ASTRO-2023-C020

Attitude Control for Momentum-biased Transformable Spacecraft under Solar Radiation Pressure

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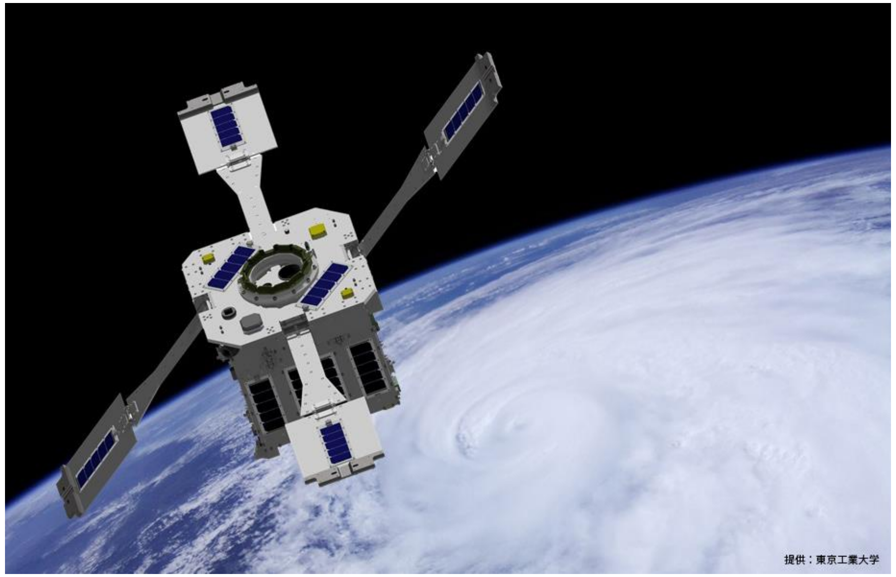
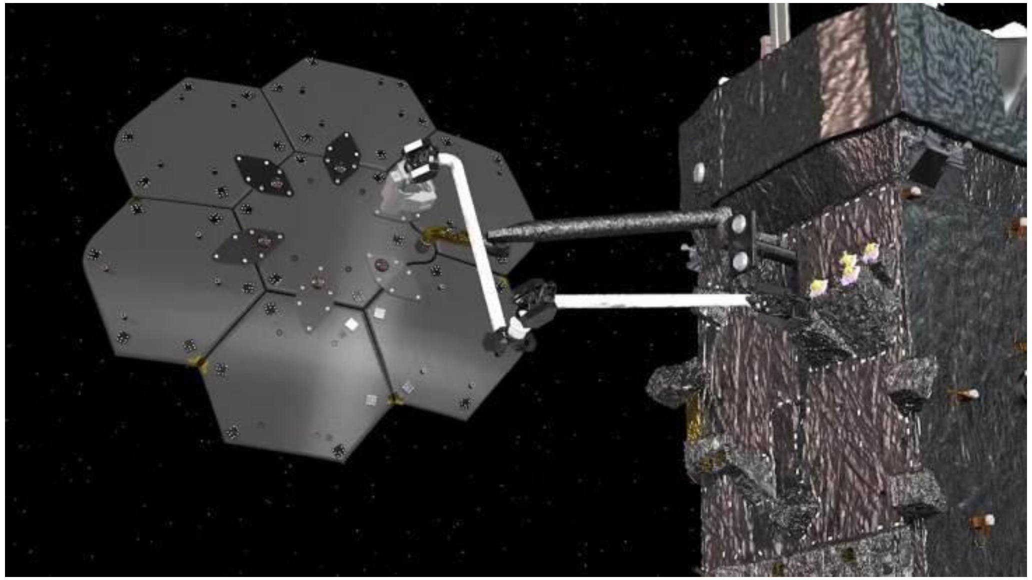
Introduction
- Current & future space robot missions

- Dynamic structural reconfiguration for agile attitude maneuver
  - Hibari (Tokyo Tech)
- Various missions for highly dexterous manipulation
  - PULSAR (DFKI), CAESAR (DLR) , OSAM-1 (NASA)
  - On-orbit repairing, assembling, refueling…
- Future space explorer, service satellite would be more versatile, dexterous

Hibari (Tokyo Tech, 2021)

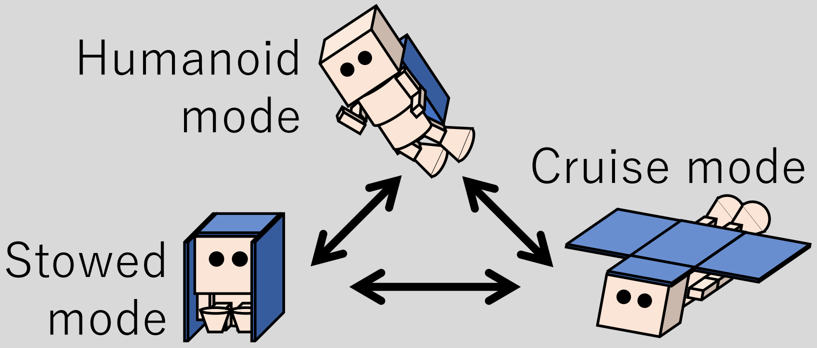


OSAM-1（NASA)



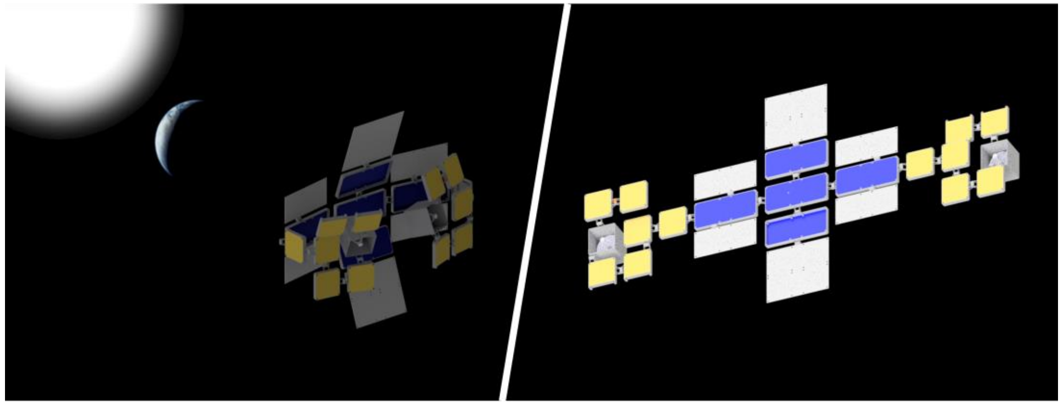
Introduction
- Transformable spacecraft

Multi-DoF & reconfigurable free-flying space robot



Characteristics
1. **Adaptive** structure reconfiguration
2. **Economical** orbit & attitude control
3. **Dexterous** multi-tasking with redundant control DoF

→ Provide new possibilities for future space missions

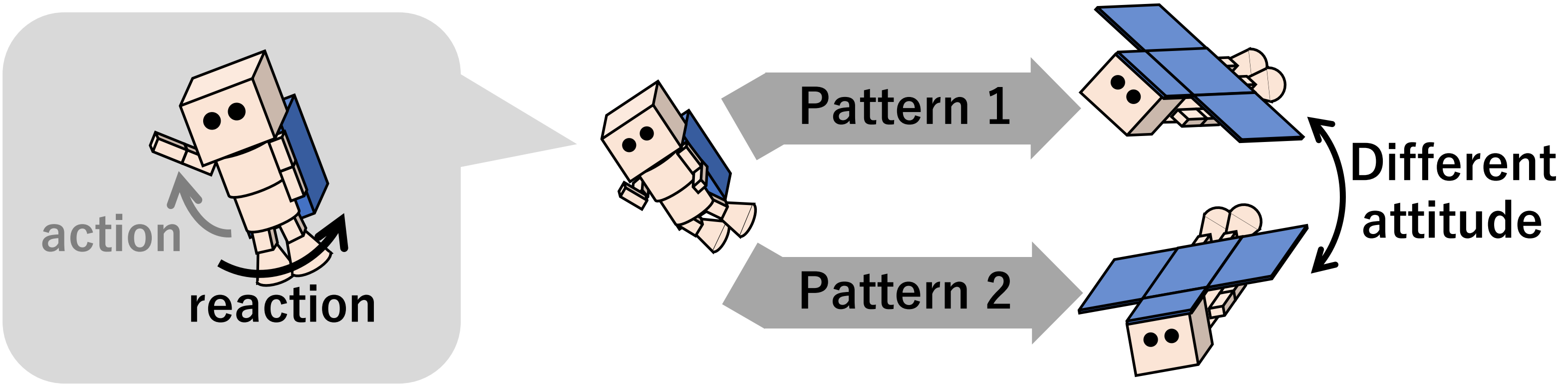


Transformer (JAXA & Universities in Japan)

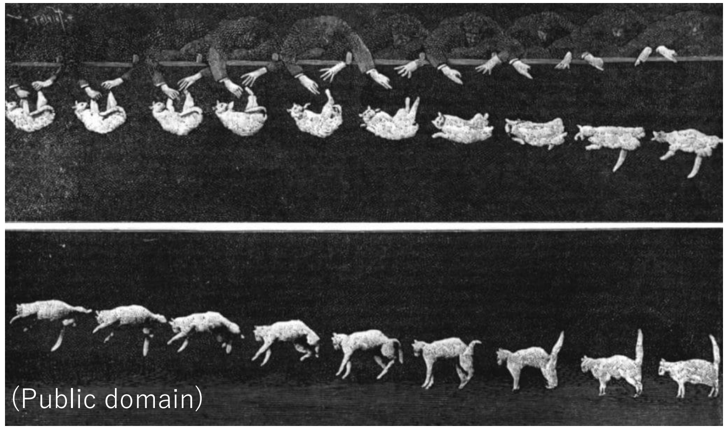
Specifications
- ✓ Engineering demonstrator
- ✓ Small class (~ 300 kg in wet mass)
- ✓ 27 body components with 18 actuatable joints
- ✓ Dynamic structure reconfiguration

Introduction
- Nonholonomic attitude control

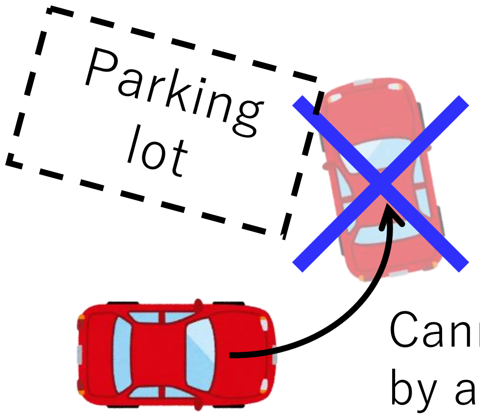
Angular momentum conservation is **nonholonomic**



