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線状爆発における特性法を用いた解析について

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Characteristics Method Applied to a Line Source Explosion*

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and Satoru OGAWA[*3]

Abstract

Cylindrical blast waves generated in a real air is investigated by means of the method of characteristics under the assumption of a line source explosion. The effect of dissociation on shock wave propagation in air is evaluated by utilizing an effective isentropic index. The effect of dissociation is remarkable in the velocity profiles behind shock front for initial shock strengths between M_0=4 and 8. Various mesh sizes on numerical computation are tested to determine the trajectory of the shock front in the time-space domain. In the case of perfect gases without dissociation, the 51 points on a constant time line are enough to integrate the characteristic equations. For dissociating gases, the 201 points are necessary to continue stable computations. Pressure profiles at a long time elapse after explosion reveal that the pressure in the vicinity of the explosion center gradually decreases below the initial pressure of the atmosphere different from similarity solutions.

Key words; Point source explosion, Characteristics, Dissociating air,Isentropic Index.

1. Introduction

A standard method for solving the equations of unsteady one-dimensional flows is to render two independent variables to a single one by combining a space variable r with a time t. In the past there are two analytical methods, which are similarity solutions based on the dimensional analysis and the method of characteristics for a system of hyperbolic equations in the Eulerian system. In the last few decades, remarkable advances in available computer codes together with hardware enable us numerically to simulate various complex flows. The formulation of the problems is finally completed so as to satisfy the boundary conditions imposed on both explosion center and shock front discontinuity. The present problem is a type of source flow, so that the continuity equation includes the source term as an inhomogeneous term. Such the source term has a singular property. Only the solutions on a point source explosion, in this case, give mathematically exact solutions. Physical meaning of the

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point explosion is that at the explosion center, the symmetry condition for flow velocity must be satisfied. Typical solutions were proposed by Sedov, Taylor, von Neumann and Korobeinikov. Across a shock front in high temperature air, the dependent variables as flow velocity u, density ρ , and pressure p have many possible interpretations depending on the thermodynamic situation under consideration. In this case the energy equation should be described by an appropriate caloric equation of state relating the internal energy with the pressure and the density, even if the substance behaves as a perfect gas. In general cases, transition process across the shock front is important. In the present analysis, however, a simple thermodynamic model is introduced to analyze high temperature air flows by applying an isentropic index as a function of temperature and density or pressure in stead of constant ratio of specific heats. In addition, only local equilibrium flows are considered without taking into count the relaxation process. Otherwise we cannot specify the degree of dissociation.

Though the blast wave theory was much developed relevant to the blast-damage of atomic bombs since the nineteen-forties, the most of explosion problems are of considerable theoretical and practical interest. The flow between a body surface and a conical shock wave generated around hypersonic vehicles can be analyzed by applying a method of similarity solutions (1). The method is utilized under the assumption that the flow variables change with time in such a manner that their distributions with respect to coordinate variables always remain similar in time. Taylor (2), von Neumann (3), and Sedov (4) obtained self-similarity solutions, separately, when the initial pressure in front of the shock front can be ignored in comparison to the highly compressed gas just behind the shock front. Thus the self-similar solutions are available for infinitely strong shock waves. However, when the shock wave travels at a position far from explosion center, the effect of initial pressure on the decay of shock strength becomes significant. Then the blast wave theory for the finite strength of shock waves received a good deal of attention since nineteenth-fifties. Various similarity solutions are obtained by Korobeinikov Mil'nikova, and Ryazanov(5), Stanukovich(6), Chernyi(7), Sakurai(8), and Oshima(9). The problems of detonation waves are also investigated by Levin and Chernyi(10), Korobeinikov(11), and Lee(12). Their investigations were available only for gases with constant specific heats. The blast wave theory in high temperature air with dissociation was firstly investigated by Higashino(13). Though the similarity methods are generally applied to solve the gas-dynamics of explosion, these investigations are valid, only if the location of shock wave front is given by the power law of time, as pointed out by Guderley(14). In reality, when the blast wave is sufficiently weakened and approaches to sound wave, the overpressure behind the shock front steadily decreases. Finally the pressure behind the shock front sufficiently falls off in the regular manner and the pressure distribution decreases below the initial pressure of the atmosphere at a long time elapsed after explosion. Such characteristic phenomena of pressure profiles cannot be analyzed by using the similarity method. It is impossible analytically to solve a set of partial differential equations for blast wave problems, even if variable transformations are formulated for a point or a line source explosion. In the present analysis, we will propose a new approach exactly to solve the problem including the effect of dissociation in a real air by means of the method of characteristics.

As the blast wave travels in the air far away from the explosion source, the similarity assumption based on the power law of time cannot be valid. To obtain realistic distributions of pressure at long time elapsed after explosion, Higashino(15) solved the problem by applying the method of characteristics. In regard to the system of hyperbolic equations, a set of partial differential equations can be reduced to a family of ordinary differential equations, since the independent variables, r and t, are suitably combined to give only one parameter along the characteristic lines. Each Mach wave or wave equation can transmit information only at a finite constant velocity which represents a propagation speed of disturbances traveling along every characteristic direction in space. Inspection of equations reveals that there are three characteristic curves or Mach waves in the flow field of the source flow. Though the solutions computed by the characteristics method give analytically exact ones, the computations on non-linear propagation of blast waves depend fairly on the mesh size. Therefore the effect of the numerical grid size on computations is tested and discussed in the present analysis.

