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## Mixed-Fidelity Efficient Global Optimization Applied to Design of Supersonic Wing

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### Abstract

Multidisciplinary design optimizations are practical key technology for the efficient design of a supersonic transport (SST). However, the computational cost should be expensive with high fidelity flow solver. Thus, the surrogate models such as the Kriging method is one of the promising technique. On the other hand, the computational cost still expensive, because a lot of CFD runs is required to achieve global search with high-fidelity solver. In this study, to develop higher efficient global exploration method, it was considered that a fusion of the database by a low cost/ low fidelity solver and a high cost/ high fidelity solve using Kriging model. A test problem and a design problem of the wing of SST was carried out to investigate the efficiency of the proposed method. In the design of the SST, the linear potential solver and the structured Euler solver was employed. According to the result, the total computational cost was drastically reduced while the same optimum solution can be explored as a single-fidelity optimization.

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**Keywords:** Multi-Fidelity Design, Kriging Method, Supersonic Aircraft

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### 1. Introduction

Recently, the several efficient design techniques are proposed for real world aircrafts based on numerical simulations. In Ref [1], the Efficient Global Optimization (EGO) which was the optimization based on Kriging surrogate models [2] applied to real world design problems with less computational cost. EGO was effective for global search with by heuristic methods, like genetic algorithm (GA) [3]. Thus, EGO process could search the global optimum with a little evaluation cost. Obayashi, et al. proposed Multi-Objective Design Exploration (MODE)[4].

On the other hand, the computational cost still expensive, because the lot of CFD runs is required to achieve global search with high-fidelity solver. Thus, mixed-fidelity approach is promising method for acquiring higher efficient. In this paper, two Kriging models about aerodynamic performance were constructed: One was based on a low-fidelity/low-cost full solver, and another was based on a high-fidelity/high-cost solver. After two regression models were constructed, the difference between two models was estimated. Based on the difference, additional design samples were selected and evaluated by high fidelity solver for improvement global model. The proposed design procedure is expected to solve the next generation SST design, e.g. supersonic business jet (SSBJ)[5].

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In this study, EGO with mixed fidelity approach is applied to the wing and the nose design of the silent supersonic transport (SST) [6][7] as shown in Fig. 1(a), which is designed at Japan Aerospace Exploration Agency (JAXA).

The present design problem focuses on the design of the type of supersonic wing that can help achieve a low sonic boom intensity ( $\Delta P$  defined in Fig. 1(b)). In EGO process, several sample designs are obtained by design of experiment (DOE) [1][3]. Aerodynamic performances of obtained sample designs are evaluated using CAD-based automatic panel analysis (CAPAS) and a structured mesh based Euler solver developed at JAXA [6][7].

After the sampling process, Kriging based functional analysis of variance (ANOVA) [1][4][8] are also used to investigate the efficiency of proposed method. ANOVA, which is one of the multivariate analysis methods, is used to acquire quantitative information about the contributions of the design variables to every objective functions.

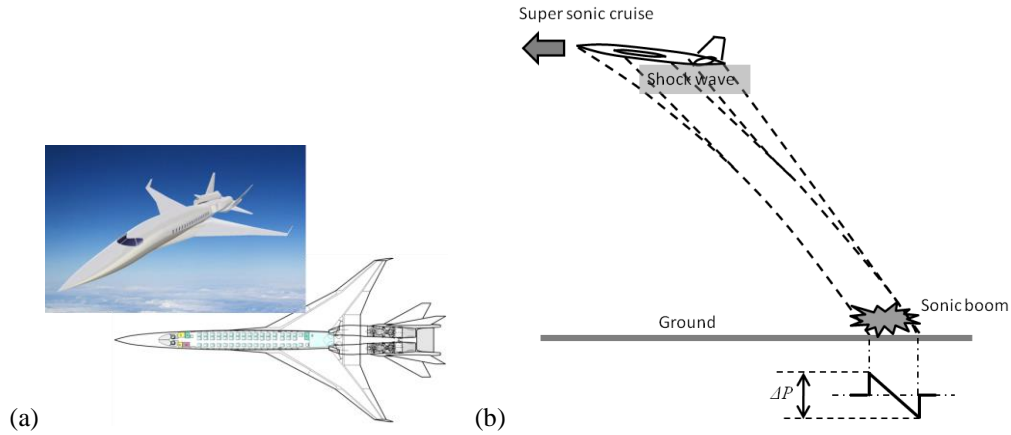


Fig. 1. Illustration of pressure particles for (a) upstream inlet condition in high temperature fields and (b) downstream moving water front in low temperature field.