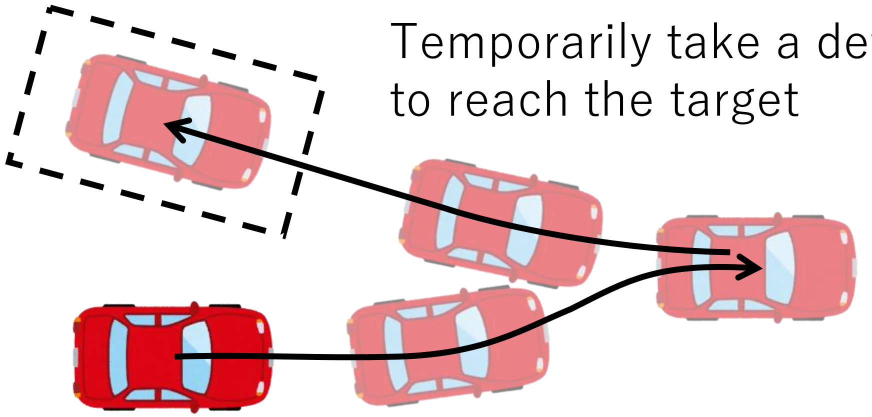
e.g.) Falling cat



e.g.) Car parking



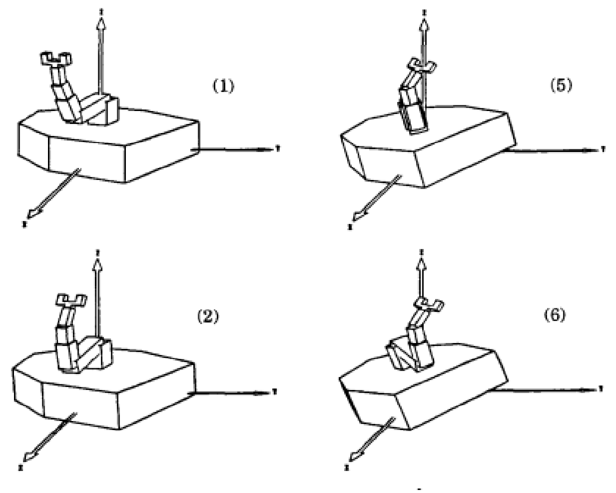
Cannot reach the target
by a time-invariant feedback



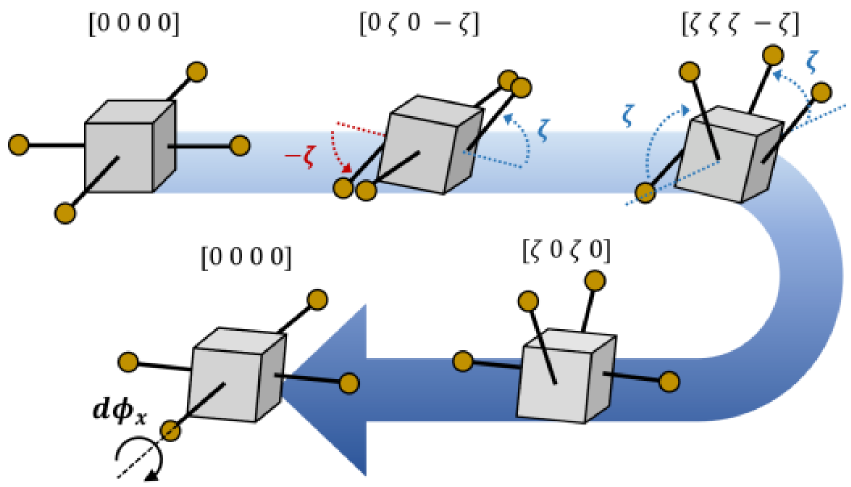
Temporarily take a detour route
to reach the target

Introduction
- Previous studies of nonholonomic attitude control

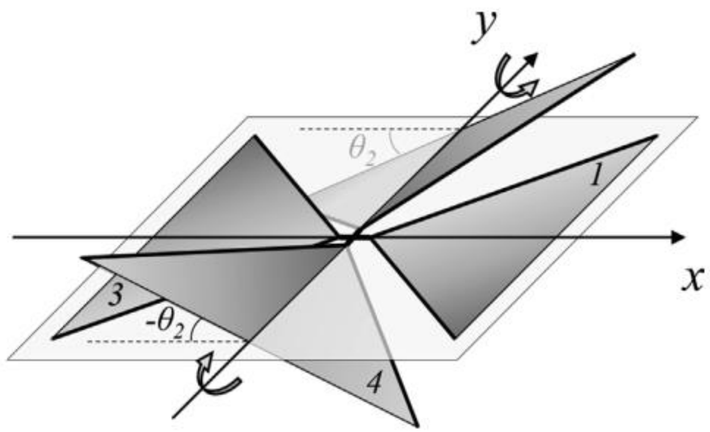
- Robotic arm, variable shape mechanism, solar array drive assemblies (SADA)…
- Model dependent control law
  - Only valid for symmetric, low-DoF models
  - Planar motion
- Body reconfiguration is not considered in most studies



Nakamura & Mukherjee, 1993



Watanabe et al., 2016

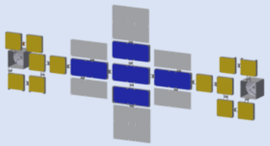


Gong et al., 2022

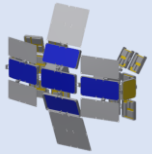
Introduction
- Authors' previous studies

- ✓ General spacecraft model
- ✓ Satisfied joint angle limits & self-collision avoidance
- ✓ Achieved target body configuration & target attitude simultaneously
- ✓ Total joint travel is largely reduced by optimization

Initial config.



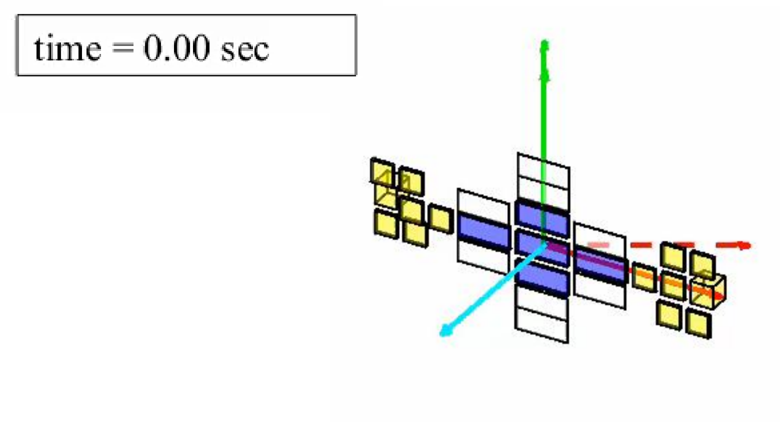
Target config.



Stroke num: 5

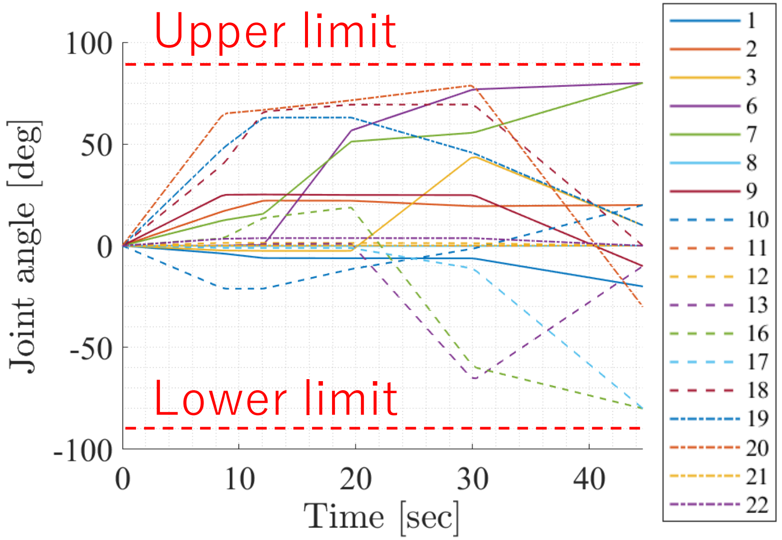
Initial attitude
$[0 \quad 0 \quad 0]$ deg

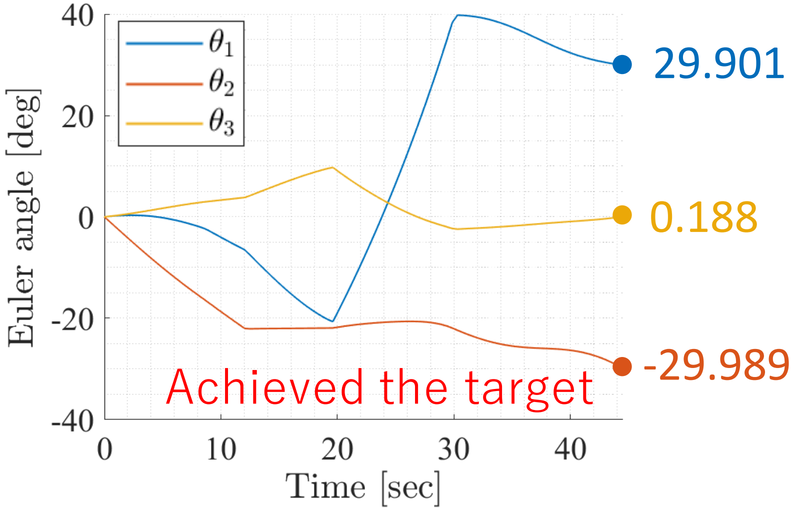
target attitude
$[30 \quad -30 \quad 0]$ deg in 2-1-3 Euler angle



X Y Z Current attitude

X Y Z Target attitude





Introduction
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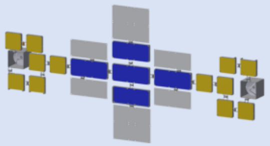
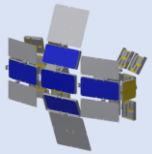
Initial config.



Target config.



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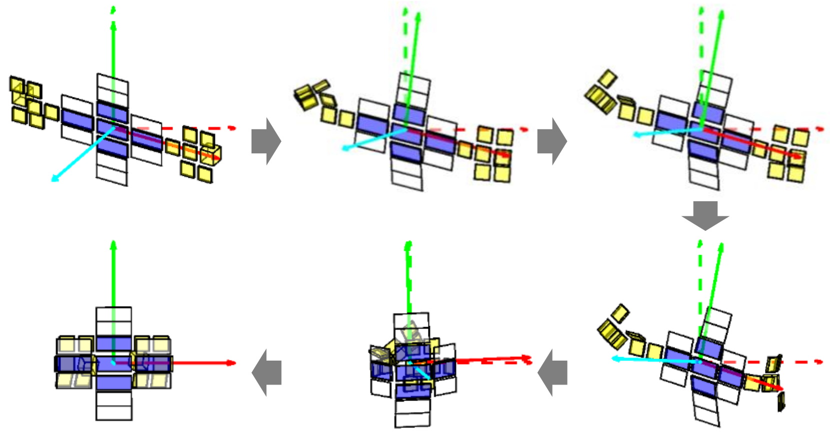
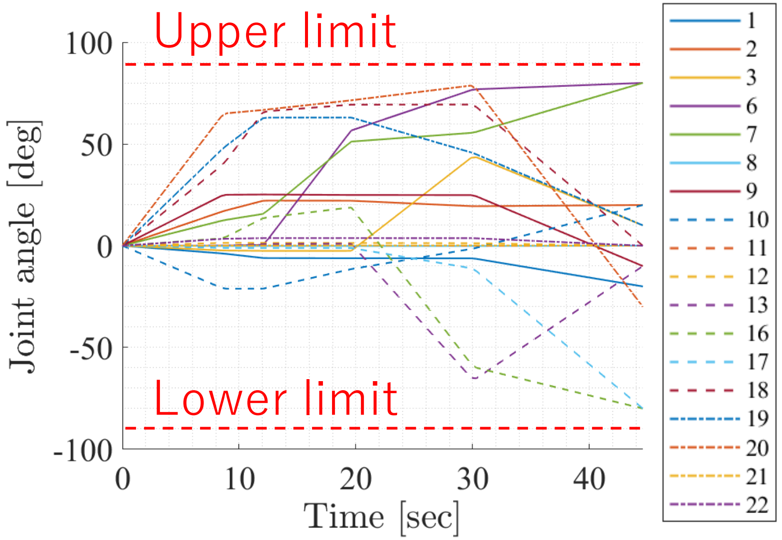
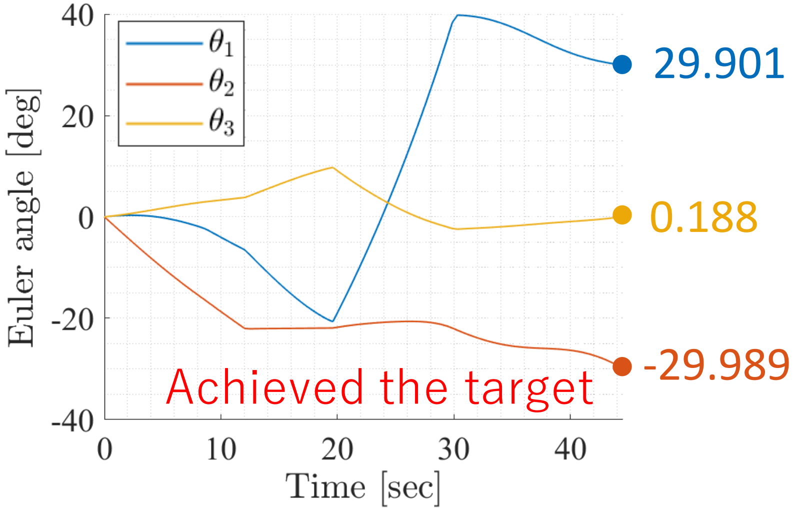


