ISSN 1349-1113
JAXA-RR-06-029E

JAXA Research and Development Report

Robust H_2 Controller Synthesis for LPV Systems and Its Application to Model-Following Controller Design : Design Examples for Aircraft Motions

Masayuki SATO

March 2007

Japan Aerospace Exploration Agency

JAXA-RR-06-029E

JAXA Research and Development Report

Robust H_2 Controller Synthesis for LPV Systems and Its Application to Model-Following Controller Design: Design Examples for Aircraft Motions

Masayuki SATO

Flight Systems Technology Center
Institute of Aerospace Technology

March 2007

Japan Aerospace Exploration Agency

Robust H_2 Controller Synthesis for LPV Systems and Its Application to Model-Following Controller Design: Design Examples for Aircraft Motions*

LPV システムに対するロバスト H_2 制御器設計と そのモデルフォロ制御設計への応用：航空機運動の設計例*

Masayuki SATO*1
佐藤　昌之 *1

ABSTRACT

This paper addresses the design problem of robust model-following controllers for linear time-invariant systems with parametric uncertainties. To design such controllers, we apply a design method for robust static output H_2 controllers for Linear Parameter-Varying (LPV) systems using parameter-dependent Lyapunov functions to a previously proposed model-following controller design. We design several model-following controllers for the lateral/directional motions of an aircraft, and demonstrate their performance by numerical simulations and flight tests.

Keywords : Model-following controller, robust H_2 problem, static output controller, aircraft motions, flight test.

1 INTRODUCTION

In the last decade, much research has been conducted on analysis and synthesis for Linear Parameter-Varying (LPV) systems. Conventionally, parameter-independent Lyapunov functions have been used (1) and references therein), however, they resulted in excessively conservative analysis and synthesized controllers; that is, not tight analysis and synthesis. This leads to the use of parameter-dependent Lyapunov functions to reduce the conservatism. In accordance with this suggestion, analysis and synthesis methods using parameter-dependent Lyapunov functions have been proposed [2-11]. Parameter-dependent Lyapunov functions generally lead to LPV controllers (i.e. gain-scheduled controllers), not Linear Time-Invariant (LTI) controllers (i.e. robust controllers); since change-of-variables method sets substituted variables, which represent controller state-space matrices, to be parameter-dependent [2-4], and variable elimination method also sets controllers to be parameter-dependent even if Lyapunov functions are set to be parameter-independent [6]. Unfortunately, the implementation of gain-scheduled controllers requires real-time calculation of state-space matrices of controllers, and thus substantial on-board computing power. Moreover, if the scheduling parameters are not measurable or can only be obtained with a delay, then implementing gain-scheduled controllers is impossible. On the other hand, implementing robust controllers is reasonable even in such cases. For these reasons, robust controller design for LPV systems using parameter-dependent Lyapunov functions has been desired.

Feron *et al.* have proposed a design method for robust dynamic output controllers for LPV systems via parametrically affine Linear Matrix Inequalities (LMIs), using parametrically affine Lyapunov functions [5]. However, their formulation includes a rank condition for static output controller design. Although there exist some effective algorithms to tackle this

---

*　平成 19 年 3 月 1 日受付（received 1 March, 2007）
*1　Flight Systems Technology Center, Institute of Aerospace Technology（総合技術研究本部　飛行システム技術開発センター）

intractable problem (rank condition) (e.g.[12]), it is difficult to restrict controllers to be some special forms, such as diagonal matrices. As another robust controller design method, robust state feedback controller design using biquadratic Lyapunov functions[7] has been proposed[8, 9]. These proposed methods are also formulated via parametrically affine LMIs, and therefore they obtain global optimal solutions for their formulations. However, the performance of controllers using the methods may be worse than that of controllers using parameter-independent Lyapunov functions because parameter-dependent Lyapunov functions are restricted to be special forms. Furthermore, these methods cannot be extended to static output controller design. As another design method of robust state feedback controllers for LPV systems, extended formulation[10] or dilated formulation[11] have been proposed. However, these methods cannot be easily extended to static output controller design. For static output controller design, the design problem for LTI systems has been formulated via two LMIs and one rank condition by Iwasaki and Skelton[13], but their method, i.e. elimination method, cannot be applied to the problem of robust static output controller design for LPV systems because it generally gives LPV controllers (i.e. gain-scheduled controllers) for LPV systems, as mentioned above.

In this paper, we propose a design method for robust static output H_2 controllers for LPV systems using parameter-dependent Lyapunov functions under the condition that stabilizing controllers are given. Our proposed method uses an iterative algorithm that monotonically converges to an optimal solution. Although the optimized controllers are not generally global optimal solutions, a candidate solution to the problem is easily obtained. The iterative algorithm used in this paper has been originally proposed by Shimomura *et al.*[14, 15] to design multi-objective controllers for LTI systems with uncommon Lyapunov variables. Although the condition that stabilizing controllers are given seems to be restrictive, if the plant dynamics are well known (e.g. mass-spring systems or aircraft motions) then stabilizing controllers can be designed with some effort.

We also apply our proposed method to a previously proposed model-following controller design[16] to overcome its drawbacks in its design, and we design several model-following controllers for aircraft motions. In this design, matrices to be designed are restricted to be special forms; block diagonal matrices, however, the proposed method can be applied and obtain candidates for optimal model-following controllers. We demonstrate their performance by numerical simulations and flight experiments.

This manuscript is organized as follows. We first describe our proposed design method for robust static output H_2 controllers, and demonstrate its effectiveness with a simple numerical example. We next show its application to model-following controller design, then show the design examples for the lateral/directional motions of an aircraft, and we finally show the performance of those controllers by numerical simulations and flight experiments.

Hereafter, X^T denotes the transpose of a matrix X, $\mathrm{He}(Y)$ denotes $Y+Y^T$, Tr denotes matrix trace, $\mathcal{E}(*)$ denotes an expectation operator, and 0 and I_n respectively denote a zero matrix of an appropriate dimension and an n-dimensional identity matrix.

2 ROBUST H_2 CONTROLLER DESIGN

We first give the formulation of robust H_2 controller synthesis, and then show our method for it.

2.1 Problem Formulation

We consider the following LPV system

$$\begin{cases} \dot{x} = A(\theta)x + B_1(\theta)w + B_2(\theta)u \\ z = C_1(\theta)x \qquad\qquad\quad + D_{12}(\theta)u, \\ y = C_2(\theta)x \end{cases} \tag{1}$$

where $x \in \mathcal{R}^n$ is the state vector with $x=0$ at $t=0$, $w \in \mathcal{R}^{n_w}$ is the disturbance input vector, $u \in \mathcal{R}^{n_u}$ is the control input vector, $z \in \mathcal{R}^{n_z}$ is the controlled output vector, $y \in \mathcal{R}^{n_y}$ is the measurement output vector. Matrices are all of appropriate dimensions. The vector $\theta = [\theta_1 \cdots \theta_\kappa]^T$ is a time-varying vector consisting of k time-varying or time-invariant parameters which cannot be measured on time; that is, they represent time-varying or time-invariant uncertainties of the plant. The ranges of θ_i and $\dot{\theta}_i$ are assumed to be known in advance and their variations are assumed to lie in convex regions $\mathcal{B}_\theta$ and $\mathcal{B}$:

$$\theta(t) \in \mathcal{B}_\theta,\ (\theta(t), \dot{\theta}(t)) \in \mathcal{B},\ \forall t \geq 0,$$

where $\dot{\theta} = [\dot{\theta}_1 \cdots \dot{\theta}_k]^T$, $(\theta, \dot{\theta})$ denotes an ordered pair of the direct product of two sets representing the existence regions of θ and $\dot{\theta}$, and $\dot{\theta}_i$ denotes the derivative of θ_i with respect to time.

We now make the following assumption.

Assumption 1 $R(\theta) := D_{12}(\theta)^T D_{12}(\theta) > 0,\ \forall\ \theta \in \mathcal{B}_\theta$

From the fact that $R(\theta) > 0$ corresponds to the positivity of a weighting function of control inputs in Linear Quadratic Regulator (LQR) problem, Assumption 1 is not a special assumption.

We look for the following LTI controller with $K \in \mathcal{R}^{n_u \times n_y}$.

$$u = Ky \tag{2}$$

Applying controller (2) to system (1), we obtain the following closed-loop system.

$$\begin{cases} \dot{x} = A_{cl}(\theta)x + B_1(\theta)w \\ x = C_{cl}(\theta)x \end{cases}, \tag{3}$$

where $A_{cl}(\theta) = A(\theta) + B_2(\theta)\ KC_2(\theta)$ and $C_{cl}(\theta) = C_1(\theta) + D_{12}(\theta)\ KC_2(\theta)$.

