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Controller design using Lyapunov redesign for spacecraft flying in libration point orbits

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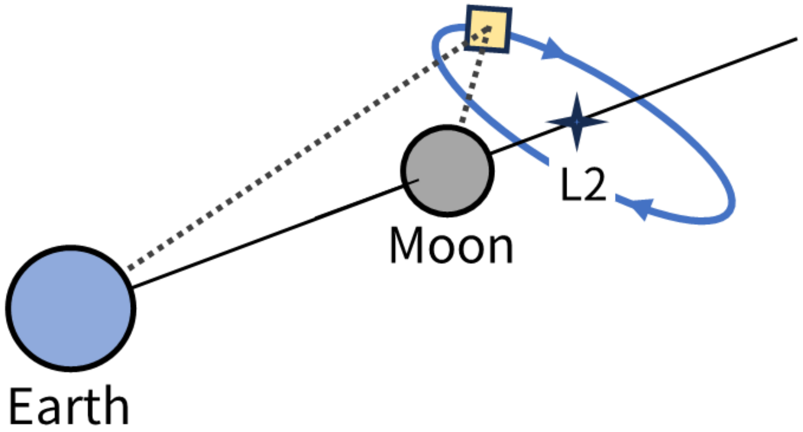
Introduction (1/2)

- Libration point orbits (LPOs)
  - Two celestial bodies (Earth and Moon) can be continuously observed.
  - Orbit can be maintained with a small amount of fuel.



Libration point orbits

- Items to be considered for controller design [1]
  - Computation costs of controllers
  - Errors in model and observation
  - **Disturbances** in Cislunar space

→ **"Robust LPO maintenance" (continuous control)**

JWST flies in an LPO
(Credit: NASA)

[1] Maksim Shirobokov et al., (2017), Journal of Guidance, Control, and Dynamics, Vol. 40

Introduction (2/2)

- Previous studies on **"Robust LPO maintenance" (continuous control)** [1]~[3]
  - Method based on dynamical structure
    - ✓ Chattering attenuation sliding mode control (CASMC) [2]
    - → **Detailed information** is necessary
  - Method to track nominal orbit
    - ✓ Periodic discrete linear time varying H∞ [3]
    - → **High-order (detailed) expression** of nominal orbit is necessary

- Objective

Design a simple robust control law to maintain LPOs

[1] Maksim Shirobokov et al., (2017), Journal of Guidance, Control, and Dynamics, Vol. 40
[2] Mai Bando et al., (2022), Frontiers in Space Technologies, Sec. Space Propulsion, Vol. 3
[3] Jayant Kulkarni et al., (2006), IEEE Transactions on Control Systems Technology

Outline

- Introduction
- Equations of motion in the CR3BP
- Controller design
- Numerical simulations
- Conclusion

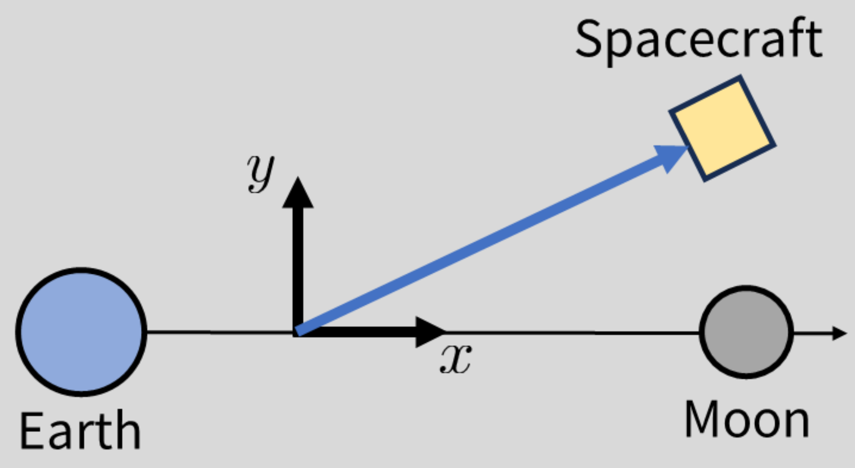
Normalized equations of motion in the CR3BP

Normalized EOM in the CR3BP [4]

$$\ddot{x} - 2\dot{y} - x = -\frac{1-\mu}{r_1^3}(x+\mu) - \frac{\mu}{r_2^3}(x-1+\mu) + u_x + d_x$$

$$\ddot{y} + 2\dot{x} - y = -\frac{1-\mu}{r_1^3}y - \frac{\mu}{r_2^3}y + u_y + d_y$$

Control (u_x, u_y), disturbance (d_x, d_y)



Rotating frame of CR3BP
(Origin: Center of mass)

Mass ratio $\mu := \frac{m_2}{m_1 + m_2} \sim 0.01215$

$r_1 = \sqrt{(x+\mu)^2 + y^2}, r_2 = \sqrt{(x-1+\mu)^2 + y^2}$

Earth position $(-\mu, 0)$, Moon position $(1-\mu, 0)$

L2 libration point position $(l_2, 0), l_2 \sim 1.16$

※ $(\cdot)_1$ represents Earth, $(\cdot)_2$ represents Moon

State space representation after translation of x-axis

[4] Mireles James, (2006) Celestial Mechanics Notes Set 4: The Circular Restricted Three Body Problem

State space representation in the CR3BP

- The EOM divided into linear system and nonlinear term [5]

$$\dot{x} = Ax + B(u + f + d)$$

Linear terms ($Ax + B(u$) — Nonlinear and disturbance terms ($f + d$)