X Y Z Current attitude

X Y Z Target attitude

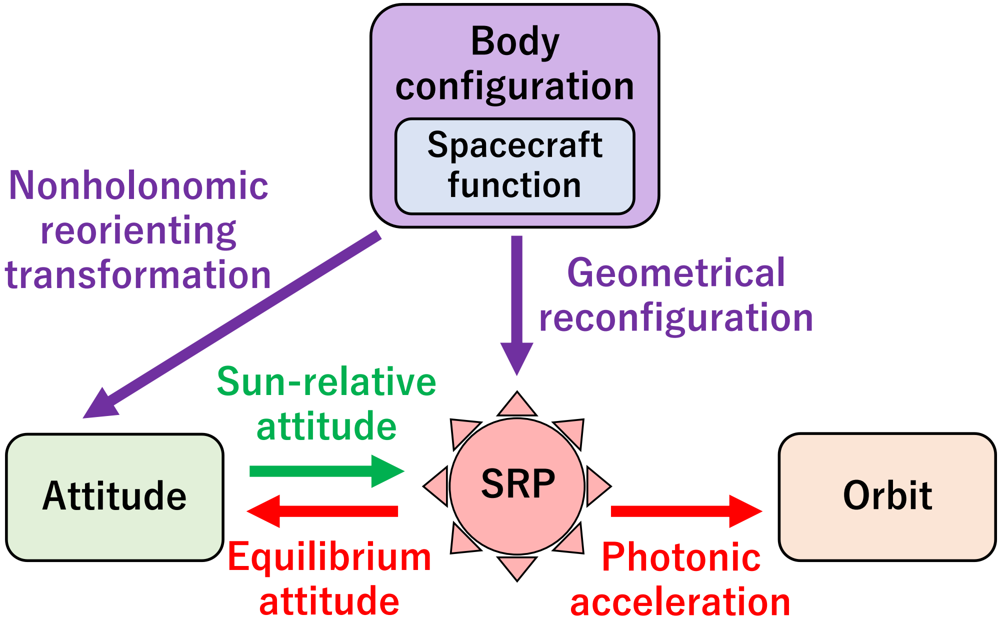


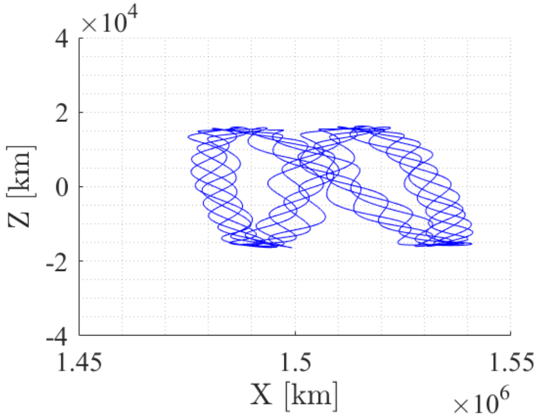


Introduction
- Transformable solar sailing

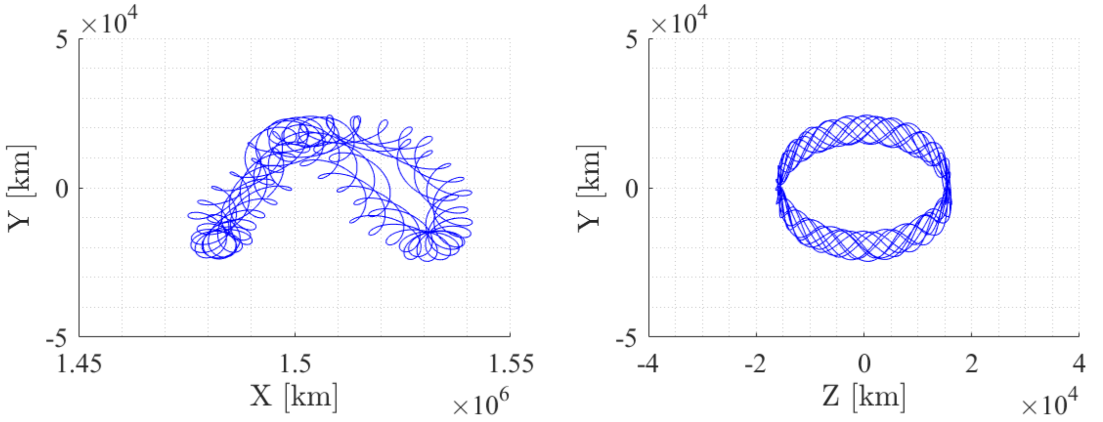
NRT can achieve:
- ✓ Surface geometry reconfiguration
- ✓ Attitude reorientation

One of the best application: **Solar sailing**





Eight-like shape: Earth influence
+
Small fluctuation: Moon influence

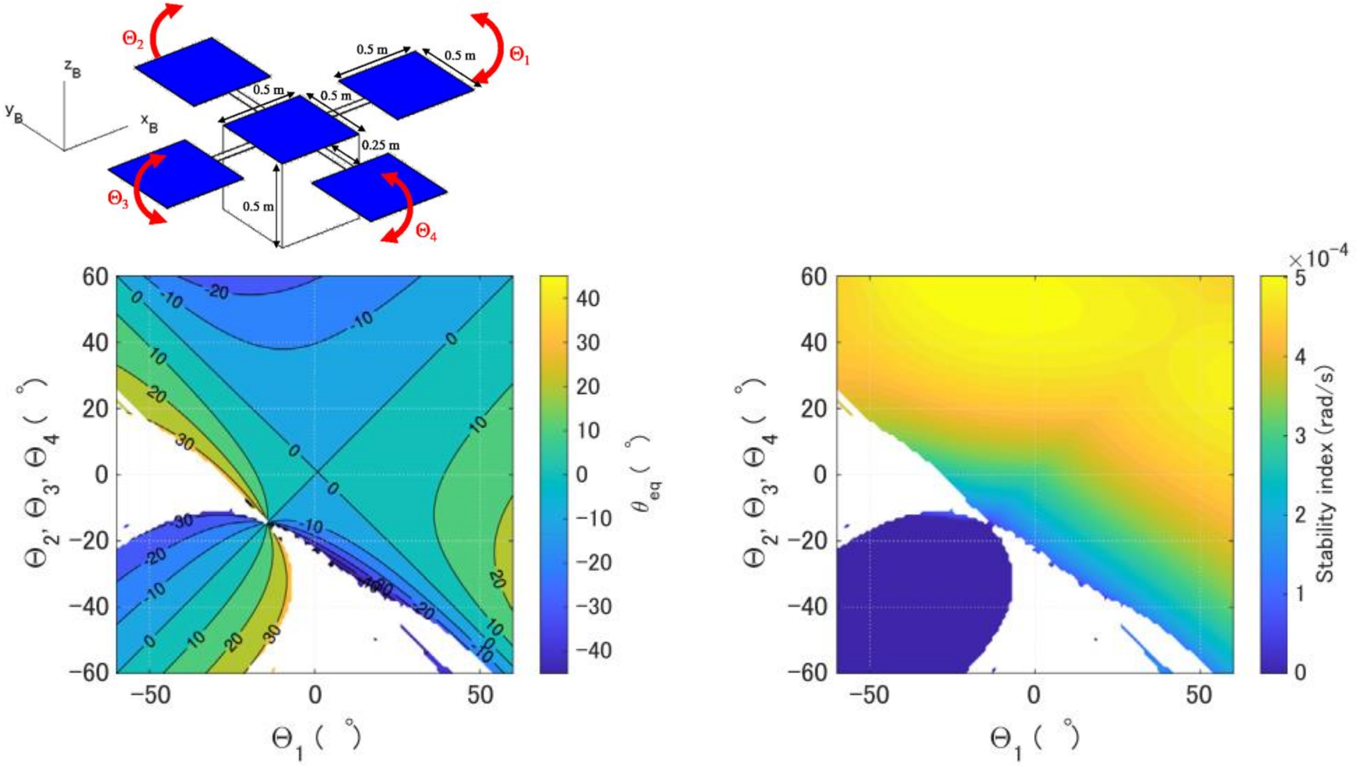


Example of artificial orbit designed in ephemeris model

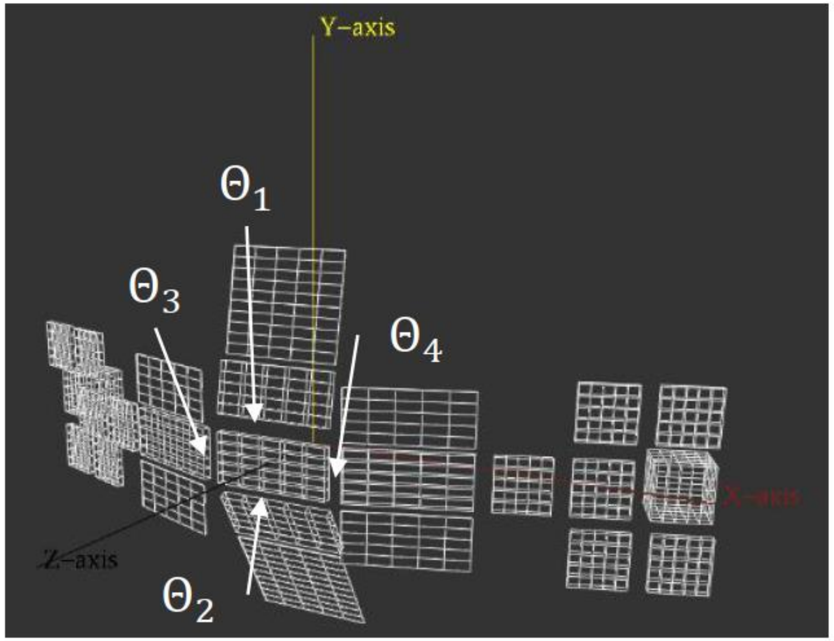
<u>Related conference paper (IAC 2022)</u> Toshihiro Chujo et al.
"Orbit-Attitude Integrated Control on Small-Amplitude Periodic Orbit around Sun-Earth L2 in Transformer Mission"