If the closed-loop system is stable with given controller gain K, then the following lemma, which is related to H_2 performance, holds. Hereafter, $\dot{P}(\theta)$ denotes $\sum_{i=1}^{k} \dot{\theta}_i \frac{\partial P(\theta)}{\partial \theta_i}$.

Lemma 1 [17] *If there exist a continuously differentiable positive definite matrix $P(\theta) \in \mathcal{R}^{n \times n}$ and a positive definite matrix $N \in \mathcal{R}^{n_w \times n_w}$ such that (4) and (5) hold, then (6) holds for $w=\delta(0)\,w_0$, where $\delta(\cdot)$ is Dirac's delta function and w_0 is a random variable satisfying $\mathcal{E}(w_0 w_0^T) = I_{n_w}$.*

$$\begin{bmatrix} N & B_1(\theta)^T P(\theta) \\ P(\theta)B_1(\theta) & P(\theta) \end{bmatrix} > 0,\ \forall \theta \in \mathcal{B}_\theta \tag{4}$$

$$\begin{bmatrix} \mathrm{He}\{P(\theta)A_{cl}(\theta)\} + \dot{P}(\theta) & C_{cl}(\theta)^T \\ C_{cl}(\theta) & -I_{n_z} \end{bmatrix} < 0,\ \forall (\theta, \dot{\theta}) \in \mathcal{B} \tag{5}$$

$$\sup_{(\theta,\dot{\theta})\in\mathcal{B}} \mathcal{E}\left(\int_0^\infty z^T z dt\right) < \mathrm{Tr}(N) \tag{6}$$

If we optimize $\mathrm{Tr}(N)$, then we obtain an upper bound of the attenuation performance of impulse disturbance $w=\delta(0)\,w_0$, which is given as the left-hand side of (6).

If LPV system (1) is an LTI system, then LMIs (4) and (5) respectively become as follows.

$$N - B_1^T P B_1 > 0 \tag{7}$$

$$\mathrm{He}\{PA_{cl}\} + C_{cl}^T C_{cl} < 0 \tag{8}$$

Minimizing $\mathrm{Tr}(N)$ subject to (7) and (8) is a standard H_2 performance analysis for LTI systems. Thus, Lemma 1 is reasonably extended from H_2 performance analysis for LTI systems to one for LPV systems.

From Lemma 1, H_2 performance analysis for LPV system (3) with given controller gain K is formulated as follows.

Problem 1 *Calculate J such that*

$$J = \inf_{P(\theta)} \mathrm{Tr}(N) \text{ subject to (4) and (5).}$$

From Lemma 1 , robust H_2 controller synthesis with controller (2) is formulated as follows:

Problem 2 *Find controller* $K \in R^{n_u \times n_y}$ *such that*

$$J = \inf_{P(\theta),K} \mathrm{Tr}(N) \text{ subject to (4) and (5)}.$$

2.2 Proposed Method

In Problem 2, inequality (5) is a Bilinear Matrix Inequality (BMI) in terms of $P(\theta)$ and K, and is not tractable. One way to address this problem is to minimize $\mathrm{Tr}(N)$ with variables $P(\theta)$ and K alternately, similarly to D-K iteration in μ synthesis. However, we cannot reduce $\mathrm{Tr}(N)$ at the step where K is a design variable, because (4) includes only $P(\theta)$. This iterative algorithm therefore does not work well, and another algorithm is needed.

After some manipulations to (5), we obtain the following inequality, which is equivalent to (5):

$$\begin{bmatrix} X_{11}(\theta) - M(\theta)^T R(\theta)^{-1} M(\theta) & C_2(\theta)^T K^T + M(\theta)^T R(\theta)^{-1} \\ KC_2(\theta) + R(\theta)^{-1} M(\theta) & -R(\theta)^{-1} \end{bmatrix} < 0, \ \forall (\theta, \dot{\theta}) \in \mathcal{B} \tag{9}$$

where $X_{11}(\theta) = \mathrm{He}\{P(\theta)A(\theta)\} + \dot{P}(\theta) + C_1(\theta)^T C_1(\theta)$ and $M(\theta) = D_{12}(\theta)^T C_1(\theta) + B_2(\theta)^T P(\theta)$.

Inequality (9) has a multiplied term of $P(\theta)$ in the upper-left block; $-M(\theta)^T R(\theta)^{-1} M(\theta)$. We cannot apply Schur complement to this term because it is negative definite. To address this problem, we apply the iterative algorithm of Shimomura *et al.*, which has been proposed to design multi-objective controllers; H_2/H_∞ controllers [14] and strictly positive real H_2 controllers [15].

For the multiplied term of $P(\theta)$, the following relation holds for any $L(\theta)$.

$$-M(\theta)^T R(\theta)^{-1} M(\theta) \le \mathrm{He}\{L(\theta)M(\theta)\} + L(\theta)R(\theta)L(\theta)^T, \ \forall \theta \in \mathcal{B}_\theta \tag{10}$$

Because the following relation holds for any $L(\theta)$.

$$\tilde{L}(\theta)^T R(\theta) \tilde{L}(\theta) \ge 0, \ \forall \theta \in \mathcal{B}_\theta$$

where $\tilde{L}(\theta) = L(\theta)^T + R(\theta)^{-1} M(\theta)$. If $L(\theta)$ is set as $-M(\theta)^T R(\theta)^{-1}$, then inequality (10) becomes an equality.

We now consider an alternative to Problem 2 replacing $-M(\theta)^T R(\theta)^{-1} M(\theta)$ with $\mathrm{He}\{L(\theta)M(\theta)\} + L(\theta)R(\theta) L(\theta)^T$ under the condition that $L(\theta)$ is given.

Problem 3 *Find a controller gain* $K \in \mathcal{R}^{n_u \times n_y}$ *such that*

$$\tilde{J} = \inf_{P(\theta),K} \mathrm{Tr}(N) \tag{11}$$

subject to (4) and

$$\begin{bmatrix} X_{11}(\theta) + \mathrm{He}\{L(\theta)M(\theta)\} + L(\theta)R(\theta)L(\theta)^T & C_2(\theta)^T K^T + M(\theta)^T R(\theta)^{-1} \\ KC_2(\theta) + R(\theta)^{-1} M(\theta) & -R(\theta)^{-1} \end{bmatrix} < 0, \ \forall (\theta, \dot{\theta}) \in \mathcal{B}, \tag{12}$$

where $X_{11}(\theta)$ *and* $M(\theta)$ *have the same definitions as in (9).*

Problem 3 is formulated via only LMIs in terms of $P(\theta)$ and K, thus it can be solved numerically.

After these preliminaries, we now propose an iterative algorithm to solve Problem 2 under the condition that a stabilizing controller for (1) is given.

Algorithm 1

Step 1 *Solve Problem 1 for (3) with given* K, *and set* $i = 0$, $\hat{J}_i = J$.

Step 2 *Solve Problem 3 with* $L(\theta)$ *defined as*

$$L(\theta) = -\{C_1(\theta)^T D_{12}(\theta) + P(\theta)B_2(\theta)\} R(\theta)^{-1}$$

where $P(\theta)$ *is obtained in the preceding step, and set* $i = i+1$, $\hat{J}_i = \tilde{J}$.

Step 3 *Solve Problem 1 with K that is obtained in Step 2, and set* $\hat{J}_i = J$.

Step 4 *If* $\hat{J}_{i-1} - \tilde{J}_i < \varepsilon$ *is satisfied with a sufficiently small positive number* ε *then stop the iteration, otherwise return to Step 2.*

This algorithm solves analysis problem (Problem 1) and transformed synthesis problem (Problem 3) alternately.

For Algorithm 1, the following relation holds.

Theorem 1 *For Algorithm 1, the following relation holds*:

$$J \le \cdots \le \hat{J}_i \le \tilde{J}_i \le \cdots \le \hat{J}_1 \le \tilde{J}_1 \le \hat{J}_0.$$

Proof 1 *We first show* $\tilde{J}_i \le \hat{J}_{i-1}$. *Assume that there exist* $P(\theta)$, K, *and* N *that satisfy (4) and (5) in Problem 1. Since the left-hand side of (12) is the same as the left-hand side of (9) with* $L(\theta)$ *defined in Step 2, inequality (12) always holds with* $P(\theta)$ *and* K *that are obtained in the preceding Problem 1. Therefore, Problem 3 is always solvable. Moreover,* $\tilde{J}_i \le \hat{J}_{i-1}$ *holds due to the freedom of K in Problem 3.*

Next, we show $\hat{J}_i \le \tilde{J}_i$. *Assume that there exist* $P(\theta)$, K, *and* N *that satisfy (4) and (12) in Problem 3. From the relation of (10), the following relation holds.*

$$\textit{Left-hand side of (9)} \le \textit{Left-hand side of (12)} < 0$$

Therefore, (5) always holds with $P(\theta)$ *and* K *that are obtained in the preceding Problem 3; that is, Problem 1 is always solvable. Moreover,* $P(\theta)$ *obtained in Problem 3 is not generally the same as* $P(\theta)$ *that is obtained in Problem 1 because Problem 3 is not a problem that calculates J defined in Problem 1. Therefore* $\hat{J}_i \le \tilde{J}_i$ *holds due to the freedom of* $P(\theta)$ *in Problem 1. This completes the proof.*

If the controller gain K is supposed to be a special form, e.g. a diagonal matrix, then we can easily restrict K to be such form. This is shown in Sections 3 and 4.

