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# One-Dimensional Model of Self-Organization in Dusty Plasma

By

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**Abstract :** Dust particles in plasma are self-organized due to strong Coulomb coupling. In order to explain the self-organization mechanism of dust particles in plasma, one-dimensional theoretical model is introduced. This model predicts variation of the Gibbs free energy against the distance between the dust particles. From the calculation results of the free energy variation, it is found that the attractive-repulsive force exists in the case of small particles. As the particle size increases, the force changes from the attractive-repulsive force to the repulsive one. In the attractive-repulsive region, particle distance can be estimated, though the distance estimation is difficult in the repulsive region. Hence, the distance dependency on the particle radius is investigated in the attractive-repulsive region. From the dependency, it is found that the distance is proportional to the square root of the radius. In addition, it is found that the distance is proportional to the fourth power root of the coupling parameter.

**Key words :** dusty plasma, self-organization, one-dimensional model, free energy, Coulomb coupling parameter, attractive-repulsive force, repulsive force, particle distance

## 1. Introduction

It was predicted that strongly coupled plasma (SCP) behaved like liquid or solids<sup>1,2)</sup>. However, it is difficult to obtain the SCP in laboratories by using usual plasma. The successful observations of the SCP were reported only about ten years ago by using dusty plasma<sup>3-8)</sup>. These experiments showed that dust particles were regularly self-organized like atoms in crystals. In addition, several structures of the self-organization were also shown. These structures are the same as structures of real crystals, that is, a simple-hexagonal (sh), body-centered cubic (bcc) and face-centered cubic (fcc) structures. Hence, the self-organization is called plasma crystals or Coulomb crystals.

Plasma crystals have a potential superiority to research the physics of the SCP since it is easy to obtain the stable SCP in laboratories. The plasma crystals are also suitable model systems for researching fundamental physics such as critical point phenomena<sup>9)</sup>, phase transition<sup>10-12)</sup> or phonon propagation<sup>13-15)</sup>. In addition, the plasma crystals contribute to the research of the astrophysics such as interstellar and interplanetary dust particles<sup>16,17)</sup>, or planetary rings<sup>18)</sup>.

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Although the dusty plasma has many advantages in many scientific disciplines as mentioned above, it is still unclear whether the self-organization is caused by attractive-repulsive force or by only repulsive force. In many papers, the latter force is described as the essential force in dusty plasma. In this case, the dusty plasma is confined or packed by potential formed around the dusty plasma. This means that the confinement potential must be stronger as the dusty plasma pressure increases. On the other hand, if the attractive-repulsive force is essential to the self-organization in dusty plasma, no confinement potential is basically required. Therefore, to clarify whether the self-organization mechanism is the attractive-repulsive force or the repulsive force is essential scientific issue in the research field of the dusty plasma. In addition, if the attractive-repulsive force existed in the dusty plasma, this force should work like bonding force between atoms. Therefore, in this case, the plasma crystals should be an ideal model system for researching crystal growth mechanisms such as morphological instability, nucleation or step kinetics.

Thus, in order to explain the self-organization, we introduce a one-dimensional model of dusty plasma by considering the Coulomb interaction. The introduced model expresses the difference of the Gibbs free energy between before and after the self-organization. The one-dimensional model should differ from a two- or three-dimensional model by the factor of  $1/r$ . However, this factor should not obstruct essential understandings of the self-organization mechanisms. Since the one-dimensional model has a great feature that the Coulomb interaction can be expressed as a linear combination of potential profiles around a single dust particle, influence of each term in the model on the change of the Gibbs free energy is quantitatively investigated. This will help to understand the essence of the self-organization mechanisms. By using the one-dimensional model, the change of the free energy is discussed against the particle distance. In addition, the scaling law is discussed by basing on the calculation results of the one-dimensional model.

## 2. Model

### 2.1. Power flow

In order to introduce a model of the difference of the Gibbs free energy between before and after the self-organization, we consider power flow of the dusty plasma in the steady state. The power flow is shown in Fig. 1. First, the power is supplied from a power source and is absorbed by the plasma. Then the absorbed power is consumed through three processes. The first is power losses, which are caused by the plasma and the dust particles escaping from the system. The second is plasma production and heating. The last is heating of dust particles.

The power losses, the first process, do not affect the system energy at all. Therefore, this is neglected to estimate the change of the Gibbs free energy. The dust heating power, the third process, is the only power to give kinetic energy to the particles. The increased kinetic energy is the driving force of the self-organization. The kinetic energy of the particles is consumed in approaching each other against increasing repulsive force. The repulsive force is formed since the inside electric field decreases as the particles approach each other as compared with the outside electric field. This is described later in 2.4. Finally, the kinetic energy is negligibly small and the self-organization is formed. This means that the kinetic energy is consumed in decreasing entropy. The residual process, the plasma production and heating, must be also counted. This contributes for increasing stored energy of plasma. The kinetic energy of the dust particles and the stored energy, which will give the change of the Gibbs free energy, are indicated as black-colored arrows in Fig. 1.

### 2.2. Stored energy before self-organization

The minimum self-organization structure is composed by two particles. In addition, a self-organized structure composed by numbers of the particles can be easily estimated from the summation of the energy of the minimum

structure. Therefore, the two particles are not only the simplest case but also the essential case. Thus the stored energy under the condition that two particles exist in the plasma is assumed to make a one-dimensional model of the self-organization. The configuration for modeling is shown in Fig. 2.

