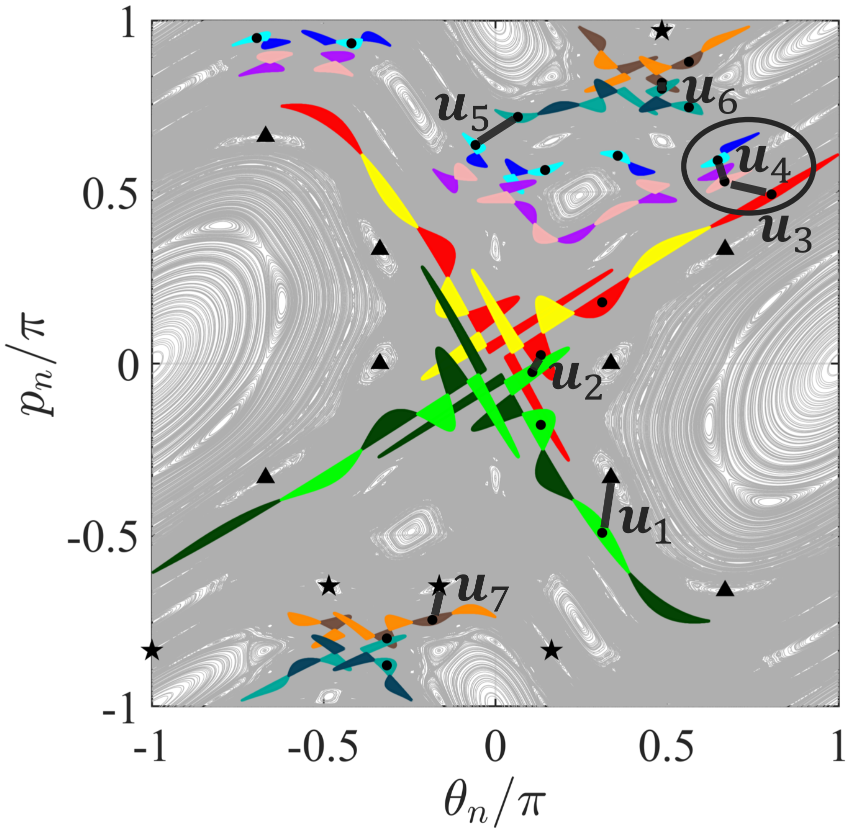


Optimal transfer with lobes ($J = 2.1712$)

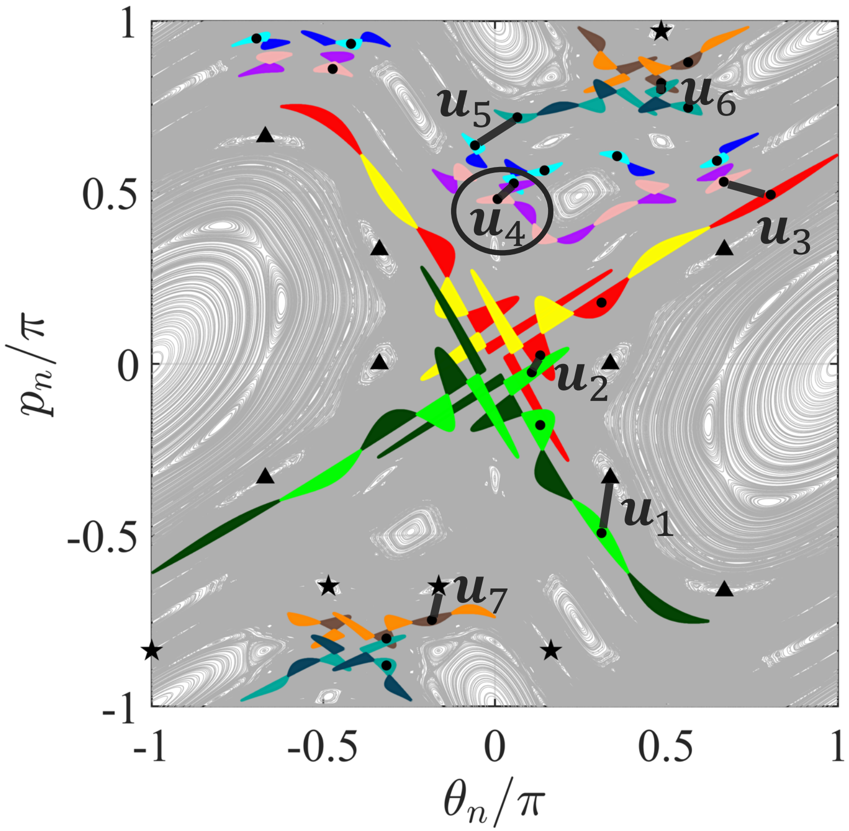
Results

Graph Analysis

- Example: $u^* = 0.52$, $r^* = 0.02$



Optimal transfer ($J = 2.1673$)