Remark 1 *Although inequalities (4), (5), and (12) are LMIs in terms of decision variables,* $P(\theta)$ *and* K, *they are not generally affine functions with respect to parameters, so we must grid the range of* θ_i *and solve these inequalities at the grid points, similarly to* 2), 4).

2.3 Numerical Example

To illustrate the effectiveness of Algorithm 1, we introduce the following numerical example.

$$A(\theta) = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -2\theta & \theta & 0 & 0 \\ \theta & -\theta & 0 & 0 \end{bmatrix},\ B_1 = \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix},\ B_2 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix},\ C_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix},\ D_{12} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix},\ C_2 = [0\ \ 0\ \ 0\ \ 1]$$

We set $0.5 \le \theta \le 1.0$ and $|\dot{\theta}| \le 0.2$. For this example, Routh-Hurwitz criteria for the same plant with fixed θ indicates that the scalar feedback gain K must be negative.

We design a robust static output controller (2) using Algorithm 1. In this example we grid the range of θ at ten points linearly and solve LMI conditions at these points, and set ε as 1.0×10^{-4}.

We first design the controller with $P(\theta) = P_0 + \theta P_1$ and set two initial stabilizing gains as -0.65 and -1.7. Figure 1 shows the values of $\mathrm{Tr}(N)$ and K versus the number of iterations. Next, we design the controller with $P(\theta) = P_0 + \theta P_1 + \theta^2 P_2$ and set two initial stabilizing gains as -0.25 and -10, which are respectively farther from the optimal values than -0.65 and -1.7. Figure 2 shows the values of $\mathrm{Tr}(N)$ and K versus the number of iterations. These figures show that the proposed method works well; that is, Theorem 1 holds. Figure 3 shows the values of $\mathrm{Tr}(N)$ versus the variations of K using Problem 1; that is, H_2 performance analysis. This figure illustrates that the optimized solutions obtained using the proposed method are very close to the global optimal solutions for this problem.

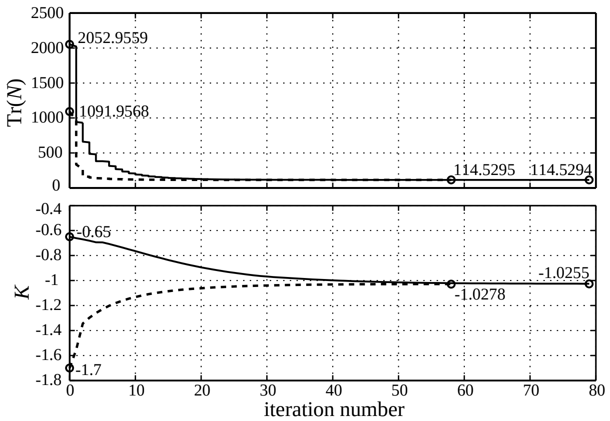

Figure 1 Tr(N) and controller gain K versus iteration number using $P(\theta) = P_0 + \theta P_1$

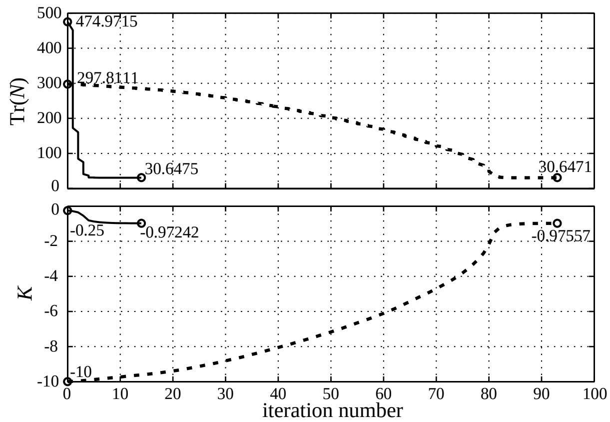

Figure 2 Tr(N) and controller gain K versus iteration number using $P(\theta) = P_0 + \theta P_1 + \theta^2 P_2$

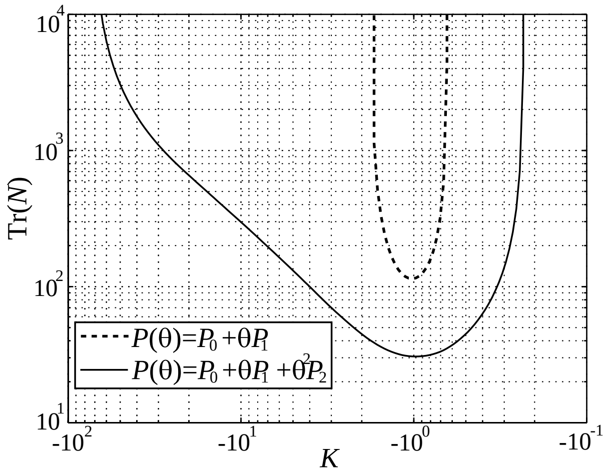

Figure 3 Tr(N) versus controller gain K

3 APPLICATION TO MODEL-FOLLOWING CONTROLLER DESIGN

In the previous section, we introduce an algorithm to design robust static output H_2 controllers with ease. In this section, we apply the algorithm to the design of the model-following controller proposed by Kawahata[16]. This is an explicit model-following controller and flight experiments using this controller demonstrated that it had good performance[18]. However, the design method has some drawbacks, which will be pointed out later on, we therefore apply the robust H_2 controller synthesis to overcome the drawbacks.

3.1 Previous Design Methods

We first describe previously proposed design methods of the model-following controller in 16).

A nominal plant G_p and a stable model to be followed G_m are given below as (13) and (14) respectively.

$$G_p : \begin{cases} \dot{x}_p = A_p x_p + B_p u_p \\ y_p = C_p x_p \end{cases}, \tag{13}$$

$$G_m : \begin{cases} \dot{x}_m = A_m x_m + B_m u_m \\ y_m = C_m x_m \end{cases}, \tag{14}$$

where $x_p \in \mathcal{R}^{n_p}$ and $x_m \in \mathcal{R}^{n_m}$ respectively denote the state vectors of the plant and model with $x_p = 0$ and $x_m = 0$ at $t = 0$, $u_p \in \mathcal{R}^{n_{u_p}}$ and $u_m \in \mathcal{R}^{n_{u_m}}$ respectively denote the input vectors to the plant and model, $y_p \in \mathcal{R}^{n_{y_p}}$ and $y_m \in \mathcal{R}^{n_{y_m}}$ respectively denote the output vectors of the plant and model, and matrices A_p, A_m, etc. are supposed to have appropriate dimensions.

The following assumptions are made:

Assumption 2 *The numbers of plant inputs, plant outputs, and model outputs are all l; that is,* $n_{u_p} = n_{y_p} = n_{y_m} = l$.

If plant has redundant inputs, then it does not satisfy this assumption. In such case, one method to satisfy the assumption is to select the plant inputs so that the assumption is satisfied.

We further made the following assumption.

Assumption 3 *The relative degrees of the model outputs are equal to those of the respective plant outputs. The relative degrees* σ_i *of the plant outputs are defined as follows:*

$$\sigma_i := \min\{h : C_{pi}A_p^{h-1}B_p \neq 0,\, h \geq 1\},\ i = 1,\cdots,l,$$

where C_{p_i} *denotes the ith row of* C_p. *The relative degrees of the model outputs are defined in the same way.*

If the relative degrees of model outputs are less than those of plant outputs, then we require the derivatives of model inputs when designing the model-following controller proposed in 16). In such case, we can satisfy the assumption by adding strictly proper filters to model inputs (unfortunately, we need the state variables of the filters to implement the subsequent controllers), or by making some further assumptions so that the assumption is satisfied (see the design example in Section 4). If the relative degrees of model outputs are greater than those of plant outputs, then the following formulation is a little bit different. However, we have no need to use the derivatives of model inputs when designing the model-following controller proposed in 16).

