Orbital Dynamics of Magnetoplasma Sail

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Abstract : The orbital dynamics of magnetoplasma sail (MPS) is investigated focusing on the required acceleration for escape from the solar system. One of the main features of the MPS propulsion system is that the thrust in confined to anti-sun direction. This motivates the study of the orbital motion under an outward continuous radial acceleration as a function of the distance from the central body. Two representative cases in terms of the MPS power system are analyzed; one is the constant radial acceleration case, and the other assumes the radial acceleration inversely proportional to the square of the radial distance. The former corresponds to constant power case such as radioisotope thermoelectric generator (RTG) system, while the latter corresponds to the case assuming solar paddle for the MPS power system.

Key words : magnetoplasma sail, radial thrust.

1. Introduction

One of the main features of the MPS propulsion system is that the thrust is confined to anti-sun direction. This motivates the study of the orbital motion under an outward continuous radial acceleration as a function of the distance from the central body. This classical problem was investigated by several researchers in the past (Ref. 1-11). An analytical solution is available when the radial acceleration is applied to a spacecraft under the influence of the gravity field of the central body. Tsien[1] and Battin[2] showed that there exists a critical value for the constant radial acceleration above which the spacecraft will achieve escape conditions. When the acceleration is below the critical value, the spacecraft attains a maximum distance along an outbound trajectory. If the radial acceleration is maintained beyond the maximum distance, the spacecraft spirals back to the initial radius along an inbound trajectory. Boltz[3] treated the radial acceleration case where the acceleration is inversely proportional to the square of the distance from the central body using the equation of motion described by the velocity and flight path angle. Prussing and Coverstone-Carroll[4] provided a nonlinear radial spring interpretation for the constant radial acceleration problem with an energy well approach. Akella[5] focused on the property of the constant radial acceleration case and shows that while the time intervals for the outbound and inbound trajectories are identical, the trajectories themselves are very different from one another. Broucke and Akella[6] described the general types of solutions of the constant continuous outward radial acceleration. They used numerical integrations and concepts such as the theory of periodic orbits and Poincare's

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charachteristic exponents.

Yamakawa[7] investigated the orbital motion under a continuous outward radial acceleration as a function of the distance from the central body analytically. The attainable maximum radial distance was given as a function of the radial acceleration, and numerical experiments were carried out showing some periodic orbits under radial acceleration. Formulating the interplanetary trajectories under radial thrust as an optimal control problem, Yamakawa[8] treated the escape from the solar system considering the maximization of the terminal orbital energy, while conserving the orbital angular momentum. The achievable hyperbolic excess velocity was studied in terms of the available maximum radial acceleration and the transfer angle.

Because an MPS spacecraft obtains momentum from the solar wind, a guidance law is indispensable for the planetary explorer under the variable solar wind circumstance. This requires a detailed solar wind prediction model and the simultaneous use of chemical propulsion for guidance maneuver. In Ref. 9, a guidance scheme was proposed for orbital motion under continuous outward radial acceleration that is inversely proportional to the square of the radial distance from the Sun. The maximum attainable radial distance of the outbound trajectory was investigated, and a guidance scheme for achieving this target maximum distance was established under radial acceleration disturbances. The scheme not only provides a control law for continuous radial acceleration, but also yields the amount and timing of impulsive maneuvers required to satisfy the guidance requirement at the terminal point.

Ref. 10 analyzed the characteristics of trajectories of low thrust propulsion systems whose thrust direction is restricted. Low thrust propulsion mission performance was represented by three parameters; initial thrust/mass ratio, thrust angle and specific impulse. It was assumed that electrical power provided to the propulsion system is inversely proportional to the square of heliocentric distance, and the spacecraft mass decreases as the fuel is expended. Hyperbolic excess velocity with respect to the Earth was assumed 0 km^2/s^2 at the Earth departure. This corresponds to the marginal escape from the Earth's magnetosphere followed by the activation of the magneto-plasma sail engine. It was shown that the MPS is suitable for missions to Pluto and beyond if the MPS realizes high acceleration (i.e., 1.5 mN/kg) even if the thrust direction is restricted to outward radial direction.

In Refs. 11 and 12, a preliminary spacecraft design was conducted based on the Jupiter flyby mission as an example. The spacecraft weighs 1,000 kg at launch and the fuel mass (Xe) is 430 kg. The thrust/power ratio was assumed as 1N / 4kW. The electric power for plasma supply is generated from the solar cell paddle, which generates 8.0 kW at 1.0 AU. The high gain antenna is pointed toward the earth for communication, and the solar cell paddle is mounted on a single-axis gimbal and is pointed to the Sun to get enough electric power. The plasma is supplied from two plasma sources. By changing the attitude of spacecraft with respect to the Sun, the direction of generated thrust (steering angle) is controlled.