2. Basic equations for unsteady one-dimensional flows

In case of cylindrically or spherically symmetric flows all quantities depend only on one coordinate distance r from an axis or a point chosen as origin. Then the conservation equation of momentum for unsteady flows without transport phenomena is expressed in the Euler coordinates as

$$\rho\left(\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial r}\right)+\frac{\partial p}{\partial r}=0 \quad . \qquad (1)$$

Here the flow velocity u is also only one component directed away from the symmetry point and ρ and p, are density and pressure, respectively. The equation of mass conservation is written in the divergence form as

$$\frac{\partial\rho}{\partial t}+u\frac{\partial\rho}{\partial r}+\rho\frac{\partial u}{\partial r}+j\frac{\rho u}{r}=0 \quad . \qquad (2)$$

Here the numerical constant j is given as j=0 for a planer flow, j=1 for a cylindrical flow, and j=2 for a spherical flow. The last term appeared in the right hand side of eq. (2) is a source term which has a singular property at r=0. A simple plane flow of j=0 differs from a cylindrically or a spherically symmetric flow. Equation (2) is conveniently rewritten to obtain characteristic formulae as

$$\frac{\partial p}{\partial t}+u\frac{\partial p}{\partial r}+\rho a^2\frac{\partial u}{\partial r}=-j\frac{\rho u}{r}a^2 \ , \qquad (3)$$

Here the derivatives of the density ρ is replaced by those of the pressure p through sound speed a .

The conservation equation of energy is described in various forms. Since there are no compression shocks in the flow region behind the incident blast wave, the entropy may be preserved. Therefore changes in the thermodynamic state of each particle obey isentropic process. In such thermodynamic situation the energy equation can be written by entropy S as $dS=0$. Furthermore, if we express the entropy as a function of temperature T and pressure p or density ρ , then the energy equation may be written in the following expression as

$$\frac{\partial}{\partial t}\left(p\rho^{-\Gamma}\right)+u\frac{\partial}{\partial r}\left(p\rho^{-\Gamma}\right)=0 \ , \qquad (4)$$

Here Γ is an effective isentropic index which is identical to the ratio of specific heats for ideal gases without dissociation. In many cases the value of the isentropic index Γ is defined according to the various criteria in thermodynamic states and is generally given by the ratio of specific heats at constant pressure to constant volume. It is also possible to determine the value from the experimental point of view by measuring the velocity of sound. However, the classical kinetic theory of heat has thrown light on many important points in regards to specific heats for real gases in high temperature. Furthermore, we can take into account even the effect of rotational and vibrational energy mode of high temperature molecules in virtue of the quantum theory [16], [17], [18], [19]. In fact, the internal energy and enthalpy of molecules can be given as a function of temperature and density or pressure with the help of the kinetic theory of gases. We can thus define an isentropic index Γ as a function of temperature and pressure in stead of the ratio of constant specific heats [20], even for dissociating gases. Thus high enthalpy air composed of 80% nitrogen and 20% oxygen can be described as a function of temperature T and pressure p, respectively, since the specific internal energy and enthalpy are given by explicit functions based on a kinetic theory of molecules. The classical definition of the isentropic index is simply defined by the ratio of specific heats as

$$\Gamma = \frac{C_p}{C_V} \ ; \ C_p = \left(\frac{\partial h}{\partial T}\right)_p \ , \ C_V = \left(\frac{\partial e}{\partial T}\right)_V \ . \qquad (5)$$

However, we sometimes introduce the following isentropic index given by the ratio of the specific enthalpy to the internal energy as

$$\Gamma = \frac{h(T,p)}{e(T,p)} \ . \qquad (6)$$

When the gas temperature is not so high [20], the value of Γ calculated by eq.(5) is almost the same as the ratio of specific heats evaluated by eq.(6). In practice, a caloric equation of state may be expressed by introducing such the effective isentropic index Γ in stead of the specific heat ratio as

$$p\rho^{-\Gamma} = const. \qquad (7)$$

It follows from eq. (7) that a sound speed a is given in terms of pressure p and density ρ as

$$.\left(\frac{\partial p}{\partial \rho}\right)_S = \Gamma\frac{p}{\rho} = a^2 \ . \qquad (8)$$