Introduction
- Previous studies of transformable solar sailing

- Highly nonlinear relationship of attitude and body configuration under SRP
  - Only valid for symmetric, low-DoF models



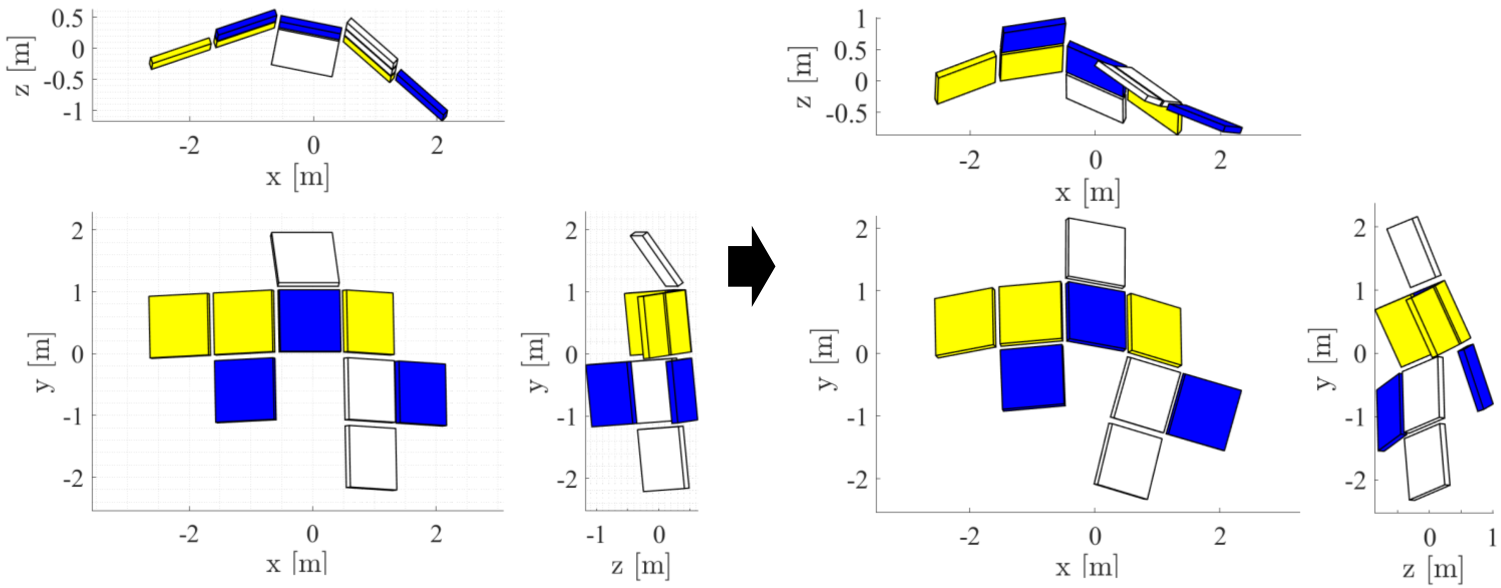
Chujo, Acta Astronautica, 2022



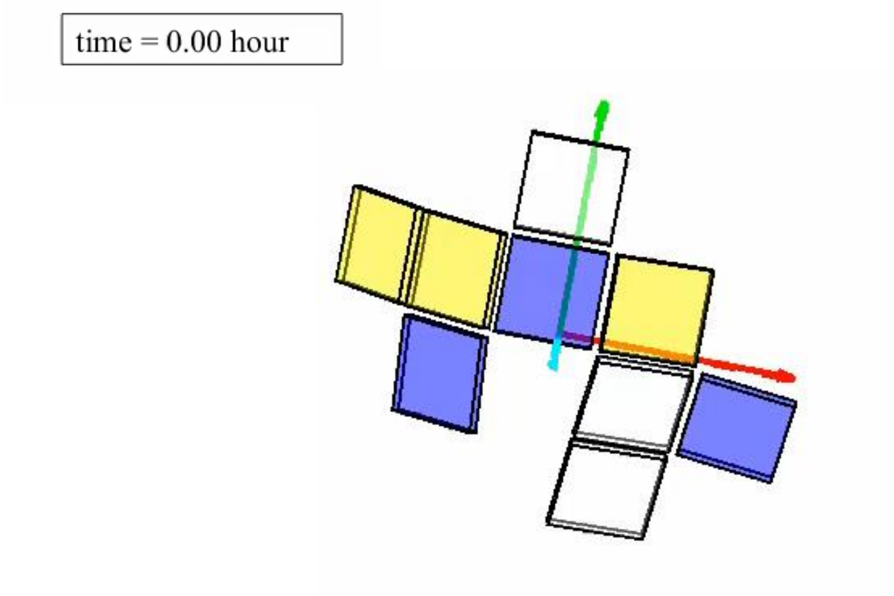
Chujo et al., 2022

Introduction
- Authors' previous studies

- Applicable to general transformable spacecraft
- Optimization of body configuration to achieve desired SRP force & torque
- Attitude stabilization with only joint actuation



Joint angle optimization



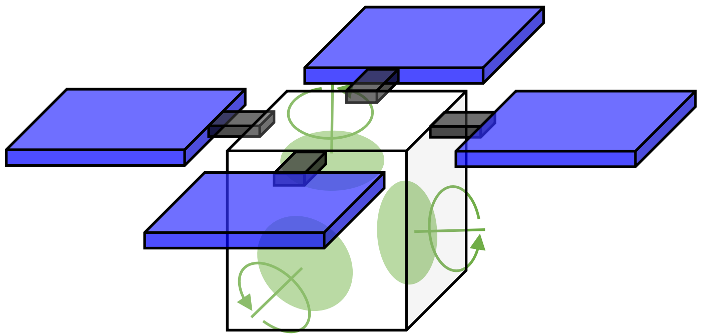
Attitude stabilization

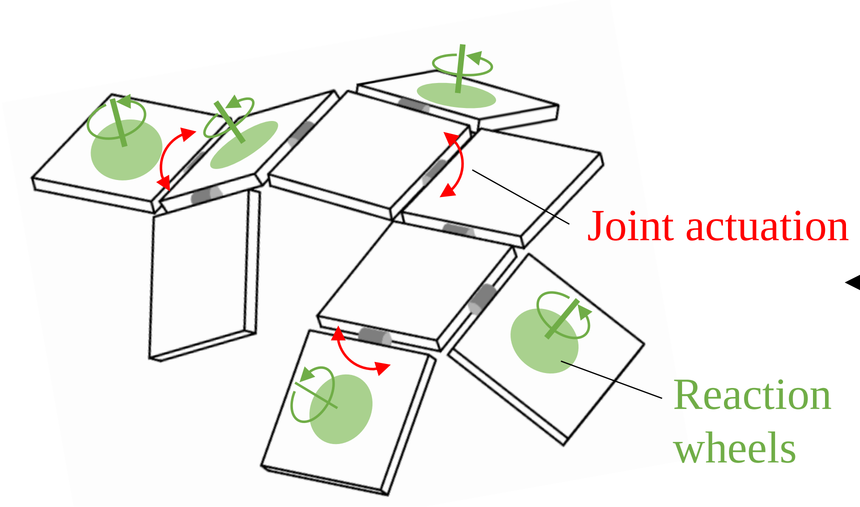
Related article: Kubo & Chujo, Astrodynamics, 2023 (in press)

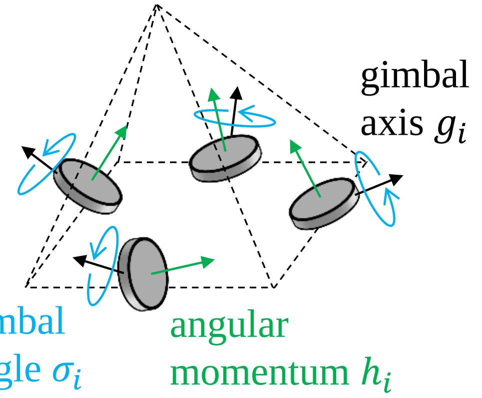
Introduction
- Zero-momentum vs biased-momentum

Zero-momentum
- Pure nonholonomic attitude reorientation without gyroscopic rigidity
- Low attitude stability

Biased momentum
- High attitude stability
- Reconfigurability of reaction wheels in body frame
- Fast reorientation might be possible leveraging gyroscopic effect
- General formulation has not been provided





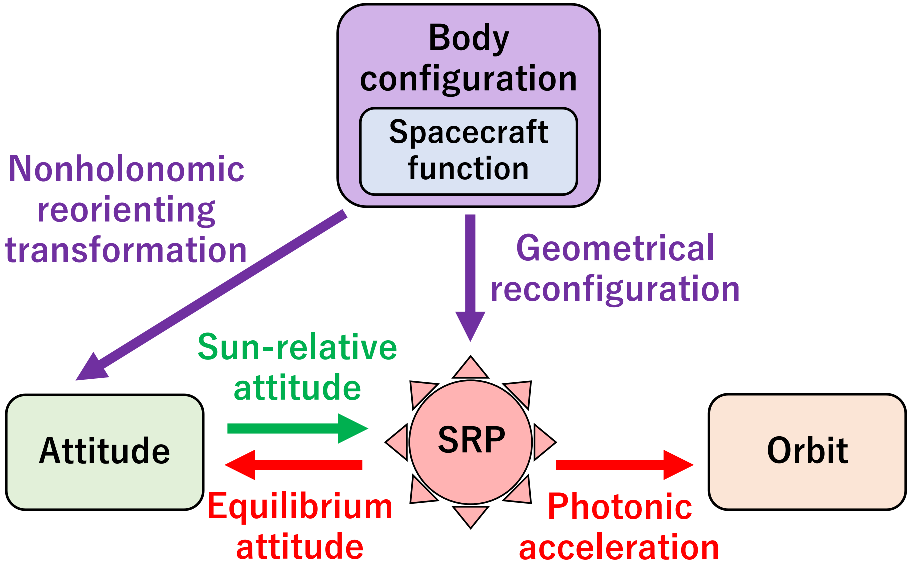


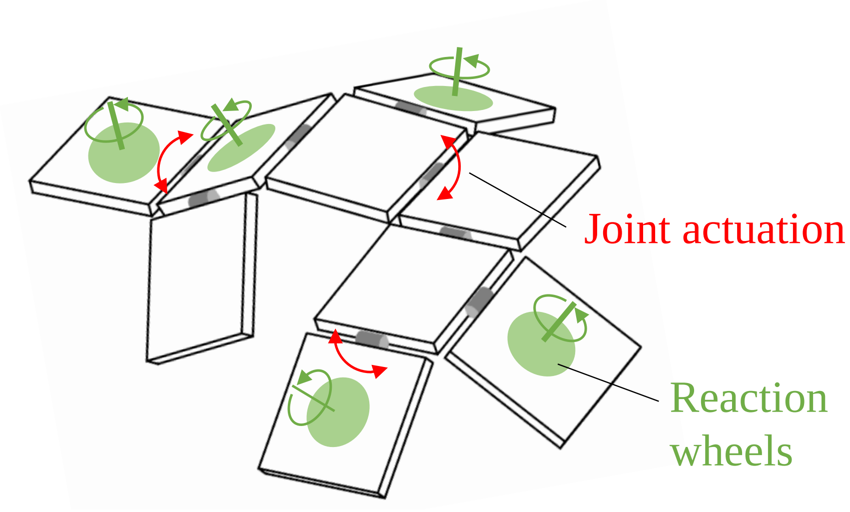
Control moment gyro (CMG)