## 2. Overview of Efficient Global Optimization

The procedure of the present design (Fig. 2) is as follows: First,  $N$  samples are decided by Latin hypercube sampling (LHS) [1][4][8] which is one of the space filling methods, and sample designs are evaluated for the construction of Kriging surrogate models. Then,  $n$  additional designs are added as sample points, and model accuracy is improved by constructing Kriging models using  $N+n$  samples.  $n$  additional points are decided by expected improvement (EI) maximization [1][3] discussed below. MOGA is applied to solving this maximization problem. This process is iterated until improvement of objective functions becomes little. Finally, the non-dominated front can be investigated, and data mining techniques can also be applied to obtain the information of the design problem. The detail of each procedure is described in the following sections.

### 2.1. Kriging model

Kriging model expresses the value  $y(x_i)$  at the unknown design point  $x_i$  as:

$$y(x_i) = \mu + \varepsilon(x_i) \quad (i = 1, 2, \dots, m) \quad (1)$$

where,  $m$  is the number of design variables,  $\mu$  is a constant global model and  $\varepsilon(x_i)$  represents a local deviation from the global model. The correlation between  $\varepsilon(x_i)$  and  $\varepsilon(x_j)$  is strongly related to the distance between the two corresponding point,  $x_i$  and  $x_j$ . In the model, the local deviation at an unknown point  $x$  is expressed using stochastic processes. Some design points are calculated as sample points and interpolated with Gaussian random function as the correlation function to estimate the trend of the stochastic process [9].

### 2.2. Selection of additional samples

Once the models are constructed, the optimum point can be explored using an arbitrary optimizer on the model. However, it is possible to miss the global optimum, because the surrogate model includes uncertainty at the predicted point. Therefore, this study introduced EI values as the criterion. EI for maximization problem can be calculated as follows:

$$E[I(x)] = (\hat{y} - f_{\max})\Phi\left(\frac{\hat{y} - f_{\max}}{s}\right) + s\phi\left(\frac{\hat{y} - f_{\max}}{s}\right) \quad (2)$$

EI for minimization problem can be calculated as follows:

$$E[I(x)] = (f_{\min} - \hat{y})\Phi\left(\frac{f_{\min} - \hat{y}}{s}\right) + s\phi\left(\frac{f_{\min} - \hat{y}}{s}\right) \quad (3)$$

where  $f_{\max}$  and  $f_{\min}$  are the maximum/minimum values among sample points and  $\hat{y}$  is the value predicted by Eq. (1) at an unknown point  $x$ .  $\Phi$  and  $\phi$  are the standard distribution and normal density, respectively. EI considers the predicted function value and its uncertainty, simultaneously. Thus, the solution that has a large function value and a large uncertainty may be a promising solution. Therefore, by selecting the point where EI takes the maximum value, as the additional sample point, robust exploration of the global optimum and improvement of the model can be achieved simultaneously as shown in Fig. 3(a) because this point has a somewhat large probability to become the global optimum.

In this study, a multi-objective problem is considered. Thus, to decide additional samples, EI values corresponding to each objective function should be simultaneously maximized. This study employed a Non-dominated Sorting Genetic Algorithms-II (NSGA2) to obtain non-dominated solutions by solving the multi-objective problems. To improve each Kriging model, several non-dominated solutions can be used as additional sample designs. Here, K-means clustering method is applied to clustering of non-dominated solutions, and sample designs which are the closer to the centroid of each cluster are selected as additional samples (Fig. 3(b)).

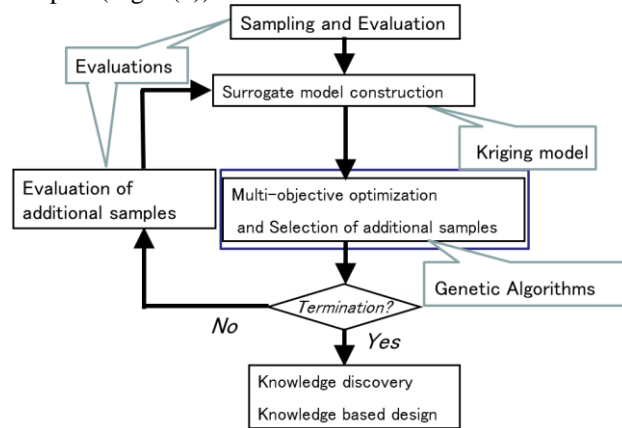


Fig. 2 Procedure of original Efficient Global Optimization [1, 2].