Under these assumptions, the model-following controller in 16) is given in (15). (See 16) for further details.)

$$u_p = -K_x x_p + K_{xm} x_m + K_{um} x_m, \tag{15}$$

where K_x, K_{xm}, and K_{um} are defined as follows.

$$K_x = B_p^{*-1}(A_p^* + KM_p), \qquad K_{xm} = B_p^{*-1}(A_m^* + KM_m),$$

$$K_{um} = B_p^{*-1}B_m^*,$$

$$A_p^* = \begin{bmatrix} C_{p_1}A_p^{\sigma_1} \\ \vdots \\ C_{p_l}A_p^{\sigma_l} \end{bmatrix}, \qquad A_m^* = \begin{bmatrix} C_{m_1}A_m^{\sigma_1} \\ \vdots \\ C_{m_l}A_m^{\sigma_l} \end{bmatrix},$$

$$B_p^* = \begin{bmatrix} C_{p_1}A_p^{\sigma_1-1}B_p \\ \vdots \\ C_{p_l}A_p^{\sigma_l-1}B_p \end{bmatrix}, \qquad B_m^* = \begin{bmatrix} C_{m_1}A_m^{\sigma_1-1}B_m \\ \vdots \\ C_{m_l}A_m^{\sigma_l-1}B_m \end{bmatrix},$$

$$M_p^* = \begin{bmatrix} \begin{bmatrix} C_{p_1} \\ \vdots \\ C_{p_1}A_p^{\sigma_1-1} \end{bmatrix}^T & \cdots & \begin{bmatrix} C_{p_l} \\ \vdots \\ C_{p_l}A_p^{\sigma_l-1} \end{bmatrix}^T \end{bmatrix}^T, \quad M_m^* = \begin{bmatrix} \begin{bmatrix} C_{m_1} \\ \vdots \\ C_{m_1}A_m^{\sigma_1-1} \end{bmatrix}^T & \cdots & \begin{bmatrix} C_{m_l} \\ \vdots \\ C_{m_l}A_m^{\sigma_l-1} \end{bmatrix}^T \end{bmatrix}^T$$

Matrix K is arbitrarily assigned under the restriction that it is a block-diagonal matrix.

The model-following controller (15) has the following properties:

i. If the plant initial states and model initial states are all zeros, then the plant outputs coincide with the model outputs for arbitrary model inputs.

ii. If the initial plant states are not zeros or there are perturbations to the plant, then the model-following errors converge according to the characteristic equations of the errors assigned by K. Furthermore, the poles of the closed-loop plant and the roots of the characteristic equations of the errors are the same, and $n - z$ of these poles or roots are placed by K, where n denotes the degree of the plant and z denotes the number of invariant zeros of the plant.

From the second property, Kawahata has suggested placing the poles of the closed-loop system so as to decrease model-following errors and disturbance effects on the plant, but he has showed no way to achieve this under the condition that there exist uncertainties and perturbations to the plant. Miyazawa[19] has proposed a design method of decreasing disturbance effects under such condition, which is shown in the following.

We first describe the analysis of disturbance attenuation performance under the condition that there exist neither uncertainties nor perturbations to the plant, we then describe a design problem minimizing disturbance attenuation performance under the condition that there exist uncertainties and perturbations to plant.

For a given K, its disturbance attenuation performance combined with control input effort is defined as (16) for an appropriate $x_p = x_{p0}$ at $t=0$; that is, the plant state is to be driven to x_{p0} from 0 by disturbances and the inputs and states of model are set to be zeros. In this definition, $Q = Q^T \geq 0$ and $R = R^T > 0$ are assumed.

$$J := \mathcal{E}\left(\int_0^\infty x_p^T (Q + K_x^T R K_x) x_p dt\right) \tag{16}$$

If we obtain K minimizing J defined in (16), then the controller (15) with K is a model-following controller with optimal disturbance attenuation. However, this formulation does not consider uncertainties such as modeling errors, and the obtained model-following controller is not practical since the stability of the closed-loop system is apt to be broken due to uncertainties of plant.

Now we describe the problem minimizing disturbance attenuation performance under the condition that there exist uncertainties and perturbations to plant. Miyazawa has proposed to use the Multiple Delay Models and Multiple Design Points (MDM/MDP) concept[20, 21] in the evaluation of disturbance attenuation; that is, multiple delays (17) are set at plant inputs to estimate phase uncertainties in the high frequency range or time delays of plant system, and multiple design points (18) are used to represent uncertainties, i.e. modeling errors and perturbations to plant (see Figure 4).

$$u_i = \frac{-T_{ij}/2s+1}{T_{ij}/2s+1} u_{i_c}, \; i=1,\cdots,l, \; j=1,\cdots,m \tag{17}$$

$$\dot{x}_{p_k} = A_{p_k} x_{p_k} + B_{p_k} u_{p_k}, \; k=1,\cdots,q, u_{p_k} = \left[u_1 \; u_2 \cdots u_l\right]^T \tag{18}$$

Here, u_i, u_{i_c}, T_{ij}, and m respectively denote the ith plant input, the command of the ith plant input, the j th delay time of the i th plant input, and the number of delay models. Matrices A_{p_k} and B_{p_k} denote the state-space realizations of the k th plant, and q denotes the number of design points. In this formulation, we assume that there exist perturbations and uncertainties only in the A and B matrices of plant. If there exist perturbations and uncertainties in the C matrix, we can satisfy the assumption by adding strictly proper filters to plant outputs.

Given an appropriate gain K, let s denote $m^l \times q$, and $J_1, \cdots, J_s$ respectively denote the performance of the 1st, $\cdots$, sth plant models, with given K and matrices A_p^*, B_p^* and M_p derived from the nominal plant. If we obtain K minimizing $J_f :=$ max $\{J_1, \cdots, J_s\}$, then controller (15) with obtained K is a model-following controller with optimal disturbance attenuation under the condition that the plant is expressed as the nominal plant model with some uncertainties described as multiple delay models (17) and multiple design points (18).

The above design sequence is summarized below.

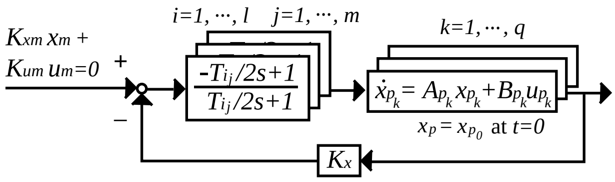

Figure 4 Block diagram of model-following controller design using MDM/MDP concept

Sequence 1

Step A: *Calculate* A_p^*, B_p^*, M_p, A_m^*, B_m^*, *and* M_m *from nominal plant (13) and model (14).*

Step B: *Create* $m^l \times q$ *plant models from multiple delay models (17) and multiple design points (18). Then minimize* J_f *with* K *using some numerical optimization algorithm.*

Step C: *Calculate* K_x, K_{xm}, *and* K_{um} *with optimal* K.

3.2 Proposed Method using Robust H_2 Controller Synthesis for LPV Systems

The above-mentioned design method produces good controllers as shown in 19). However, the method only minimizes disturbance attenuation performance under the condition that model inputs are always zeros; that is, it does not evaluate model-following errors under the condition that there exist model inputs. For practicality, we must minimize model-following error performance, such as $\int_0^\infty (y_p - y_m)^T S (y_p - y_m) dt$ with $S = S^T \geq 0$, as well as disturbance attenuation performance under the condition that there exist uncertainties and perturbations to plant and model inputs are arbitrary. Furthermore, the above design method has some drawbacks in the evaluation of the performance: It does not consider the whole range of uncertainties or perturbations to plant; we only minimize the worst case among the selected plant models; and the performance is evaluated for some LTI systems even if the plant models consisting of the MDM/MDP models are a single time-varying system. If the MDM/MDP models are expressed as the nominal LTI plant with parametric uncertainties, the robust H_2 problem in Section 2 is applicable to overcome these drawbacks with a slight revision. While the model-following performance index, $\int_0^\infty (y_p - y_m)^T S (y_p - y_m) dt$, has been previously reported in many papers (e.g. 22) and references therein), here, unlike the previous literature, we evaluate the model-following error performance for not an LTI plant but for an LTI plant with parametric uncertainties.

The multiple design points usually represent models of plant at various different equilibriums, and these models can then be expressed as an LPV system. In the next section, we show how to express delay models as an LPV system with a design example. In this section, we show a design method for a model-following controller (15) in which the robust H_2 problem is applied to overcome the drawbacks mentioned above. The design sequence differs from Sequence 1 only in Step B.