Introduction
- Purpose

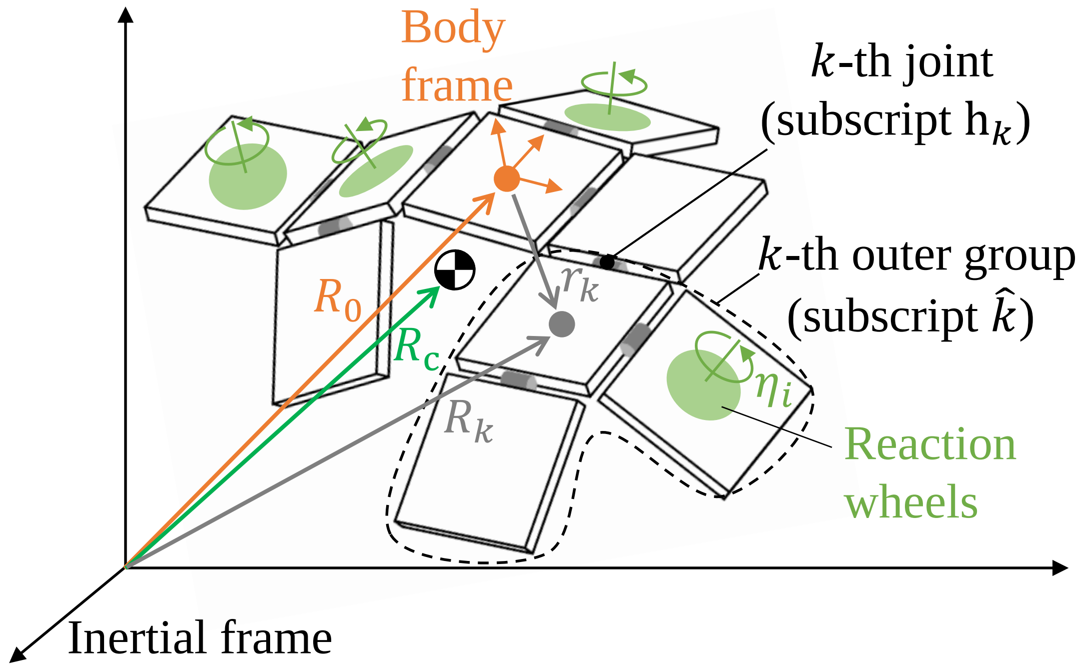
✓ Control law applicable to general transformable spacecraft
  • General mathematical formulation independent of spacecraft model

✓ Attitude control of biased-momentum transformable spacecraft under SRP
  • Attitude reorientation
  • Attitude stabilization





Formulations
- Definitions



- No loop structure
- Multiple rigid bodies

ϕ	Euler angles of body 0
ω	Angular velocity of body 0
θ	Joint angle vector ∈ R^m
η	Wheel momentum vector ∈ R^l
D_η	Wheel axes matrix ∈ $R^{3\times l}$
T	External torque
$M_{\omega\omega}$	Generalized mass matrix for ω
$M_{\omega\theta}$	Generalized mass matrix for $\dot{\theta}$

Total angular momentum
$$h = \underbrace{M_{\omega\omega}\omega}_{\text{Whole-spacecraft rotation}} + \underbrace{M_{\omega\theta}\dot{\theta}}_{\text{Joint rotation}} + \underbrace{D_\eta \eta}_{\text{Wheel rotation}}$$

Formulations
- Attitude equations

Equation of motion for attitude

Gyroscopic rigidity

$$T = \omega^{\times} M_{\omega\omega}\omega + \left(\frac{\partial M_{\omega\omega}}{\partial\theta}\dot{\theta}\right)\omega + M_{\omega\omega}\dot{\omega} + \omega^{\times} M_{\omega\theta}\dot{\theta} + \left(\frac{\partial M_{\omega\theta}}{\partial\theta}\dot{\theta}\right)\dot{\theta} + M_{\omega\theta}\ddot{\theta} + \omega^{\times} D_{\eta}\eta + \frac{\partial D_{\eta}}{\partial\theta}\dot{\theta}\eta + D_{\eta}\dot{\eta}$$

Difference of moment of inertia

Gyroscopic effect of wheels driven by joint actuation

$$\simeq M_{\omega\omega}\dot{\omega} + M_{\omega\theta}\ddot{\theta} + \omega^{\times} D_{\eta}\eta + \frac{\partial D_{\eta}}{\partial\theta}\dot{\theta}\eta + D_{\eta}\dot{\eta} \qquad (\omega, \dot{\theta} \ll 1)$$

SRP force & torque → **Function of joint angles & attitude**

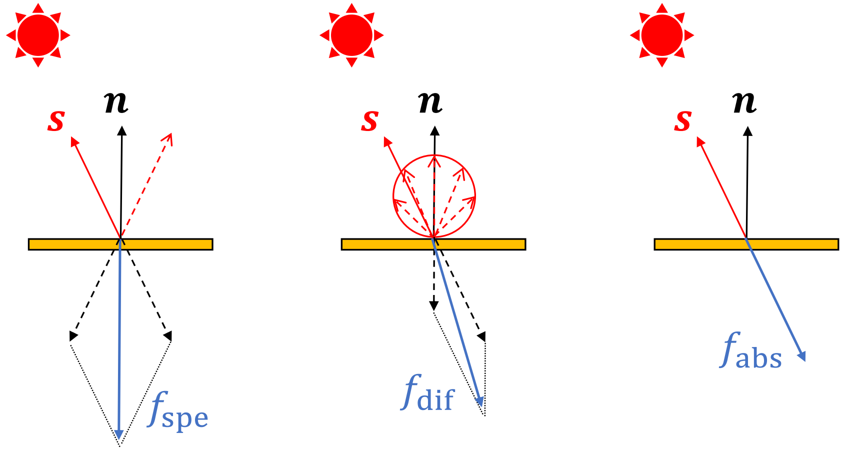
$$T = \sum_i T_i = \sum_i r_{ic}^{\times} F_i = \sum_i (R_i - R_c)^{\times} F_i$$

$$F_i(s, n_i) = -PA_i \left(n_i^{\top} s\right) \left\{ (C_{\mathrm{abs}i} + C_{\mathrm{dif}i}) s + \left(\frac{2}{3} C_{\mathrm{dif}i} + 2\left(n_i^{\top} s\right) C_{\mathrm{spe}i} \right) n_i \right\}$$

$$= -(p_{\mathrm{n}1i} n_{1i} + p_{\mathrm{n}2i} n_{2i} + p_{\mathrm{s}i} s_i) A_i$$

$$\text{where} \quad p_{\mathrm{n}1i} = 2PC_{\mathrm{spe}i}, \quad p_{\mathrm{n}2i} = \frac{2}{3} PC_{\mathrm{dif}i}, \quad p_{\mathrm{s}i} = P(C_{\mathrm{abs}i} + C_{\mathrm{dif}i}),$$

$$n_{1i} = \left(n_i^{\top} s\right)^2 n_i, \quad n_{2i} = \left(n_i^{\top} s\right) n_i, \quad s_i = \left(n_i^{\top} s\right) s$$



Specular   Diffusive   Absorption

Formulations
- Linearized equations

Linearization around equilibrium state $\phi = \tilde{\phi} + \delta\phi,\ \theta = \tilde{\theta} + \delta\theta,\ \eta = \tilde{\eta} + \delta\eta,\ T = \tilde{T} + \delta T$

$$\frac{\partial T}{\partial \phi}\delta\phi + \frac{\partial T}{\partial \theta}\delta\theta \simeq M_{\omega\omega}C_\omega\delta\ddot{\phi} + M_{\omega\theta}\delta\ddot{\theta} - (D_\eta\tilde{\eta})^\times C_\omega\delta\dot{\phi} + \frac{\partial D_\eta}{\partial \theta}\tilde{\eta}\delta\dot{\theta} + D_\eta\delta\dot{\eta}$$

$$\leftrightarrow \quad \delta\ddot{\phi} = L_\phi\left(\frac{\partial T}{\partial \phi}\delta\phi + \frac{\partial T}{\partial \theta}\delta\theta - M_{\omega\theta}\delta\ddot{\theta} + (D_\eta\tilde{\eta})^\times C_\omega\delta\dot{\phi} - \frac{\partial D_\eta}{\partial \theta}\tilde{\eta}\delta\dot{\theta} - D_\eta\delta\dot{\eta}\right)$$

$$\text{where} \quad L_\phi = C_\phi M_{\omega\omega}^{-1}, \qquad \dot{\phi} = C_\phi\omega, \qquad \omega = C_\omega\dot{\phi}$$

Linearized equation of motion of the whole system

$$\frac{\mathrm{d}}{\mathrm{d}t}\begin{bmatrix}\delta\phi\\ \delta\theta\\ \delta\dot{\phi}\\ \delta\dot{\theta}\\ \delta\eta\end{bmatrix} = \begin{bmatrix} O_{3\times3} & O_{3\times m} & U_{3\times3} & O_{3\times m} & O_{3\times l}\\ O_{m\times3} & O_{m\times m} & O_{m\times3} & U_{m\times m} & O_{m\times l}\\ L_\phi\frac{\partial T}{\partial\phi} & L_\phi\frac{\partial T}{\partial\theta} & L_\phi(D_\eta\tilde{\eta})^\times C_\omega & -L_\phi\frac{\partial D_\eta}{\partial\theta}\tilde{\eta} & O_{3\times l}\\ O_{m\times3} & O_{m\times m} & O_{m\times3} & O_{m\times m} & O_{m\times l}\\ O_{l\times3} & O_{l\times m} & O_{l\times3} & O_{l\times m} & O_{l\times l}\end{bmatrix}\begin{bmatrix}\delta\phi\\ \delta\theta\\ \delta\dot{\phi}\\ \delta\dot{\theta}\\ \delta\eta\end{bmatrix} + \begin{bmatrix} O_{3\times m} & O_{3\times l}\\ O_{m\times m} & O_{m\times l}\\ -L_\phi M_{\omega\theta} & -L_\phi D_\eta\\ U_{m\times m} & O_{m\times l}\\ O_{l\times m} & U_{l\times l}\end{bmatrix}\begin{bmatrix}\delta\ddot{\theta}\\ \delta\dot{\eta}\end{bmatrix}$$

$$\leftrightarrow \quad \frac{\mathrm{d}x}{\mathrm{d}t} = Ax + Bu$$

Formulations
- Equilibrium state

Equilibrium attitude & joint configuration can be obtained by the optimization problem below:

$$\underset{\phi,\theta}{\text{minimize}} \quad f(\phi,\theta) = -\left\| \mathrm{Im}\left(\sqrt{\Lambda}\right)\right\|_\infty$$

↑ Faster attitude oscillation

$$\text{subject to} \quad c(\phi,\theta) = \max\left(\mathrm{Re}\left(\sqrt{\Lambda}\right)\right) \le 0,$$