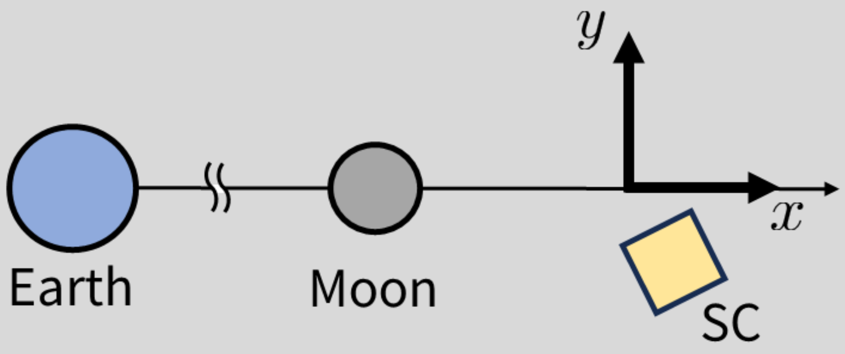
$$x = \begin{bmatrix} x & y & \dot{x} & \dot{y} \end{bmatrix}^\top, \; u = \begin{bmatrix} u_x & u_y \end{bmatrix}^\top, \; f = \begin{bmatrix} f_x & f_y \end{bmatrix}^\top$$

$$A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 2\alpha + 1 & 0 & 0 & -2 \\ 0 & 1 - \alpha & 2 & 0 \end{bmatrix}, B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Earth position $(-\mu - l_2, 0)$, Moon position $(1 - \mu - l_2, 0)$

L2 libration point position $(0, 0)$

※ $\alpha > 0$ (Const.)



Rotating frame of CR3BP
(Origin: L2 point)

Mode decomposition: x into z

Mode decomposition ($x \rightarrow z$)

◻ State equation after the mode decomposition

[2]

$$\dot{z} = \tilde{A}z + \tilde{B}(u + f + d)$$

Linear terms: $\tilde{A}z + \tilde{B}(u$ — Nonlinear and disturbance terms: $f + d)$

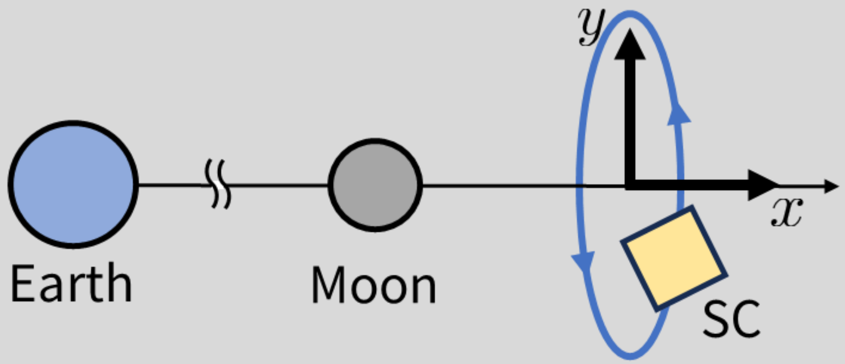
$$z = \begin{bmatrix} z_{\mathrm{c1}} & z_{\mathrm{c2}} & z_{\mathrm{s}} & z_{\mathrm{u}} \end{bmatrix}^\top,\ u = \begin{bmatrix} u_x & u_y \end{bmatrix}^\top,\ f = \begin{bmatrix} f_x & f_y \end{bmatrix}^\top$$

$$\tilde{A} = \begin{bmatrix} 0 & Q_2 & 0 & 0 \\ -Q_2 & 0 & 0 & 0 \\ 0 & 0 & -Q_3 & 0 \\ 0 & 0 & 0 & Q_3 \end{bmatrix},\ \tilde{B} = \begin{bmatrix} 0 & \frac{P_1}{P_1 - P_2} \\ -\frac{Q_2}{R} & 0 \\ \frac{Q_3}{2R} & -\frac{P_2}{2(P_1 - P_2)} \\ -\frac{Q_3}{2R} & -\frac{P_2}{2(P_1 - P_2)} \end{bmatrix}$$

$$f_x = l_2 - 2\alpha x - \frac{1-\mu}{r_1^3}(x + l_2 + \mu) - \frac{\mu}{r_2^3}(x + l_2 - 1 + \mu)$$

$$f_y = \alpha y - \frac{1-\mu}{r_1^3} y - \frac{\mu}{r_2^3} y$$

※ $P_\bullet,\ Q_\bullet,\ R > 0$ (Const.)



Rotating frame of CR3BP
(Origin: L2 point)

[2] Mai Bando et al., (2022), Frontiers in Space Technologies, Sec. Space Propulsion, Vol. 3

Formulation as a stabilization problem

In-plane motion modes

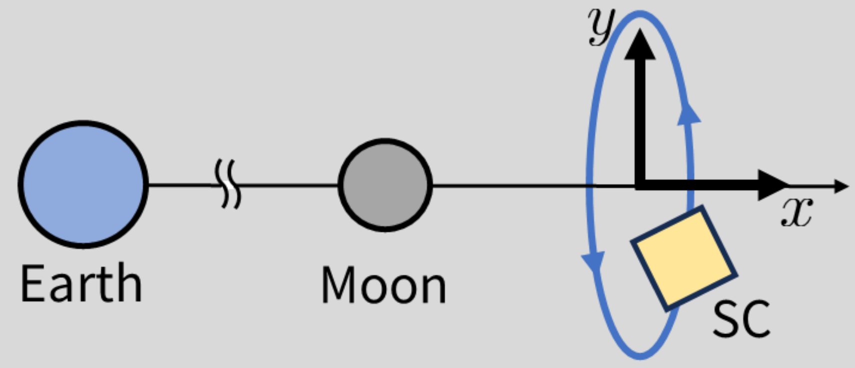
$$\frac{\mathrm{d}}{\mathrm{d}t}\begin{bmatrix} z_{\mathrm{c}1} \\ z_{\mathrm{c}2} \end{bmatrix} = \begin{bmatrix} 0 & Q_2 \\ -Q_2 & 0 \end{bmatrix}\begin{bmatrix} z_{\mathrm{c}1} \\ z_{\mathrm{c}2} \end{bmatrix} + \begin{bmatrix} 0 & \frac{P_1}{P_1-P_2} \\ -\frac{Q_2}{R} & 0 \end{bmatrix}\begin{bmatrix} u_x \\ u_y \end{bmatrix}$$