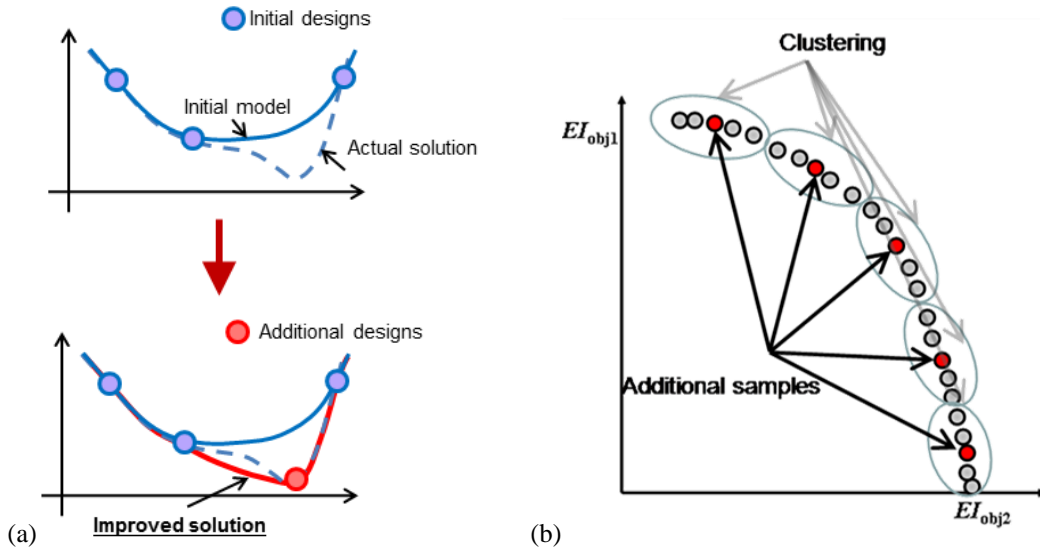


Fig. 3 (a) Image of improvement of the model, (b)K-means clustering and selection of additional samples from non-dominated solutions based on EI maximization.

### 3. EGO with Mixed-Fidelity Approach

In this study, the improvement of the efficiency of EGO by means of mixed-fidelity approach is proposed. This method introduce two flow solver; one is high-fidelity but expensive Euler simulation (solver A), and another is low-fidelity but low-cost the potential solver (solver B). High-fidelity solver is solved to acquire the highly accurate flowfield for promising designs after the global trend is observed by low-fidelity solver. The global optimum point is explored through the EI maximization ( $EI_A$ ) and the correction of the high-fidelity model by the maximum error point between the solutions obtained by two solvers. The design problem can be expressed as follows. The procedure of the proposed approach is shown in Fig. 4.

$$\begin{cases} \text{Maximize} & EI_A \\ \text{Maximize} & |\hat{y}_A - \hat{y}_B| \end{cases} \quad (4)$$