A system whose state-space realization is given as $\{A_p(\theta), B_p(\theta), [C_p\ 0], 0\}$ represents plant model with multiple delay models (17) and multiple design points (18), where $A_p(\theta)$, $B_p(\theta)$, and $[C_p\ 0]$ are augmented from the nominal ones by delay models. We now formulate the robust model-following controller design problem for plant expressed as $\{A_p(\theta), B_p(\theta), [C_p\ 0], 0\}$ and model (14) with controller (15) where A_p^*, B_p^*, M_p, A_m^*, B_m^*, and M_m are obtained from the nominal plant model. The performance for evaluating model-following errors and disturbance attenuation is set as follows with $e = y_p - y_m$:

$$J = \mathcal{E}\left(\int_0^\infty \left(e^T S e + x_p^T Q x_p + u_p^T R u_p\right) dt\right) \tag{19}$$

for random model inputs (u_m) and gusts (w_g) at $t = 0$. In this expression, $S = S^T \geq 0$, $Q = Q^T \geq 0$, and $R = R^T >$ are assumed. We may also consider the servo problem for model-following errors. However, human operators (pilots) in the control loop behave like servo controllers, so we do not consider the servo problem. Finding an optimal block-diagonal matrix K for J defined in (19) is a robust H_2 controller synthesis with block-diagonal static output controller for the following generalized plant.

$$\left[\begin{array}{c|c|c} A(\theta) & B_1(\theta) & B_2(\theta) \\ \hline C_1 & D_{11} & D_{12} \\ \hline C_2 & & \end{array}\right] = \left[\begin{array}{cc|cc|c} A_p(\theta) - B_p(\theta) B_p^{*-1} [A_p^*\ \ 0] & B_p(\theta) B_p^{*-1} A_m^* & B_g & B_p(\theta) B_p^{*-1} B_m^* & B_p(\theta) B_p^{*-1} \\ 0 & A_m & 0 & B_m & 0 \\ \hline [Q^{1/2}\ \ 0] & 0 & 0 & 0 & 0 \\ [S^{1/2} C_p\ \ 0] & -S^{1/2} C_m & 0 & 0 & 0 \\ [-R^{1/2} B_p^{*-1} A_p^*\ \ 0] & R^{1/2} B_p^{*-1} A_m^* & 0 & R^{1/2} B_p^{*-1} B_m^* & R^{1/2} B_p^{*-1} \\ \hline [-M_p\ \ 0] & M_m & & & \end{array}\right] \tag{20}$$

In (20), state variables are $[x_p^T\ x_d^T\ x_m^T]^T$, disturbance inputs are $[w_g^T\ u_m^T]^T$, and control input is u_p, where x_d denotes the state variables of delay models and w_g denotes the disturbance input to plant. Matrices $[A_p^*\ 0]$, $[Q^{1/2}\ 0]$, $[S^{1/2} C_p\ 0]$, $[-R^{1/2} B_p^{*-1}$

$A_p^* \, 0]$, and $[-M_p \, 0]$ are appropriately augmented by delay models. Matrix B_g represents the disturbance input matrix of plant with an appropriate dimension.

However, the generalized plant (20) has D_{11} term that prevents us from defining its H_2 problem. Since disturbance inputs $[w_g^T \; u_m^T]^T$, consisting of disturbance inputs to plant and model inputs, do not have meaningful signals in high frequency range, we add a strictly proper filter $F(s)$ whose state-space representation is given as $\{A_f, B_f, C_f, 0\}$ to $[w_g^T \; u_m^T]^T$. We then have the following generalized plant with the notation in (20):

$$\left[\begin{array}{cc|c|c} A(\theta) & B_1(\theta)C_f & 0 & B_2(\theta) \\ 0 & A_f & B_f & 0 \\ \hline C_1 & D_{11}C_f & & D_{12} \\ \hline C_2 & 0 & & \end{array}\right] \tag{21}$$

We can now define robust H_2 controller synthesis for this generalized plant. In this representation, we confirm that Assumption 1 is satisfied: $D_{12}(\theta)^T D_{12}(\theta) = B_p^{*-T} R B_p^{*-1} > 0$.

After the above preliminaries, the proposed design method for the robust optimal model-following controller is given below.

Sequence 2

Step A: *Calculate A_p^*, B_p^*, M_p, A_m^*, B_m^*, and M_m from nominal plant (13) and model (14).*

Step B *Make a parametric state-space realization expressed as $\{A_p(\theta), B_p(\theta), [C_p \, 0], 0\}$ from MDM/MDP models of the plant, and set a filter $F(s)$ and matrix B_g appropriately. Then find robust H_2 optimal gain K for (21) using Algorithm 1.*

Step C: *Calculate K_x, K_{xm}, and Kum with optimal K.*

Remark 2 *In (20), if we set $S = 0$ then J defined in (19) becomes $\mathcal{E}\left(\int_0^\infty (x_p^T Q x_p + u_p^T R u_p) dt\right)$. Furthermore, if we set $u_m = 0$, then $x_m = 0$ holds since the dynamics of model are calculated by on-board computers, and the generalized plant (20) becomes as follows after eliminating x_m, u_m, and e.*

$$\left[\begin{array}{c|c|c} A_p(\theta) - B_p(\theta)B_p^{*-1}\left[A_p^* \;\; 0\right] & B_g & B_p(\theta)B_p^{*-1} \\ \hline \left[Q^{1/2} \;\; 0\right] & 0 & 0 \\ \left[-R^{1/2}B_p^{*-1}A_p^* \;\; 0\right] & 0 & R^{1/2}B_p^{*-1} \\ \hline \left[-M_p \;\; 0\right] & & \end{array}\right] \tag{22}$$

The design problem for an optimal model-following controller (15) in which the performance index is one defined in (16) for $x_{p0} = \delta(0) \, B_g \, w_g$, where $\delta(\cdot)$ is Dirac's delta function and the plant is expressed as $\{A_p(\theta), B_p(\theta), [C_p \, 0], 0\}$, is a robust H_2 controller synthesis for (22). Then the performance index used in 19) is confirmed to be a special case of our performance index and our design problem is confirmed to be a more general framework than one in 19).

4 Model-following Controllers for Lateral/Directional Motions of MuPAL- α

To demonstrate a practical application of our proposed design method, we design several model-following controllers for the lateral/directional motions of MuPAL- α [23], one of JAXA's research aircraft (based on a Dornier Do-228 turboprop commuter aircraft), and present the results of their evaluation by numerical simulations and flight experiments.

4.1 Design

The nominal plant model of MuPAL- α is a linearized model at straight level flight at true air speed TAS=66.5 m/s and altitude H=1520 m. For the delay models, first-order Padé-approximate delay models (17) are assumed at aileron and rudder inputs. For the design points, we set steady sideslip flight at $\beta = \pm 5, \pm 10$ deg at the same airspeed and altitude, where β denotes the sideslip angle. Let T_a and T_r respectively denote delay time parameters of the aileron and rudder delay models and they are set as time-invariant parameters, which are not measurable on time. The variations of these parameters are set as in (23) below from the characteristics of the modeled actuators (0.16 s), noise filters (0.02 s), the cycle time of MuPAL- α 's flight control computer (0.02 s), the dead time of the system (0.08 s), and a margin (0.05 s). Matrices that represent the maximum deviation of plant models from the nominal plant model are expressed as in (24) where δ is set as a time-varying parameter, which is not measurable, and its range is normalized as in (23). Here, the parameter vector θ is given as $[1/T_a\ 1/T_r\ \delta]^T$.

$$\begin{aligned} &T_a \in [0.02, 0.33] \\ &T_r \in [0.02, 0.33] \\ &\delta \in [-1, 1] \end{aligned} \tag{23}$$

$$A_p(\theta) = \begin{bmatrix} -0.1781 & 6.0791 & 9.7633 & -65.6230 & 0 & 2.8900 \\ -0.0575 & -3.8100 & 0 & 1.3430 & -10.7500 & 1.1870 \\ 0 & 1.0000 & 0 & 0.0944 & 0 & 0 \\ 0.0253 & -0.0628 & 0 & -0.4750 & 0.3450 & -2.2200 \\ 0 & 0 & 0 & 0 & -2/T_a & 0 \\ 0 & 0 & 0 & 0 & 0 & -2/T_r \end{bmatrix} + \delta \begin{bmatrix} 0.0185 & 0.2416 & 0.0021 & 1.0038 & 0 & 0 \\ 0.0037 & 0 & 0 & 0.0150 & 0 & 0.0010 \\ 0 & 0 & 0 & 0.0023 & 0 & 0 \\ 0.0062 & 0.0050 & 0 & 0 & 0.0030 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$B_p(\theta) = \begin{bmatrix} 0 & 2.8900 \\ -10.7500 & 1.1870 \\ 0 & 0 \\ 0.3450 & -2.2200 \\ 4/T_a & 0 \\ 0 & 4/T_r \end{bmatrix} + \delta \begin{bmatrix} 0 & 0 \\ 0 & 0.0010 \\ 0 & 0 \\ 0.0030 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}, \quad C_p = \begin{bmatrix} 1.0000 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 180/\pi & 0 & 0 & 0 \end{bmatrix} \tag{24}$$

The aircraft to be followed are a Boeing 747 at TAS=67.4 m/s and H=0 m and Lockheed Jetstar at TAS=136.3 m/s and H=0 m. State-space realizations of models are calculated from 24). The state variables of the plant and models are $[v$[m/s] p[rad/s] ϕ[rad] r[rad/s]$]^T$, control input variables are $[\delta_a$ [rad] δ_r [rad]$]^T$, and output variables to be followed are $[v$[m/s] ϕ[deg]$]^T$. Here, v denotes side velocity, p and r denote roll and yaw rates respectively, p denotes roll angle, and δ_a and δ_r denote aileron and rudder angles respectively.