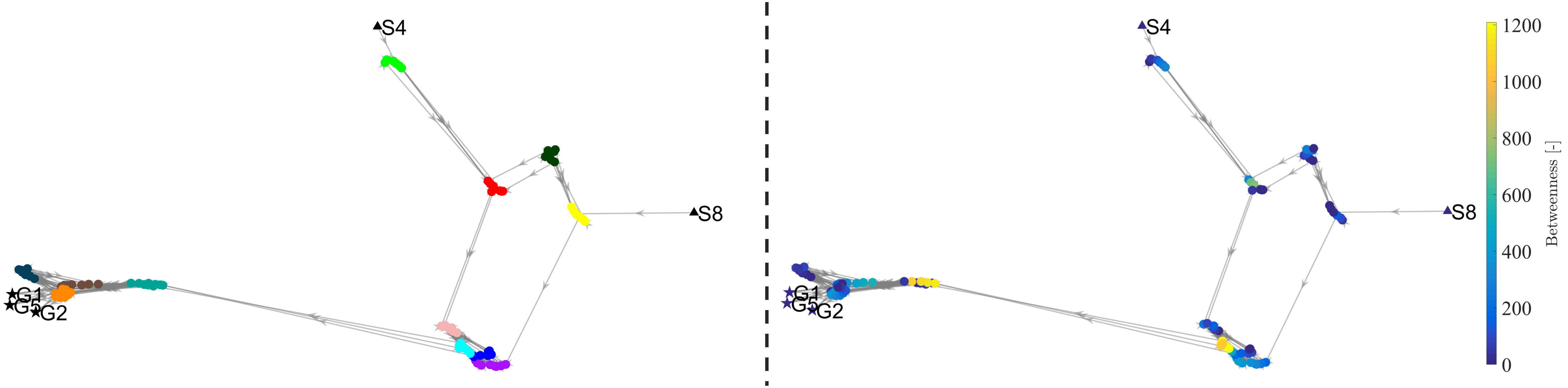


Optimal transfer with lobes ($J = 2.1712$)

Considerations

Graph Analysis

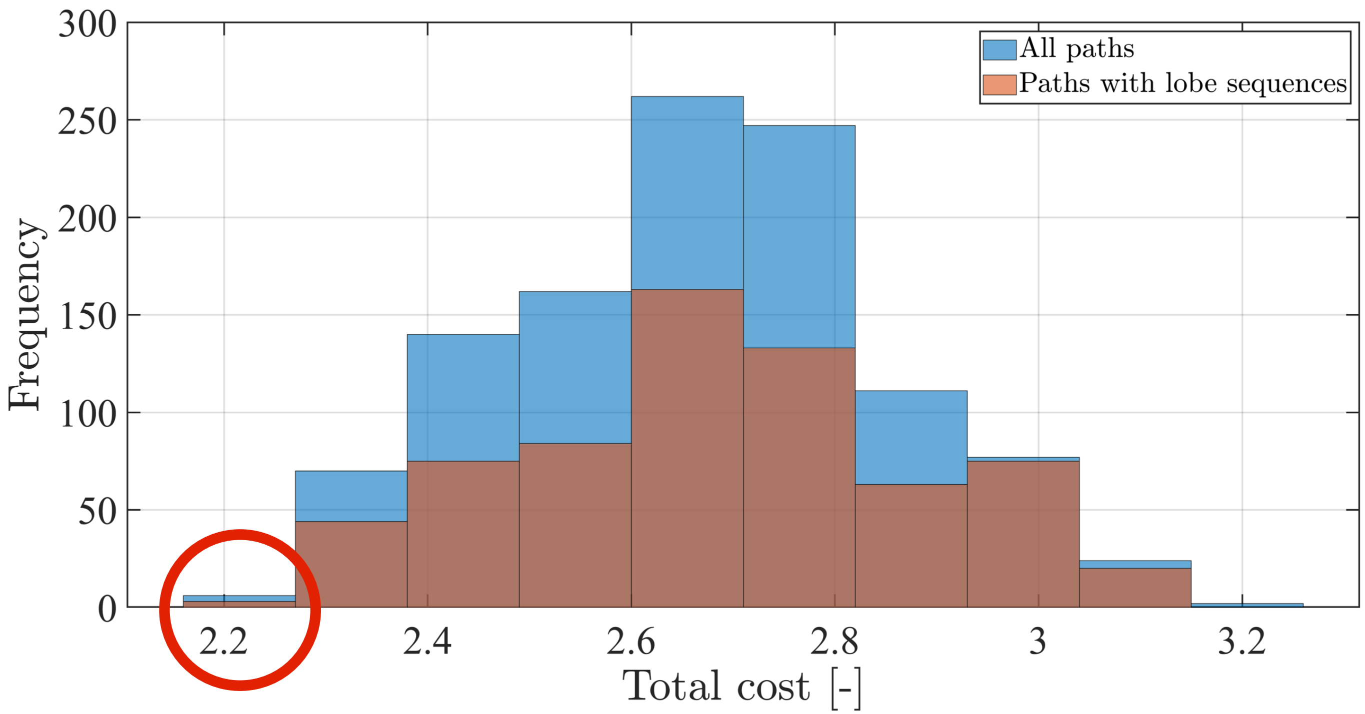
- Betweenness: Measure how often each graph node appears on a shortest path between two nodes