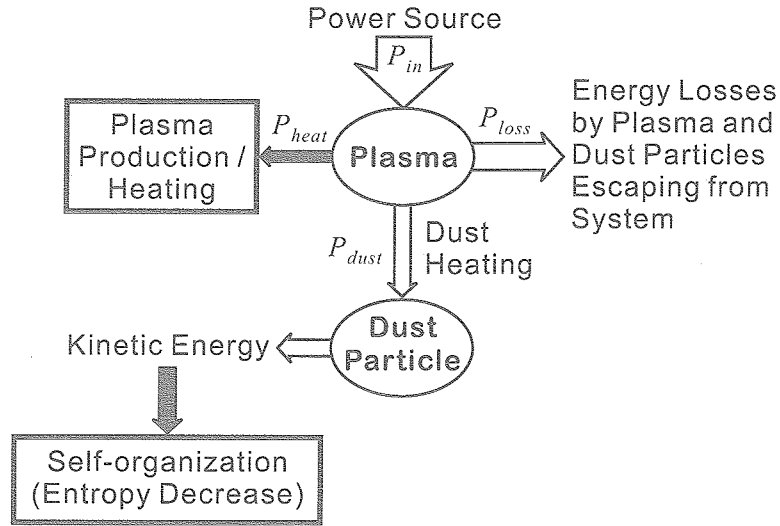


Fig.1 Power flow of dusty plasma for considering a model. In this paper, it is assumed that the black-colored power flow changes the Gibbs free energy in the steady state.

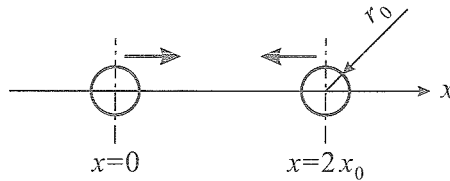


Fig.2 Configuration for one-dimensional modeling.  $r_0$  and  $x_0$  represent the particle radius and the half distance between the centers of the particles, respectively.

The non-organization means that the two particles are far away from each other and can move individually. Namely, the potential profile around the particle is the same as that of a single particle in plasma. In this paper, the one-dimensional Descartes coordinate system is used. Hence the potential is expressed as

$$\phi_1(x) = -(\phi_p - \phi_0) \exp\left(-\frac{x-r_0}{\lambda_D}\right) + \phi_p \quad (x \geq r_0), \quad (1)$$

where  $\phi_1$  is the potential around the particle,  $\phi_p$  the plasma potential,  $\phi_0$  the potential on the particle surface,  $r_0$  the particle radius,  $\lambda_D$  the Debye length. Since the potential profile is the Yukawa-type potential in the two- or three-dimensional coordinate system or in the spherical symmetric system, the Yukawa potential may be more strict expression than eq. (1). However, the difference between  $\phi_1$  and the Yukawa potential is only the factor of  $1/r$ . This means that the behavior of eq. (1) is basically the same as the Yukawa potential. Therefore, our one-dimensional model brings essential understanding of actual dusty plasmas though quantitative predictions such as a particle

distance may have a certain error by using our one-dimensional model as compared with experimental measurements in actual dusty plasmas.

The stored energy related to the kinetic energy  $E_1$  is described as

$$E_1 = 4 \int_{r_0}^{\infty} n_{e_0} \exp\left[-\frac{q_e(\phi_1 - \phi_p)}{k T_e}\right] \frac{1}{2} k T_e dx + 4 \int_{r_0}^{\infty} n_{i_0} \frac{1}{2} k T_i dx, \quad (2)$$

where  $n_{e_0}$  is electron density where the potential is equal to the plasma potential,  $n_{i_0}$  ion density,  $T_e$  electron temperature and  $T_i$  ion temperature,  $q_e$  the electron charge. The Maxwellian is assumed as the velocity distribution function. The factor of 4 means that there are two particles, which have plus and minus sides.

The stored energy related to the potential energy  $W_1$  is described as

$$W_1 = 4 \int_{r_0}^{\infty} n_{e_0} \exp\left[-\frac{q_e(\phi_1 - \phi_p)}{k T_e}\right] q_e (\phi_1 - \phi_p) dx + 4 \int_{r_0}^{\infty} n_{i_0} q_i (\phi_1 - \phi_p) dx, \quad (3)$$

where  $q_i$  is the ion charge. The total stored energy  $U_1$  in this case is the summation of  $E_1$  and  $W_1$ , that is,

$$U_1 = E_1 + W_1. \quad (4)$$

### 2. 3. Stored energy after self-organization

It is assumed that the potential profile between the two particles after the self-organization is described as the summation of the each potential profile, that is,

$$\begin{aligned} \phi_2 = & -(\phi_p - \phi_0) \exp\left(-\frac{x - r_0}{\lambda_D}\right) - (\phi_p - \phi_0) \exp\left(-\frac{2x_0 - r_0 - x}{\lambda_D}\right) \\ & + \phi_p + (\phi_p - \phi_0) \exp\left(-\frac{2x_0 - 2r_0}{\lambda_D}\right) \quad (r_0 \leq x \leq 2x_0 - r_0) \end{aligned}, \quad (5)$$

where  $x_0$  is the half distance between the center of the particles. Such the potential summation is a feature of the expression in the one-dimensional Descartes coordinate. The potential profile at the other sides is the same as the profile expressed in eq. (1). Therefore, the stored energy related to the kinetic energy  $E_2$  is described as

$$E_2 = 2 \int_{r_0}^{x_0} n_{e_0} \exp\left[-\frac{q_e(\phi_2 - \phi_p)}{k T_e}\right] \frac{1}{2} k T_e dx + 2 \int_{r_0}^{x_0} n_{i_0} \frac{1}{2} k T_i dx + \frac{E_1}{2}. \quad (6)$$

In addition, the stored energy related to the potential energy  $W_2$  is described as

$$W_2 = 2 \int_{r_0}^{x_0} n_{e_0} \exp\left[-\frac{q_e(\phi_2 - \phi_p)}{k T_e}\right] q_e (\phi_2 - \phi_p) dx + 2 \int_{r_0}^{x_0} n_{i_0} q_i (\phi_2 - \phi_p) dx + \frac{W_1}{2}. \quad (7)$$