3. Characteristic formulations for the fundamental equations

To formulate characteristic equations as the wave function of distance r and time t, new variables $\xi = \xi(r,t)$, $\eta = \eta(r,t)$, are introduced [21),(22),(23),(24),(25]. When the independent variables in the basic equations, (1), (3) are transformed into the new variables, the following equations can be obtained with unknown derivatives with respect to ξ *and* η , as,

$$\left(\frac{\partial \xi}{\partial t} + u\frac{\partial \xi}{\partial r}\right)\frac{\partial p}{\partial \xi} + \rho a^2\frac{\partial \xi}{\partial r} = -\left(\frac{\partial \eta}{\partial t} + u\frac{\partial \eta}{\partial r}\right)\frac{\partial p}{\partial \eta} - \rho a^2\frac{\partial \eta}{\partial r} - j\frac{\rho u}{r}, \qquad (9)$$

$$\frac{1}{\rho}\frac{\partial \xi}{\partial r} + \left(\frac{\partial \xi}{\partial t} + u\frac{\partial \xi}{\partial r}\right)\frac{\partial u}{\partial \xi} = -\frac{\partial p}{\partial \eta}\frac{1}{\rho}\frac{\partial \eta}{\partial r} - \left(\frac{\partial \eta}{\partial t} + u\frac{\partial \eta}{\partial r}\right)\frac{\partial u}{\partial \eta} \ . \qquad (10)$$

The problem is then reduced to solve the partial differential equations, (9) and (10) with the energy equation (4), so as to satisfy the initial conditions between the explosion center at r=0 and the shock front at r=R. Since ξ and η are chosen arbitrary and independent each other, the characteristic directions are determined from the conditions that the determinants for each variable ξ *and* η are zero at every point in the (r,t) plane. Thus the following conditions are established to specify the characteristic directions. It should be noticed here that the equation (9) has the source term, so that the variables ξ *and* η are generally independent each other, only if the source term is vanished. This condition is satisfied from the boundary condition at r=0. Then we can get the following conditions for specifying the characteristic lines as eigenvalues.

$$\begin{bmatrix} \frac{\partial \xi}{\partial t}+u\frac{\partial \xi}{\partial r} & \rho a^2 \frac{\partial \xi}{\partial r} \\ \frac{1}{\rho}\frac{\partial \xi}{\partial r} & \frac{\partial \xi}{\partial t}+u\frac{\partial \xi}{\partial r} \end{bmatrix} = 0, \quad \begin{bmatrix} \frac{\partial \eta}{\partial t}+u\frac{\partial \eta}{\partial r} & \rho a^2 \frac{\partial \eta}{\partial r} \\ \frac{1}{\rho}\frac{\partial \eta}{\partial r} & \frac{\partial \eta}{\partial t}+u\frac{\partial \eta}{\partial r} \end{bmatrix} = 0 \ ; \ j\frac{\rho u}{r} = 0 \ at \ r = 0 \ . \quad (11)$$

Two characteristic curves for $\xi = const.$ and $\eta = const.$ are obtained from eqs.(11) as

$$\frac{d\xi}{dt} = \frac{\partial \xi}{\partial t} + (u \pm a)\frac{\partial \xi}{\partial r}, \quad \frac{d\eta}{dt} = \frac{\partial \eta}{\partial t} + (u \pm a)\frac{\partial \eta}{\partial r}, \quad \frac{dr}{dt} = u \pm a \ . \quad (12)$$

Here we used the sound velocity a defined by eq.(8). The Mach wave traveling with propagation speed of $u + a$ from an arbitrary point to the right is compression waves which constitute the right running family, while a left running family traveling with the speed of $u - a$ constitutes the family of expansion waves in the (r,t) plane.

When the derivatives of ξ and η with respect to time t are to be remained finite, consistency conditions on pressure p, and velocity u may be complied with the characteristic curves. By multiplying the momentum equation by sound speed a and adding to or subtracting from the continuity equation, we can get the constitutive equations along corresponding characteristic curves in stead of eqs. (1), (2) as

$$\begin{aligned} \left[\frac{\partial u}{\partial t} + (u+a)\frac{\partial u}{\partial r}\right] + \frac{1}{\rho a}\left[\frac{\partial p}{\partial t} + (u+a)\frac{\partial p}{\partial r}\right] &= -\frac{jua}{r}\Delta t \ , \\ \left[\frac{\partial u}{\partial t} + (u-a)\frac{\partial u}{\partial r}\right] + \frac{1}{\rho a}\left[\frac{\partial p}{\partial t} + (u-a)\frac{\partial p}{\partial r}\right] &= \frac{jua}{r}\Delta t \ . \end{aligned} \quad (13)$$

Therefore the conservation equations of mass and momentum are transformed into two wave equations. Thus the problem is reduced to integrate the following consistency equations in the characteristic forms on every constant time line as

$$du \pm \frac{1}{\rho a}dp = \mp j\frac{au}{r}dt \ , \quad along \ \frac{dr}{dt} = u \pm a \ on \ C_{\pm} \ . \quad (14)$$

Another characteristic curve is obtained directly from the energy equation. Since equation (4) takes already a characteristic form, the characteristic equation is easily derived along individual particle-pass trajectories as,

$$\frac{p}{p'} = \left(\frac{a}{a'}\right)^{\Gamma/(\Gamma-1)} \quad , \; along \;\; \frac{dr}{dt} = u \; on \;\; C_0 \quad . \qquad (15)$$

The system of these ordinary differential equations (14), (15) constitutes the fundamental equations for cylindrical or spherical flows on explosions. They are integrated under the given initial conditions that are specified at the explosion center, r=0, and at the shock front, r=R.