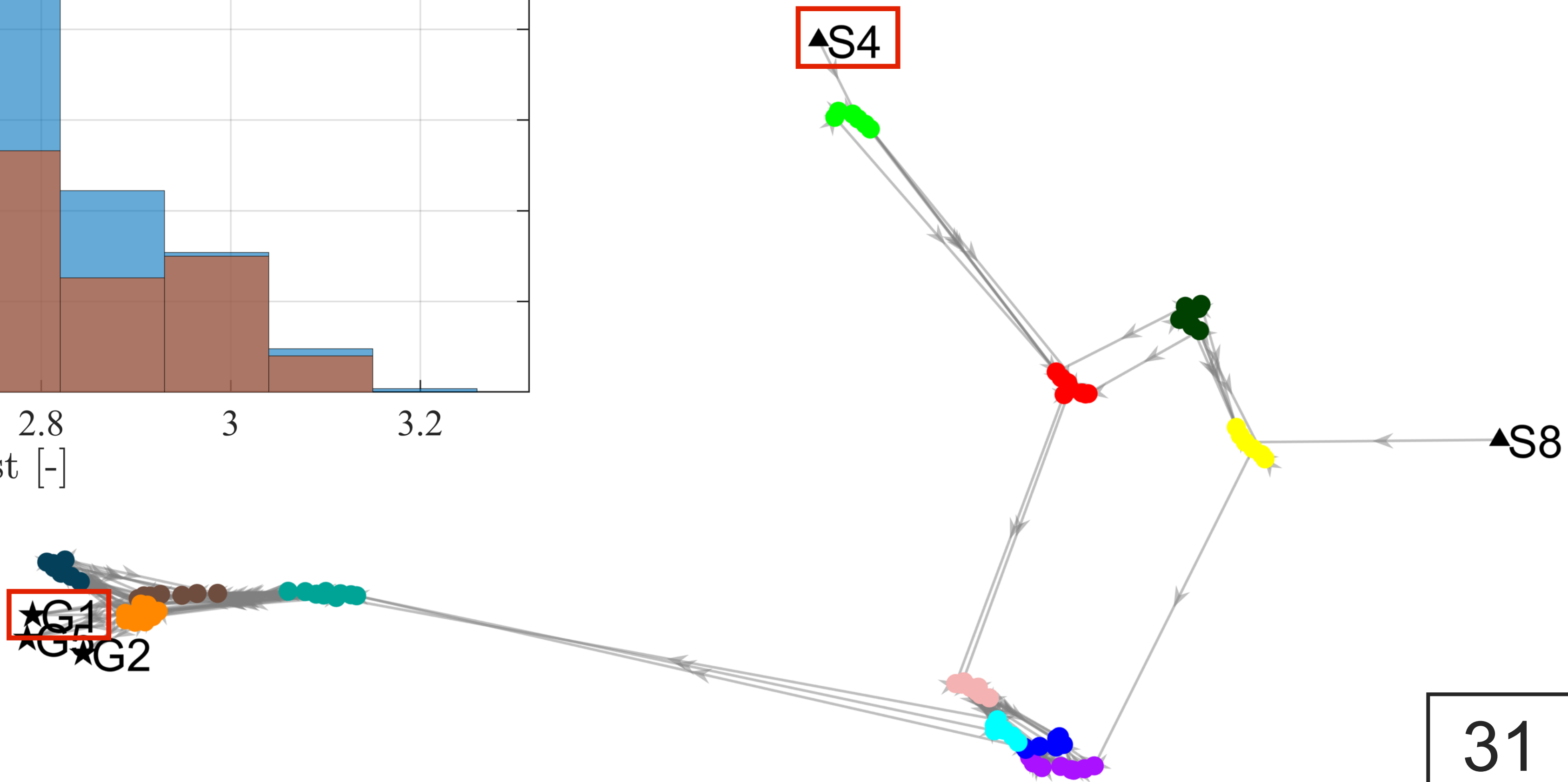


Considerations

Graph Analysis

- Example: $u^* = 0.52$, $r^* = 0.02$





Graph Analysis