⇒ Simple harmonic motion in the unforced system

Stable and unstable modes

$$\frac{\mathrm{d}}{\mathrm{d}t}\begin{bmatrix} z_{\mathrm{s}} \\ z_{\mathrm{u}} \end{bmatrix} = \begin{bmatrix} -Q_3 & 0 \\ 0 & Q_3 \end{bmatrix}\begin{bmatrix} z_{\mathrm{s}} \\ z_{\mathrm{u}} \end{bmatrix} + \begin{bmatrix} \frac{Q_3}{2R} & -\frac{P_2}{2(P_1-P_2)} \\ -\frac{Q_3}{2R} & \frac{P_2}{2(P_1-P_2)} \end{bmatrix}\begin{bmatrix} u_x \\ u_y \end{bmatrix}$$

⇒ Unstable mode diverges in the unforced system



Rotating frame of CR3BP
(Origin: L2 point)

Orbit maintenance problem for LPOs

⇕

Stabilization problem of the unstable mode [2]

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Controller design (1/3)

Approach

$$u = u_1 + u_2$$

Step 1. Stabilization of the linear system [2]: input u_1

Step 2. Compensation of nonlinear and perturbation terms: additional input u_2

Step 1. Stabilization of the linear system [2]: input $u_1 = \begin{bmatrix} u_{1,x} & u_{1,y} \end{bmatrix}^\top$

$$\frac{\mathrm{d}}{\mathrm{d}t}\begin{bmatrix} z_\mathrm{s} \\ z_\mathrm{u} \end{bmatrix} = \begin{bmatrix} -Q_3 & 0 \\ 0 & Q_3 \end{bmatrix}\begin{bmatrix} z_\mathrm{s} \\ z_\mathrm{u} \end{bmatrix} + \begin{bmatrix} \frac{Q_3}{2R} & -\frac{P_2}{2(P_1-P_2)} \\ -\frac{Q_3}{2R} & \frac{P_2}{2(P_1-P_2)} \end{bmatrix}\begin{bmatrix} u_x \\ u_y \end{bmatrix}$$

$$\dot{z}_\mathrm{u} = Q_3 z_\mathrm{u} - \frac{Q_3}{2R}u_{1,x} - \frac{P_2}{2(P_1 - P_2)}u_{1,y}$$

$$= -Q_3\left(\frac{k_x}{2R} - 1\right) z_\mathrm{u}$$

$$u_{1,x} = k_x z_\mathrm{u}, \;\; u_{1,y} = 0 \;\; (k_x > 2R)$$

Controller design (2/3) – Step 2

Lyapunov redesign [6]

Design additional inputs so that the closed-loop system is asymptotically stable in the presence of disturbances.

$$\frac{\mathrm{d}}{\mathrm{d}t}\begin{bmatrix} z_{\mathrm{s}} \\ z_{\mathrm{u}} \end{bmatrix} = \begin{bmatrix} -Q_3 & 0 \\ 0 & Q_3 \end{bmatrix}\begin{bmatrix} z_{\mathrm{s}} \\ z_{\mathrm{u}} \end{bmatrix} + \begin{bmatrix} \frac{Q_3}{2R} & -\frac{P_2}{2(P_1-P_2)} \\ -\frac{Q_3}{2R} & -\frac{P_2}{2(P_1-P_2)} \end{bmatrix}\begin{bmatrix} u_x + f_x + d_x \\ u_y + f_y + d_y \end{bmatrix}$$

$$u_x = k_x z_{\mathrm{u}} + u_{2,x}, \quad u_y = u_{2,y}$$

$$= \begin{bmatrix} -Q_3 & \frac{Q_3}{2R}k_x \\ 0 & -Q_3\left(\frac{k_x}{2R} - 1\right) \end{bmatrix}\begin{bmatrix} z_{\mathrm{s}} \\ z_{\mathrm{u}} \end{bmatrix} + \begin{bmatrix} \frac{Q_3}{2R} & -\frac{P_2}{2(P_1-P_2)} \\ -\frac{Q_3}{2R} & -\frac{P_2}{2(P_1-P_2)} \end{bmatrix}\begin{bmatrix} u_{2,x} + f_x + d_x \\ u_{2,y} + f_y + d_y \end{bmatrix}$$

Nominal system

Nonlinear and **unknown** disturbance terms

[6] Hassan Khalil, (2014), Nonlinear Systems 3rd ed, pp.579 – 589

Controller design (3/3) – Step 2

Lyapunov function

Consider stabilization of two modes $z_\mathrm{s},\ z_\mathrm{u}$

$$V(z_\mathrm{s}, z_\mathrm{u}) = \frac{1}{2Q_3}\left\{(z_\mathrm{s}+z_\mathrm{u})^2 + \frac{2R}{k_x - 2R}(1+q_\mathrm{u})\right\} > 0$$

$$\left(\dot{V}(z_\mathrm{s}, z_\mathrm{u}) = -(z_\mathrm{s}^2 + q_\mathrm{u} z_\mathrm{u}^2) < 0\right)$$

q_u : relatively weight parameter of z_u

Additional input derived by the Lyapunov redesign

$z = \begin{bmatrix} z_\mathrm{s} & z_\mathrm{u} \end{bmatrix}^\top$

$$u_2 = -\eta \frac{\left(\frac{\partial V}{\partial z}\tilde{B}\right)^\top}{\left|\frac{\partial V}{\partial z}\tilde{B}\right|},\quad \left(\frac{\partial V}{\partial z}\tilde{B}\right)^\top = \begin{bmatrix} -\frac{1}{k_x - 2R}(1+q_\mathrm{u})z_\mathrm{u} \\ -\frac{P_2}{Q_3(P_1 - P_2)}\left\{z_s + \frac{R}{k_x - 2R}\left(\frac{k_x - R}{R} + q_\mathrm{u}\right)z_\mathrm{u}\right\} \end{bmatrix}$$