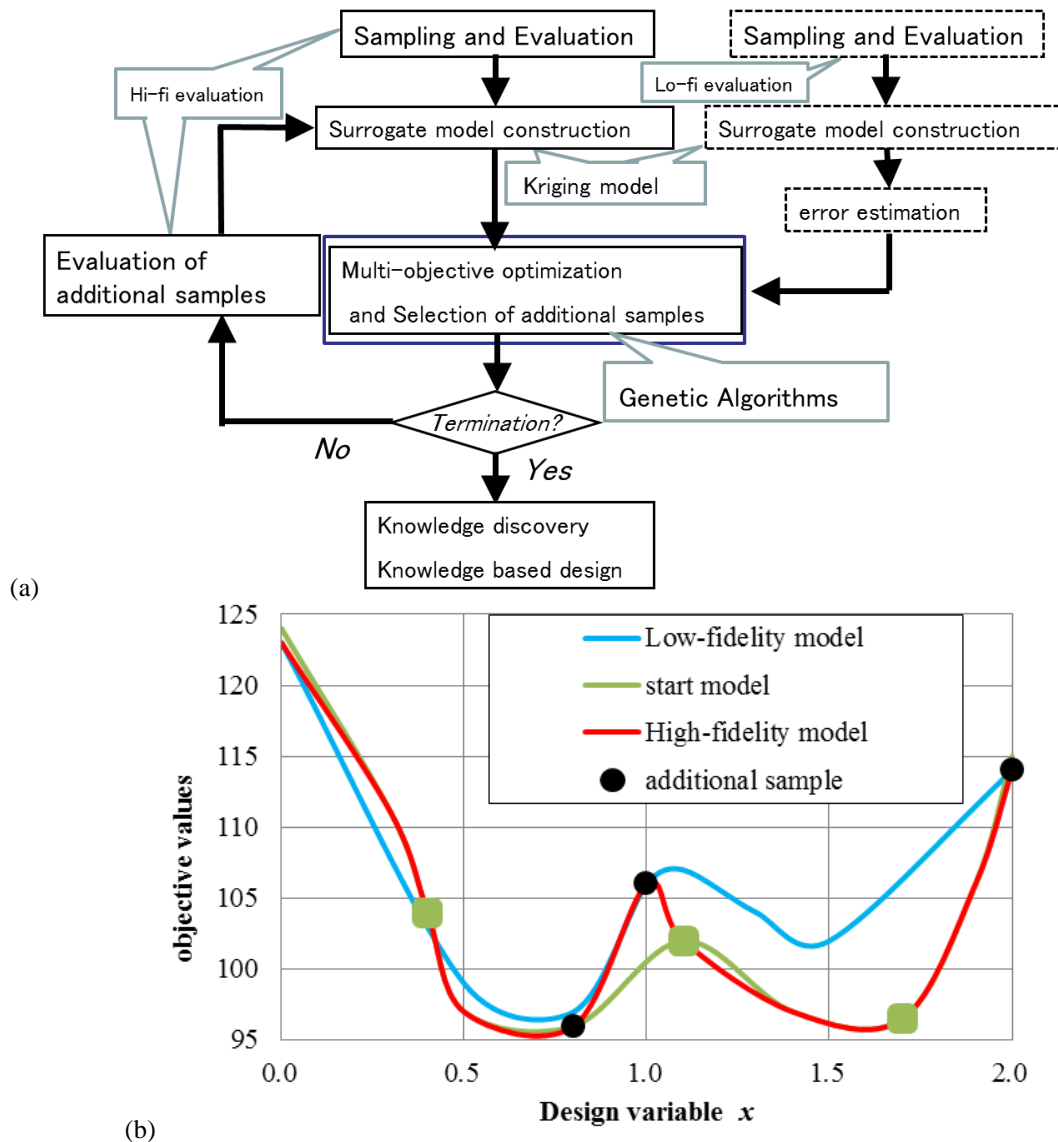


Fig. 4 (a) Procedure of mixed-fidelity approach, (b) Image of the improvement of high-fidelity model by means of mixed-fidelity approach.

#### 4. Investigation of Mixed Fidelity Approach by Solving test Function

##### 4.1. Definition of test function

To investigate the efficiency of the proposed method, the minimization problem of a test function is carried out. In this study, Brannin function (Fig. 5(a)) expressed below is used as a test problem.

$$f(\mathbf{x}) = \left( x_2 - \frac{5}{4\pi^2} x_1^2 + \frac{5}{\pi} x_1 - 6 \right)^2 + 10 \left( 1 - \frac{1}{8\pi} \right) \cos(x_1) + 10$$

$$-5 \leq x_1 \leq 10 \quad 0 \leq x_2 \leq 15$$
(5)

It has two independent variables, and minimum value is 0.398 at three points,  $\mathbf{x} = X1(-\pi, 12.275)$ ,  $X2(\pi, 2.275)$ , and  $X3(9.424, 2.475)$ . In this investigation, Eq. (5) is used as a solver A. To carry out the mixed fidelity exploration, the low-fidelity model is defined based on Eq. (5) as following.

$$f(\mathbf{x}) = \left( x_2 + \frac{5}{\pi} x_1 - 6 \right)^2 + 10 \left( 1 - \frac{1}{8\pi} \right) \cos(x_1) + 10$$

$$-5 \leq x_1 \leq 10 \quad 0 \leq x_2 \leq 15$$
(6)

Eq. (6) is defined by eliminating the fourth order term (Fig. 5(b)). Such elimination is similar to elimination the viscous term in developing the flow solver. (That is to say that the difference between Navier-Stokes equation and Euler equation.) In this investigation, Eq. (6) is considered as a solver B.

