Direction of Arrival and Magnitude Estimation by The Prony Method with Consideration of Damping Factor

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1. Introduction

This paper uses the Prony method to estimate both the direction of arrival (DOA) and the magnitude of radio waves with an equi-spaced linear array antenna. Because the Prony method has been reported to be not applicable for estimations in the presence of noise [1], in this study, I improve this method and then validate the improved approach by numerical simulation.

2. DOA and Magnitude Estimation by the Prony Method

When p radio waves with magnitude w_m arrive at an equi-spaced linear array antenna from direction θ_m, the outputs of the antenna form a data sequence which is expressed as follows:

$$D[n]=\sum_{i=1}^{p} w_i e(\theta_i)\exp\left\{-j(n-1)\frac{2\pi}{\lambda}d\sin\theta_i\right\}, n=1,2,\cdots,N \quad (1)$$

Here λ is the wave length and N, d, and $e(\theta)$ are the number, spacing, and field pattern of the element, respectively. Because Eq. (1) is an exponential sequence, $N/2$ pairs of amplitude *Amp*(*m*) and the argument *Arg*(*m*) of the exponentials are obtained by the Prony method. *Arg*(*m*) is expressed as follows:

$$Arg(m)=\alpha_m + j\beta_m . \quad (2)$$

Here α_m is the damping factor of the *m*th exponential. From β_m and the argument of the exponential in Eq. (1), θ_m is calculated as follows:

$$\theta_m = \sin^{-1}\left(-\frac{\lambda}{2\pi d}\beta_m\right). \quad (3)$$

Then, w_m is calculated as follows:

$$w_m = \frac{Amp(m)}{e(\theta_m)} . \quad (4)$$

This is the principle for estimating the DOA and magnitude of radio waves by the standard Prony method.

3. Improved Method and Results

In principle, the values of α_m that correspond to radio waves are zeros. However, in reality, they are not always zeros and may have small values. Conversely, α_m values that correspond to noise have arbitrary values. Thus, some exponentials that correspond to noise can be excluded for the range of α_m, to improve the Prony method such that it will work when noise is present

To validate this improvement, I performed a numerical simulation. Fig. 1 shows *Amp*(*m*) and *Arg*(*m*) values from 40 trials obtained by the Prony method when two radio waves with equi-amplitude and phase arrive at an antenna from the directions of 10 and 14 degrees. In this case, the parameters were as follows: N = 20, $d = 0.5\lambda$, a uniformly illuminated element of 0.3λ length, *SNR* = 10 *dB*, and 10 snapshots. Here the horizontal and vertical axes are the damping factor and the DOA estimation error, respectively, and the radius of the circle is expressed as *Amp*(*m*). Large circles with small DOA estimation errors can be seen in the region where the damping factor is small. These are the exponentials that correspond to radio waves, and the others are those corresponding to noise. Thus, the exponentials with small damping factors are used and the influence of noise is reduced. Figs. 2 and 3 show the estimation results by the standard and improved Prony methods, respectively, for 10 trials with the same parameters as in Fig. 1. The improved method used exponentials of $|\alpha_m| \le 0.04$. A large output at approximately -10 degrees in Fig. 2 does not exist in Fig. 3, and Fig. 3 also has fewer outputs corresponding to noise than Fig. 2.

4. Conclusions

I improved the Prony method by the consideration of the damping factor and demonstrated that the improved approach can work in the presence of noise.

Reference

[1]Rahman, Kai-Bor, "Total least squares approach for frequency estimation using linear prediction," IEEE Trans. Acoustics, Speech & Signal Processing, vol.35, no.10, pp.1440-1454, 1987

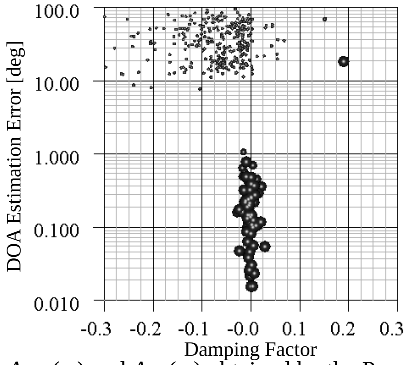

Fig. 1 *Amp*(*m*) and *Arg*(*m*) obtained by the Prony method

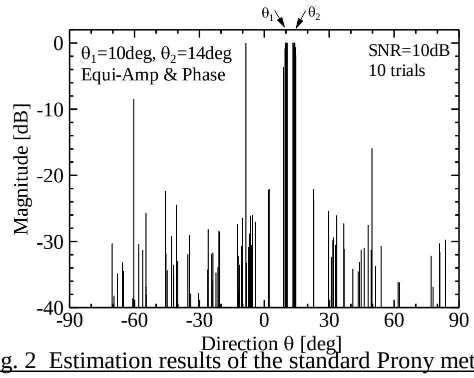

Fig. 2 Estimation results of the standard Prony method

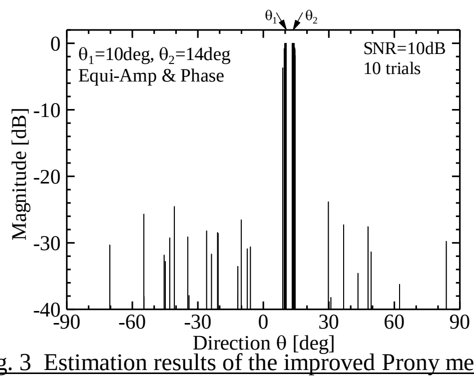

Fig. 3 Estimation results of the improved Prony method