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Detection of frequency-mode-shift during thermoacoustic combustion oscillations in a staged aircraft engine model combustor

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We conduct an experimental study using time series analysis based on symbolic dynamics to detect a precursor of frequency-mode-shift during thermoacoustic combustion oscillations in a staged aircraft engine model combustor. With increasing amount of the main fuel, a significant shift in the dominant frequency-mode occurs in noisy periodic dynamics, leading to a notable increase in oscillation amplitudes. The sustainment of noisy periodic dynamics during thermoacoustic combustion oscillations is clearly shown by the multiscale complexity-entropy causality plane in terms of statistical complexity. A modified version of the permutation entropy allows us to detect a precursor of the frequency-mode-shift before the amplification of pressure fluctuations. *Published by AIP Publishing*. https://doi.org/10.1063/1.5003912

I. INTRODUCTION

Combustion-driven thermoacoustic instability, referred to as thermoacoustic combustion oscillations, leads to serious structural damage in practical combustors, affecting the aircraft propulsion and land-based gas-turbine engine performance. Some review articles[1,2] and a book[3] have given an encompassing overview of the physical mechanisms stemming from the closed-loop interaction between an unsteady acoustic wave and heat-release fluctuations during thermoacoustic combustion oscillations in various types of confined combustor. To avoid the onset of thermoacoustic combustion oscillations, detection methodologies for thermoacoustic combustion oscillations and the characterization of a rich variety of dynamic behavior have become of much interest in the fields of applied mathematics and nonlinear physics.[4–9] The suppression of thermoacoustic instability has recently been attempted by Biwa *et al.*[10]

Time series analysis based on dynamical systems theory and complex networks is becoming useful for accomplishing two aims: one is to offer a deeper physical interpretation of nonlinear combustion dynamics, which is difficult to understand from only power-spectrum analysis, and the other is to develop substitute detectors for capturing the occurrence of unstable combustion states such as thermoacoustic combustion oscillations and blowout. Nair *et al.*[11] have shown that the Kaplan-Glass method[12] is useful for capturing the possible presence of chaotic dynamics in combustion noise. The presence of chaos during combustion noise was further proved by Tony *et al.*[7] They have also reported the importance of information entropy in recurrence plots for characterizing the dynamic behavior of pressure fluctuations during low- to high-amplitude oscillations.[13] Unni *et al.*[14] used a symbolic time series analysis to detect an impeding thermoacoustic combustion oscillations. Murugesan and Sujith[15] have discussed the importance of network properties such as the degree, average path length, and clustering coefficient in the natural visibility graph for the early detection of impending thermoacoustic combustion oscillations. Our recent studies[16–18] on nonlinear combustion dynamics have also revealed the possibility of employing dynamical systems theory and complex networks in new engineering applications to prevent the onset of blowout. Our primary interest in this study is to show the feasibility of symbolic dynamics as a possible detection method for thermoacoustic combustion oscillations in different types of combustor discussed in previous studies.[4,16–18]

Lean premixed combustion is a highly promising technique that can effectively reduce nitrogen oxide (NOx) emissions. However, it has the inherent problem that it is difficult to form a stable combustion state under a wide range of engine operating conditions. To overcome this problem, Japan Aerospace Exploration Agency (JAXA) has developed a new type of fuel stage combustor incorporating pilot and main stages.[19–21] The main purpose of this study is to develop a pragmatic methodology based on symbolic dynamics to detect a precursor of frequency-mode-shift during thermoacoustic combustion oscillations in a staged aircraft engine model combustor with increasing amount of main fuel. Bandt and Pompe[22] proposed the permutation entropy based on symbolic dynamics as a means of evaluating the degree of randomness of dynamic behavior. It represents the information entropy considering the probability distribution of possible existing rank order patterns in a time series. The multiscale complexity-entropy causality plane (CECP) incorporating a scale-dependent approach in terms of statistical complexity has recently been shown to ensure good performance in distinguishing between deterministic and stochastic dynamics in noisy time series data.[23] These quantities are used in a rapidly increasing number of fields in nonlinear science. In this study, we investigate the dynamic behavior of thermoacoustic combustion oscillations with a frequency-mode-shift using the multiscale complexity-entropy causality plane. We propose a modified version of the permutation entropy to

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detect a precursor of dominant frequency-mode-shift during thermoacoustic combustion oscillations. This paper is organized as follows. Section II gives a brief description of the experimental apparatus and conditions. A methodological framework of time series analysis based on symbolic dynamics and statistical complexity is given in Sec. III. Section IV presents the results and discussion. A summary is provided in Sec. V.