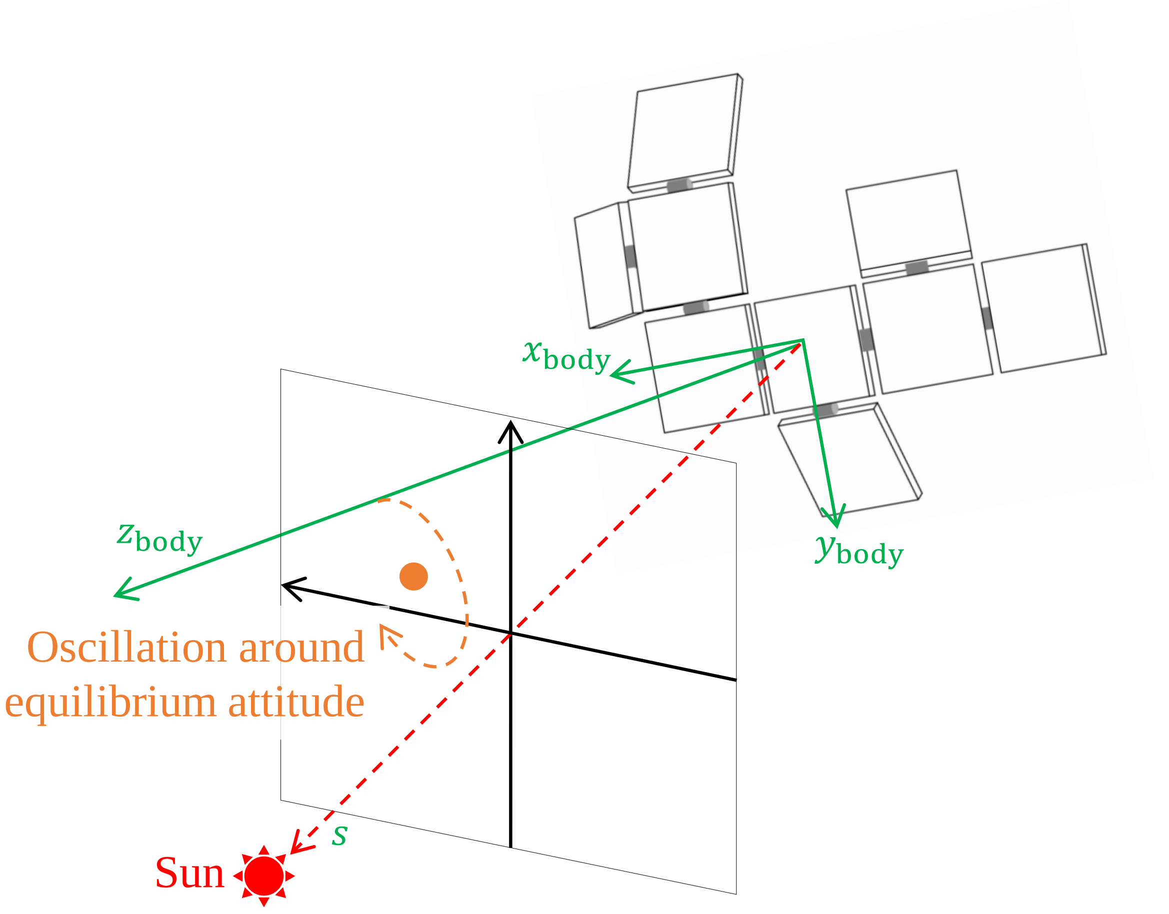
↑ Suppress attitude divergence

$$c_{\mathrm{eq}}(\phi,\theta,F_{\mathrm{targ}},T_{\mathrm{targ}}) = \begin{bmatrix} F_{\mathrm{targ}} - F \\ T_{\mathrm{targ}} - T \end{bmatrix} = 0, \quad \theta_{\mathrm{lb}} \le \theta \le \theta_{\mathrm{ub}}$$

↑ Obtain target SRP force & torque

where Λ is eigenvalues of $L_\phi \frac{\partial T}{\partial \phi}$

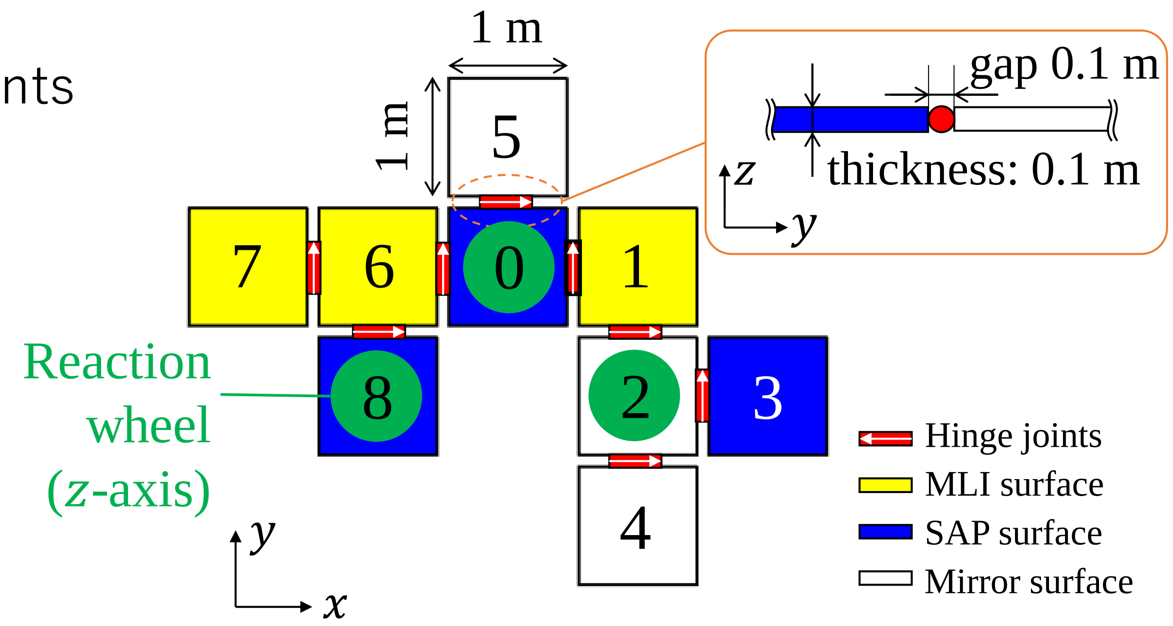
($=$ Eigenvalues of natural attitude oscillation under SRP)



Simulation
- Conditions

- Spacecraft model
  - 9 flat panels connected with 8 actuatable joints
  - 10 kg for each panel (90 kg in total)
  - Random joint configuration
  - Random surface material distribution
  - Nominal wheel momentum 0.3 Nms

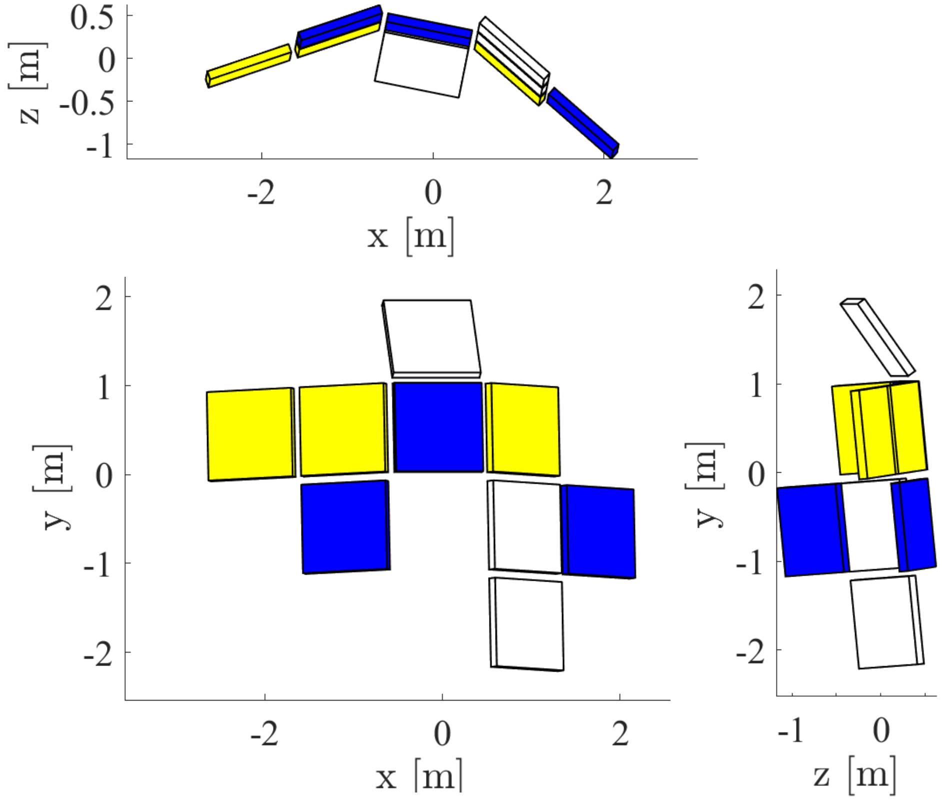
- SRP at 1.01AU (SEL2 halo orbit)



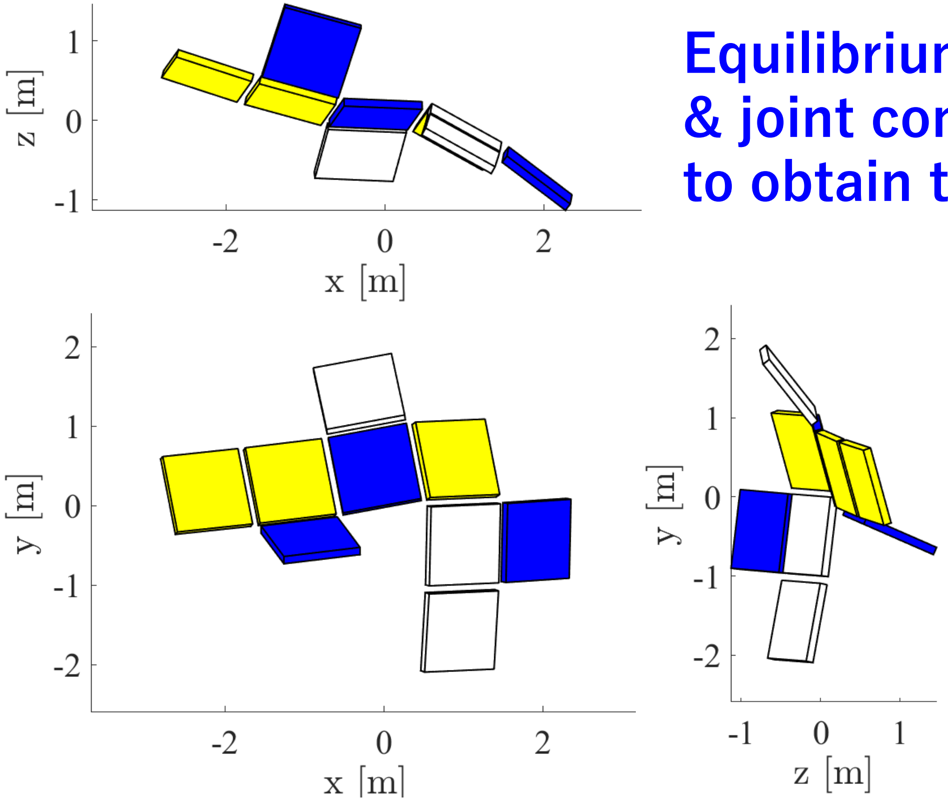
Material	C_{spe}	C_{dif}	C_{abs}
MLI	0.375	0.255	0.370
SAP	0.086	0.060	0.854
Mirror	1.0	0.0	0.0

