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### **A Comparative Evaluation of High-Elevation Angle Orbits for Medical Applications**

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**Japan Aerospace Exploration Agency**

# A Comparative Evaluation of High-Elevation Angle Orbits for Medical Applications

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## ABSTRACT

The Japan Aerospace Exploration Agency, JAXA (formerly National Aerospace Laboratory of Japan, NAL) has been studying the possibility of remote medical treatment using communication satellites. To transmit stable, high-bit-rate movies and precise images of a patient from an ambulance via satellite during transportation to a medical center, a high satellite elevation angle is required in order to avoid shadowing from high buildings. A number of satellite constellations can be proposed for such a system, consisting of three or more satellites. In this paper, several types of orbits such as 24-hour orbits, figure-eight orbits, Tundra orbits, 16-hour orbits, 12-hour orbits, Molniya orbits, or other interesting orbits are compared by considering coverage duration, the required number of satellites, perturbations, link budgets, launch costs, and so on.

**Key Words:** Orbit, Satellite Constellation, High elevation angle

## 概 要

宇宙航空研究開発機構(旧航空宇宙技術研究所)では平成12年度の科学技術振興調整費に採択された「高度衛星・通信技術を医療に応用するための研究開発」を東海大学医学部並びに他の機関と共同で行い、高仰角の軌道に衛星を打ち上げて移動体(具体的には救急車)から動画像を送信し、救急司令センターからの確な医療行為指示を受けて患者の救命率向上を目指すシステムについて検討してきた。本論文では、そのための高仰角軌道として、24時間周期軌道、8の字軌道、16時間軌道、12時間軌道やその他の興味深い軌道について、可視時間や必要衛星数、擾乱、軌道投入コストなどの比較について述べる。

## Symbols

$a$	: semi-major axis	$r$	: radius
$a_e$	: radius of the Earth	$r_s$	: heliocentric orbital radius of the Earth
$e$	: orbital eccentricity	$u$	: eccentric anomaly
$f$	: true anomaly	$\eta$	: orbital parameter ( $\eta = \sqrt{1-e^2}$ )
$G$	: gravitational constant	$\mu_e$	: gravitational coefficient ( $\mu_e = Gm_e$ )
$i$	: orbital inclination	$\omega$	: argument of perigee
$m_e$	: mass of the Earth	$\Omega$	: right ascension of ascending node
$m_s$	: mass of the Sun	$\Omega'$	: longitude of ascending node
$n$	: mean motion	$\rho$	: orbital parameter ( $\rho = \int ndt$ )
$p$	: semi-latus rectum	$\varphi$	: latitude
$P_n(x)$	: the Legendre polynomial of degree $n$	$\psi$	: longitude
$P_n^m(x)$	: the associated Legendre functions of degree $n$ and order $m$	$\sigma$	: mean anomaly at epoch

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## 1. INTRODUCTION

Since 2000, the JAXA (formerly NAL), the National Institute of Information and Communications Technology, NICT (formerly Communications Research laboratory), the Tokai University Institute of Medical Sciences and other related organizations have been studying real-time patient motion picture transmission systems from ambulances via satellite to emergency centers for accurate diagnosis by medical doctors, in order to improve lifesaving during patient transportation to hospital<sup>1)</sup> (Fig. 1).

For medical treatment, it is considered that an information transmission rate of about 6 Mbps would be required. At present, however, the available communication speed using mobile phones is about some hundreds of bps to some kbps, and so it is impossible to transmit images of a sufficient quality to allow a reliable diagnosis to be made. The cost to build new ground antennas would be enormous; for example, it was calculated that US\$2.278 billion would be required only for three big cities in Japan, Tokyo, Osaka and Nagoya, which is greater than the cost of launching and operating three communications satellites<sup>2)</sup>. Moreover, communications satellite can cover all of Japan and can be used not only in major cities but also on isolated islands and secluded places in the mountains. Another merit of satellite communications is that satellites are immune to disasters on the ground such as earthquakes and fires, and in fact they are expected to be used for acquiring information in the event that such disasters occur.

One of the problems of satellite communication is shadowing from trees or buildings along streets during transportation. The elevation angle of a geo-stationary satellite is less than 50 degrees in Japan and stable data transmission is impossible since communication links are easily lost. On the other hand, at an elevation angle of about 70-80 degrees, communication links are possible even in large cities where high buildings line the streets. Thus the purpose of this study is to search for orbits which can maintain a high elevation angle for a sufficiently long period time and to consider how to design a constellation based on this.