Thus the total stored energy  $U_2$  is described as

$$U_2 = E_2 + W_2. \quad (8)$$

### 2. 4. Entropy change

The kinetic energy given by the plasma to the particles is consumed in approaching to each other. Of course, the kinetic energy may be used to be apart from each other. However, such the particles will escape from the system. Therefore, this corresponds to the energy loss and should not be counted as the entropy-related energy. In order to

obtain the entropy change, the electric fields in the case of the sole particle and in the case of the self-organized particles are described first. In the former case, the field is described as

$$\left. \frac{\partial \phi_1}{\partial x} \right|_{x=r_0} = \frac{\phi_p - \phi_0}{\lambda_D}. \quad (9)$$

On the other hand, in the latter case, the field is

$$\begin{aligned} \left. \frac{\partial \phi_2}{\partial x} \right|_{r=r_0} &= \left[ \frac{\phi_p - \phi_0}{\lambda_D} \exp\left(-\frac{x-r_0}{\lambda_D}\right) - \frac{\phi_p - \phi_0}{\lambda_D} \exp\left(\frac{x-X+r_0}{\lambda_D}\right) \right]_{x=r_0}, \\ &= \frac{\phi_p - \phi_0}{\lambda_D} - \frac{\phi_p - \phi_0}{\lambda_D} \exp\left(\frac{2r_0 - X}{\lambda_D}\right) \end{aligned} \quad (10)$$

where the centers of the two particles are assumed to be located at  $x = 0$  and  $x = X$ . Therefore, the force given by the electric field is described as

$$\begin{aligned} F &= Q \left( -\frac{\partial \phi_1}{\partial x} + \frac{\partial \phi_2}{\partial x} \right) = Q \left[ -\frac{\phi_p - \phi_0}{\lambda_D} + \frac{\phi_p - \phi_0}{\lambda_D} - \frac{\phi_p - \phi_0}{\lambda_D} \exp\left(\frac{2r_0 - X}{\lambda_D}\right) \right] \\ &= \frac{Q(\phi_p - \phi_0)}{\lambda_D} \exp\left(\frac{2r_0 - X}{\lambda_D}\right) \end{aligned} \quad (11)$$

at  $x = r_0$ . Since it is reported that the dust particles have the negative charges in many experiments, eq. (11) represents the repulsive force. However, as mentioned in subsection E, whether the absolute value of  $Q$  is negative may depend on the temperature. If low temperature but high plasma potential is achieved, the force becomes attractive force and thus condensation may occur. In this paper,  $Q$  is assumed to be negative.

The kinetic energy is consumed in approaching the particles against the repulsive force. This consumed energy is described as

$$\begin{aligned} \Delta W &= 2 \int_a^{2x_0} \frac{Q(\phi_p - \phi_0)}{\lambda_D} \exp\left(\frac{2r_0 - X}{\lambda_D}\right) dX = -2Q(\phi_p - \phi_0) \left[ \exp\left(\frac{2r_0 - X}{\lambda_D}\right) \right]_a^{2x_0}, \\ &= -2Q(\phi_p - \phi_0) \exp\left(\frac{2r_0 - 2x_0}{\lambda_D}\right) + 2Q(\phi_p - \phi_0) \exp\left(\frac{2r_0 - a}{\lambda_D}\right) \end{aligned} \quad (12)$$

where  $a$  is the coordinate being far enough from  $2x_0$ . The  $\Delta W$  corresponds to the entropy decrease,  $\Delta(ST)$ .

## 2. 5. Potential on particle surface

The potential of the particle surface  $\phi_0$  is described here. The dust particles electrically float from the ground potential. This means that ion current density  $j_i$  is equal to electron current density  $j_e$  on the particle surface. Thus,  $j_i$  and  $j_e$  are expressed as

$$j_i = Z e n_{i0} \left( \frac{k T_i}{8 \pi m_i} \right)^{1/2} \exp \left[ -\frac{e Z (\phi_0 - \phi_p)}{k T_i} \right], \text{ and} \quad (13)$$

$$j_e = e n_{e0} \left( \frac{k T_e}{8 \pi m_e} \right)^{1/2} \exp \left[ \frac{e (\phi_0 - \phi_p)}{k T_e} \right], \quad (14)$$

respectively. In these equations,  $m_i$  and  $m_e$  represent ion mass and electron mass, respectively. By using eqs.

(13) and (14),

$$\phi_0 \left( \frac{eZ}{kT_i} + \frac{e}{kT_e} \right) - \phi_p \left( \frac{eZ}{kT_i} + \frac{e}{kT_e} \right) = \ln \left( \frac{Z n_{i_0}}{n_{e_0}} \right) + \frac{1}{2} \ln \left( \frac{T_i m_e}{T_e m_i} \right) \quad (15)$$

is obtained. Here the relation of  $Z n_{i_0} \approx n_{e_0}$  is substituted to eq. (15). In addition, the relation of  $T_i \approx T_e = T_0$  is assumed in this paper. Then eq. (15) is rewritten as

$$(\phi_0 - \phi_p) \frac{e(Z+1)}{kT_0} = \frac{1}{2} \ln \left( \frac{m_e}{m_i} \right). \quad (16)$$

Therefore, the potential on the particle surface is described as

$$e\phi_0 = \frac{1}{2} \frac{kT_0}{(Z+1)} \ln \left( \frac{m_e}{m_i} \right) + e\phi_p. \quad (17)$$

The first term on the right side of eq. (17) is negative since  $m_e$  is much less than  $m_i$ . Namely, the potential on the particle surface is relatively negative as compared with the plasma potential. However, whether the potential is absolutely negative or not may depend on the temperature.