II. EXPERIMENTS

In this study, we employ a staged aircraft engine model combustor developed in the JAXA TechClean project. Details of our experimental system have been reported in previous studies.[19–21] As shown in Fig. 1, our experimental system mainly consists of a burner system, a combustion chamber, and an exhaust nozzle. The burner system consists of two burners: one is an external main burner with three swirlers and the other is an internal pilot burner with two counter-rotating swirlers. Both burners are co-axially located. The main inner and outer swirlers are in co-rotation with the pilot outer swirler, while the main middle swirler is in counter-rotation with the main inner swirler and outer swirlers. The main fuel is injected from the inner wall of the main mixer through simple holes. The pilot fuel is injected by a prefilmer between two counter-rotating swirlers. The diameter of the burner at the inlet of the combustor is 60 mm, and a fluidic element is mounted at the burner's entrance. Kerosene is employed as the fuel in this study. We supply compressed air preheated by an electric heater to the combustion chamber through the two burners. We vary the air-flow split ratio of the pilot and main mixers using fluidic-elements: One fluidic-element outlet is connected to the pilot mixer and the other is connected to the main inner swirler. Note that active control by the fluidic element is beyond the scope of this study because we focus on the characterization and detection of thermoacoustic combustion oscillations.

Combustion experiments are conducted using a high-pressure and high-temperature combustion test facility at JAXA. The combustion chamber is placed in a high-pressure casing. Glass windows are mounted on the side walls of the combustion chamber and high-pressure casing. The inlet air temperature and pressure are 760 K and 700 kPa, respectively. The pressure fluctuations are measured by a pressure transducer (Kulite Semiconductor Products, Model XTEH-10L-190-1000A) as an important physical quantity representing the combustion state. Note that a pressure transducer is commonly preferable for monitoring the combustor status in practical combustion systems. In this study, the pressure transducer is located on the wall of the combustion chamber, 31 mm from its inlet. The sampling frequency of the pressure fluctuations is 40 kHz. The amount of pilot fuel is set to 1.8 g/s. The amount of main fuel is gradually changed from 7.5 to 9.3 g/s so as to induce well-developed thermoacoustic combustion oscillations with both a dominant frequency-mode-shift and notable oscillation amplitudes in a staged aircraft engine model combustor as a representative case.

III. METHODOLOGICAL FRAMEWORK OF TIME SERIES ANALYSIS BASED ON SYMBOLIC DYNAMICS AND STATISTICAL COMPLEXITY

Here, we briefly review the methodologies employed in this study to quantify the dynamic behavior of pressure fluctuations.

A. Permutation entropy

The idea of entropy, which can be represented as the rate of production of information, is of fundamental importance for treating complex nonlinear phenomena. Information entropy considering permutation patterns represented by the rank order relations among the values of a time series, referred to as the permutation entropy,[22] has recently received a lot of attention.[24] The permutation entropy based on the concept of symbolic dynamics enables us to capture significant changes in the randomness of dynamic behavior and has recently been adopted for the analysis of various types of combustion and flame front instability.[7,17,25–27] To estimate the permutation entropy in a similar manner to in Ref. 26, we first consider all $D!$ possible permutations of successive data points in a time series consisting of $\mathbf{p}(t)\ (=(p'(t), p'(t+1), \ldots, p'\ (t+D-1))$, where p' are the pressure fluctuations and D is the embedding dimension. After obtaining the probability distribution of each permutation pattern $p(\pi_i)(i=1,2,\ldots,D!)$, we estimate the permutation entropy $H_p[\mathbf{P}]$ normalized by the maximum permutation entropy $H_{p,\max}\ (=\log_2 D!)$,

$$H_p[\mathbf{P}] = \frac{-\sum_{\pi_i} p(\pi_i)\log_2 p(\pi_i)}{H_{p,\max}}. \tag{1}$$

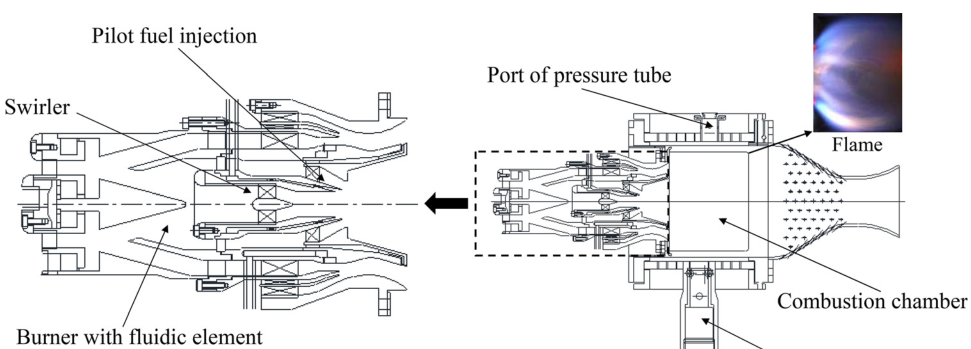

FIG. 1. Experimental apparatus.