4. Boundary conditions

In case of nuclear explosions, the explosion energy is confined in a very small vessel comparing with an ordinary chemical explosive and is released in the surrounding air within a short time interval. When an amount of initial energy per unit length or per unit area is deposited along a line or a plane in the air, a propagating blast wave followed by outward flow can be generated around the explosion center. The decay rate of the finite strength of the shock wave may be physically restricted by the initial energy density of the explosion. On the other hand, there exists another boundary condition at the explosion center to be used regardless the explosion energy. The flow velocity is always directed out ward and is evaluated only on the distance from the center of explosion. From the mathematical point of view, the mass flow at the explosion center, r=0, must satisfy $\rho u = 0$. However, if there is no mass density at the explosion center, blast waves cannot be generated. Therefore, the initial pressure and density must always be finite from the beginning at t=0. In addition, it is natural to consider that the flow velocity at the center is naught regardless time by virtue of symmetrical property of the flow. Thus the physical condition of the explosion energy can be replaced by a mathematical model at the explosion center. This simple model is called a point source model for explosion named by Korobeinikov [1],

$$u(0,t) = 0. \qquad (16)$$

The basic equations (9) and (10) have two independent characteristic variables, ξ and η, only if the equation (16) is satisfied as a boundary condition on every constant time line. The point source model is applicable to various problems for source flows including high temperature gas flows with chemical reactions.

On the other hand, the flow of the matter across the shock front must satisfy the conditions of balance for mass, momentum, energy and entropy. The balance statements per unit area of of shock front surface give the normal shock conditions across the shock discontinuity as,

$$\frac{p}{p_0} = 1 + \frac{2\left(M_s^2 - 1\right)}{\Gamma + 1} \ , \ \frac{\rho}{\rho_0} = \frac{\left(\Gamma + 1\right) M_s^2}{\left(\Gamma - 1\right) M_s^2 + 2} \ , \ \frac{u}{U} = \frac{2}{\Gamma + 1}\left(1 - \frac{1}{M_s^2}\right). \quad (17)$$

Here U is shock wave velocity propagating at the distance r=R and Ms is the shock Mach number defined by using a sound velocity a_0 in front of the shock front defined as

$$U = \frac{dR}{dt} \ , \quad M_s = \frac{U}{a_0} \ . \quad (18)$$

5. Numerical Computations

As in the previous analysis [20], the real gas effects on blast waves are analyzed by introducing an effective isentropic index. Fundamental differential equations, (14), and (15) in the physical characteristics forms, are replaced by the finite difference formulae as

$$\Delta u = \mp \frac{1}{\rho a} \Delta p \mp \frac{au}{r} \Delta t \ , \quad along \ \frac{\Delta r}{\Delta t} = u \pm a; \ C_{\pm} \ , \qquad (19)$$

$$\frac{p}{p'}\left(\frac{a}{a'}\right)^{2\Gamma/(\Gamma-1)} \quad along \quad \frac{\Delta r}{\Delta t} = u \ ; \ C_0 \ , \qquad (20)$$

Here Δ represents a finite difference width between two adjacent points on a constant time line, namely, time increment. Independent variables, r and t, are space and time coordinates. Dependent variables ρ, u and p express the density, velocity, and pressure of flow, respectively. Variables p' and a' should be evaluated for every particle path line and have constant values along the particle path. The pursuit of available solution along the characteristic curves is possible for the linear system of hyperbolic equations [26], [27],[28],[29],[30]. The intersection of two families of characteristic curves may not occur at equal intervals in either time t or distance r, except possibly on the initial time line. An advantageous method numerically computing a whole flow region is to seek the solutions satisfying the characteristic equations on a constant time line bounded between the center of explosion and the shock discontinuity. When the initial shock strength Ms and the distribution of flow variables as pressure, density, and velocity are specified on a constant time line at arbitrary position and time, the flow variables on the next constant time line can be analyzed. In practice, the finite difference equations (19) are numerically integrated on the constant time line as a two point boundary value problem. In the present analysis, to begin computation with a given strength of shock Mach numbers, Ms=M_0, the initial values of pressure, density, and flow velocity on the constant time line $t = t_0$, are estimated by applying the method of quasi similarity solutions by Oshima(9). The solutions at every equidistance points on the constant time line are computed by similar manner to the method of similarity. Thus the flow variables on every constant time line can be successively computed by step by step in accordance with $t = t_0 + n\Delta t,\ n = 1,2,3,---,n.$ The CFL condition is used as usual for giving every time step. The detailed procedures for practical computations are described in the previous paper (15).

6. Results and Discussion

Before starting the characteristic computations, the properties of the thermodynamic state variables in high temperature air are evaluated and compared to the results by Hansen (16) and Sischa (17). Their results agreed well with the present thermodynamic computations. A thermodynamic state is generally given by local thermodynamic conditions as a function of temperature and pressure or density. In the present analysis, because the flow behind decaying shock front is always unsteady, it is impossible to take into account relaxation processes on chemical reactions behind shock front different from shock tube problems. Therefore computations are carried out under a local equilibrium assumption. In the present analysis the initial conditions for pressure and temperature or density are taken to be a standard atmosphere at rest.