##### 4.2. Results

Figure 6 shows the comparison of the convergence history for each minimum point,  $X1$ ,  $X2$ , and  $X3$  obtained by the original EGO and the proposed mixed-fidelity approach. Comparing Fig. 6(a) and (b), the proposed method carried out better convergence than the original EGO. Figure 7 shows the comparison of the sampling results and the surrogate models constructed based on the obtained samples by means of each method. According to the Fig. 7, the similar surrogate model can be obtained by means of the proposed method compared with the original EGO. Additionally, higher diversity can be maintained than that of the original EGO in explore process. These result suggest that the proposed method can carried out higher efficient exploration than the original EGO.

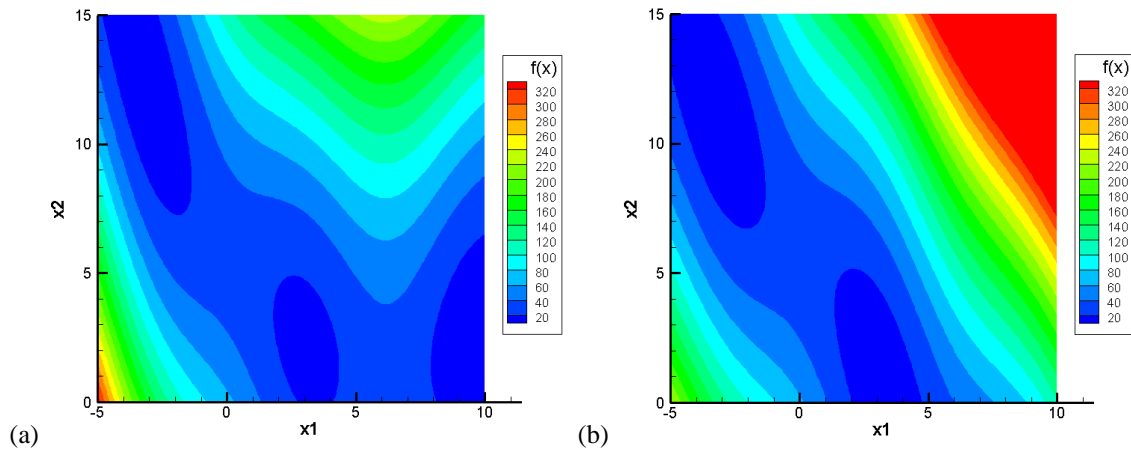


Fig. 5 Exact solution of test function, (a)Original Brannin function (Eq. 5), (b)Brannin function with elimination highest order term (Eq. 6).

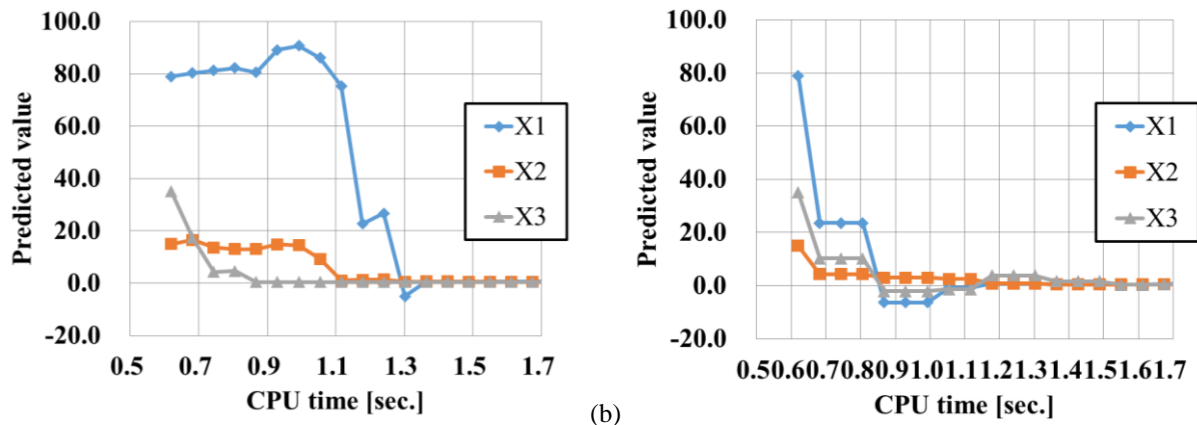


Fig. 6 Convergence history of finding three minimum point. (a)Optimization by original EGO, and (b)Optimization by EGO with mixed-fidelity approach.