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Here, H_p ranges from 0 to 1. $H_p = 0$ corresponds to a monotonically increasing or decreasing process, while $H_p = 1$ corresponds to a completely random process.

B. Multiscale complexity-entropy causality plane

The multiscale complexity-entropy causality plane (CECP), consisting of the Jensen-Shannon statistical complexity and the permutation entropy, is becoming useful for distinguishing between the deterministic dynamics and pure stochastic dynamics in finite time series contaminated with an additive uncorrelated noise.[23] The importance of the multiscale CECP has been studied by Zunino *et al.*[23] using noisy dynamical systems. In fact, the utility of the multiscale CECP has been shown in a recent experimental study on flame front instability induced by buoyancy-swirl coupling.[27] On the basis of the idea considered by Lopez-Ruiz *et al.*,[28] the Jensen-Shannon statistical complexity C_{JS} is defined as the product of the disequilibrium Q_{JS} and the permutation entropy H_p

$$C_{JS}[\mathbf{P}] = Q_{JS}[\mathbf{P}, \mathbf{P_e}] H_p[\mathbf{P}], \tag{2}$$

$$Q_{JS}[\mathbf{P}, \mathbf{P_e}] = \frac{\log_2(D!)}{Q_{JS,max}} \{H_p[(\mathbf{P} + \mathbf{P_e})/2] - H_p[\mathbf{P}]/2 - H_p[\mathbf{P_e}]/2\}, \tag{3}$$

$$Q_{JS,max} = -\frac{1}{2}\left\{\frac{D!+1}{D!}\log_2(D!+1) - 2\log_2(2D!) + \log_2(D!)\right\}. \tag{4}$$

Here, $\mathbf{P}_e$ $(= 1/D!, 1/D!, \ldots, 1/D!)$. Note that for the computation of the multiscale CECP, the D-dimensional phase space consists of $\mathbf{p}(t)$ $(= (p'(t), p'(t+\tau), \ldots, p'(t+(D-1)\tau)))$ to consider the variations in the embedding time delay τ of the phase space. The multiscale CECP represents the evolution of the complexity considering both the randomness and the disequilibrium as a function of the embedding time delay.

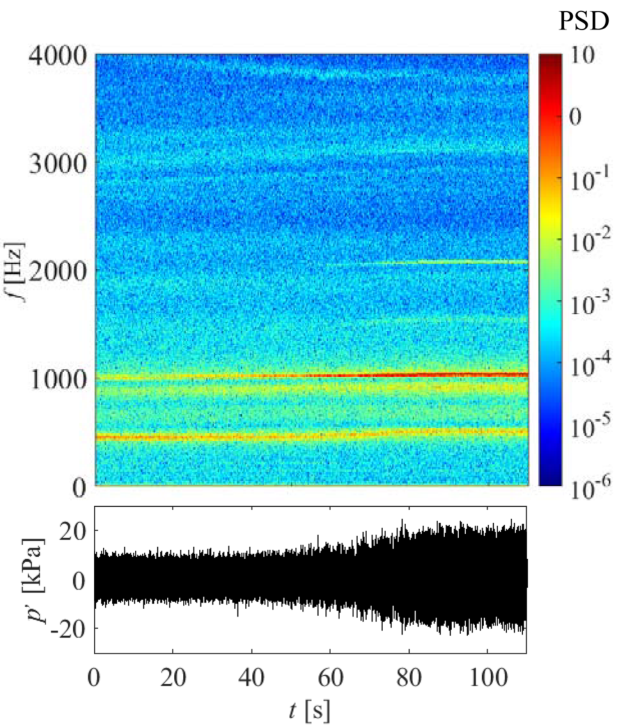

FIG. 2. Temporal evolutions of pressure fluctuations and the corresponding power spectrum during thermoacoustic combustion oscillations.