We now design the model-following controllers according to Sequence 2 as follows.

Step A in Sequence 2

The plant system has an invariant zero, so the model-following control law (15) cannot place all poles arbitrarily, and as a result the feedback gains are very large and the subsequent H_2 performance for LPV plant is very poor. We therefore make the following assumption.

Assumption 4 *The elements of the first row of B_p are zeros.*

Under this assumption the relative degree of v is two in the plant and one in the models. The model-following controller in 16) is therefore different from (15), and requires derivatives of the model inputs. We therefore make the same assumption for the models.

Assumption 5 *The elements of the first row of B_m are zeros.*

Under these assumptions, matrix K in (15) is restricted as $\begin{bmatrix} k_1 & k_2 & 0 & 0 \\ 0 & 0 & k_3 & k_4 \end{bmatrix}$ and can place all poles. Matrices A_p^*, B_p^*, M_p, A_m^*, B_m, and M_m are then calculated under these assumptions.

Step B in Sequence 2

We next calculate robust H_2 optimal K by applying Algorithm 1. Hereafter Assumptions 4 and 5 are not assumed.

Matrices to be set are set as follows.

$$S = 10\times\begin{bmatrix}1 & 0\\ 0 & (180/\pi)^2\end{bmatrix},$$

$$Q = 10\times\begin{bmatrix}1 & 0\\ 0 & (180/\pi)^2 I_3\end{bmatrix}$$

$$R = (180/\pi)^2 I_2\,,$$

$$B_g = 5\times\begin{bmatrix}1 & 0 & 0 & 0\\ 0 & 0 & \pi/180 & 0\end{bmatrix}^T,$$

$$F(s) = \begin{cases}\mathrm{diag}\left(\frac{1}{0.2s+1}I_2,\ \frac{10\times\pi/180}{0.2s+1}I_2\right), & \text{B747 model}\\ \mathrm{diag}\left(\frac{1}{0.2s+1}I_2,\ \frac{5\times\pi/180}{0.2s+1}I_2\right), & \text{Jetstar model}\end{cases}$$

Then, the initial values of K is the best of optima for the following problems with 4^4 pairs set as initial gains, which are the combinations of setting k_i (i=1, ⋯, 4) to be 1, 4, 7 or 10, using a numerical optimization algorithm in 25). As shown in section 2.3, our proposed algorithm works well. However, the algorithm produces a candidate of global minimum; that is, generally speaking, it produces a local minimum, which heavily depends on the initial gain. Thus, we use MDM/MDP method to obtain the initial gains of our proposed method.

Problem 4 *Find a controller K such that*

$$\min_K \max\{J_1, \cdots, J_8\},$$

where J_i is the square of H_2 performance for (21) with setting $T_a = 0.02$ or 0.33, $T_r = 0.02$ or 0.33, and $\delta = -1$ or 1.

The generated values are shown as initial values in Tables 1 and 2. Using these initial values, we carry out Algorithm 1 with $P(\theta) = P_0 + \frac{1}{T_a}P_a + \frac{1}{T_r}P_r$ and ε set as 1×10^{-4}, where we set $P(\theta)$ to be independent of the parameter δ because the entries of the δ dependent matrices in (24) have small values. In accordance with Remark 1, the ranges of $1/T_a$ and $1/T_r$ are gridded at ten points linearly, and we design and analyze at these points. The optimized gains generated with the proposed method are shown as optimized values in Tables 1 and 2. In this optimization process, neither Assumptions 4 nor 5 are assumed. Therefore, the optimized gains are for nominal LTI plant with parametric uncertainties with no assumptions, not for the assumed plant model.

Although we cannot confirm whether the optimized gains are the global optima, for the Jetstar model, the optimized gains differ considerably from the initial gains, and the performance using the optimized gains is much better than that resulting from the initial gains. This implies that identifying an LPV system as a system composed of many LTI systems may lead to very poor performance.

Table 1 Initial and optimized gains for B747 model

	Initial values	Optimized values
k_1	4.3974	4.2570
k_2	3.1293	2.9465
k_3	10.515	9.6351
k_4	8.0106	7.0830
$\hat{J}_i$	704.63 ($i = 0$)	698.30 ($i = 41$)

Table 2 Initial and optimized gains for the Jetstar model

	Initial values	Optimized values
k_1	0.7630	3.4680
k_2	0.5517	3.4390
k_3	8.0882	12.365
k_4	8.1012	7.5256
$\hat{J}_i$	143093 ($i = 0$)	15557 ($i = 112$)

Table 3 Controller for B747 model

K_x	$\begin{bmatrix} 6.5316\times10^{-3} & -3.1697\times10^{-1} & -9.3180\times10^{-1} & -2.8268\times10^{-1} \\ 1.5231\times10^{-2} & -1.3301\times10^{-1} & -3.5945\times10^{-1} & -1.0876 \end{bmatrix}$
K_{xm}	$\begin{bmatrix} 4.1993\times10^{-3} & -5.7417\times10^{-1} & -9.3136\times10^{-1} & -2.4104\times10^{-1} \\ 2.6223\times10^{-2} & -7.2073\times10^{-2} & -3.5472\times10^{-1} & -1.2585 \end{bmatrix}$
K_{um}	$\begin{bmatrix} 2.2427\times10^{-2} & 2.6356\times10^{-3} \\ 9.6271\times10^{-3} & 7.0872\times10^{-2} \end{bmatrix}$

Table 4 Controller for Jetstar model

K_x	$\begin{bmatrix} 5.9796\times10^{-3} & -3.5867\times10^{-1} & -1.1976 & -3.0714\times10^{-1} \\ 9.1796\times10^{-3} & -1.3743\times10^{-1} & -4.8096\times10^{-1} & -1.3130 \end{bmatrix}$
K_{xm}	$\begin{bmatrix} 3.1620\times10^{-3} & -5.7999\times10^{-1} & -1.1975 & -3.2330\times10^{-1} \\ -3.1694\times10^{-3} & -1.9264\times10^{-3} & -4.8031\times10^{-1} & -2.8398 \end{bmatrix}$
K_{um}	$\begin{bmatrix} 2.8333\times10^{-1} & 1.7797\times10^{-2} \\ -1.0031\times10^{-1} & -1.7126 \end{bmatrix}$

Step C in Sequence 2

We calculate K_x, K_{xm}, and K_{um} for the B747 and Jetstar models with the respectively optimized K's. They are shown in Tables 3 and 4.

4.2 Numerical Simulations

We conduct numerical simulations to evaluate the performance of the designed controllers. In these simulations, neither Assumption 4 for the plant nor 5 for the models are assumed.

To compare the performance of controllers evaluating model-following error performance and those that do not, we design a controller (15) minimizing the H_2 performance for (21) with the same Q, R and $F(s)$ in the previous subsection, and $S=0$ and $u_m=0$; that is, a robust H_2 controller that evaluates only disturbance attenuation performance. The initial values of K in Algorithm 1 are obtained similarly in the previous subsection and the optimized K is obtained using Algorithm 1 setting $P(\theta)=P_0+\frac{1}{T_a}P_a+\frac{1}{T_r}T_r$ and ε set as 1.0×10^{-4}. The initial and optimized values are shown in Table 5. The gains do not change in accordance with models since this problem minimizes only disturbance attenuation performance of plant.