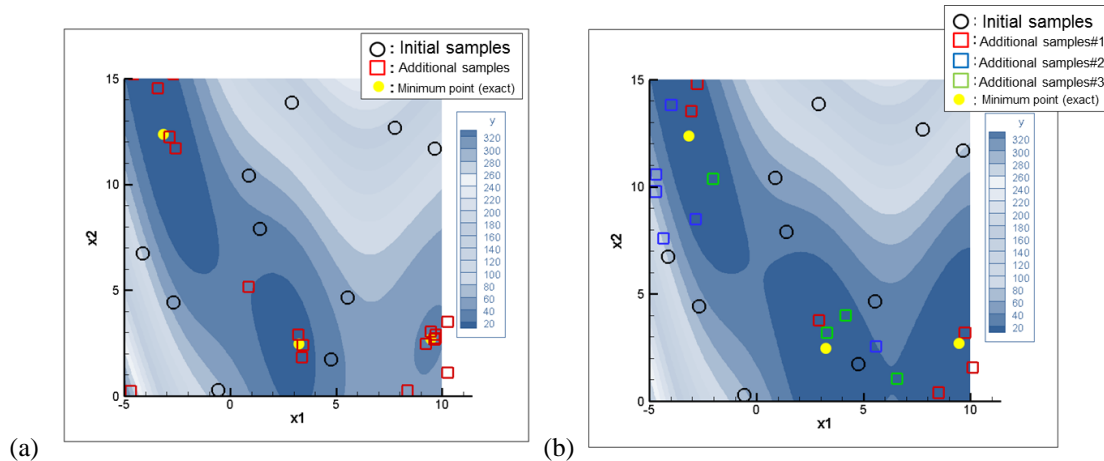


Fig. 7 Sampling results and surrogate models based on these samples. (a)Sampling by original EGO, (b) Sampling by EGO with mixed-fidelity approach.

## 5. Design Result of Lower-Boom Supersonic Aircraft

In this study, the proposed method is compared with the original EGO with solving the sonic boom minimization problem of JAXA's concept as shown in Fig. 1(a). The aerodynamic performance is evaluated by means of full potential solve at Mach 1.6. The wing and the fuselage geometry are designed here. The design space is shown in Table. 1. The parameterization of the fuselage nose is also illustrated in Fig. 8.

### 5.1. Comparison of sampling result between original EGO and EGO with mixed fidelity approach

Figure 9 shows the comparison of the sampling result. In original EGO, the total of 64 high fidelity CFD run was required to find the global optimum designs. As this result, the global exploration has been finished 620 hours. On the other hand, the global optimum designs could be found with the total of 32 high fidelity CFD runs and the total of 64 low fidelity solver runs by means of the proposed mixed fidelity approach. As this result, the global exploration by the proposed approach has been finished 350 hours. According to this result, the proposed mixed fidelity approach could drastically reduce the design cost.

### 5.2. Investigation of sampling results by means of analysis of variance

The main purpose of the present study is to obtain the global information of the design space efficiently. If the similar trend could be obtained, the contribution ratio to the objective function should be similar. To investigate it, Kriging model based Analysis of Variance (ANOVA) [1, 2] is applied to the sampling results as shown in Fig. 10.

According to the Fig. 10,  $dv4$  and  $dv3$  have predominant effect in each figure. In addition,  $dv2$ , 4, 5, and 9 also have effect to the objective function. This result suggests that the model obtained by the mixed fidelity approach has similar tendency as the model obtained by EGO.

### 5.3. Design example

Figure 11 shows the comparison of flowfield around the aircraft which achieve minimum sonic boom intensity obtained by EGO and the present approach. According to this figure, almost same flowfield could be obtained. This result suggested that the proposed method can find almost same optimum design compared with the original EGO. Figure 12 shows the comparison of the wave form of the sonic boom. Each result obtained by EGO and the present design shows the similar wave foam.