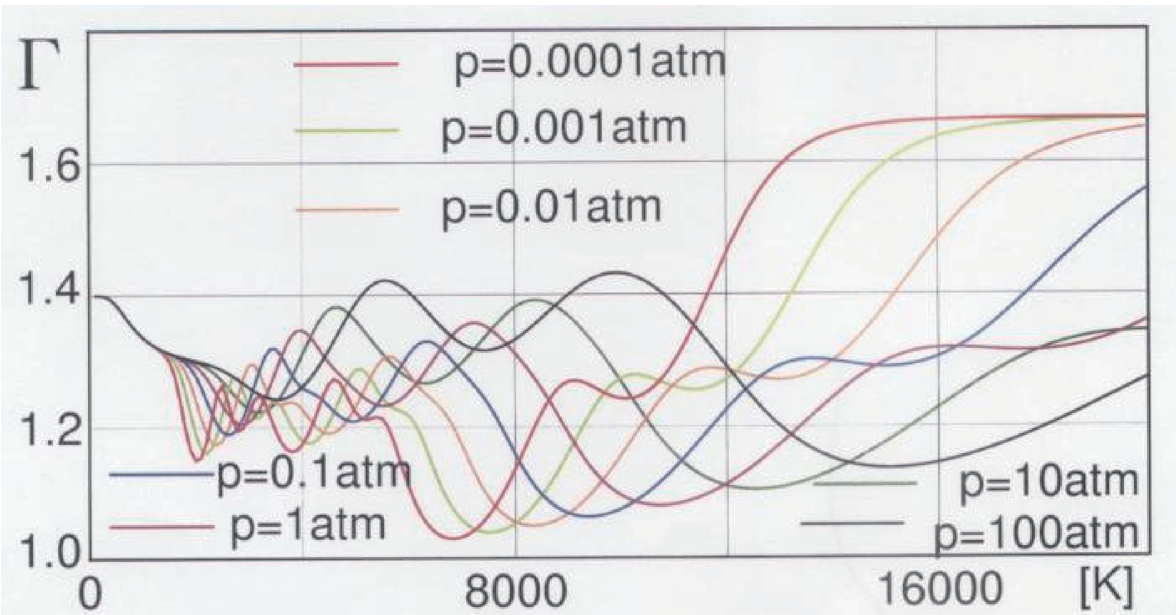

Fig.1 Effective isentropic index vs. temperature; Eq.(6) .

Numerical computations were carried out for cylindrically symmetric flows with the initial shock Mach numbers between $Ms=M_0=3$ and $M_0=9$. In case of fairly strong blast waves, nonlinear effects of shock compression are significant, so that the blast wave cannot decay linearly. In such flow regions, the results by means of the method of characteristics may depend on the mesh size as pointed by Guderley(31). However, since there are no reasonable methods (28) to evaluate the accuracy of computations, testing in computations was carried out by adopting 51, 101, and 201 points on constant time lines. Computations based on the 51 points on the constant time line were almost sufficient to integrate the characteristic equations for the ideal gases without dissociation. Since the degree of dissociation is a strong function of temperature as well as density, the value of the isentropic index violently varies due to small change in temperature. The 201 data points on the constant time line may be necessary to continue stable computations for the case of dissociating air. Especially for shock waves stronger than the initial shock Mach number of nearly $Ms=M_0=4$, we experienced sometimes numerical instabilities during iteration procedure to compute the isentropic index. For such computations, we used to the formula defined by eq. (6) for the isentropic index. The variation of such isentropic indexes against temperature is shown in Fig.1 for comparison. This figure shows that the change in such the isentropic index is almost the same, when the temperature is less than 8000K.

The real gas effects of high temperature air were recognized between $Ms=M_0=4$ and 8. The typical profiles of pressure, density, and flow velocity behind shock front for $Ms=M_0=4$ and $M_0=7$ are shown in Fig.2, and Fig.3. The physical values in these figures are normalized by the values just behind the shock front. The profiles behind shock front almost coincide with those calculated by means of the similarity method (20).

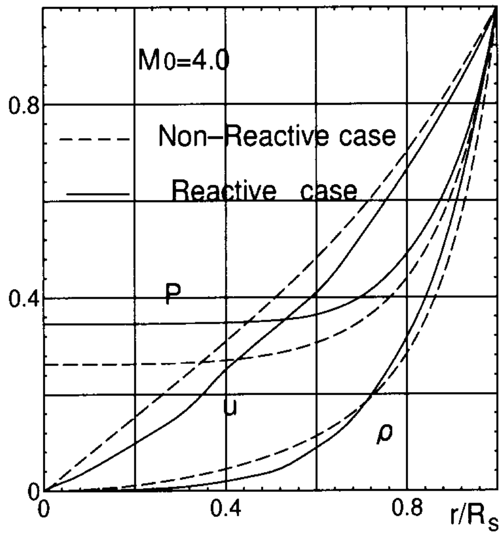

Fig.2 Initial shock strength M_0=4.