IV. RESULTS AND DISCUSSION

Figure 2 shows the temporal evolutions of pressure fluctuations and the corresponding power spectrum. We observe that when $t = 40$ s, two distinct peaks appear at approximately 450 and 1000 Hz. The former instability mode corresponds to the acoustic fundamental mode in the longitudinal direction of the combustor, while the latter mode corresponds to its second-harmonic mode. The second-harmonic mode becomes predominant during thermoacoustic combustion oscillations with an increasing amount of main fuel, leading to a notable increase in the oscillation amplitudes. As will be shown later, the permutation entropy is strongly affected by the frequency of the sine wave (see Fig. 4). This makes the use of the permutation entropy difficult when treating thermoacoustic combustion oscillations with multiple dominant peaks in the power spectrum, which provides us with a new and important issue in the potential practical use of the permutation entropy. Therefore, in this study, we propose a modified version of the permutation entropy for detecting a precursor of the significant shift in the dominant frequency-mode before the amplification of pressure fluctuations ($t \approx 50$ s).

Figure 3 shows the variations in the Jensen-Shannon statistical complexity C_{JS} and the permutation entropy H_p in terms of the embedding delay time τ, together with the $H_p - C_{JS}$ plane. $H_p(C_{JS})$ for pressure fluctuations at $t = 10$–12 s first reaches almost unity (zero) at $\tau = 90$. These extreme values appear at intervals of 90 with increasing τ. This value corresponds to the period of the acoustic fundamental mode. We observe the movement of the trajectory from the left side to the right side on the $H_p - C_{JS}$ plane as τ is increased. The important point here is the significant reverse motion of the trajectory in the extracted region of the $H_p - C_{JS}$ plane, indicating the possible presence of noisy periodic dynamics.[23] $H_p(C_{JS})$ at $t = 70$–72 s also takes a value of almost unity (zero), similarly to at $t = 10$–12 s, but the extreme values first appear at $\tau = 39$. This value corresponds to the period of the second-harmonic mode. The trajectory of the $H_p - C_{JS}$ plane nearly corresponds to that obtained for a stochastically driven van der Pol oscillator.[23] This means that the pressure fluctuations at $t = 70$–72 s represent the noisy periodic dynamics. A similar trend becomes prominent at $t = 100$–102 s, showing the strong presence of noisy periodic dynamics. The multiscale CECP is valid for showing the presence of noisy periodic dynamics during thermoacoustic combustion oscillations with a significant shift in the dominant frequency-mode.

Figure 4 shows the variation in the permutation entropy H_p as a function of the frequency of the sine wave $(= \sin(2\pi ft)$, where f is frequency). We clearly observe that f strongly affects H_p. This means that the permutation entropy changes in response to the frequency when multiple dominant peaks are formed in the power spectrum during thermoacoustic combustion oscillations and the frequency range is different from the expected dominant frequency. On this

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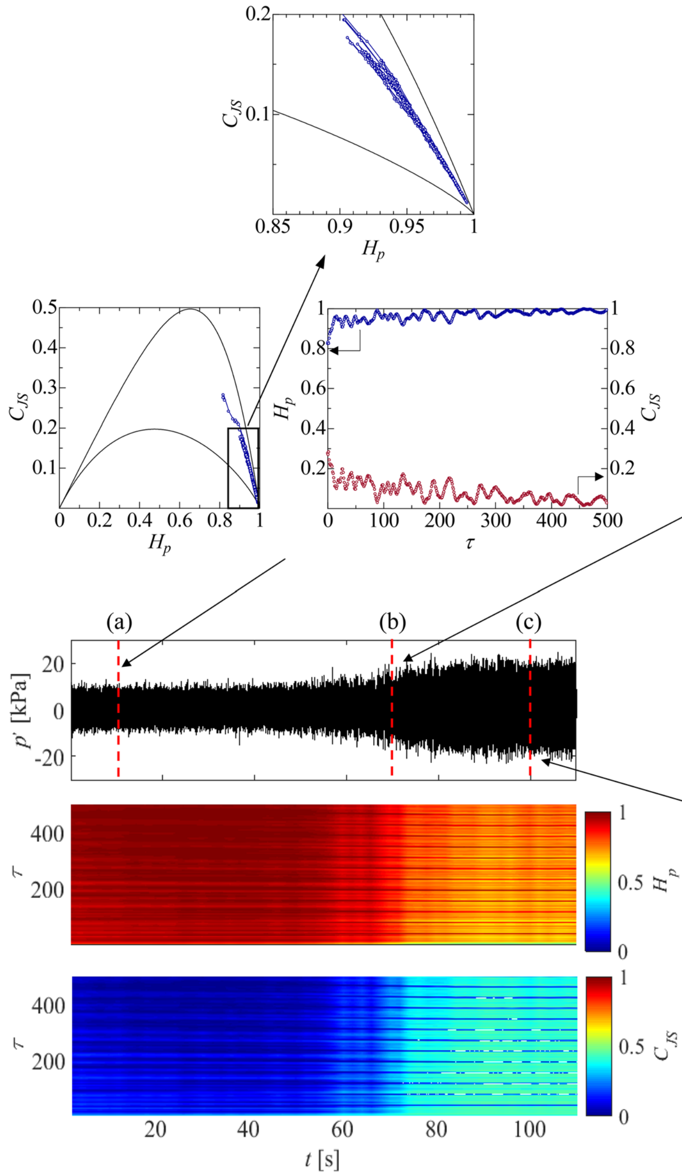