### 2. 6. Change of Gibbs free energy

By using eqs. (4), (8) and (12), the difference of the Gibbs free energy between before and after the self-organization is described as

$$\begin{aligned} \Delta G = & 2 \int_{r_0}^{x_0} n_{e_0} \exp \left[ -\frac{q_e(\phi_2 - \phi_p)}{kT_e} \right] \left[ \frac{1}{2} kT_e + q_e(\phi_2 - \phi_p) \right] dx + 2 \int_{r_0}^{x_0} n_{i_0} \left[ \frac{1}{2} kT_i + q_i(\phi_2 - \phi_p) \right] dx \\ & - 2 \int_{r_0}^{\infty} n_{e_0} \exp \left[ -\frac{q_e(\phi_1 - \phi_p)}{kT_e} \right] \left[ \frac{1}{2} kT_e + q_e(\phi_1 - \phi_p) \right] dx - 2 \int_{r_0}^{\infty} n_{i_0} \left[ \frac{1}{2} kT_i + q_i(\phi_1 - \phi_p) \right] dx. \quad (18) \\ & - 2Q(\phi_p - \phi_0) \exp \left( \frac{2r_0 - 2x_0}{\lambda_D} \right) + 2Q(\phi_p - \phi_0) \exp \left( \frac{2r_0 - a}{\lambda_D} \right) \end{aligned}$$

## 3. Results and Discussion

Since eq. (18) is nonlinear integral equation, we carry out the numerical calculation to obtain the  $\Delta G$ . To calculate eq. (18), we must estimate charges on the particle surface. The charge on the surface is described as

$$Q = -4\pi r_0^2 \varepsilon_0 \frac{\phi_p - \phi_0}{\lambda_D}. \quad (19)$$

Equation (19) indicates that there is the minimum  $r_0$  since the minimum  $Q$  exists and is equal to the elementary charge. If the particle radius is less than the minimum  $r_0$ , the particle cannot maintain the surface charge. In such case, the dust particle should be a neutral particle and thus the dusts separate from the system.

In order to calculate the eq. (18), we must define one-dimensional density though the density is generally the three-dimensional. The dimension is converted by using the following equation of

$$n_{1d} = \left( \frac{4\pi n_{3d}}{3} \right)^{\frac{1}{3}}, \quad (20)$$

where  $n_{1d}$  is the density with the one dimension and  $n_{3d}$  is that with the three dimension.

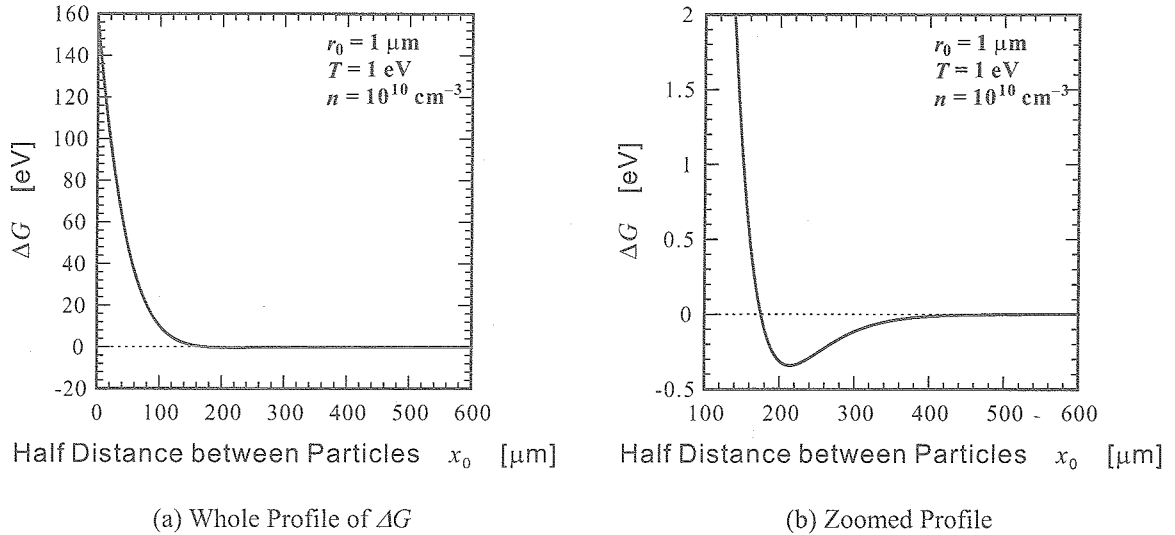


Fig.3  $\Delta G$  dependency on the half distance between the centers of the dust particles in the case of the particle radius of  $1 \mu\text{m}$ . The thin dashed line represents the origin of the vertical axis.

Here, we substitute the density of  $10^{10} \text{ cm}^{-3}$ , the temperature of  $1 \text{ eV}$ , the plasma potential of  $1 \text{ V}$  to eq. (18). These parameters are in the range of the typical parameters in low temperature plasma and high pressure neutral gas environment. In addition, since the infinity is not suitable for the numerical integral, the  $1000\lambda_D$  instead of the infinity, which is long enough, is used. We assume that the initial  $x_0$  is  $1000\lambda_D$ . Therefore, the total length of the system is  $4000\lambda_D$ . The calculation result in the case of the particle radius of  $1 \mu\text{m}$  is shown in Fig. 3. Figure 3 (a) shows the  $\Delta G$  dependency on the half distance between the centers of the particles,  $x_0$ . From this figure, it is found that the maximum  $\Delta G$  exists at  $x_0 = 1 \mu\text{m}$ , that is, the distance from one particle surface to another is zero. In addition, it seems that the  $\Delta G$  approximates to 0 as  $x_0$  increases. To know the detailed  $\Delta G$  variation, the  $\Delta G$  dependency is zoomed and is shown in Fig. 3 (b). From this figure, it is found that the  $\Delta G$  has the minimum value of about  $-0.34 \text{ eV}$  at about  $x_0 = 214 \mu\text{m}$ . This profile indicates the attractive-repulsive force, which is similar to the atomic bonding. The profile means that the self-organization occurs if the kinetic energy of the dust particle is less than  $0.34 \text{ eV}$  at the distance of  $428 \mu\text{m}$  between the centers of the two particles. Figure 4 is the result in another case of the particle radius of  $10 \mu\text{m}$ . This figure shows that no minimum  $\Delta G$  exists. This means that only repulsive force exists. Therefore, the spontaneous organization never occurs. In order to obtain a regular pattern formation, confinement potential is required to pack the particles within a certain area. Although other results in the cases of other radii are not shown, the change from the attractive-repulsive force to the repulsive force occurs gradually and smoothly by changing the particle radius in a wide range of plasma parameters. In order to understand the behavior of the minimum  $\Delta G$ , the  $\Delta G$  dependency on the particle radius is investigated and is shown in Fig. 5. From Fig. 5, it is found that the absolute value of the minimum  $\Delta G$  steeply decreases as the radius increases. The  $\Delta G$  is almost 0 at about the radius of  $3 \mu\text{m}$ . Since the absolute value of  $\Delta G$  must be larger than the kinetic energy of the particle to trap the