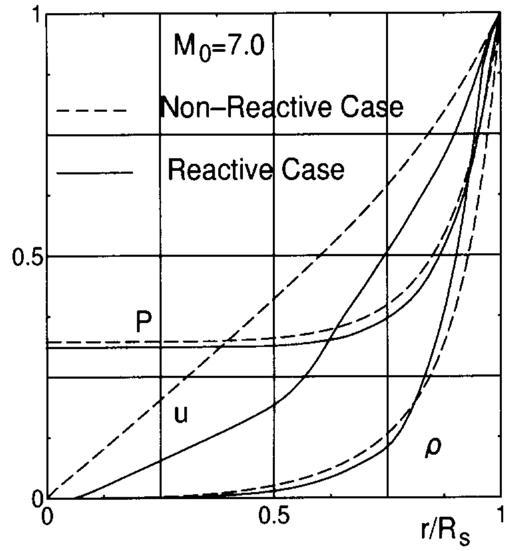

Fig3. Initial shock strength M_0=7.

(Profiles behind a shock front on pressure p, density ρ , and flow velocity u)

The stronger effects of dissociation on the flow velocity was apparently seen in the case of $Ms=M_0=7$ than 4. The distributions of flow velocity and pressure behind shock front may significantly affect the decay rate of blast waves. Trajectories of shock front propagating in the (r,t) plane are shown in Fig. 4 for the case of $Ms=M_0=8$. Although the decay coefficients [20] between $M_0=Ms=4$ and 8 for the dissociating gases significantly vary, the non-uniform effect of the blast decay was not explicitly appeared in the shock trajectories as in Fig.5, and the shock front decays monotonously in (r,t) plane. However the blast waves in real air decay faster than shock waves in the ideal gas without dissociation. In every computation, the shock wave trajectories are curved line at the beginning but gradually approached to straight ones. It means that the blast wave non-linearly propagates at first, so that the effect of mesh sizes on the propagating shock should be significant. However, the 51 points for the ideal gas and 201 points for the real gas are sufficient in every computation. Only the discrepancy of numerical results depending on the mesh sizes comes from nonlinear properties of gas-dynamics. As a whole we cannot find noticeable differences in the shock wave trajectories except for close to the explosion center, where the nonlinear effect of gas-dynamics is important. Therefore, it is recommended that in the vicinity of the explosion center, we should utilize the similarity solutions together with the symmetry condition of flow. In contrast to the flow close to the explosion center, the shock wave approaches to sound wave as limiting case. In this flow region, the similarity method is inadequate, since the similarity solution cannot tend to acoustic wave. The basic concept of similarity assumption is that the shock wave decay with time obeys power law of time.

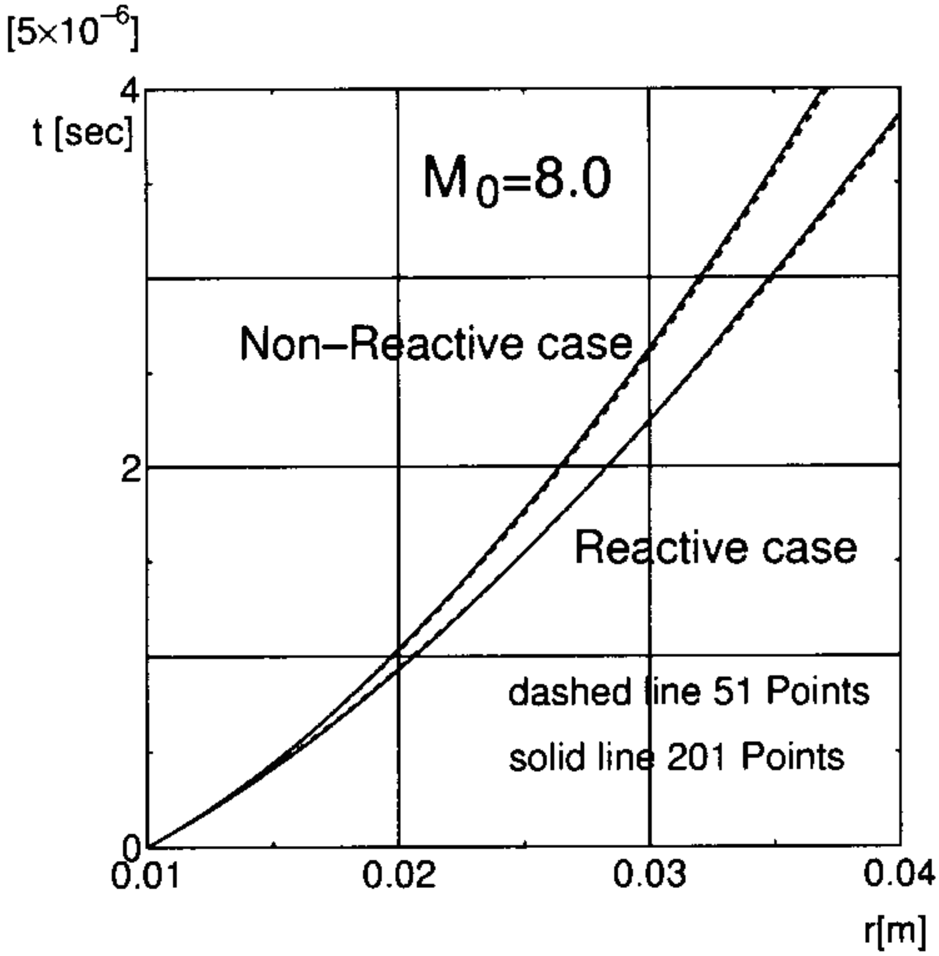

Fig.4 Shock wave trajectory in (r,t) plane; Initial shock strength Ms= M_0=8 .