η: Parameter that determines the size of additional input

[6] Hassan Khalil, (2014), Nonlinear Systems 3rd ed, pp.579 – 589

Outline

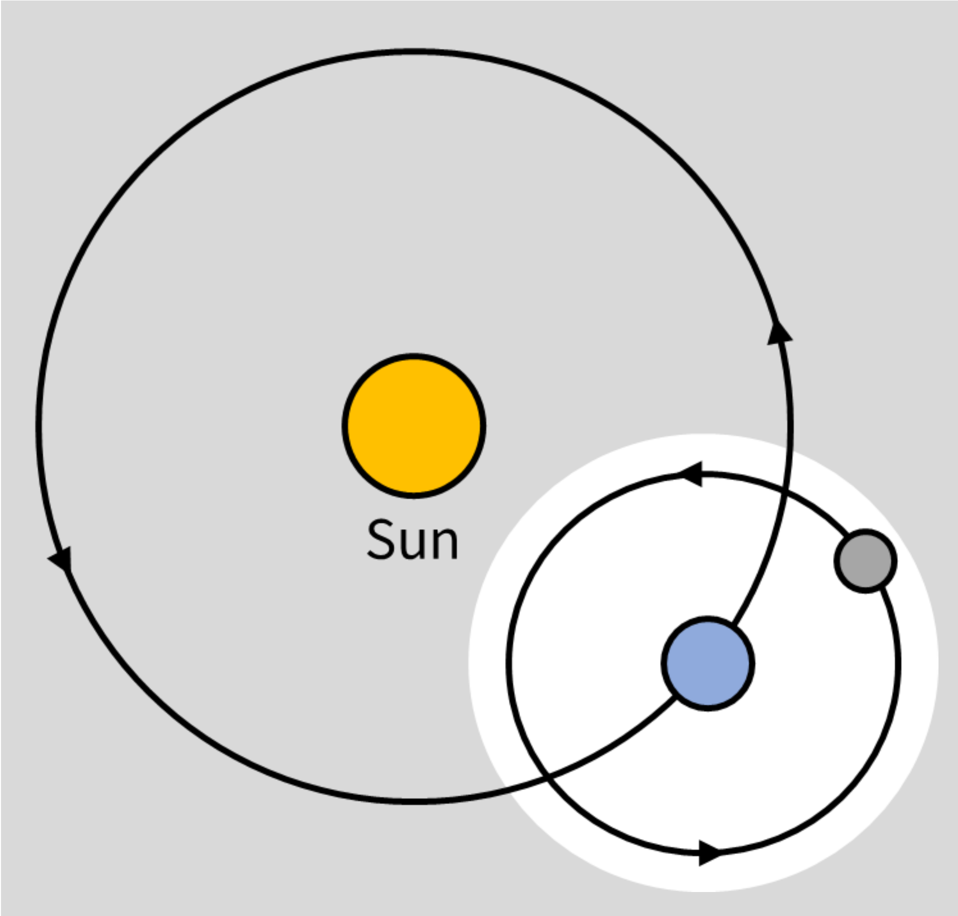
Disturbances in Cislunar space (1/3)

Disturbances in Cislunar space

Basic disturbance at L2 point in Cislunar space [7]

Disturbance	Size [mN/kg]
Earth	7.36×10^{-2}
Moon	3.51×10^{-2}
Sun	3.72×10^{-2}
LNGP	6.72×10^{-7}
Sun radiation	4.61×10^{-8}

LNGP :
Lunar Gravity and Topography Recovery Potential



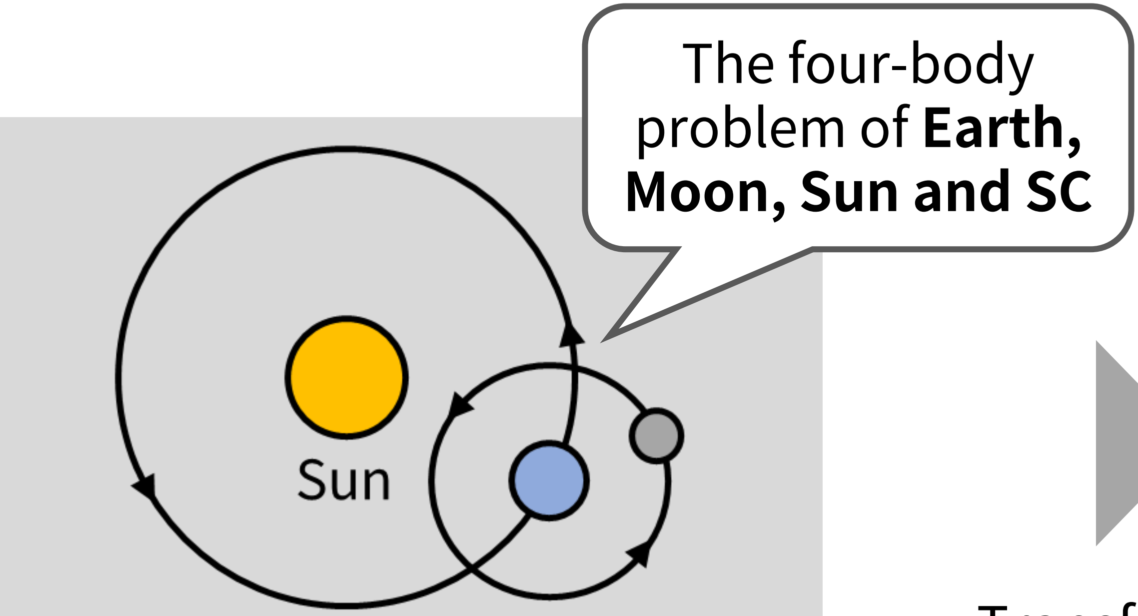
[7] Du Chongrui et al., (2020), IOP Conference Series: Materials Science and Engineering, 984

