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The Mode-Selection Mechanism of Cross-Flow Instability in Three-Dimensional Boundary Layers
三次元境界層における横流れ不安定のモード選択機構

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The Mode-Selection Mechanism of Cross-Flow Instability in Three-Dimensional Boundary Layers*

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Abstract

Linear stability theory predicts that the most unstable disturbance of crossflow instability in three-dimensional boundary layers is a time dependent mode rather than a stationary mode. Stationary rather than traveling mode has been predominantly observed in natural disk flow, spinning in still air, which is known to be a typical example of three-dimensional boundary layers. The present paper discusses this mode selection mechanism of crossflow instability on the basis of reviewing both experimental and theoretical research and making linear stability calculations inclusive of a wall-curvature term as well as streamline-curvature and non-parallel terms. This reveals that the wall curvature plays a key role in the mode selection mechanism of three-dimensional boundary layers and that initial amplitudes of unstable disturbances are of principal importance.

Keywords: three-dimensional boundary layer, cross-flow instability, streamline-curvature instability, traveling mode, stationary mode

概　　要

線形安定理論は、三次元境界層における横流れ不安定に対して時間依存型モードは定在モードに比べてより不安定であると予測している。三次元境界層の典型的な流れ場である静止流体中で回転する円盤流では定在モードが進行波モードに比べて優勢的に観察されているけれども、その理由については合理的な説明がなされていない。本論文では、これまでの実験的観察と理論的研究のリビューに加えて、壁面曲率項、流線曲率項及び非平行性を含めた安定計算を実施して、その選択機構について議論している。この結果、壁面曲率項は横流れ不安定のモード選択機構に重要な役割を演じる、と同時に撹乱の初期振幅も重要であることが明らかにされている。

1. Introduction

Modern high-speed aircraft have sweptback wings to reduce the effects of shock waves forming on the wing. In this configuration, however, the chord-wise pressure gradient on the wing is different from the direction of external streamlines and a 'cross flow' is induced in the direction from the wing tip to the root near the leading edge and vice versa near the trailing edge. The crossflow (henceforth referred to as C-F) velocity profile normal to the wing surface has an inflection point somewhere, because the velocity becomes zero both on the wall and away from the wall. Due to the inflectional instability, three-dimensional boundary layers transition at much lower Reynolds numbers than two-dimensional boundary layers. According to linear stability theory (see Itoh, 1985, 1996a[1,2]), unsteady C-F disturbances have a larger growth rate than steady disturbances fixed relative to the wing surface, but many experimental observations show a dominant appearance of stationary mode rather than traveling mode, except for the experiments by Takagi and Itoh (1994, 1995, 1996, 1998)[3-6]. This disagreement between experiment and

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theory still remains one of the unsolved problems on three-dimensional boundary layer instability. The present paper aims to shed new light on this formidable problem through not only reviewing existing literature regarding C-F instability, but also adding linear stability calculations in consideration of a wall-curvature term as well as streamline-curvature and non-parallel terms.

2. Overview of earlier research on crossflow instability

Much attention has been paid to three-dimensional boundary layer transition since Gray (1952) [7] found that C-F instability dominates the flow along a swept wing. In his flight tests, a row of striations aligned with the local flow direction were visually observed in the laminar boundary layer near the attachment line of a swept wing. Anscombe and Illingworth (1952) [8] soon reconfirmed this observation in wind tunnel experiments and also found that sweep angle has a destabilizing effect on the boundary layer; that is, the transition location moves forward as sweep angle is increased. Gregory and Walker (see Gregory, Stuart and Walker (1955) [9]) used rotating disk flow in a quiescent fluid as an example of a canonical three-dimensional boundary layer, which exemplified C-F instability. By using china clay on the disk surface they were able to observe similar striations, stationary relative to the surface in the laminar region. These streaks are considered an action of longitudinal vortices originating from crossflow instability. They reported that the number of vortices was about 30 and their inclination to the azimuth was roughly 13°.