Table 1. Design space

|            | Design parameter   | Design space |       | Unit      |
|------------|--|--------------|-------|-----------|
|            |  | upper        | lower |           |
| <i>dv1</i> | Twist angle at root  | 0.0          | 2.0   | degree    |
| <i>dv2</i> | Twist angle at tip   | -4.0         | 0.0   | degree    |
| <i>dv3</i> | Camber at root (25% chord)                                   | 0.0          | 5.0   | %chord    |
| <i>dv4</i> | Camber at root (75% chord)                                   | -3.0         | 2.0   | %chord    |
| <i>dv5</i> | Camber at kink (25% chord)                                   | -2.0         | 2.0   | %chord    |
| <i>dv6</i> | Camber at kink (75% chord)                                   | -2.0         | 2.0   | %chord    |
| <i>dv7</i> | Upper surface displacement of fuselage (25% fuselage length) | -0.1         | 0.1   | %fuselage |
| <i>dv8</i> | Lower surface displacement of fuselage (25% fuselage length) | -0.1         | 0.1   | %fuselage |
| <i>dv9</i> | Side surface displacement of fuselage (25% fuselage length)  | -0.1         | 0.1   | %fuselage |

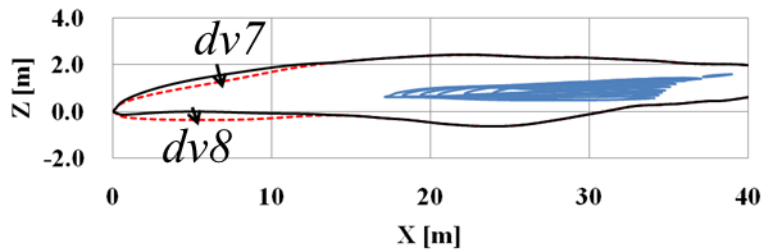


Fig. 8 Illustrations of design parameters at fuselage nose.

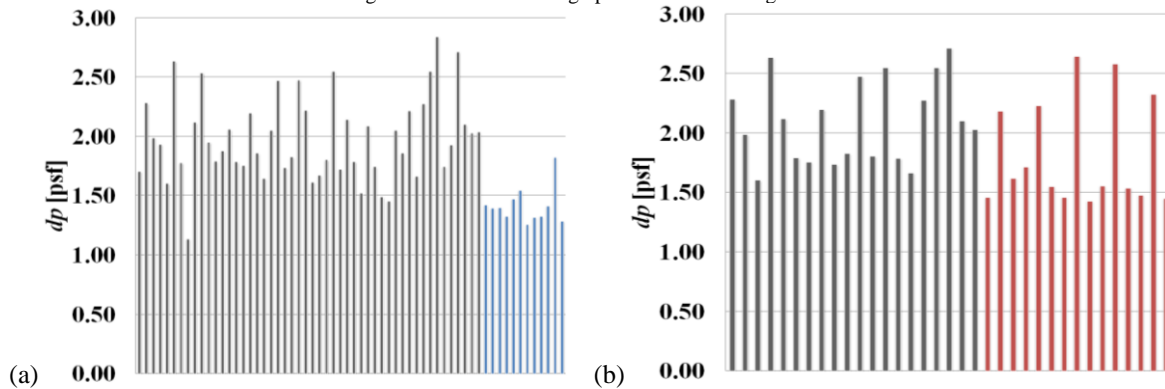


Fig. 9 Comparison of the design samples. (a) EGO, and (b) EGO with mixed fidelity approach.