Disturbances in Cislunar space (2/3)

- Disturbance model – Bicircular model

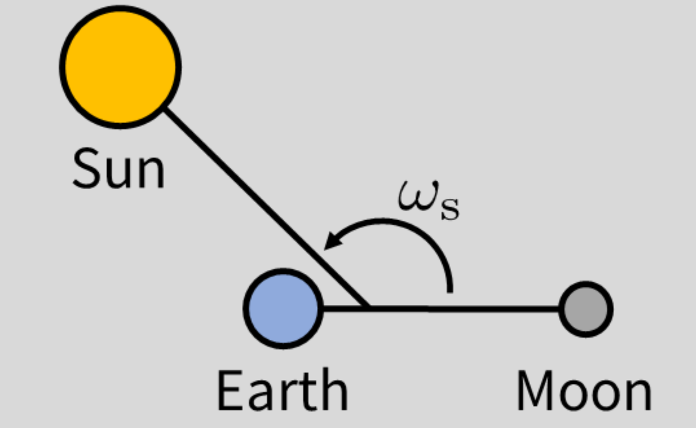
$$d \leftarrow d_{\mathrm{s}} = \begin{bmatrix} d_{\mathrm{s},x} & d_{\mathrm{s},y} \end{bmatrix}^{\top}$$

└ sun



Bicircular model [8]
(Restricted four-body problem)

Transform to rotating frame in Earth–Moon system



Bicircular model [7][8]
(Earth-Moon rotation frame)

ω_{s} = Relative angular velocity of the Sun

[7] Du Chongrui et al., (2020), IOP Conference Series: Materials Science and Engineering, 984
[8] Wang Sang Koon et al., (2011) , Dynamical Systems, the Three-Body Problem and Space Mission Design

Disturbances in Cislunar space (3/3)

Formulation of Sun Gravity Disturbance (SGD)

$$d_s = \begin{bmatrix} d_{s,x} & d_{s,y} \end{bmatrix}^\top$$

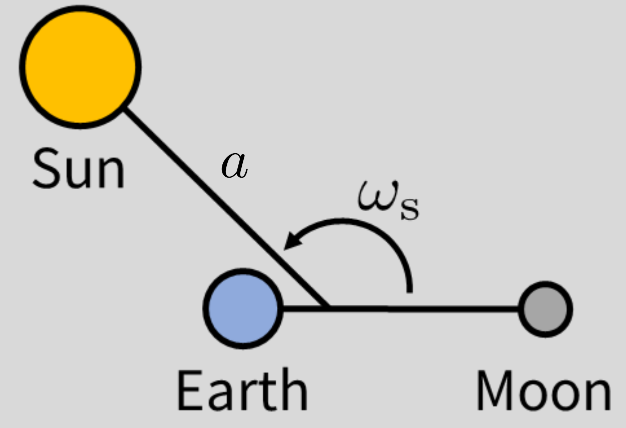
$$d_{s,x} = \frac{m_s}{r_{s-sc}^3}(x - a\cos\theta_s) + \frac{m_s}{a^2}\cos\theta_s$$

$$d_{s,y} = \frac{m_s}{r_{s-sc}^3}(y - a\sin\theta_s) + \frac{m_s}{a^2}\sin\theta_s$$

$$\theta_s = \omega_s t + \theta_{s,0},\ \ \omega_s = -0.925,\ \ \theta_{s,0} = 0$$

$$m_s = 3.29 \times 10^5,\ \ a = 3.89 \times 10^2$$

r_{s-sc} : Distance between Sun and SC



Bicircular model [7][8]
(Earth-Moon rotation frame)

ω_s = Relative angular velocity of the Sun

[7] Du Chongrui et al., (2020), IOP Conference Series: Materials Science and Engineering, 984
[8] Wang Sang Koon et al., (2011) , Dynamical Systems, the Three-Body Problem and Space Mission Design

Numerical simulations (1/3)

Reference
MUSES-C $0.063\ \mathrm{mN/kg}$

Conditions

$$x_0 = \begin{bmatrix} x & y & \dot{x} & \dot{y} \end{bmatrix}^\top = \begin{bmatrix} 1538 & 1153 & 0 & 0 \end{bmatrix}^\top \ [\mathrm{km}, \mathrm{km/s}]$$