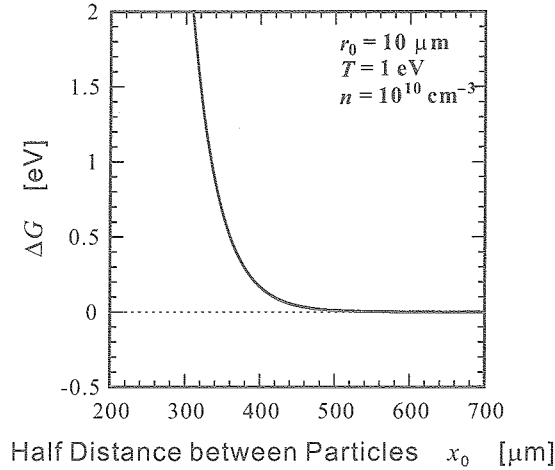


Fig.4  $\Delta G$  dependency on the half distance between the centers of the dust particles in the case of the particle radius of  $10\mu\text{m}$ . The thin dashed line represents the origin of the vertical axis.

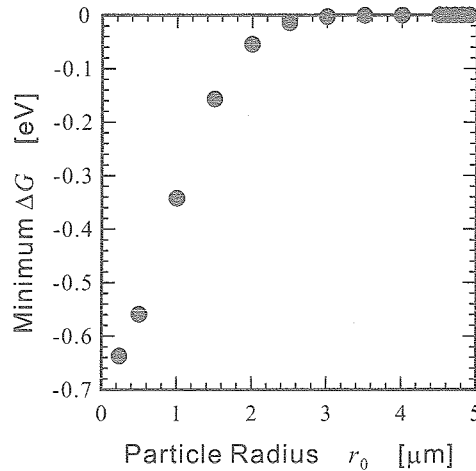


Fig.5  $\Delta G$  dependency on the particle radius. The closed circles represent the numerically calculated  $\Delta G$ .

particle, the self-organization may not occur at the radius of more than about  $1.5\mu\text{m}$ . In the range of more than about  $5\mu\text{m}$ , the negative  $\Delta G$  is not obtained in this condition. Namely, the strict threshold changing from the attractive-repulsive force to the repulsive force is about  $5\mu\text{m}$  in this case.

In order to investigate the reason of the negative  $\Delta G$  occurrence, the behaviors of the terms corresponding to the changes of the kinetic energy, the potential energy and the entropy, that is,  $E_2 - E_1$ ,  $W_2 - W_1$ , and  $\Delta(ST)$ , are obtained. These terms in the case of the particle radius of  $1\mu\text{m}$  are shown in Fig. 6. Figure 6 (a) shows the kinetic energy dependency on the particle distance. The thin solid line, the thin dashed line and the bold solid line represent  $E_2$ ,  $E_1$  and  $\Delta E = E_2 - E_1$ , respectively. The value of the  $\Delta E$  takes the vertical axis on the right side in the figure, while the others take that on the left side. From Fig. 6 (a), it is found that the  $\Delta E$  increases as the distance decreases. This is caused by increasing the total number of the ions and the electrons because the total length of the Debye shielding decreases due to the overlap of the shielding though the system length is constant. Figure 6 (b) shows the potential energy dependency on the particle distance. The thin solid line, the thin dashed line and the bold solid line represent  $W_2$ ,  $W_1$  and  $\Delta W = W_2 - W_1$ , respectively. The value of the  $\Delta W$  takes the vertical axis on the right side in the figure,