FIG. 3. Variations in the Jensen-Shannon statistical complexity C_{JS} and the permutation entropy H_p in terms of the embedding delay time τ, together with the $H_p - C_{JS}$ plane.

basis, we here propose a modified version of the permutation entropy as

$$H_p^*[\mathbf{P}] = \frac{H_p[\mathbf{P}] - H_{p,\sin}[\mathbf{P}]}{1 - H_{p,\sin}[\mathbf{P}]}, \tag{5}$$

where H_p^* is a modified permutation entropy incorporating the effect of the frequency and $H_{p,\sin}$ is the permutation entropy for the sine wave. We set the value of $H_{p,\sin}$ using the dominant frequency in the pressure fluctuations extracted from the power spectrum distribution. Temporal evolutions of the root mean square of the pressure fluctuations $p'_{\rm rms}$, H_p, and H_p^* are shown in Fig. 5. $p'_{\rm rms}$ remains nearly unchanged until t is approximately 50 s, while H_p begins to change from $t = 42$ s. They markedly change at $68\,\mathrm{s} \le t \le 80\,\mathrm{s}$, corresponding to the significant increase in the oscillation amplitude of p'. H_p finally reaches approximately 0.4 owing to the shift to the second-harmonic mode with a stronger periodicity than the fundamental mode. In contrast, H_p^* begins to change from $t = 25$ s and markedly decreases at $68\,\mathrm{s} \le t \le 80\,\mathrm{s}$, similarly to H_p. These results indicate that our modified version of the permutation entropy is feasible for capturing the precursor of the dominant frequency-mode-shift earlier than the root mean square of the pressure fluctuations and the original permutation entropy. Figure 6 shows the relation between X and H_p^*. Here, X $(= A_p/A_e)$ is the area ratio of the power spectrum density, A_p is the sum of the power spectrum density in the range of $f_p \pm 50$ Hz, f_p is the dominant frequency in the pressure fluctuations, and A_e is the sum of the power spectrum density in the entire frequency range. Note that τ is set to $N_{\rm period}/D$, where $N_{\rm period}$ is the number of discrete data corresponding to the dominant period of pressure fluctuations. We clearly observe a formation of two isolated regimes A and B on the $X - H_p^*$ plane. The plots are always located in regime A until $t = 8$ s. When t reaches approximately 20 s, they begin to be located in the upper part of regime B owing to the

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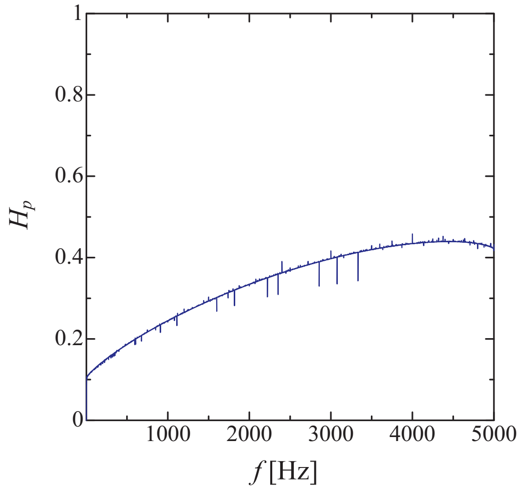

FIG. 4. Variation in the permutation entropy H_p as a function of the frequency of the sine wave f.

appearance of the dominant frequency-mode-shift, then move back and forth between the two regimes at $20\,\mathrm{s} \leq t \leq 68\,\mathrm{s}$ with increasing X owing to an increase in the periodicity of the dominant second-harmonic mode. Note that in our preliminary test, the original permutation entropy cannot divide the dynamic behavior of pressure fluctuations into those two regimes. These results clearly show that the $X - H_p^*$ plane has a potential use for monitoring the precursor of the dominant frequency-mode-shift in noisy periodic dynamics.