Stuart (Gregory, Stuart and Walker (1955)) was a pioneer in the theoretical analysis of three-dimensional boundary layer instability. His analysis shows that C-F instability is a characteristic of the inflection-point type, unlike the Tollmien-Schlichting instability, which belongs to the viscous type. Furthermore, vortex inclination is predictable but the number of vortices is four times the observed value. This disagreement concerning the number of vortices around the disk was solved in subsequent investigations (Malik, Wilkinson & Orszag 1981, Itoh 1985, Mack 1985), but new theoretical results have posed the problem that C-F instability is sensitive not only to stationary vortices but also traveling-wave disturbances.

Poll (1985) [12] performed experiments on a yawed circular cylinder to simulate flow near the leading edge of swept wings in a free-stream turbulence level lower than 0.16%. By using surface oil-flow visualization and a glue-on hot-wire cradle, both striation patters and two types of periodical disturbances were observed. In collaboration with Malik (see Malik and Poll 1984) [13], the experimental data were compared with linear stability calculations. An important result is that the stationary vortices and the observed lower-frequency disturbances (with a frequency of 1kHz) originate from C-F instability, and their characteristics show reasonably good agreement with calculations. Incidentally, Bippes and Nischke-Koskwy (1987) [14] also observed traveling mode in a swept flat plate boundary layer with a displacement body above the plate, though it was dominated by stationary vortices.

An interesting experiment on rotating disk flow was conducted by Wilkinson and Malik (1985) [15] to show the effect of an isolated roughness element placed slightly upstream of the critical point of C-F instability. As expected, they observed the dispersive development of a row of spiral C-F stationary vortices in a wedge-shaped region originating from the roughness element. It is also pointed out that stationary C-F mode is very susceptible to roughness, even as small as tiny dust and atmospheric motes, which give initial amplitudes of stationary mode. Therefore, Saric and his co-workers (1993) [16] polished their swept-wing model as much as possible to eliminate the appearance of stationary vortices in low free-stream turbulence (Tu = 0.04%). Their findings were that the transition Reynolds number is actually increased as the true root-mean-square value of surface roughness height is decreased, but the stationary mode still dominates laminar to turbulent transition.

The research group of Bippes (1991[17], see Müller, 1989[18]) examined the effect of free-stream turbulence on C-F instability using three wind tunnels with different free-stream performance, but using an identical flat plate model to sustain the same surface roughness condition. The experimental results indicate that the stationary mode rather than the traveling mode is dominant in a low turbulence environment (of the order of Tu = 0.05%) and vice versa in high turbulent free-stream (Tu = 0.3%), suggesting that free-stream turbulence gives the initial amplitude of a non-stationary mode. A similar observation was also reported by Takagi et al. (2000) [19], who conducted a rotating disk experiment introducing a traveling C-F mode through a hole by means of a loudspeaker installed underneath the disk. Instead of the laboratory frame measurements, they used a hot-wire probe adhered on the disk, sensitive to the traveling mode. Although it was primarily a traveling C-F mode with forced frequency that was observed at the incipient period of rotation, stationary vortices seemed to overwhelm the forced mode as extraneous disturbances in still air were being dissipated. This is consistent with the fact that stationary vortices are preferentially amplified in a low turbulence environment.

Takagi and Itoh (1994) made comparative experiments on yawed circular cylinder models in two different wind tunnels. The first experiment at a free-stream turbulence level of Tu = 0.12% showed a naturally amplified traveling mode in place of the stationary mode, observed in other experiments. When they used the same oil-flow technique as that of Poll, however, many striations appeared in the laminar region as a result of the action of stationary vortices. This appearance of a stationary mode is obviously due to fine grains introduced for surface flow visualization, because stationary vortices completely disappeared again after roughness elements were removed from the surface. The second experiment used another cylinder model made of acrylic pipe with a particularly smooth surface and was performed in low environmental turbulence (Tu $\approx$ 0.05%). The hot-wire scan at a constant height from the surface in the chordwise direction showed the surprising appearance of a stationary mode in the first run. However, an inspection of the model surface revealed fingerprints and other dirty spots, which were then carefully cleaned off with alcohol. Consequently, subsequent hot-wire surveys showed no signature of a stationary mode but instead a traveling mode. These observations in the circular cylinder boundary layer are in striking contrast with the aforementioned results of Bippes and Saric, observed on the flat plate or swept wing and imply that some unknown factor plays an important role in the selection mechanism between the two modes.