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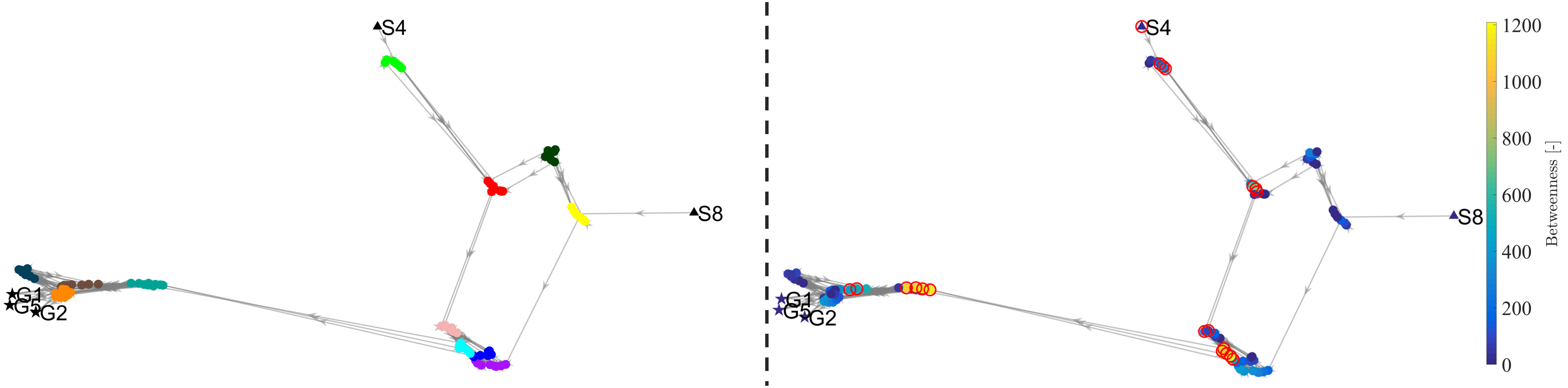
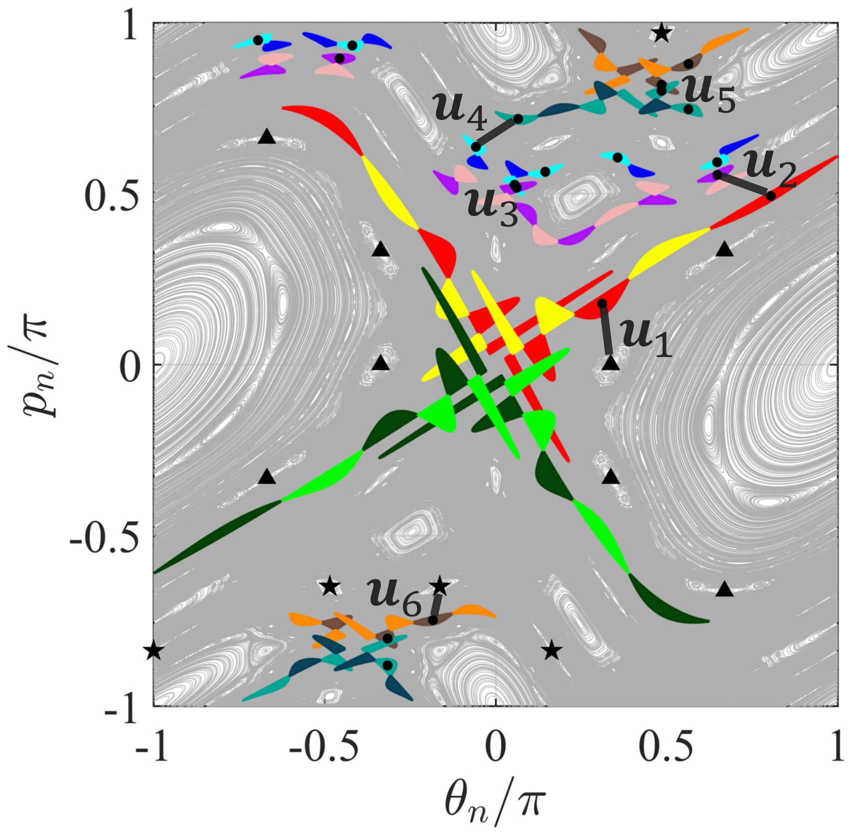