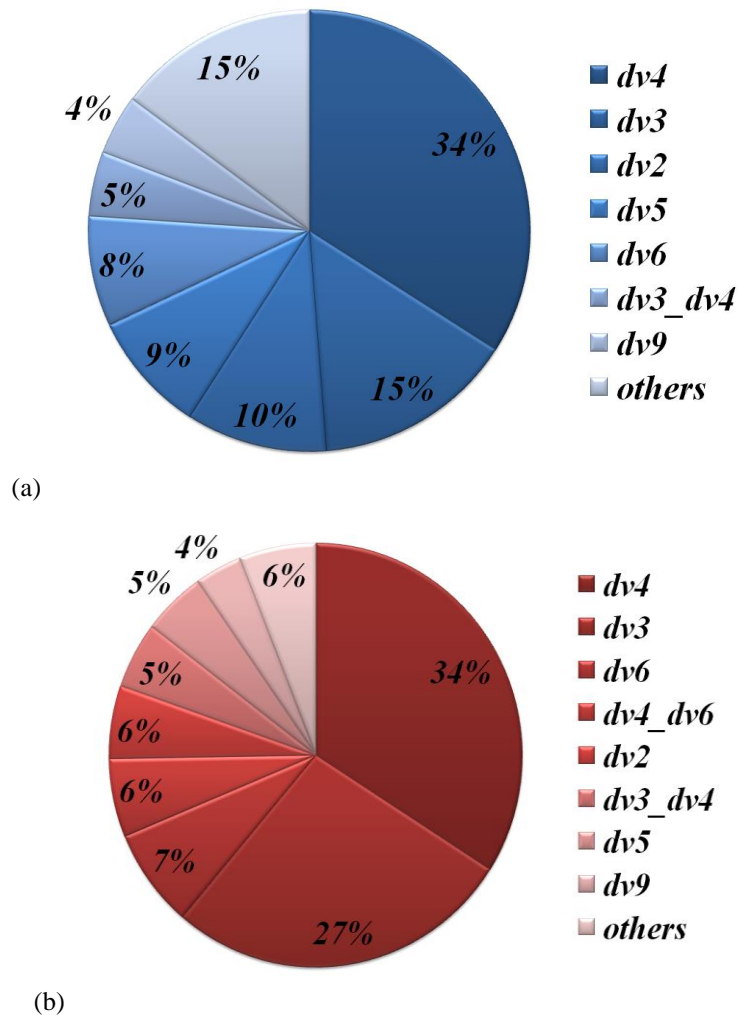


Fig. 10 Comparison of the ANOVA result. (a)EGO, and (b)EGO with mixed fidelity approach.

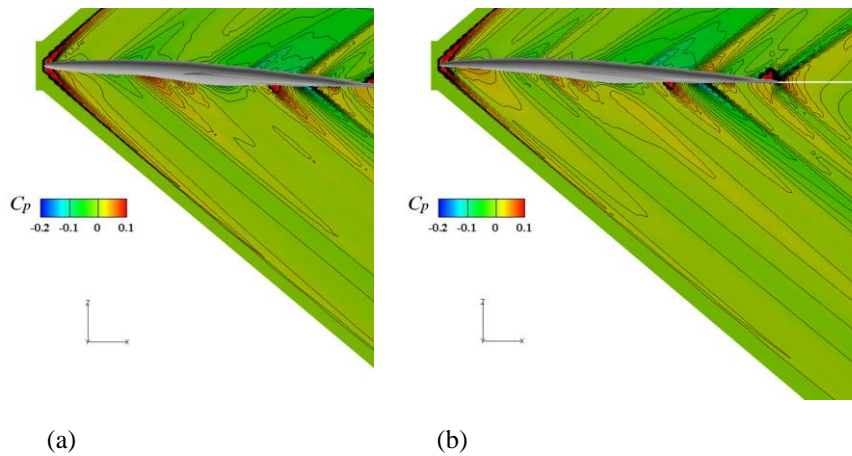


Fig. 11 Comparison of flowfield. (a)flowfield designed by EGO, and (b) flowfield designed by EGO with mixed fidelity approach.



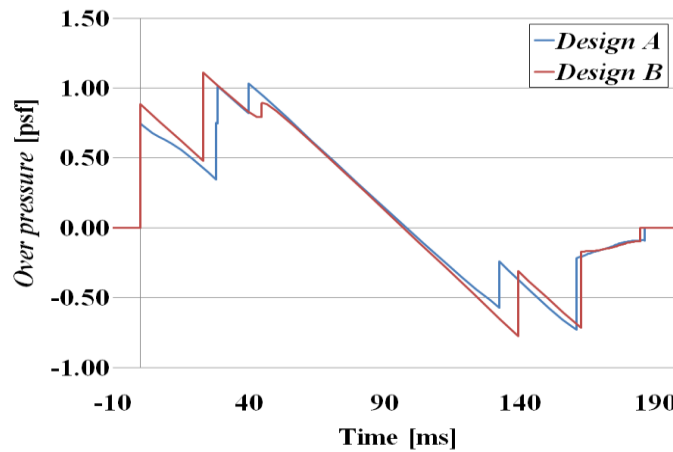


Fig. 12 Comparison of wave form of sonic boom. Design A is designed by original EGO, and Design B is flowfield designed by EGO with mixed fidelity approach.

## 6. Conclusions

In this study, the mixed fidelity approach is discussed for a low boom SST design. According to the comparison between the original EGO and the proposed method, the computational cost is drastically reduced, while the equivalent design knowledge could be obtained.

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