Simulation time: 100 days

Parameters in proposed controller

Gain in the linear control law : $k_x = 9.0 > 8.13 \sim 2R$

Size of the additional input : $\eta = 0.05\ [\mathrm{mN/kg}]$

Relatively weight parameter : $q_\mathrm{u} = 1$

Only three parameters

Control laws used for numerical calculations

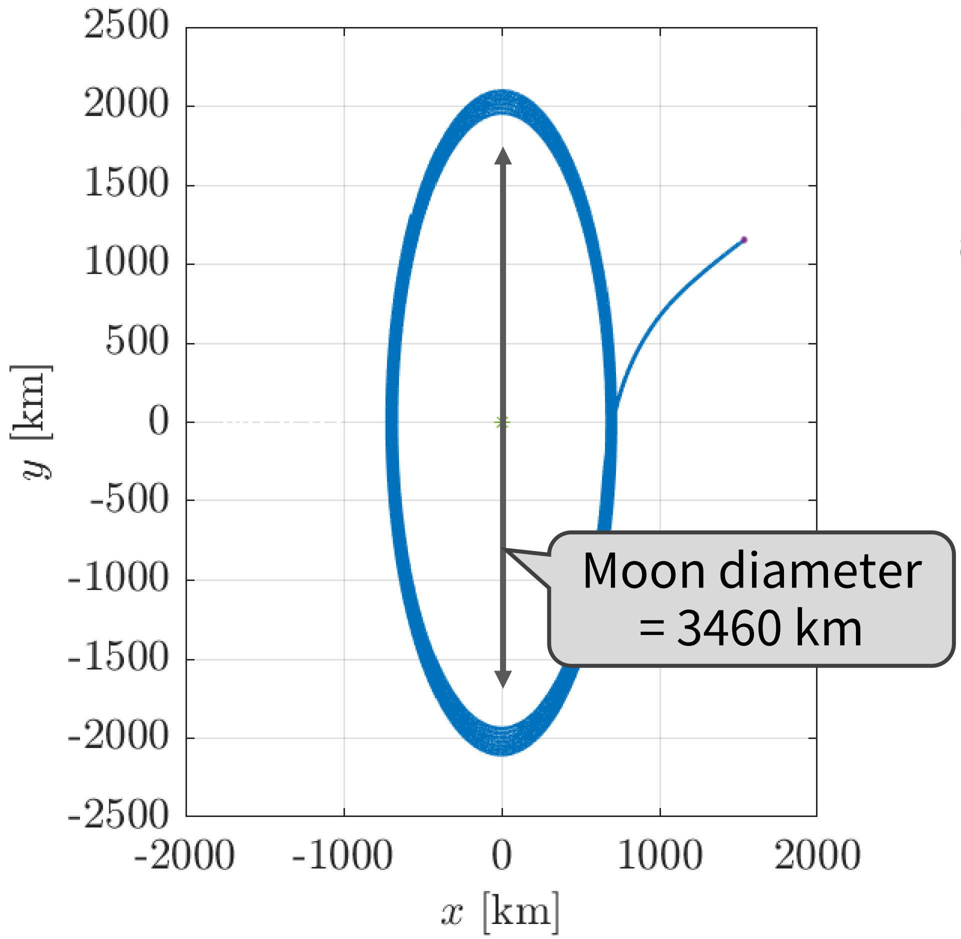
① Proposed controller: $u = u_1 + u_2$
② Linear controller: $u = u_1$

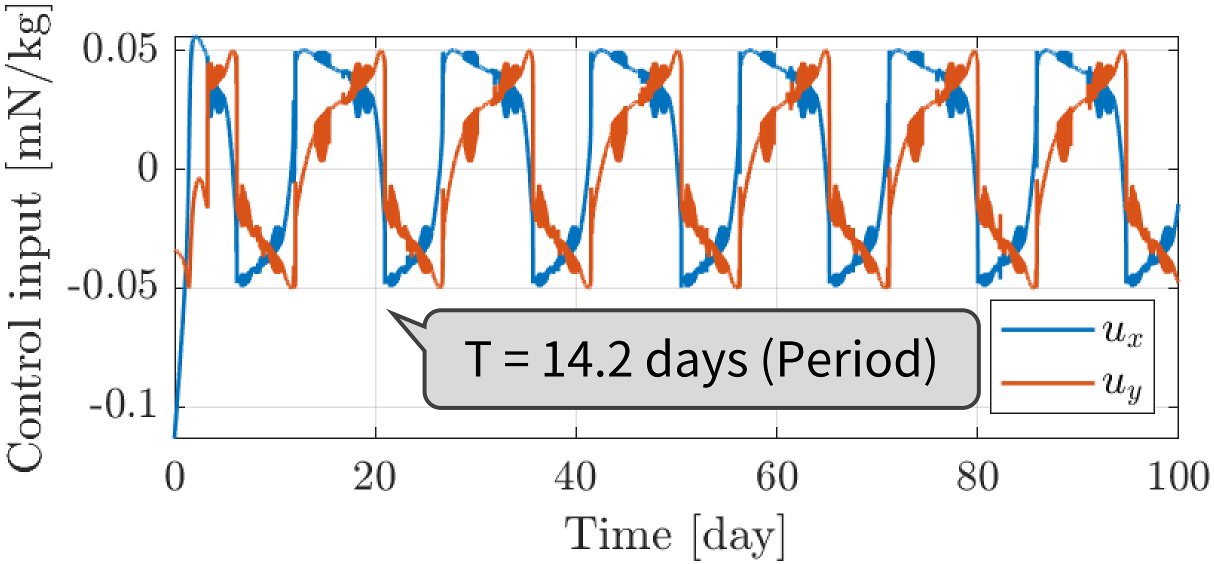
Numerical simulations (2/3)