Well-known measures such as the largest Lyapunov exponent, the correlation dimension, and the recurrence quantifications have been widely used in previous studies on thermoacoustic systems.[5,8,9,13,29–31] These measures are important for characterizing the combustion dynamics but fail to capture the subtle changes in deterministic dynamics under a small amount of additive noise.[32] The permutation entropy was proposed as a useful measure for evaluating the randomness of dynamic behavior in noisy time series data, and has been adopted for various types of combustion system.[7,25–27,33] However, as is shown in Fig. 4, the oscillation frequency in time series data strongly affects the permutation entropy. In this sense, the modified version of the permutation entropy

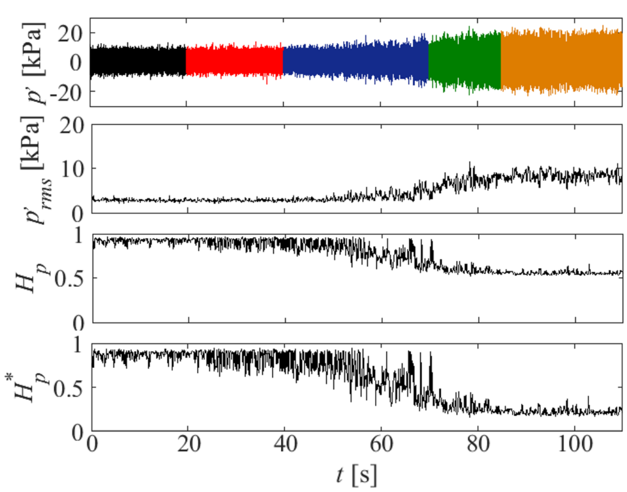

FIG. 5. Temporal evolutions of the root mean square of the pressure fluctuations p'_{rms}, H_p, and H_p^*.

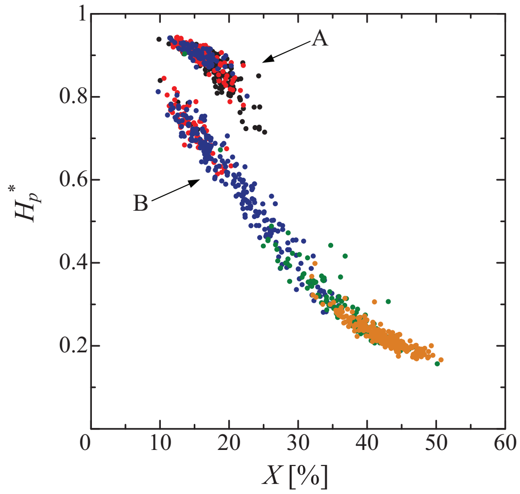

FIG. 6. Relation between X and H_p^*. Here, X $(=A_p/A_e)$ is the area ratio of the power spectrum density, A_p is the sum of the power spectrum density in the range of $f_p \pm 50\,\mathrm{Hz}$, f_p is the dominant frequency in the pressure fluctuations, A_e is the sum of the power spectrum density in the entire frequency range.

proposed in this study is valid for detecting the precursor of a significant shift in the dominant frequency-mode in noisy time series data, which is difficult to discern by only power-spectrum analysis. Finally, it would be interesting to examine (1) how the modified version of the permutation entropy changes as a result of the intermittency present during the transition from combustion noise to thermoacoustic combustion oscillations and (2) how the modified version of the permutation entropy changes when the oscillation amplitudes remain nearly unchanged.

V. SUMMARY

We have conducted an experimental study using time series analysis based on symbolic dynamics to detect a precursor of frequency-mode-shift during thermoacoustic combustion oscillations in a staged aircraft engine model combustor. With increasing amount of the main fuel, a significant shift in the dominant frequency-mode occurs in noisy periodic dynamics, leading to a notable increase in the oscillation amplitudes of pressure fluctuations. The sustainment of noisy periodic dynamics during thermoacoustic combustion oscillations is clearly shown by the multiscale complexity-entropy causality plane in terms of statistical complexity. The modified version of the permutation entropy proposed in this study enables us to detect a precursor of the significant shift in the dominant frequency-mode before the amplification of pressure fluctuations ($t \approx 50\,\mathrm{s}$), which is difficult to discern by only power-spectrum analysis.

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