3. Discussion on selection mechanism

The characteristics of C-F instability inherent in three-dimensional boundary layers have been studied in different geometric configurations such as a swept wing, swept flat plate with a displacement body, a rotating disk in a still fluid and a yawed circular cylinder. However, only the circular cylinder model seems to favor the traveling mode of C-F instability in accordance with the prediction of linear stability theory (Takagi & Itoh, 1994, 1995). The stationary mode is prone to appear on a rotating disk, swept flat plate and a swept wing, in particular, under lower turbulence conditions. Since the surface of the first two configurations is flat and the last has a very weak curvature (except for the leading edge region), in contrast to the circular cylinder, which has a large, constant wall curvature, this implies that the wall curvature plays an important role in the mode selection of C-F instability in low turbulence flow. Thus, a more detailed comparison between experimental observations and computational results obtained from linear stability theory should be made with particular attention drawn to two typical flow fields with and without body curvature, namely the yawed circular cylinder boundary layer and the rotating disk flow.

3.1 The case of a yawed circular cylinder boundary layer

By comparing their analytical calculations with the experimental results obtained by Poll (1985), Malik and Poll (1984) maintain that wall curvature has a significant effect on the flow stability. Figure 1 shows their computational results for the case of a sweep angle of 63° and $R_c = Q_\infty C/\nu = 0.88\times$

10^6, where Q_∞, C and ν are the uniform-flow velocity, the chordwise length of model including fairing plates and the kinematic viscosity, respectively. Two computations with and without wall curvature show significant differences in the growth rate of unstable disturbances and indicate that the convex wall curvature produces a strong damping effect. The effect of damping is to make the N factor of the disturbance with the greatest amplification be halved in excess of X/C = 0.15, where X is the distance from the attachment line. Another remarkable feature of this figure is that the traveling mode is markedly more amplified than the zero-frequency mode, fixed in space. The N factor for the most amplified traveling mode is a value of 5.68 at X/C = 0.17, while the zero-frequency mode has a value of 2.68. It follows that the amplitude ratio of the traveling mode to stationary mode is approximately $e^{5.68}/e^{2.68} \fallingdotseq 20$, indicating predominant growth of the time-dependent mode. In reality, Poll (1985) observed a traveling mode in agreement with this stability prediction.

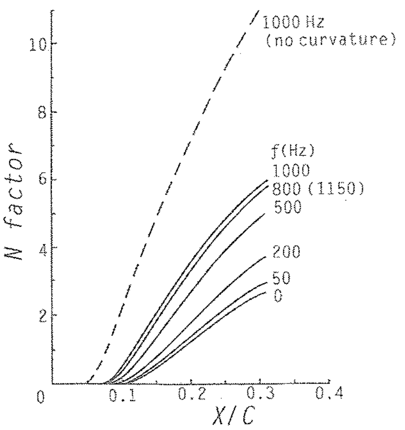

Fig.1 N factor variation with position and disturbance frequency for a yawed circular cylinder boundary layer with a fixed free-stream Reynolds number $R_c = 0.88 \times 10^6$ for the case of a sweep angle 63°.

The latest theoretical approach using the method of complex characteristics by Itoh (1996b) revealed another interesting result for wedge-shaped disturbances originating from a point source in a yawed circular cylinder boundary layer. Results of

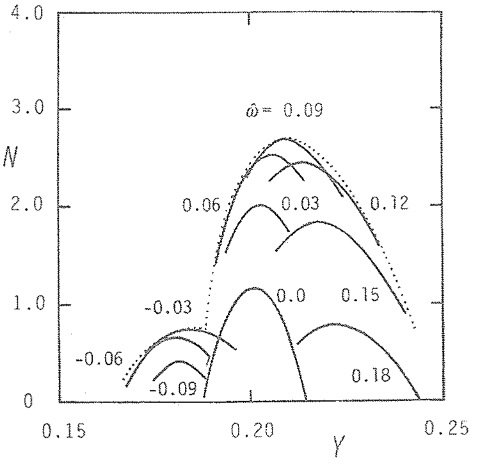

Fig.2 Variations of the total growth rate N observed at X_1=0.3 originating from a point source at X_0=0.1 with the spanwise coordinate Y for various values of the frequency $\hat{\omega}$ for a yawed circular cylinder boundary layer with a fixed free-stream Reynolds number $R_Q = 0.5 \times 10^6$ for a sweep angle of $\Lambda = 50°$.