We conduct numerical simulations for rudder doublet inputs to the models, which are the same inputs as in the subsequent flight experiments, with the optimized gains in Tables 1, 2, and 5. In these simulations, we use LTI plant models with maximum pure delays (0.33 s) for plant aileron input and plant rudder input, and we set $\delta=-1, 0, 1$. Figures 5 and 6 show time histories of v_p-v_m, $\phi_p-\phi_m$, and plant inputs δ_a and δ_r, where variables with subscript p denote plant variables and variables with subscript m denote model variables. In Figures 5 and 6, the left figures show time histories with the controllers (15) calculated with gains in Tables 1 or 2 and the right figures show time histories with the controllers (15) calculated with gains in Table 5. We confirm that the controllers evaluating model-following errors have better performance than those do not, especially in Figure 5. Although we omit motion time histories of plant and

Table 5 Initial and optimized gains for optimal disturbance attenuation

	Initial values	Optimized values
k_1	2.0396	2.2223
k_2	2.0902	2.2982
k_3	5.1908	4.9949
k_4	7.2970	6.9919
$\hat{J}_i$	63.644 ($i=0$)	63.385 ($i=65$)

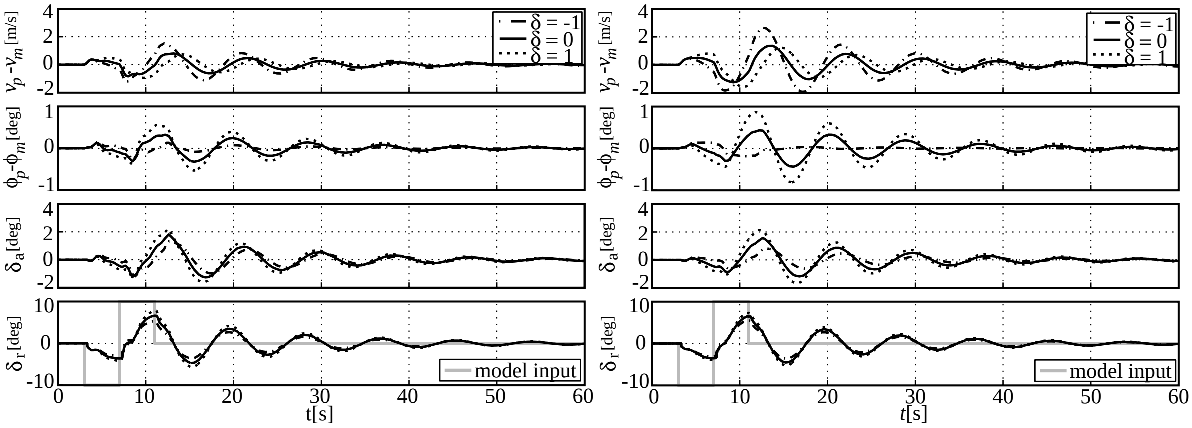

Figure 5 Numerical simulations of rudder doublet inputs to B747 model

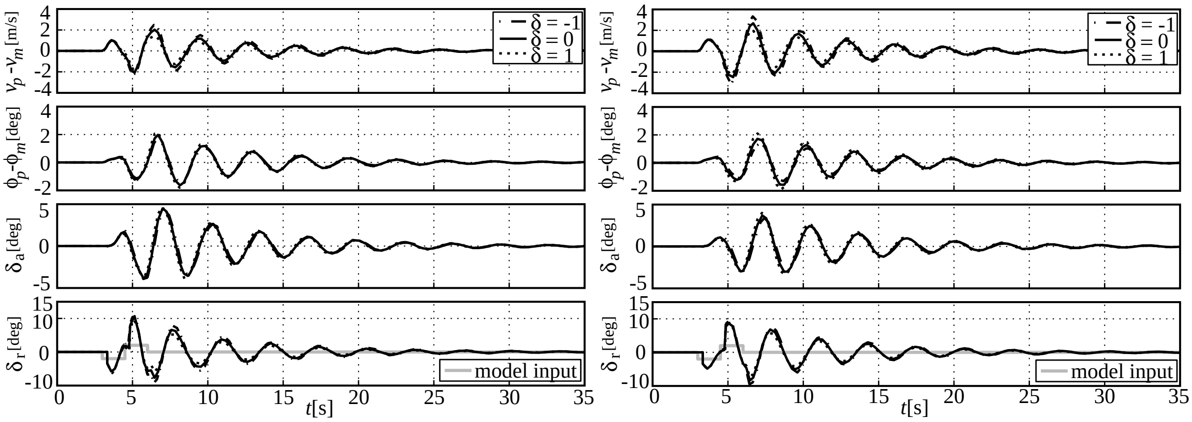

Figure 6 Numerical simulations of rudder doublet inputs to Jetstar model

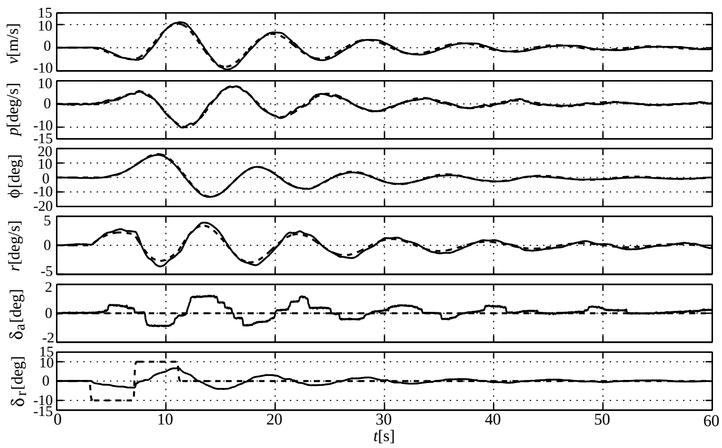

Figure 7 Time histories of rudder doublet inputs to B747 model (dotted: model, solid: MuPAL- α)

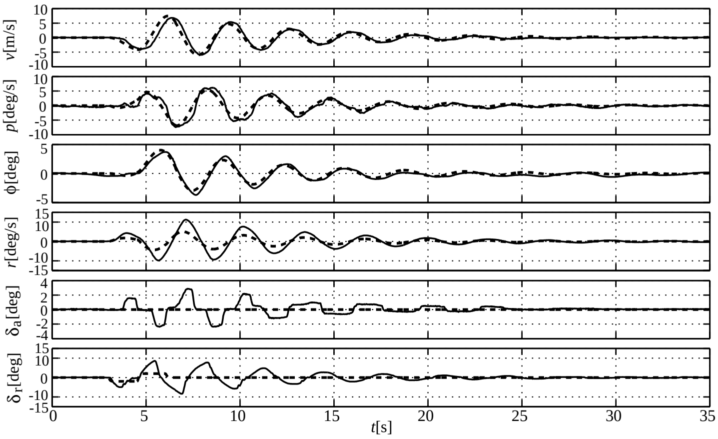

Figure 8 Time histories of rudder doublet inputs to Jetstar model (dotted: model, solid: MuPAL- α)

models, we see their time histories in the subsequent flight experiments (Figures 7 and 8). We see discrepancies in outputs, especially in Figure 6, this seems to be due to Assumptions 4 and 5 when designing A_p^*, A_m^*, etc. If we want to reduce the discrepancies, we do either of the following; not make Assumptions 4 and 5, or add strictly proper filters to model inputs. However, in our case, the former leads to worse performance than making assumptions, and the latter leads to the use of state variables of filters and subsequent numerical burden for designing controllers. Thus, in our design, Assumptions 4 and 5 are reasonable.

4.3 Flight Experiments

After the numerical simulations, the controllers designed with optimized gains in Tables 1 and 2 were verified by flight experiments. Due to prevailing weather conditions, the flight experiments were conducted under conditions slightly different conditions from the nominal: TAS=62-70 m/s, H=1300-1400 m. In these experiments, Assumption 5 for models was not assumed.

The details of on-board computers of MuPAL- α and the implementation of designed flight controllers are described in 26). Thus, we omit them in this manuscript.

Figures 7 and 8 respectively show the response of B747 and Jetstar models with rudder doublet inputs. The rudder doublets were input to excite the Dutch-roll motions of model aircraft; B747 model, Jetstar model, and MuPAL- α respectively have about 8 s, 3 s, and 4 s period Dutch-roll motions. The inputs to B747 and Jetstar models are 10 deg and 8 s, and 2 deg and 3 s duration inputs respectively. Figure 7 demonstrates that the designed controller for B747 model has good performance since there is very little discrepancy in outputs (v and ϕ). Although Jetstar model has larger natural vibration frequency than plant has, Figure 8 demonstrates that the designed controller for Jetstar model also simulate the motions of Jetstar model. In contrast to Figure 7, Figure 8 shows discrepancies in outputs (v and ϕ) and this is mainly because of Assumptions 4 and 5 which are made by us due to technical problems.

We next conducted experiments to verify the practicality of controllers with normal pilot inputs. Figures 9 and 10 respectively show steady turns of the B747 and Jetstar models. (There is no ψ (yaw angle) data of models in these figures because of the capacity limitations of the flight control computers.) These figures demonstrate that the designed model-following controllers have good performance with normal pilot inputs under the real gust conditions (e.g. especially from 120 to 140 s in Figure 9 and from 90 to 110 s in Figure 10).