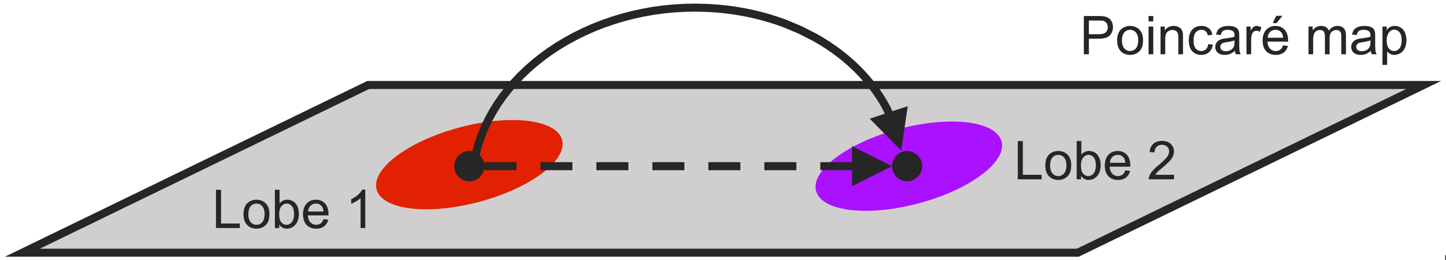
Relation to CR3BP

- The proposed method can be applied to Poincaré map in the CR3BP



Optimal transfer with lobes

- Standard map → Poincaré map
- $\boldsymbol{u}$ is yield by solving the two-point boundary value problem



Summary

Conclusion

- Developed the method to find the optimal transfer using lobe sequences in the standard map

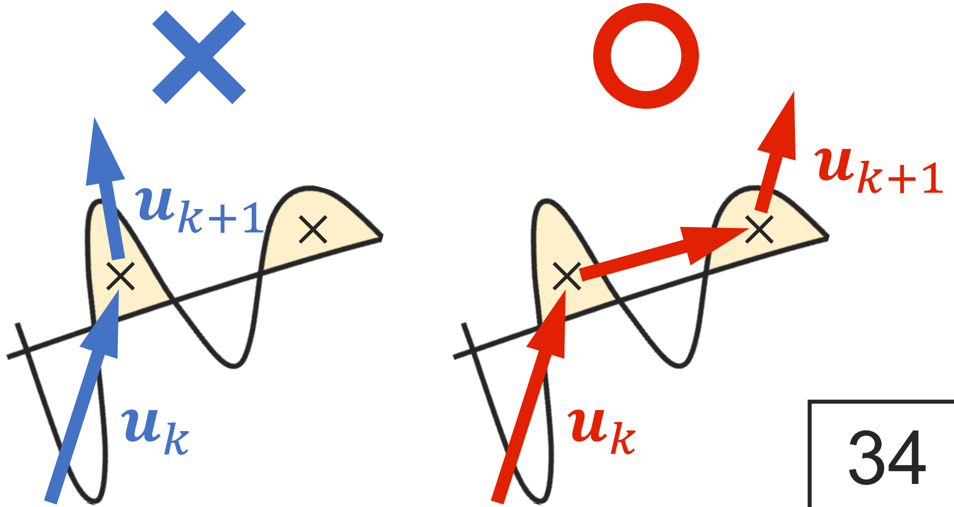
minimize $\quad J = \sum \|\boldsymbol{u}_k\|$

subject to $\quad \|\boldsymbol{u}_k\| \leq u^*$

$\quad r \geq r^*$

$+$

Use lobe sequences properly



Future Work

- Apply our method to desinging transfers in the CR3BP
- Confirm that tranfers obtained by our method become a good initial guess