The proposed orbits for communications satellite are: Molniya, Tundra, figure-eight and other highly-elliptical orbits. 24-hour continuous high elevation angle can be achieved by a three- or more satellite constellation.

Fig. 3 shows a constellation consists of five 16-hour orbits as an example. These orbits have the same shape and inclination. Their orbital planes differ, but their ground tracks coincide with each other (Fig. 4) and as seen from the ground each satellite covers the same trajectory in the sky.

Fig. 5 shows the elevation angle of these five satellites seen from Tokyo, and it shows that a 24-hour continuous high elevation angle can be achieved as satellites appear one after another.

The orbits need to be evaluated considering:

- coverage duration and the required number of satellites
- communication link budget
- orbit perturbations and the required change in velocity ( $\Delta V$ )
- the Van Allen belts
- launch cost.

In the demonstration phase of this system, a small, piggyback satellite is to be used. For such small satellites, the limited number of available communication lines is sufficient and its mission lifetime can be short, but the launch cost should be low. On the other hand, if real systems are established in future it should have a sufficient number of lines and an adequate mission lifetime, which will be achieved by large, heavy satellites. The orbit should be selected considering the above evaluation points for both the demonstration and real systems.

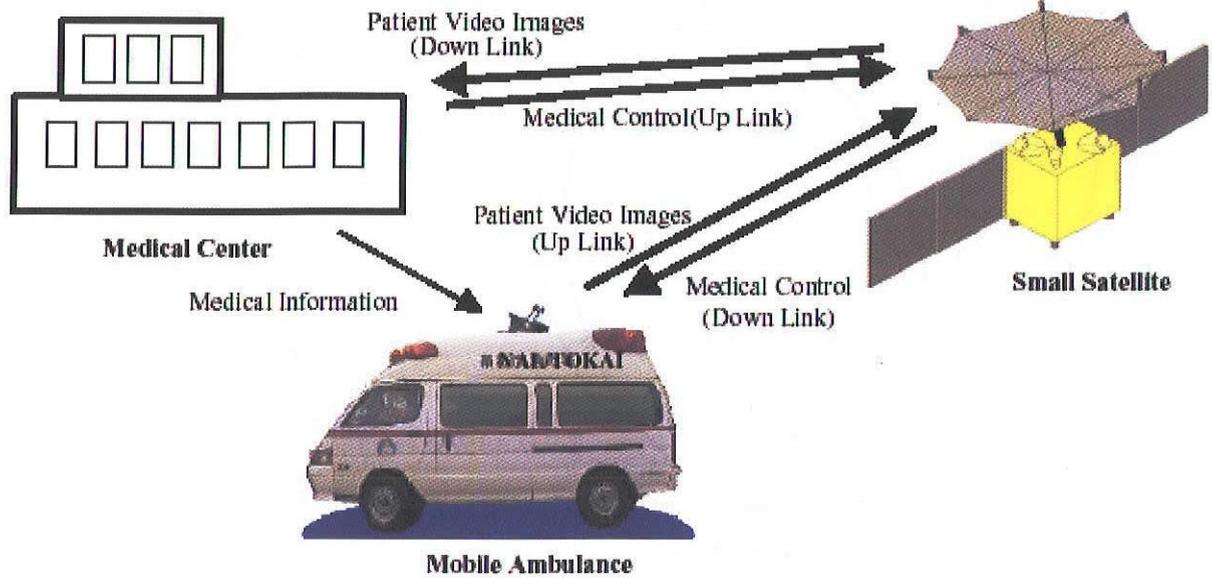


Figure 1: Overall communication links of our proposed system

Fig. 1 Medical system

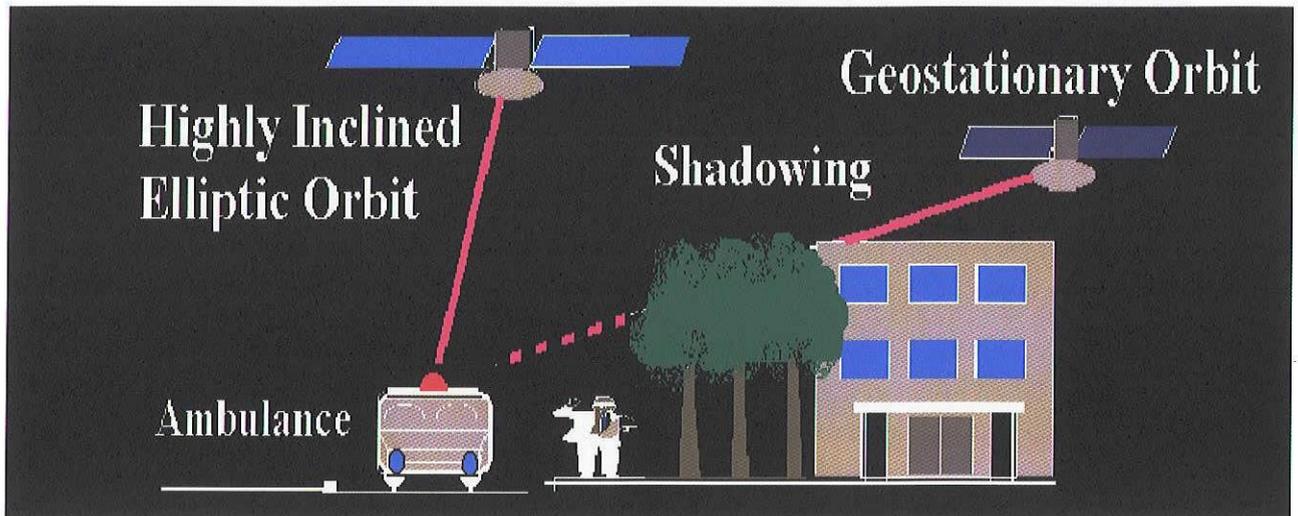


Fig. 2 Shadowing and high elevation satellite

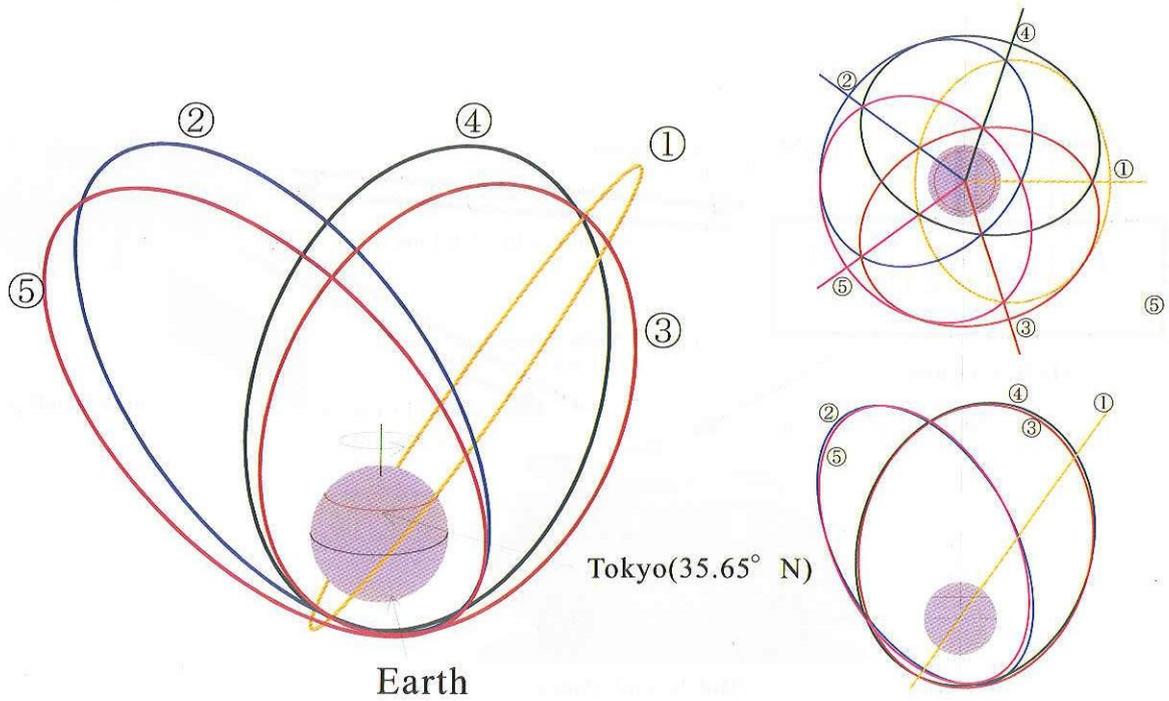


Fig. 3 Constellation of satellites

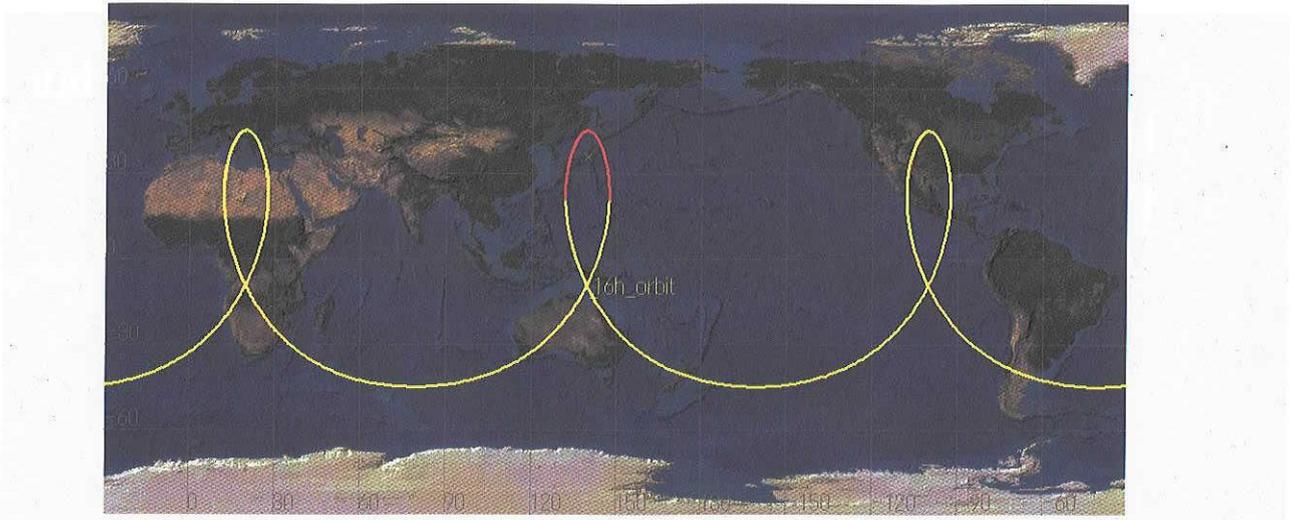


Fig. 4 ground track of 16-hour orbit

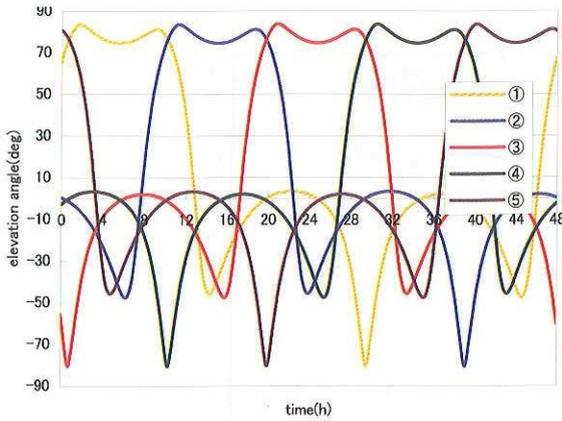


Fig. 5 change in elevation angle of five 16-hour orbits

## 2. HOW TO SEARCH ORBITS

### 2.1 Orbital period

To reduce the total mission costs, the number of satellites should be small and thus each orbit should have a repeating ground track (RGT). As a result, candidates orbital periods are:

- orbit with a 24-hour RGT  
orbital period: 24-hours, 12 hours, 8 hours, etc.
- orbit with a 48-hour RGT  
orbital period: 48-hours, 16 hours, 9.6 hours, etc.
- orbit with a 72-hour RGT  
orbital period: 72-hours, 36 hours, 18 hours, etc.

As the time period between RGTs becomes longer, the total number of required satellites increases, and so orbital periods longer than 72 hours are not considered. For the above orbital periods, a high elevation can be maintained for a long period of time if other orbital elements are properly set.

When the total number of satellites is fixed, the required coverage duration  $T_v$  is determined by the following:

$$T_v > T_r/N + 0.5,$$

where  $T_r$  is the time between repeating ground tracks, and  $N$  is the number of satellites. A redundancy period of 0.5 hours (or 30 min.) is required, since it is difficult to switch communicating satellites instantaneously during transportation by ambulance. Average transportation time is about 16 min. in Tokyo, so 30 min is considered a sufficiently long period. Thus, the required coverage duration varies with the number of satellites as follows:

-24-hour RGT

$N=3$ : 8.5 hour

$N=4$ : 6.5 hour

-48-hour RGT

$N=5$ : 10.1 hour

$N=6$ : 8.5 hour,

and so on.