Simulation
- Attitude & joint angle optimization

- Target force $F = [-0.893 \quad -0.446 \quad -4.463] \times 10^{-5}$ N, Target torque $T = [0 \quad 0 \quad 0]$ Nm
- Solver: fmincon, interior-point algorithm (MATLAB)



Initial guess



Equilibrium attitude & joint configuration to obtain target SRP thrust

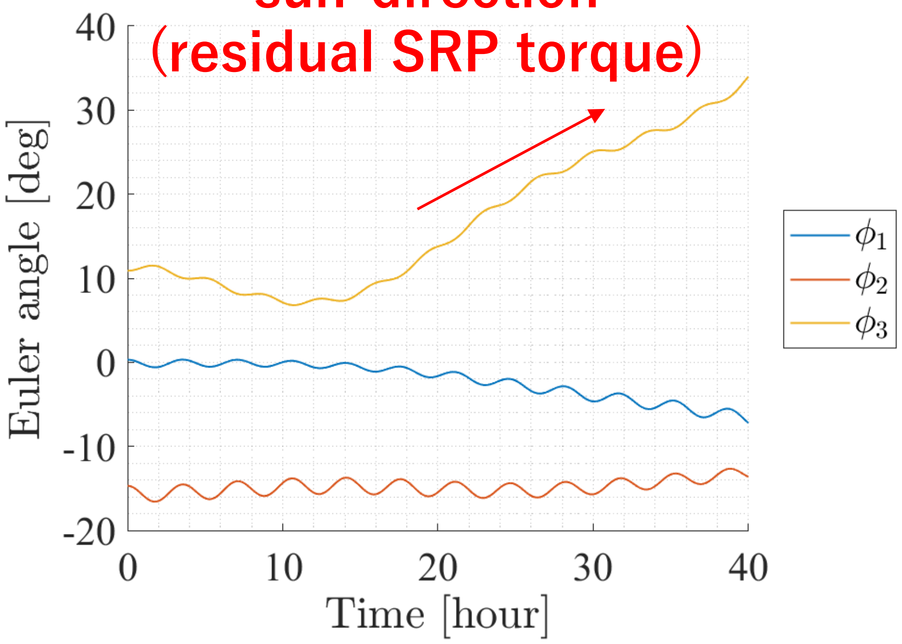
Optimized result

Simulation
- Free motion without RW momentum

Wheel momentum $\eta = [0 \quad 0 \quad 0]$, No joint feedback control

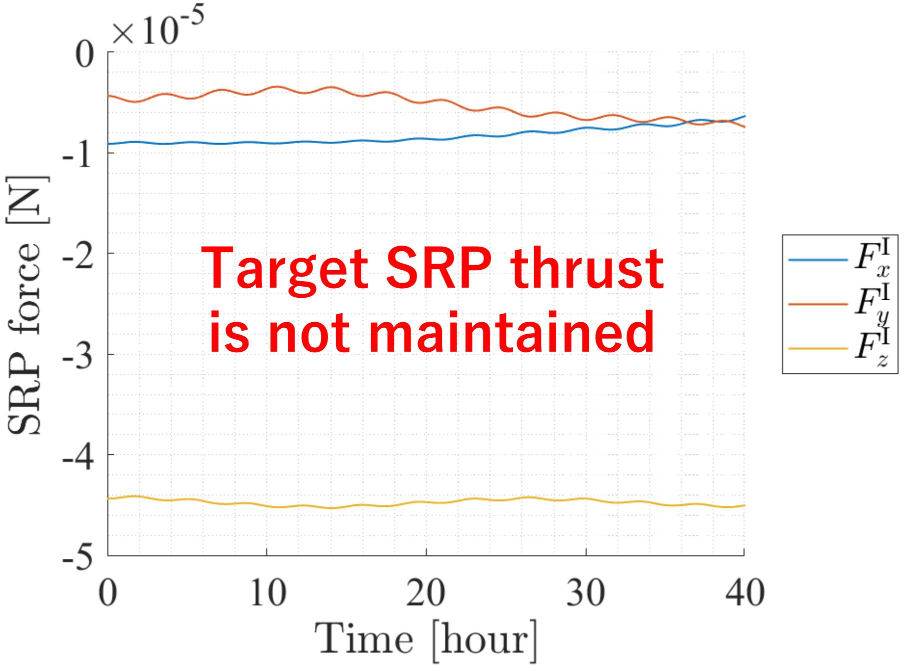
Initial attitude error $\delta\phi_0 = [0.819 \quad 0.567 \quad 0.088]$ deg, angular velocity $\omega_0 = [0 \quad 0 \quad 0]$ deg/s

Unstable around sun-direction (residual SRP torque)

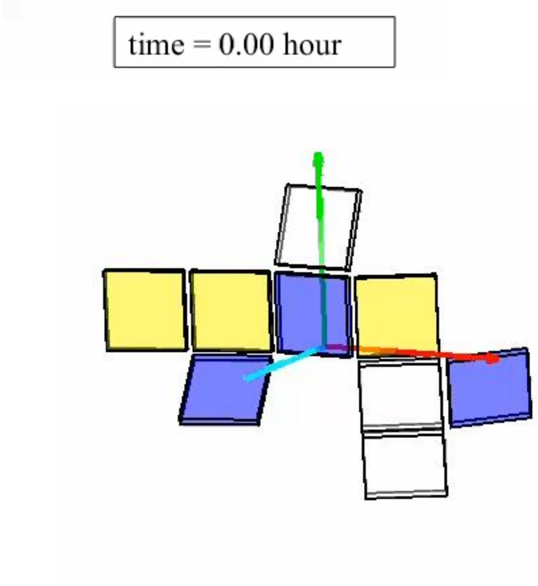


Euler angle history

Target SRP thrust is not maintained

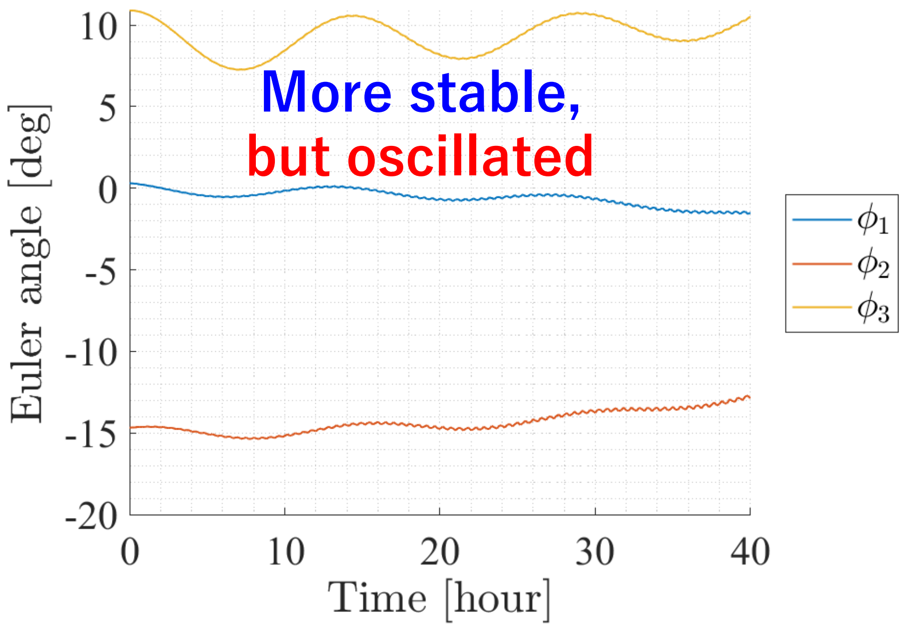


SRP thrust history

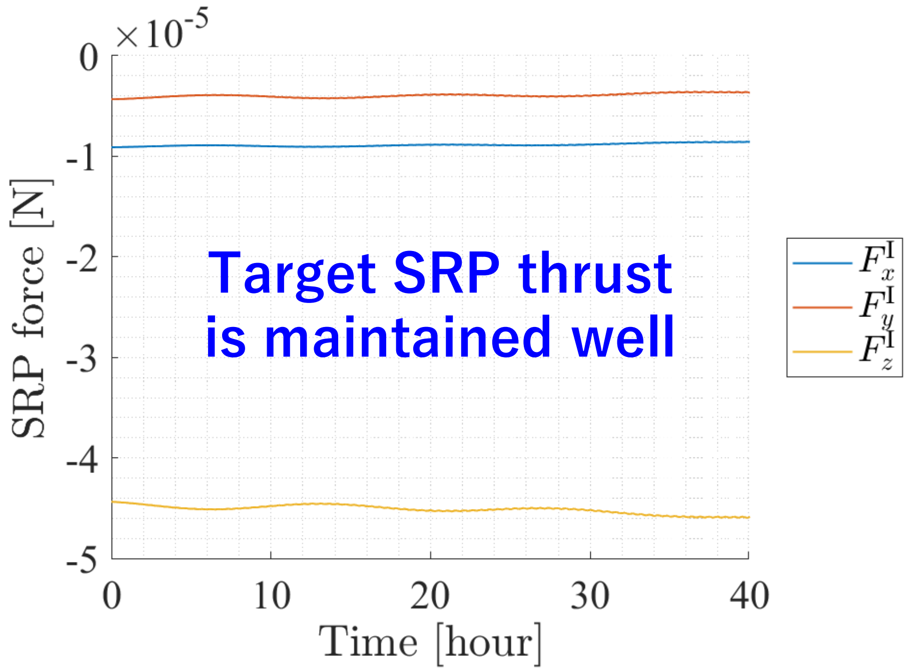


Simulation
- Free motion with RW momentum

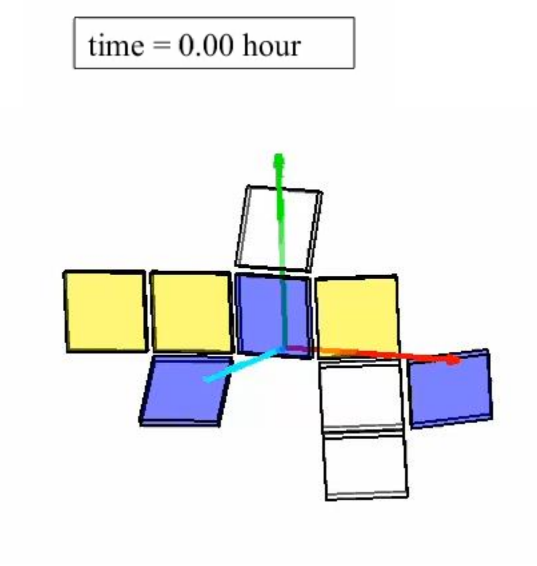
Wheel momentum $\eta = [0.3 \quad 0.3 \quad 0.3]$, No joint feedback control
Initial attitude error $\delta\phi_0 = [0.819 \quad 0.567 \quad 0.088]$ deg, angular velocity $\omega_0 = [0 \quad 0 \quad 0]$ deg/s