The results computed by means of the method of characteristics under the assumption of line source explosions give analytically exact solutions, even if the chemical reactions in high temperature air are taken into account. When the blast wave is sufficiently weakened and approaches to sound velocity at a long time elapsed after explosion, it propagates almost at constant speed in the atmosphere. In this circumstance, the decay rule of propagating blast waves may not obey the similarity analyses expressed by the power law of time. In addition, the pressure in the vicinity of explosion center may decrease due to the expansion of centered rarefaction waves. Finally the pressure in the vicinity of the explosion center becomes below the initial pressure of the atmosphere at a few millisecond elapsed after explosion as shown in Fig.5. Such property of blast waves characterized by over expansion flow cannot be analyzed by means of self -similarity analyses. In this stage, the change in flow variables become very small, at the same time, the effect of the mesh size on the blast wave analyses becomes negligibly small.

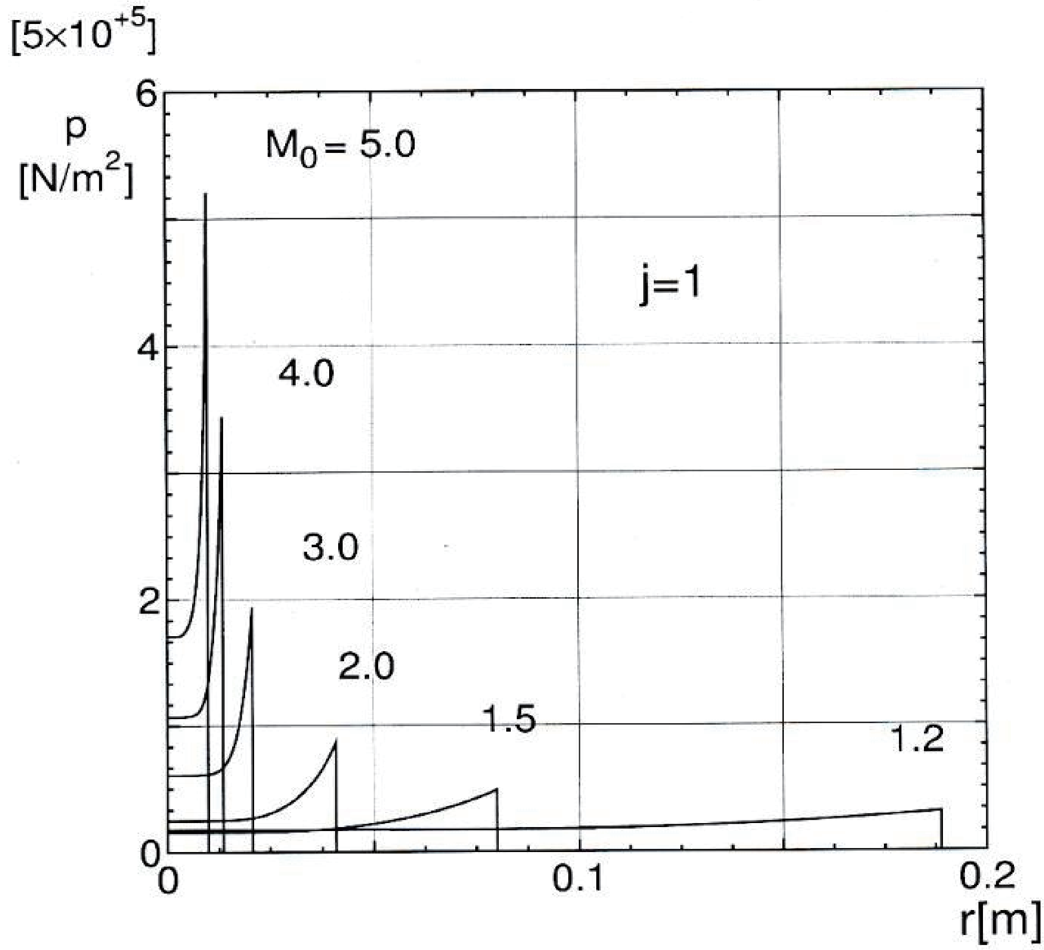

Fig.5 Pressure decay of cylindrical blast wave (j=1) vs. distance r.
Initial shock strength $Ms = M_0 = 5$

7. Concluding Remarks

The propagation of blast waves with real gas effects in high temperature air was investigated by means of the method of characteristics. The results of the pressure, density and flow velocity are almost the same as the similarity solutions. Two characteristic lines can be derived from the continuity and the momentum equations of cylindrical flows under the line source explosion. To solve axis-symmetric explosion, the initial condition for a line source model given by u(0)=0 is necessary independently to get two characteristics curves. In the case of the analyses for ideal gases without dissociation, computations adopted by 51 data points on a constant time line are sufficient to analyze the flow behind blast waves. However in the case of the dissociating air, 201 data points may be necessary to continue stable computations, since the change in effective isentropic index may strongly depend on temperature change during iteration computations.