In this paper, a spacecraft trajectory under the continuous outward radial acceleration as a function of the power of the radial distance from the central body is investigated focusing on the required acceleration to escape from the solar system. Two cases for the MPS power system are assumed: one assumes constant radial acceleration, while the other assumes radial acceleration inversely proportional to the square of the radial distance. The former corresponds to constant power case such as radioisotope thermoelectric generator (RTG) system, while the latter corresponds to the case assuming solar paddle for the MPS power system. Part of the results of the paper was presented in Ref. 13.

2. Equations of Motion

The problem considered is the motion of the spacecraft under the influence of the thrust of the spacecraft and the gravitational attraction of the central body, namely, the sun The acceleration due to the thrust of the spacecraft is confined to the outward radial direction. Therefore, the trajectory remains in a plane and described by the two degree-of-freedom equations of motion. Let position of the spacecraft at any time instant τ be given by the polar coordinates ρ and θ, where ρ is the distance from the center of attraction and θ the angular position. Then the nondimensional equations of motion for the spacecraft in polar coordinates are given by [7]

$$\frac{d^2\rho}{d\tau^2} = \rho\left(\frac{d\theta}{d\tau}\right)^2 - \frac{1}{\rho^2} + \varepsilon\frac{1}{\rho^n} \tag{1}$$

$$\frac{d}{d\tau}\left(\rho^2\frac{d\theta}{d\tau}\right) = 0 \tag{2}$$

where ε is the nondimensional acceleration at the starting instant. All the variables are normalized by the radius of the Earth's orbit, the corresponding circular velocity, and the magnitude of the gravitational attraction of the sun at the Earth's radial distance. The period of the Earth's orbit is 2π, and the reference time unit is 58.132 days. The reference distance, velocity, and acceleration units are 1.49597870 x 10^{11} m, 29,784 m/s, and 0.00593 m/s^2, respectively. Assuming that the spacecraft is initially placed at the circular orbit (i.e., Earth orbit), then the initial velocity conditions are as follows:

$$\left(\frac{d\rho}{d\tau}\right)_0 = 0 \tag{3}$$

$$\left(\frac{d\theta}{d\tau}\right)_0 = 1 \tag{4}$$

Equation (2), describing the conservation of angular momentum can then be integrated as follows.

$$\frac{d\theta}{d\tau} = \frac{1}{\rho^2} \tag{5}$$

By substituting this equation into equation (1), the equation for ρ is

$$\frac{d^2\rho}{d\tau^2} = \frac{1}{\rho^3} - \frac{1}{\rho^2} + \varepsilon\frac{1}{\rho^n} \tag{6}$$

which can be rewritten as

$$\frac{1}{2}\frac{d}{d\rho}\left(\frac{d\rho}{d\tau}\right)^2 = \frac{1}{\rho^3} - \frac{1}{\rho^2} + \varepsilon\frac{1}{\rho^n} \tag{7}$$

The result of integrating the above equation in terms of ρ from ρ_0 to ρ is where $n \neq 1$.

$$\frac{1}{2}\left(\frac{d\rho}{d\tau}\right)^2 = \frac{1}{2}\left(-\frac{1}{\rho^2}+1\right)+\left(\frac{1}{\rho}-1\right)+\left(\frac{\varepsilon}{n-1}\right)\left(-\frac{1}{\rho^{n-1}}+1\right) \tag{8}$$

3. Escape Conditions

The conditions for the spacecraft to escape from the center of attraction are derived in this section. The square of the hyperbolic excess velocity, C_3, is obtained from the results of the previous section as follows:

$$C_3 = \left(\frac{d\rho}{d\tau}\right)^2 + \left(\rho\frac{d\theta}{dt}\right)^2 - \frac{2}{\rho} \tag{9}$$

Substituting equations (5) and (8) into the above equation, C_3 becomes

$$C_3 = \frac{2\varepsilon}{n-1}\left(-\frac{1}{\rho^{n-1}}+1\right)-1 \tag{10}$$