Euler angle history

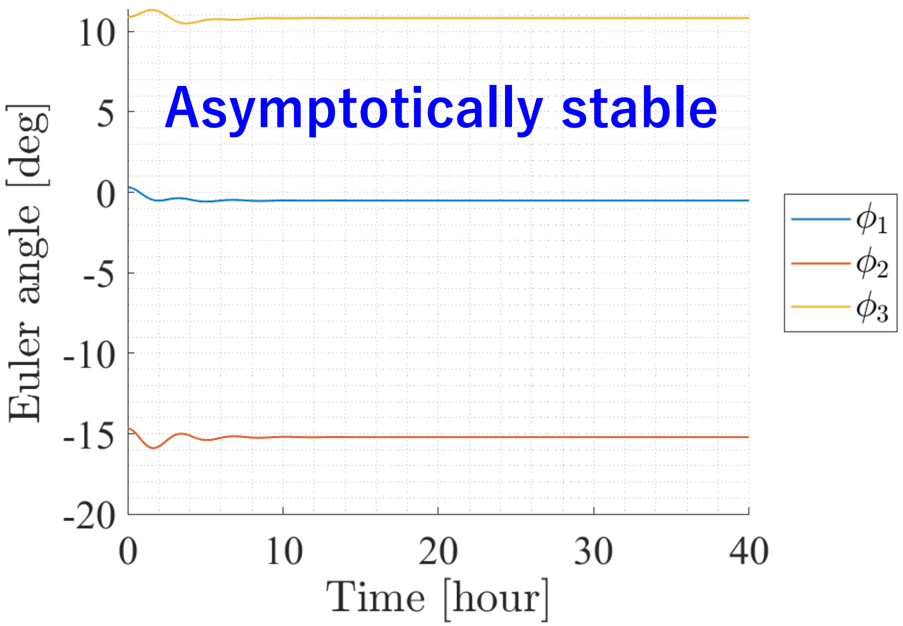


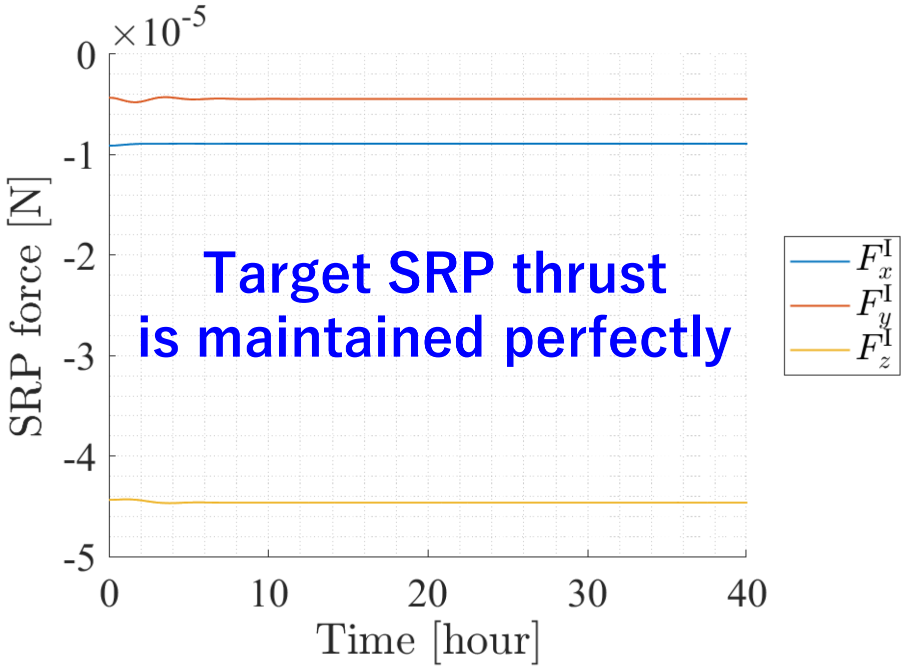
SRP thrust history

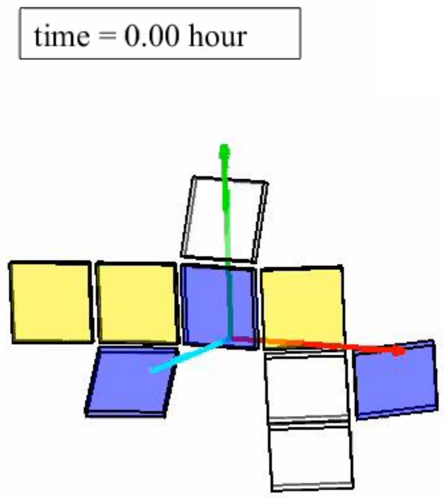


Simulation
- Joint control with RW momentum

Wheel momentum $\eta = [0.3 \quad 0.3 \quad 0.3]$, LQR joint feedback control
Initial attitude error $\delta\phi_0 = [0.819 \quad 0.567 \quad 0.088]$ deg, angular velocity $\omega_0 = [0 \quad 0 \quad 0]$ deg/s

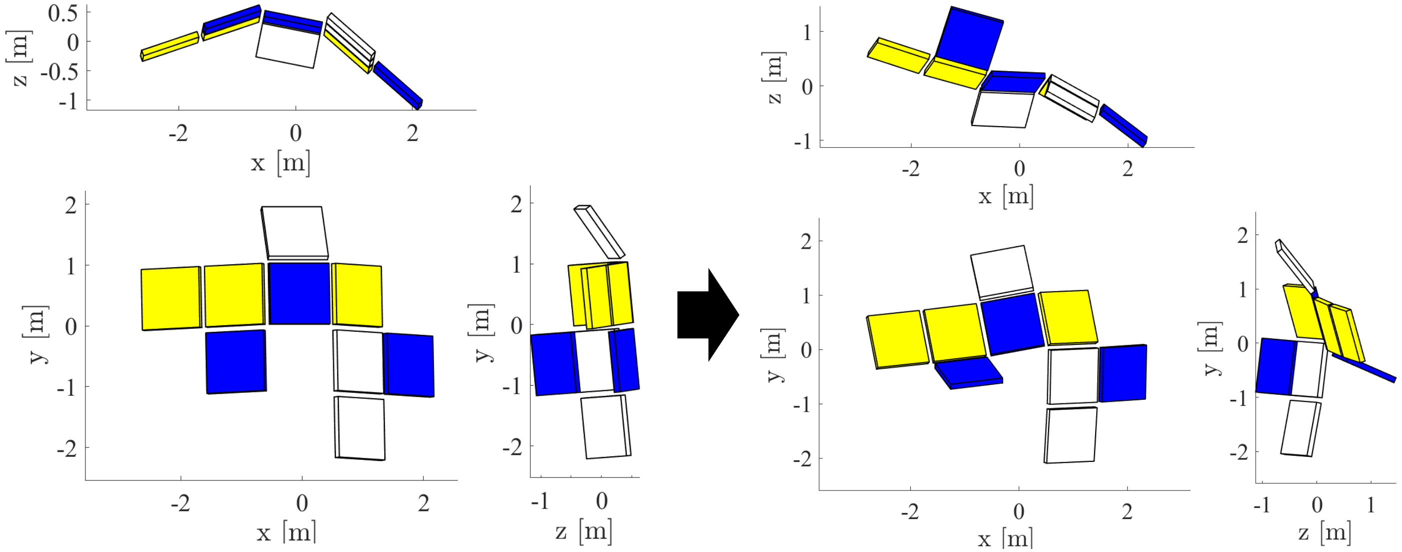


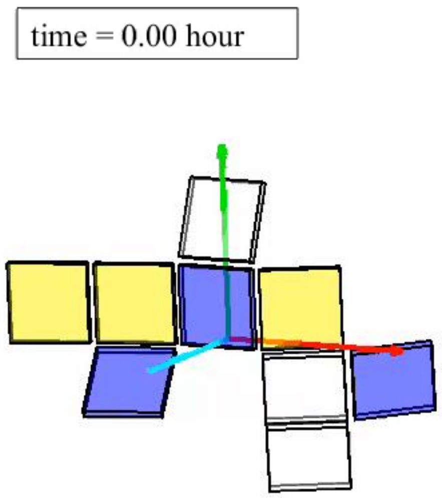




Conclusions

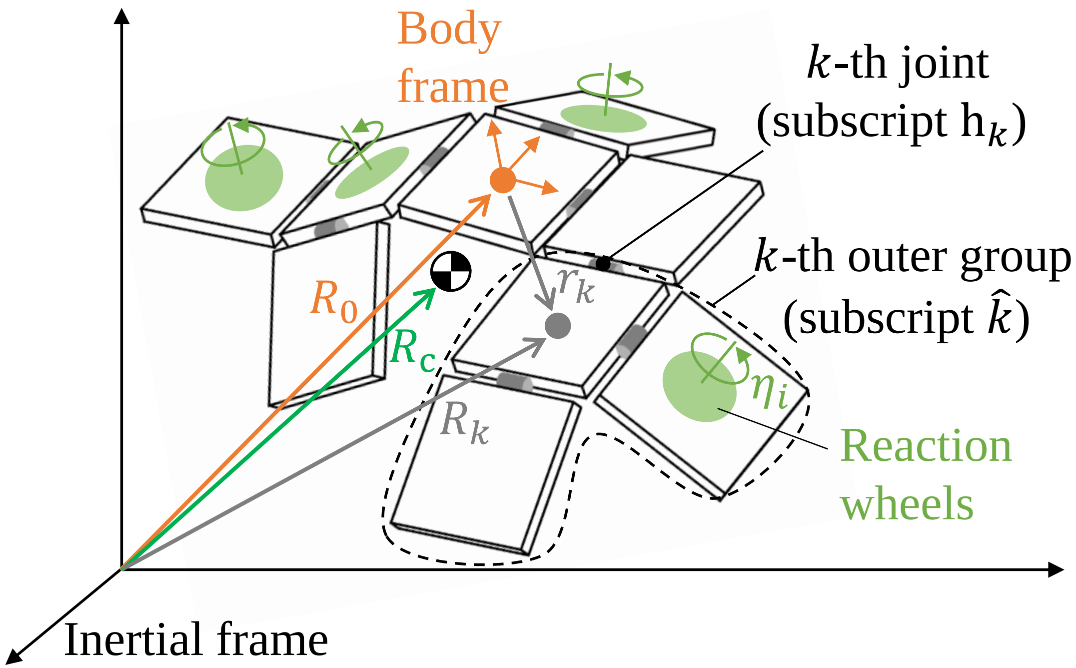
- ✓ Attitude control of biased-momentum transformable spacecraft under SRP
- ✓ General formulation applicable to arbitrary transformable spacecraft
- ✓ Attitude & joint angle optimization to obtain equilibrium attitude with target SRP thrust
- ✓ Attitude stabilization around equilibrium state with only joint actuation

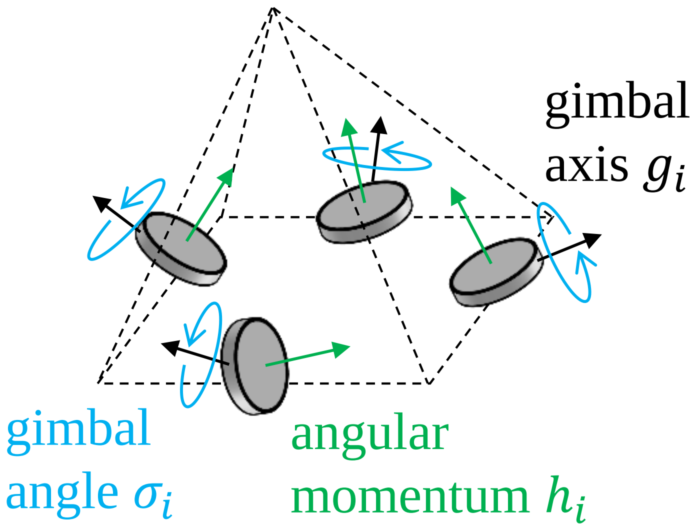




Further studies

- Optimization to obtain equilibrium attitude with best reaction wheel distribution
- Agile nonholonomic reorientation leveraging gyroscopic effect (CMG-like control)





Control moment gyro (CMG)