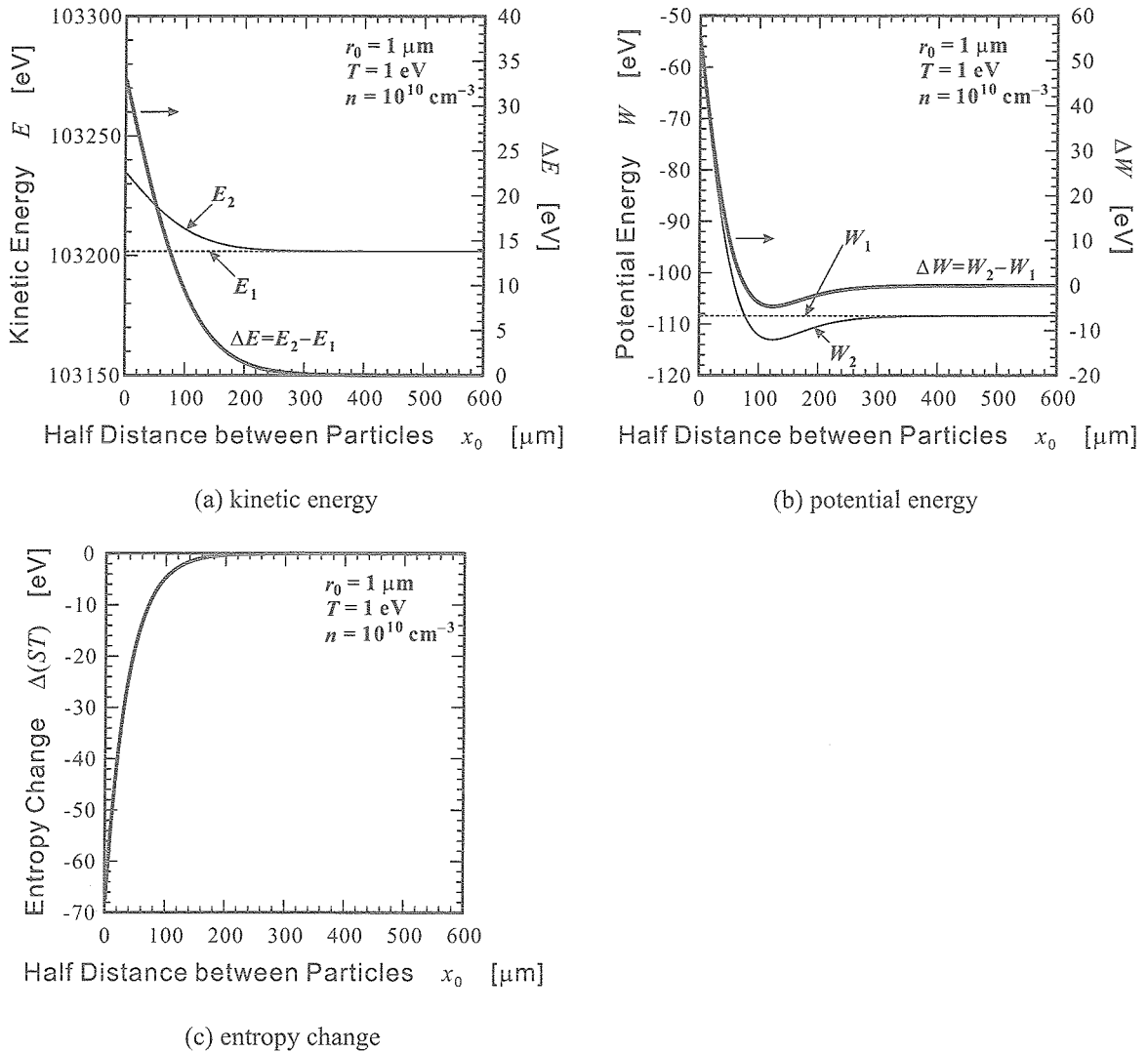


Fig.6 Dependency of terms corresponding to the kinetic energy, the potential energy and the entropy change on the half distance between the centers of the dust particles in the case of the particle radius of  $1 \mu\text{m}$ .

while the others take that on the left side. From Fig. 6 (b), it is found that the  $\Delta W$  once decreases and then increases as the distance decreases. Although this behavior differs from that of the kinetic energy, the reason of the behavior is the same, that is, the overlap of the Debye shielding. Figure 6 (c) shows the entropy change. This figure indicates that the entropy decreases as the particles get closer each other. By calculating  $\Delta E + \Delta W - \Delta(ST)$ , the  $\Delta G$  shown in Fig. 3 is obtained. The term corresponding to  $\Delta(ST)$  in the case of the particle radius of  $10 \mu\text{m}$  is shown in Fig. 7. Since the potential energy and the kinetic energy have no dependency on the particle radius in our one-dimensional model, only the entropy-related term is shown in Fig. 7. Since the surface charge  $Q$  is included in this term, this term is proportional the square of  $r_0$ . This means that the term in the  $10 \mu\text{m}$  case is 100 times larger than that in the  $1 \mu\text{m}$  case. By comparing the dependency in Fig. 7 with that in Fig. 6 (c), this ratio of 100 is clearly confirmed. This dependency of the entropy change on the particle radius is the reason that the negative  $\Delta G$  decreases as the radius increases.

Figure 8 shows that the dependency of the half distance at the minimum  $\Delta G$  on the particle radius. From this figure, it is found that the distance between the particles increases with the particle radius. This is caused by the

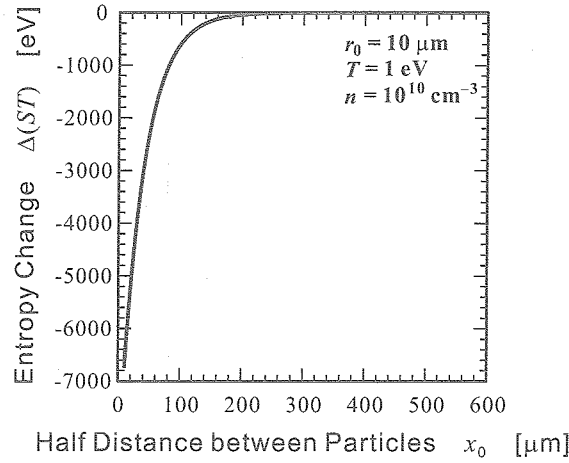


Fig.7 Dependency of terms corresponding to the entropy change on the half distance between the centers of the dust particles in the case of the particle radius of  $10\mu\text{m}$ .