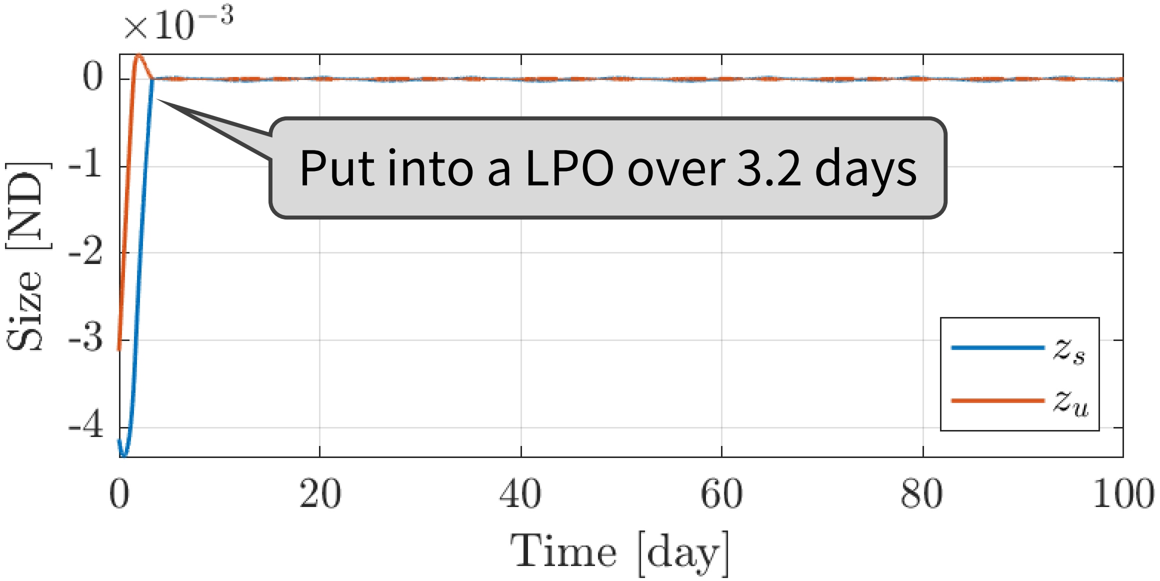
Reference
MUSES-C $0.063\ \mathrm{mN/kg}$