5 Conclusions

This paper proposes a design method for robust static output H_2 controllers for LPV systems using parameter-dependent Lyapunov functions. Although there is a special assumption that stabilizing controllers are given, the presented numerical example demonstrates that the proposed method works well. We apply the proposed method to a previously pro-

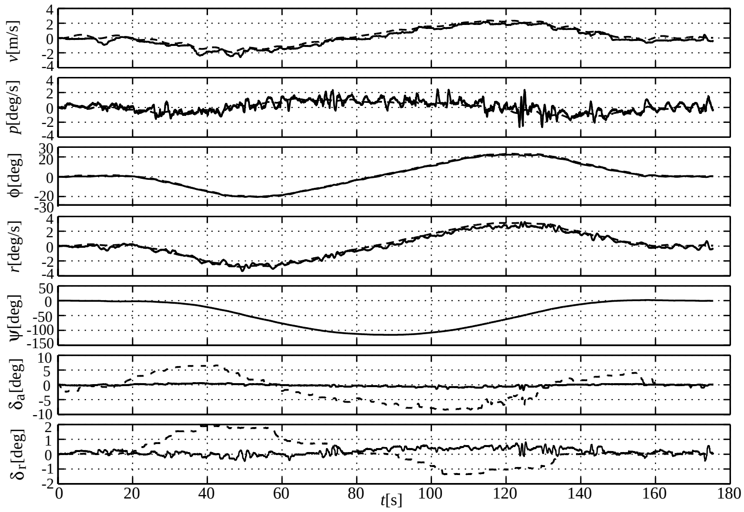

Figure 9 Time histories of steady turns of B747 model (dotted: model, solid: MuPAL- α)

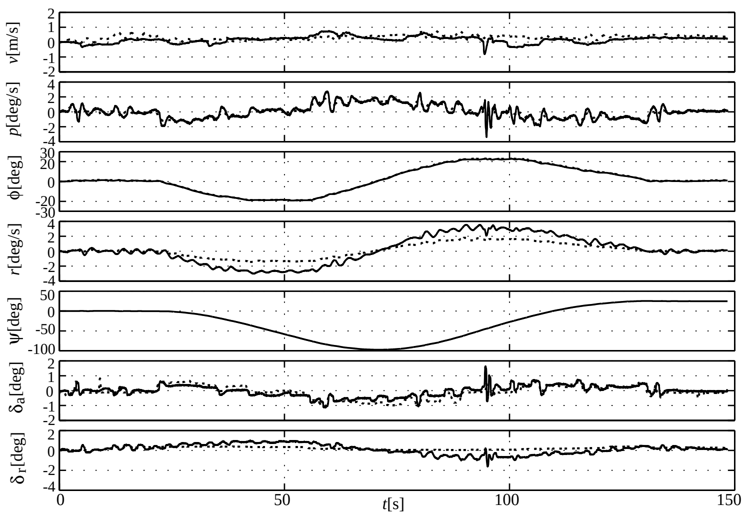

Figure 10 Time histories of steady turns of Jetstar model (dotted: model, solid: MuPAL- α)

posed model-following controller design to overcome its drawbacks. In contrast to previous methods, the proposed method minimizes the robust model-following error performance and disturbance attenuation performance for an LPV system with a process that convergences to an optimal solution. Several model-following controllers for the lateral/directional motions of an aircraft are designed using the proposed method, and numerical simulations and flight experiments confirm that these controllers have good performance.

Reference

1) S. Boyd, L.E. Ghaoui, E. Feron, and V. Balakrishnan, *Linear Matrix Inequalities in System and Control Theory*, SIAM, Philadelphia, PA; 1994.

2) P. Apkarian and R.J. Adams, "Advanced Gain-Scheduling Techniques for Uncertain Systems," *IEEE Trans. Control Systems Technology*, Vol. 6, No. 1, 1998, pp. 21–32.

3) C.W. Scherer, "Mixed H_2/H_∞ Control for Time-Varying and Linear Parametrically-Varying Systems," *Int. J. Robust and Nonlinear Control*, Vol. 6, 1996, pp. 929-952.

4) F. Wu, X.H. Yang, A. Packard, and G. Becker, "Induced $\mathcal{L}_2$-Norm Control for LPV Systems with Bounded Parameter Variation Rates," *Int. J. Robust and Nonlinear Control*, Vol. 6, 1996, pp. 983–998.

5) E. Feron, P. Apkarian, and P. Gahinet, "Analysis and Synthesis of Robust Control Systems via Parameter-Dependent Lyapunov Functions," *IEEE Trans. Automatic Control*, 1996, pp. 1041–1046.

6) P. Apkarian and P. Gahinet, "A Convex Characterization of Gain-Scheduled H_∞ Controllers," *IEEE Trans. Automatic Control*, Vol. 40, No. 5, 1995, pp. 853–864.

7) A. Trofino and C.E. de Souza, "Biquadratic Stability of Uncertain Linear Systems," *IEEE Trans. Automatic Control*, Vol. 46, No. 8, 2001, pp. 1303–1307.

8) C.E. de Souza, A. Trofino, and J. de Oliveira, "Robust H_∞ Control of Uncertain Linear Systems via Parameter-Dependent Lyapunov Functions," *Proc. CDC*, 2000, pp. 3194–3199.

9) J. de Oliveira, A. Trofino, and C.E. de Souza, "Robust H_2 Performance of LPV Systems via Parameter Dependent Lyapunov Function," *Proc. IFAC Robust Control Design*, 2000, pp. 415–420.

10) M.C. de Oliveira, J. Bernussou, and J.C. Geromel, "A New Discrete-Time Robust Stability Condition," *Systems & Control Letters*, Vol. 37, 1999, pp. 261–265.

11) Y. Ebihara and T. Hagiwara, "Robust Controller Synthesis with Parameter-Dependent Lyapunov Variables: A Dilated LMI Approach," *Proc. CDC*, 2002, pp. 4179–4184.

12) L.E. Ghaoui and S. Niculescu Ed., *Advances in Linear Matrix Inequality Methods in Control*, SIAM, Philadelphia, PA; 2000.

13) T. Iwasaki and R.E. Skelton, "All Controllers for the General H_∞ Control Problem: LMI Existence Conditions and State Space Formulas," *Automatica*, Vol. 30, No. 8, 1994, pp 1307–1317.

14) T. Shimomura and T. Fujii,"An Iterative Method for Mixed H_2/H_∞ Control Design with Uncommon LMI Solutions," *Proc. ACC*, 1999, pp. 3292–3296.

15) T. Shimomura and S.P. Pullen, "Strictly Positive Real H_2 Controller Synthesis via Iterative Algorithms for Convex Optimization," *J. Guidance, Control, and Dynamics*, Vol. 25, No. 6, 2002, pp. 1003–1011.

16) N. Kawahata,"Model-Following System with Assignable Error Dynamics and Its Application to Aircraft," *J. Guidance and Control*, Vol. 3, No. 6, 1980, pp. 508–516.

17) M. Sznaier, "Receding Horizon: An Easy Way to Improve Performance in LPV Systems," *Proc. ACC*, 1999, pp. 2257–2261.

18) M. Komoda, N. Kawahata, Y. Tsukano, and T. Ono, "VSRA In-Flight Simulator -Its Evaluation and Applications," *Proc. AIAA Flight Simulation Technologies Conference*, AIAA-88-4605-CP, 1988.

19) Y. Miyazawa,"Robust Control System Design with Multiple Model Approach and Its Application to Flight Control System," *Proc. Congress of the International Council of the Aeronautical Sciences*, 1990, pp. 1126–1135.

20) Y. Miyazawa, "Robust Flight Control System Design with Multiple Model Approach," *J. Guidance, Control and Dynamics*, Vol. 15, No. 3, 1992, pp. 785–788.

21) M. Ohno, Y. Yamaguchi, T. Hata, M. Takahama, Y. Miyazawa, and T. Izumi,"Robust Flight Control Law Design for an Automatic Landing Flight Experiment," *Control Engineering Practice*, Vol. 7, 1999, pp. 1143–1151.

22) B.L. Stevens and F.L. Lewis, *Aircraft Control and Simulation*, John Wiley & Sons, Inc., 1992.

23) K. Masui and Y. Tsukano, "Development of a New In-Flight Simulator MuPAL–α," *Proc. AIAA Modeling and Simulation Technologies Conference*, AIAA-2000-4574, 2000.

24) R. K. Heffley and W. F. Jewell, "Aircraft Handling Qualities Data," *NASA CR-2144*, 1972.

25) *Optimization Toolbox*, The Mathworks, Inc., Natick, MA; 2000.

26) MuPAL–α Development Team, "Development of MuPAL–α," *NAL TM-747*, 2000 (in Japanese).

JAXA Research and Development Report JAXA－RR－06－029E

Date of Issue : 30, March , 2007
Edited and Published by : Japan Aerospace Exploration Agency
7-44-1 Jindaiji-higashimachi, Chofu-shi, Tokyo 182-8522, Japan
URL : http://www.jaxa.jp/
Printed by : Kyoushin Co., Ltd.

Inquires about copyright and reproduction should be addressed to the Aerospace Information Archive Center, Information Systems Department, JAXA.
2-1-1 Sengen, Tsukuba-shi, Ibaraki 305-8505, Japan
phone : +81-29-868-5000 fax : +81-29-868-2956

Japan Aerospace Exploration Agency

Printed on Recycled Paper