increase of repulsive force, which is caused by the increase of the charges on the particle surface. The charge increase results in the increase of the Coulomb coupling parameter  $\Gamma$ ,

$$\Gamma = \frac{Q^2}{4\pi\epsilon_0 d kT}, \quad (21)$$

where  $d = 2x_0$ ,  $\epsilon_0$  is the dielectric constant. The dependency of the coupling parameter on the particle radius is shown in Fig. 9. The coupling parameters are obtained by assuming the kinetic energy of the dust particle of 0.1 eV. To obtain the coupling parameter  $\Gamma$  of more than a few hundreds, the particle radius of more than  $3.5\mu\text{m}$  is required. In this region,  $\Delta G$  is almost 0 as shown in Fig. 5. Therefore, experimentally observed plasma crystals may be mainly formed by the repulsive force. Electrodes for confining the dusty plasmas are required in such as case. However, as mentioned previously, the self-organization may occur under the condition the particle radius is less than  $1.5\mu\text{m}$  due to the absolute value of the  $\Delta G$  of more than 0.1 eV. Finally, the minimum  $\Delta G$  dependency on the coupling parameter is shown in Fig. 10. From the figure, it is found that the minimum  $\Delta G$  of less than  $-0.1$  eV is in the range of less than

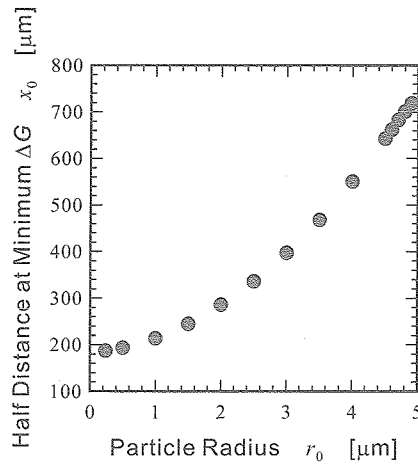


Fig.8 Dependency of the half distance at the minimum  $\Delta G$  on the particle radius. The closed circles represent the half distance obtained from the  $\Delta G$  profile.

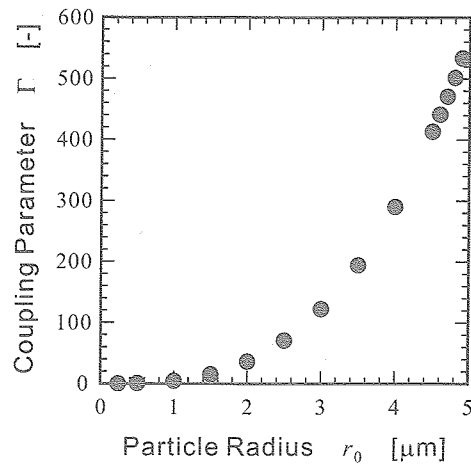


Fig.9 Dependency of the Coulomb coupling parameter on the particle radius. The closed circles represent the coupling parameter under the assumption of the dust particle energy of 0.1 eV.

a few tens of the  $\Gamma$  value. This means that the relatively weak or moderate coupling as compared with the  $\Gamma$  value of the order of a hundred, which is obtained under the condition of the radius of more than  $3\mu\text{m}$  from Fig. 9. It is summarized that the self-organization may spontaneously occur under the condition of weak or moderate coupling plasmas. This phenomenon has not been observed experimentally due to the technical difficulty of a gentle injection of the dust particles into experimental apparatus. However, if the initial kinetic energy of dust particle is small enough, the spontaneous self-organization will be observed.

Experimentally observed plasma crystals have typically two features, that is, the particle distance is a few hundreds micrometers<sup>3, 5, 10, 19)</sup> and the structure is sh, bcc or fcc<sup>3)</sup>. As mentioned above, this structure should be caused by the external confinement potential in the repulsive region. This means that the structure should be dominated by the environmental conditions such as the strength of the confinement potential, the plasma parameters and the dust density. For example, as the confinement potential is stronger or the Coulomb coupling is weaker, the fill fraction of the structure will vary from lower one to higher fill-fraction one. This should be the reason that the plasma crystals observed in laboratories have various structures such as the sh, bcc or fcc structures. The fill-fraction

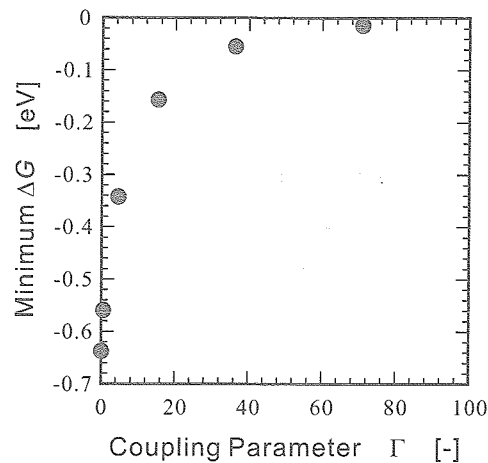


Fig.10 Dependency of the minimum  $\Delta G$  on the Coulomb coupling parameter. The closed circles represent the calculated  $\Delta G$ .

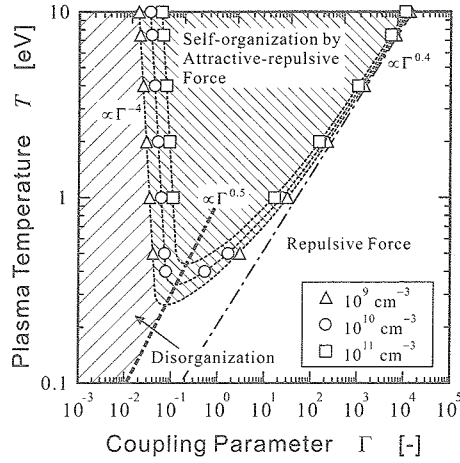


Fig.11 Temperature dependency on the Coulomb coupling parameter. Thin dashed lines represent the interpolation lines. Narrowly and widely hatchings represent the self-organization and the disorganization areas, respectively. Bold dashed line represents the line passing through the minimum of each thin dashed line. Dotted-dashed line represents the asymptote of the thin dashed lines.

of the sh, bcc and fcc structures are less than 60 %, 68 % and 74 %, respectively. Therefore, the structure will be fcc when the kinetic energy of the plasma is low, while the structure will be sh when the energy is high. Although it is difficult to quantitatively predict the particle distance by using our model but a scientifically important issue such as the structure change of the plasma crystal can be discussed by basing on our model.