- Proposed controller

$$u = u_1 + u_2$$





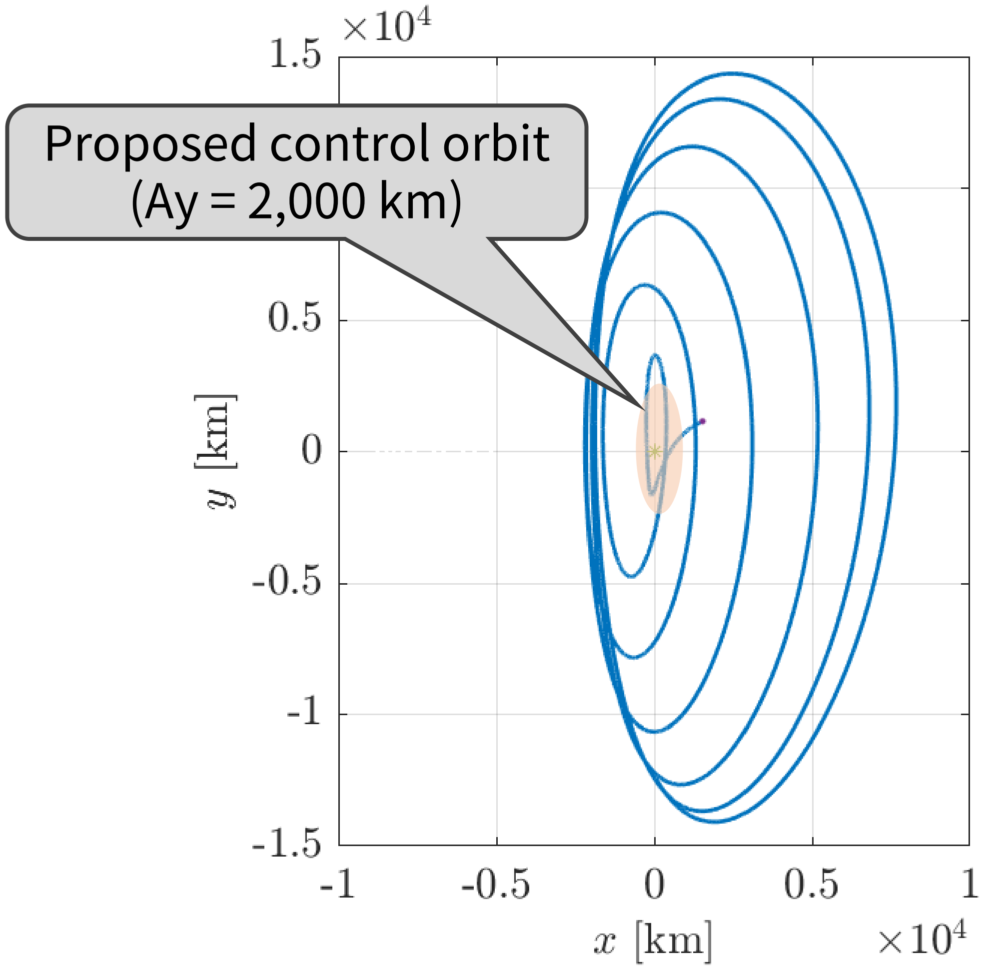


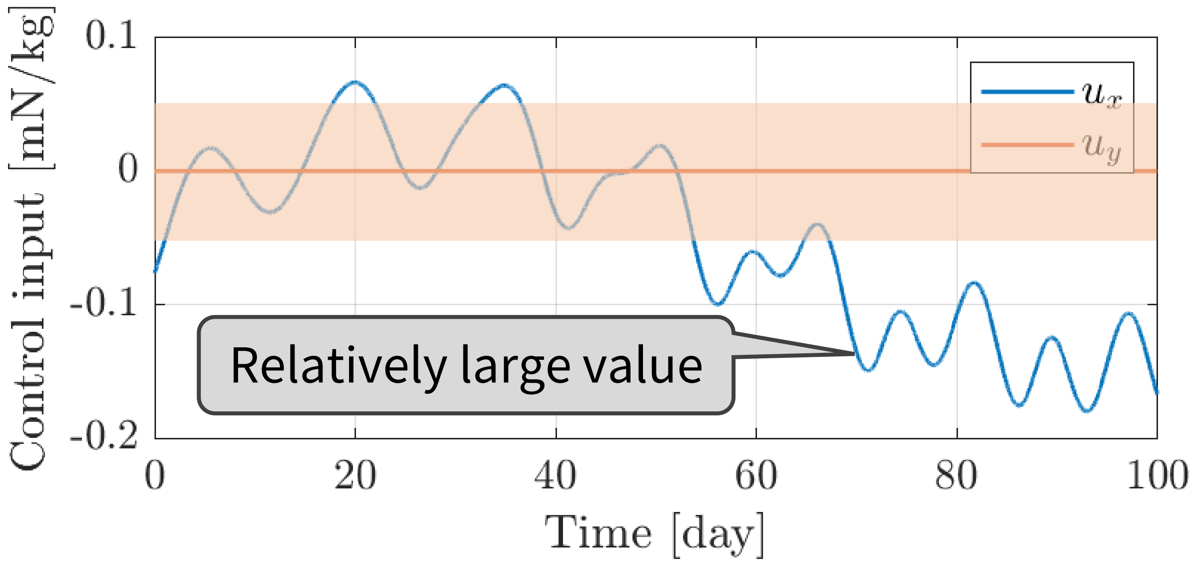
Numerical simulations (3/3)

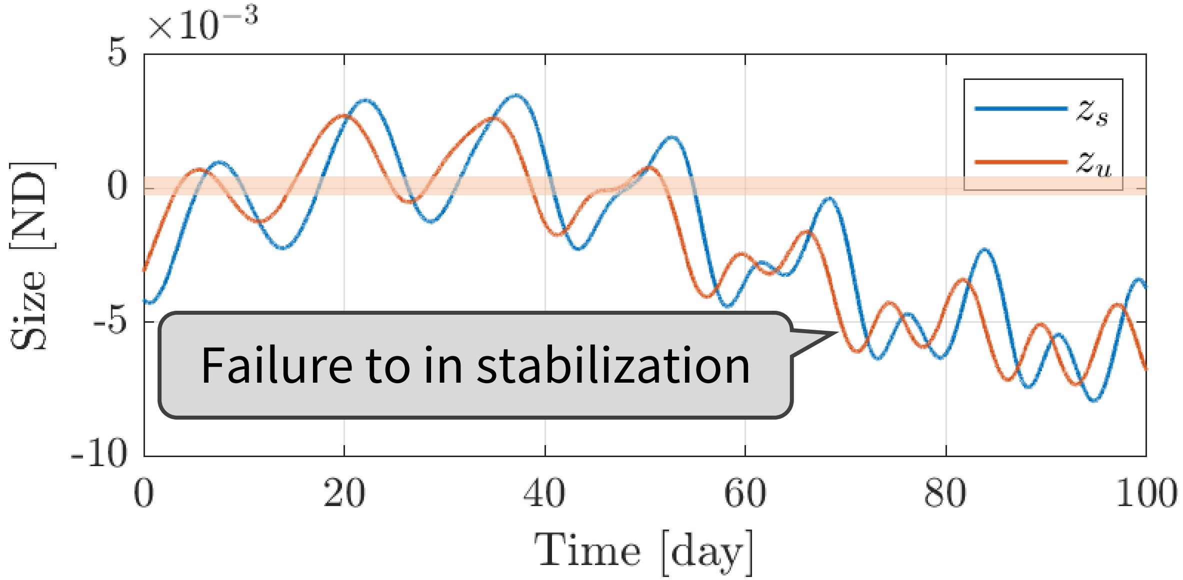
Reference
MUSES-C $0.063\ \mathrm{mN/kg}$

- Linear controller

$$u = u_1$$



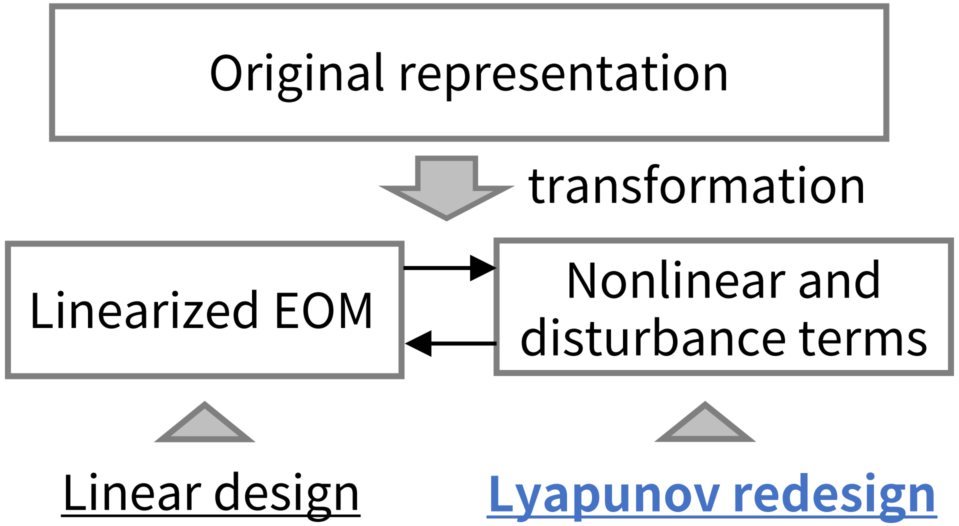




Conclusions

Objective

Design a simple robust control law to maintain LPOs



Approach

- Nominal control law: Linear design

→ **stabilization of nonperiodic linear modes**

- Additional control law: **Lyapunov redesign**

→ **compensation of nonlinear and disturbance terms**

Outcomes

- **A simple robust control law** including only three parameters
- **Achievement of orbit maintenance** under the SGD

Future work

- Incorporation of chattering avoidance structure
- Extension to formation flight