the analysis, including both non-parallelism and curvature terms, are shown in Fig. 2, where the envelope of amplitude distributions in the spanwise direction 'Y' is given for various frequencies of the disturbances, which are forced at X = 0.1 and observed at X = 0.3 on the cylinder model. Here, X denotes the non-dimensional circumferential distance (X = 0.5 at the half circumference) from the attachment line and the frequency f is presented in the non-dimensional form $\hat{\omega} = 2\pi Df / \left(Q_\infty \cos\Lambda \sqrt{R_Q}\right)$. The uniform-flow Reynolds number is given by $R_Q = Q_\infty D / (\nu \cos \Lambda) = 0.5 \times 10^6$ for the uniform-flow velocity Q_∞ = 15m/s, the cylinder diameter D = 0.5m and the sweep angle Λ = 50°. Two peaks of the envelope are seen in the figure; the major one with a positive frequency consists of C-F disturbances peaking at the spanwise distance Y = 0.21 from the origin at the forcing point and the minor peak located at the inside region of Y indicates disturbances with much smaller amplification rates due to S-C instability. The negative sign of the frequency means that the wave number vector of disturbance is opposite to its propagation direction. The largest disturbance with $\hat{\omega}$ = 0.09 has a value of N = 2.72 and the zero-frequency mode has a value of 1.16, hence the amplitude ratio of the most unstable traveling wave to the stationary mode is $e^{2.72}/e^{1.16} = 4.76$. Again, the key point for C-F instability of the circular cylinder boundary layer is the traveling mode with a much larger amplification rate than the zero-frequency disturbance. The spatial characteristics of the most unstable mode are found to be in reasonably good agreement with the experimental observations made by Takagi and Itoh (1996, 2000). Details of the theoretical description are given in Itoh (1996b)[20].

The circular cylinder model as mentioned above allows the dominant appearance of the traveling mode. It is speculated, therefore, that the convex wall curvature strongly favors the traveling mode rather than the stationary mode. According to this speculation, similar calculations based on the O-S equation were made to investigate frequency dependency of the wall-curvature effect. Figure 3 shows one peak with a positive frequency due to

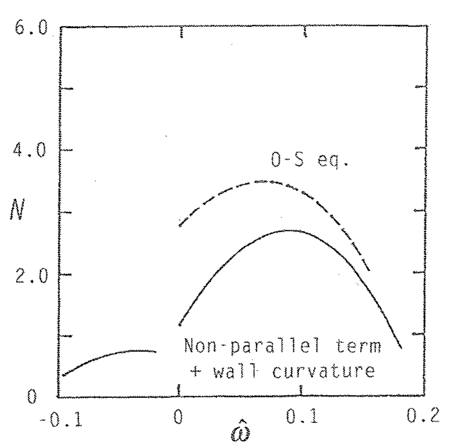

Fig.3 The solid curve represents an envelop of amplitude distribution of the whole unstable components under the same experimental conditions as in Fig. 2 and the broken curve for the case of O-S equation with no wall-curvature and non-parallel terms.

C-F instability but no smaller peak, because the O-S equation includes no wall-curvature term as well as no streamine-curvature and non-parallel terms. This figure also shows that the convex wall curvature has a stabilizing effect consistent with the result of Malik and Poll (1984) and that the O-S equation causes the relative amplitude ratio between the most unstable traveling wave and the stationary mode to be reduced from a value of 4.76 to $e^{3.5}/e^{2.78} = 2.05$.