In order to understand the self-organization in the attractive-repulsive region, the dependency of the particle distance on the Coulomb coupling parameter is investigated. The typical result is shown in Fig. 11 in the cases of the various plasma densities. This figure shows that three regions exist, that is, disorganization, attractive-repulsive and repulsive regions. It is found that the boundary between the disorganization and the attractive-repulsive regions is proportional to  $\Gamma^{-4}$ . In addition, it is found that the boundary between the attractive-repulsive and repulsive regions is asymptotic to  $\Gamma^{0.4}$  as  $\Gamma$  increases.

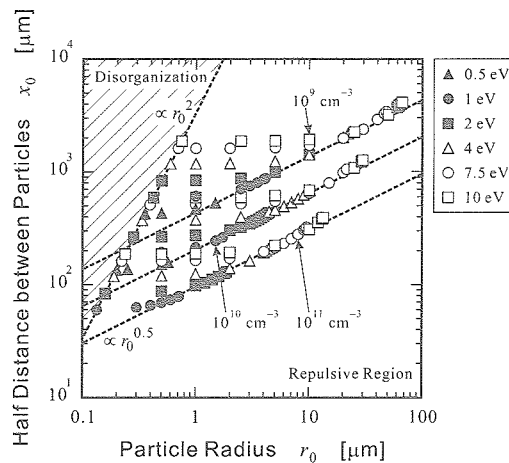


Fig.12 Dependency of half distance between particles on particle radius. Dashed lines represent scaling laws. Hatching represents the disorganization area.

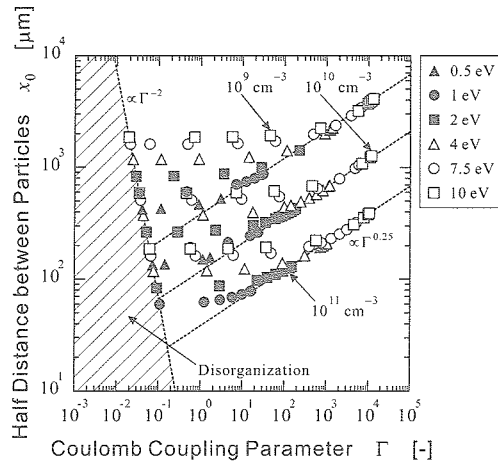


Fig.13 Dependency of half distance between particles on the coupling parameter. Dashed lines represent scaling laws. Hatching represents the disorganization area.

We investigate another scaling law. The dependency of the half distance between particles on the particle radius is shown in Fig. 12. In Fig. 12, it is found that the boundary between the disorganization region and the other regions is proportional to the square of the particle radius. In addition, the boundary between the attractive-repulsive and the repulsive regions is proportional to the square root of the radius. The dependency of the half distance on the coupling parameter is also investigated and is shown in Fig. 13. The boundary between the disorganization and the other region is proportional to  $\Gamma^{-2}$ . The boundary between the attractive-repulsive and the repulsive regions is proportional to  $\Gamma^{0.25}$ . Unfortunately, the mechanisms to dominate the power numbers of the scaling laws are not clear. In near future, theoretically and experimentally approach to understand the mechanism will be carried out.

#### 4. Conclusions

In order to investigate the mechanisms of the self-organization in the dusty plasma, the one-dimensional model is introduced. Although this model is expressed by using the Descartes coordinate, this does not prevent the essential understanding of the dusty plasma. The model expresses the difference of the Gibbs free energy between before and after the self-organization,  $\Delta G$ . By using the model, it is found that  $\Delta G$  has the minimum value when the particle radius is small. On the other hand, there is no minimum  $\Delta G$  when the radius is large. These results indicate that the self-organization mechanism changes gradually from the attractive-repulsive force to the repulsive force as the radius increases. In order to understand the reason that the minimum  $\Delta G$  exists, the contribution of the terms corresponding to the kinetic energy, potential energy and the entropy to the  $\Delta G$  is investigated. From the behavior of these terms, it is found that the negative  $\Delta G$  is caused by the variation of the potential energy. In addition, it is found that the disappearance of the negative  $\Delta G$  is caused by the increase of the absolute value of the entropy change. The dependency of the half distance at the minimum  $\Delta G$  on the particle radius is also investigated. The result shows that the distance increases with the particle radius. This is caused by the increase of repulsive force working between the particles due to the increase of the charges on the particle surface. In addition, the dependency of the Coulomb coupling parameter  $\Gamma$  on the radius is investigated. To obtain a strong coupling parameter, large radii are required. These dependencies indicate that the plasma crystals, which are obtained by using the strongly coupled plasmas, are organized by the repulsive force. This is the reason that the confinement electrodes, which are installed outside the

plasma, are required. However, the one-dimensional model indicates another self-organization mode, which is caused by the attractive-repulsive force, may exist. The attractive-repulsive force will appear under the condition of the relatively weak or moderate Coulomb coupling as compared with the coupling in the repulsive region. This means that quite a gentle injection of the dust particles into experimental apparatus will be required. If this condition is achieved, a new plasma crystal, which is spontaneously organized, will be observed. This new plasma crystal will behave like a real crystal. Therefore, the new plasma crystal will bring quite new information on scientific issues of crystal growth such as morphological instability or nucleation, or another issues of planetary physics. In the repulsive region, the plasma crystals may have several structures such as the sh, bcc or fcc structure because the structure is dominated by the environmental conditions such as the strength of the confinement potential, the plasma parameters and the dust particle density. Although the model cannot quantitatively predict the structure, the structure change is qualitatively discussed by basing on the model. By using the results of the model, we obtained several scaling laws. Such the scaling law is the first attempt in the research field of the dusty plasma. To understand the mechanism to determine the power number of the scaling law should be indispensable in order to understand the essence of the dusty plasma.

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