The effect of dissociation can be seen in the range of the initial shock strengths between $M_s=M_0=4$ and 8. The flow velocity behind the shock front decreases significantly in comparison to those without dissociation. The pressure in the vicinity of the explosion center decreases below the initial pressure of the atmosphere at few millisecond elapsed after explosion. Such property of blast pressure characterized by over expansion cannot be analyzed by means of self -similarity analyses. In this stage, the change in flow variables become very small, at the same time, the effect of the mesh size on the blast wave analyses becomes negligibly small. However, close to the explosion center, the effect of mesh size is important, since the nonlinear effect of the flow is important. It implies that the similarity method is valuable in this flow region as pointed by Guderley.

The authors prepared this manuscript with deep regret to know that Prof. H. Oguchi and also Dr. V. P. Korobeinikov have passed away. They would like to acknowledge profs. K. Oshima and H. Honma for their interests in the present work and encouragement.

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Appendix

Thermodynamic properties of air composed 20% oxygen and 80% nitrogen molecules are described by suitable equations of state. The thermal equation of state for the gas mixture [16],[17] can be written as

$$p = Z\rho RT \ , \quad Z(T,p) = (1+\alpha_N+\alpha_O) \ ,$$

Here Z and R are the compressibility factor and the gas constant for the gas mixtures. α_N and α_O are the degree of dissociation for nitrogen and oxygen gases. Here the effect of ionization is ignored for simplicity. The degree of dissociation is determined from each rate equation as

$$\frac{\alpha_i^2}{1-\alpha_i^2} = \frac{K_p(T)}{4p} = \frac{p_{Di}}{p}\frac{T}{T_{Di}}\exp\left(-\frac{T_{Di}}{T}\right) \ , \quad \text{i=N for nitrogen or i=O for oxygen}$$

In this expression, the suffix i indicates corresponding the species of the gas mixture. In this case, the degree of dissociation under locally equilibrium condition can be calculated by using the equilibrium constant of reaction K_p as

$$\alpha_O(T,p) = \frac{-0.40+\sqrt{0.16+0.20\left(1+{4p}/{K_{pO}}\right)}}{1+4p/K_{pO}}$$

$$\alpha_N(T,p)=\frac{-0.20+\sqrt{0.04+0.96\left(1+{4p}/{K_{pN}}\right)}}{1+4p/K_{pN}},$$

Specific internal energy $e(T,p)$ and enthalpy $h(T,p)$ for a gas mixture composed of 80% nitrogen and 20 % oxygen molecules can be expressed as

$$e(T,p)=\left[\frac{5}{2}+\frac{1}{2}(\alpha_O+\alpha_N)+(0.2-\alpha_O)\frac{{T_{VO}}/{T}}{\exp\left({T_{VO}}/{T}\right)-1}+(0.8-\alpha_N)\frac{{T_{VN}}/{T}}{\exp\left({T_{VN}}/{T}\right)-1}\right]RT$$
$$+\left[\frac{\alpha_O T_{VO}+\alpha_N T_{VN}}{T}+\beta\left(3+\frac{2T_j}{T}\right)\right]RT$$

$$h(T,p)=\left[\frac{7}{2}+\frac{3}{2}(\alpha_O+\alpha_N)+(0.2-\alpha_O)\frac{{T_{VO}}/{T}}{\exp\left({T_{VO}}/{T}\right)-1}+(0.8-\alpha_N)\frac{{T_{VN}}/{T}}{\exp\left({T_{VN}}/{T}\right)-1}\right]RT$$
$$+\left[(1+\alpha_O)\frac{T_{DO}}{T}+(1+\alpha_N)\frac{T_{DN}}{T}+\beta\left(5+\frac{2T_j}{T}\right)\right]RT\ ,$$

Here α_O *and* α_N are the degrees of dissociation for oxygen and nitrogen molecules. The symbols T_{VO} ,T_{VN}, T_{DO}, T_{DN} , are the characteristic temperature of vibrational energy and dissociation, for nitrogen and oxygen gases, respectively. Here β and T_j are ionization rate of the molecules and the characteristic temperature for ionization. Typical values of the characteristic temperature and pressure for nitrogen and the oxygen gases are shown in the Table 1. Another values are listed in the paper [20].

Table 1 Characteristic Values

Temperature: for oxygen T_{DO} =59000K for Nitrogen T_{DN}=113000K

Pressure: for oxygen P_{DO}=2.27x10^7 atm for nitrogen P_{DN}=6.75x!0^6 atm

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