3.2 The case of rotating disk flow

The method of complex characteristics was also applied to rotating disk flow (Itoh, 1998)[21] to describe wedge-shaped disturbances developing from a point source located at $R_0 = 250$ and observed at $R_1 = 450$, where the Reynolds number R is defined as $R = r\sqrt{\Omega/\nu}$, r being the radius from the disk center and Ω the angular velocity of the disk rotating counterclockwise. Figure 4 shows variations of the total growth rate N versus the azimuthal angle $-\phi$ (in radians) from the point source for various values of the frequency $\hat{\omega} = \omega/\Omega$, observed at the location $R_1 = 450$. We can again see two types of unstable disturbances with comparative amplitudes at this location.

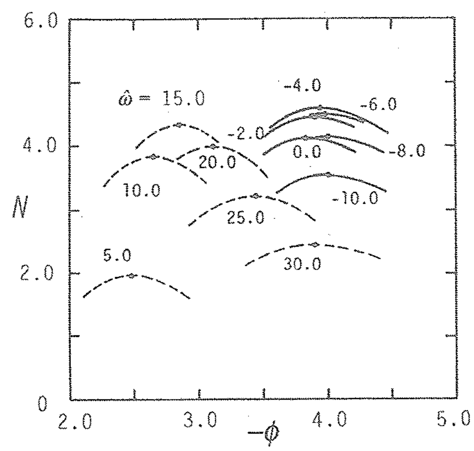

Fig.4 Variation of N factor with an azimuthal angle of $-\phi$ for various values of $\hat{\omega}$ in the case of forcing at R_0 =250 and observing at R_1 =450 in a rotating disk flow.

A group of peaks denoted by the solid curves with negative frequencies indicates a family of C-F disturbances concentrating around $-\phi = 4$ and the other family shown by dotted lines dispersed in the wide range of $-\phi$ consists of S-C disturbances with positive frequencies. The sign of the frequency indicates that C-F mode propagates in the inward direction, while the S-C disturbance directs outwards. The most unstable traveling disturbance ($\hat{\omega}$ = -4.0) and the zero-frequency mode originating from C-F instability have values of N = 4.6 and 4.0, respectively, indicating that the traveling mode is somewhat competitive with the stationary mode. In reality, the amplitude ratio of two modes results is $e^{4.6}/e^{4.0}$ = 1.8, which is less than one third the ratio compared to the circular cylinder boundary layer shown in Fig. 2. This ratio slightly increases with an increase in R and becomes comparable with the value of 2.05 obtained from the O-S equations for the circular cylinder boundary layer shown in Fig. 3.

4. Concluding remarks

By reviewing theoretical and experimental studies regarding crossflow instability of three-dimensional boundary layers some ideas have been proposed as to why stationary vortices, rather than the traveling mode, dominate laminar to turbulent transition in spite of a larger amplification rate of the latter. The review has included different geometric configurations and made linear stability calculations inclusive of a wall-curvature term as well as streamline-curvature and non-parallel terms. The difference between a yawed circular cylinder model and other configurations, such as rotating disk, swept flat plate with a displacement body and a swept wing, is due to the large constant curvature, in contrast to the others which have no or very small wall curvature in the region susceptible to crossflow instability. Detailed comparison of these cases shows that the stabilizing effect of convex wall curvature is weakened as the frequency of disturbances increases; that is, the stationary mode is stabilized significantly more than the traveling mode in convexly curved boundary layers. In the yawed circular cylinder boundary layer, the traveling mode is selectively amplified owing to its higher growth rate as well as the weaker stability effect of wall curvature and results in overwhelming stationary vortices. In a low turbulence flow, on the other hand, the initial amplitude of the traveling mode developing on the model with no or small wall curvature is much smaller than that of the stationary mode, which is very sensitive to surface roughness or imperfections of the model surface. Thus, the stationary mode appears predominantly in many practical flows. However, the traveling C-F mode can be preferentially amplified by artificially introducing a larger initial amplitude of the traveling waves compared to the stationary vortices. This is the case even in rotating disk flow where there is no wall curvature. This proposition requires further both experimental and theoretical studies focusing on the effects of the wall curvature of the model on three-dimensional boundary layer transition.

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