When the escape condition is satisfied and attains a parabolic state, C_3 becomes zero and equation (10) becomes

$$\frac{n-1}{2\varepsilon} = 1 - \frac{1}{\rho_{escape}{}^{n-1}} \tag{11}$$

where ρ_{espace} is the radial distance where the escape condition is satisfied and parabolic state is achieved. This simple equation (11) relates the following three parameters: ρ_{espace}, n, and ε, where n indicates the power index of the acceleration as a function of the radial distance, and ε corresponds to the non-dimensional acceleration defined at the initial instant. When n = 0(i.e., constant acceleration) and n = 2(i.e., inversely proportional to the square of the radial distance), the following equations (12) and (13) are simply derived from equation (11).

$$\rho_{escape} = \frac{2\varepsilon+1}{2\varepsilon}, \qquad (n=0) \tag{12}$$

$$\rho_{escape} = \frac{2\varepsilon}{2\varepsilon-1}, \qquad (n=2) \tag{13}$$

Figure1 describes the relation between nondimensional acceleration ε and ρ_{espace} based on equations (12) and (13). When starting from the same nondimensional acceleration ε, (e.g., $\varepsilon=1$), the spacecraft requires continuous acceleration from Earth departure till Mars distance ($\rho_{espace}=1.5$) in order to achieve escape for constant acceleration case (i.e., $n=0$), while the escape distance becomes larger (i.e., $\rho_{espace}=2$) for the inverse square case (i.e., $n=2$) due to the reduction of the acceleration. Figure 1 also illustrates the attainable maximum radial distance, ρ_{max}, for non-escape trajectories as a function of nondimensional acceleration ε. which are given by

$$\rho_{max} = \frac{1-\sqrt{1-8\varepsilon}}{4\varepsilon}, \qquad (n=0) \tag{14}$$

$$\rho_{max} = \frac{1}{1-2\varepsilon}, \qquad (n=2) \tag{15}$$

derived in Ref. 7. Eqs. (14) and (15) for non-escape cases are valid when $0<\varepsilon<0.125$ and $0<\varepsilon<0.5$, respectively. This condition yields that Eqs. (12) and (13) for escape cases are valid only when $\varepsilon \geq 0.125$ and $\varepsilon \leq 0.5$ respectively.

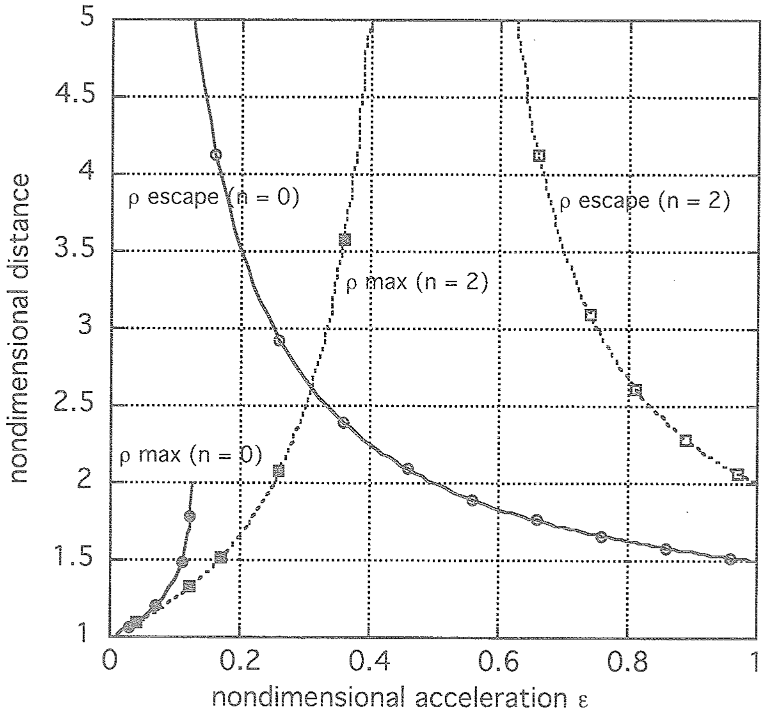

Fig 1: Relation between ε and ρ_{escape} and ρ_{max} (n = 0 and n = 2).

4. Concluding Remarks

A spacecraft trajectory under the continuous outward radial acceleration as a function of the power of the radial distance from the central body was investigated focusing on the required acceleration to escape from the solar system. Two specific cases were assumed: one assumes constant radial acceleration (i.e., RTG power), while the other assumes radial acceleration inversely proportional to the square of the radial distance (i.e., solar paddle power). Given the acceleration tendency (i.e., constant or inverse square), the relation between the nondimensional radial acceleration and the escape distance (i.e., radial distance where parabolic state is achieved) was obtained analytically.

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