### 2.2 Searching for the optimal orbital elements

After the orbital period (or semi-major-axis  $a$ ) is set, we can choose the eccentricity  $e$ , inclination  $i$ , argument of perigee  $\omega$ , right ascension of ascending node (RAAN)  $\Omega$ , and perigee passage time to specify an orbit that offers a high elevation angle for a sufficiently long period of time. Among them, the combination of RAAN and perigee passage time will determine only the time when one satellite of a constellation offering a 24-hour service is visible from a service area. Thus, there are four variables to be searched:  $e$ ,  $i$ ,  $\omega$ , and longitude of ascending node  $\Omega'$ . We have chosen to evaluate the functions as follows,

- maximize the coverage duration with a certain elevation angle (e.g. 70 deg.)
- maximize the elevation angle during certain time period (e.g. 8.5 hours).

## 3. Proposed orbits

In this section, several orbit are discussed and compared.

### 3.1. 24-hour orbit

When both the time required for the ground track to repeat and the orbital period are 24 hours, 3 satellites are sufficient to provide a continuous 24-hour service (Fig. 6). When a constellation consists of 3 satellites, that is, when an 8.5-hour coverage duration is required for each, the minimum elevation angle seen from Tokyo is 79.3 deg. Generally speaking, as the number of satellites in a constellation increases, the required coverage duration for each satellite is shortened, and the minimum elevation angle during coverage duration increases. The merits of this 24-hour orbit are that a satellite will not pass through the Van Allen belts and orbital perturbations are relatively small. However the service is limited only to Japan, as can be seen from the ground track.

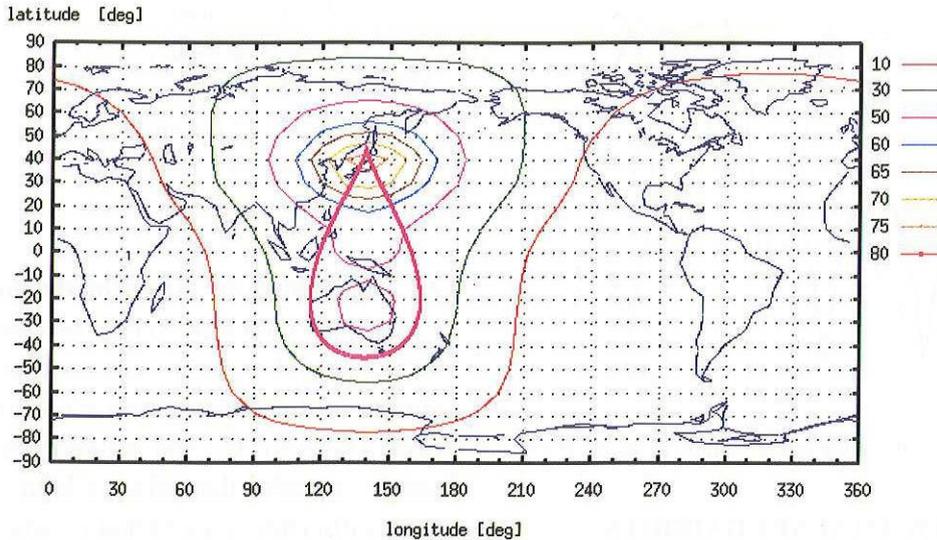


Fig. 6 ground track of 24-orbit and minimum elevation angle of 3-satellite constellation

### 3.2. Figure-eight orbit

An orbit with a 24-hour orbital period and zero-eccentricity is called a figure-eight orbit since its ground track draws a figure eight (Fig. 7). The merits of this orbit are that the distance from satellite to a ground station is relatively short, and remains almost constant since the orbit is circular. The elevation angle is 69.6 deg when it consists of three satellites (8.5-hour coverage duration is required for each), and 76.8 deg when it consists of four satellites (6.5-hour coverage duration is required for each). The same service will be available in parts of Australia where the longitude is the same as Japan, and the latitude South corresponds with the coverage latitude North. A satellite in this orbit will not pass through the Van Allen belts and orbital perturbations will be relatively small.

### 3.3 Tundra orbit

An orbit with a 24-hour period and an inclination of 63.44 deg is called a Tundra orbit (Fig. 8). The argument of the perigee does not precess when the inclination is set to this value. A satellite in this orbit (as opposed to a Molniya orbit, discussed below) will not pass through the Van Allen belts and so the satellite's lifetime is expected to be

longer. However, the apogee latitude of this orbit is high and the minimum elevation seen from Tokyo is only about 59 deg, in the case of three satellites. When it consists of four satellites, the minimum elevation angle becomes about 70 deg.

### 3.4. 12-hour orbit

When an orbit has a 12-hour orbital period and the ground track repeats every 24-hours, it has two service areas, Japan and the area located at the opposite longitude (Fig. 9). This area lies over the Atlantic Ocean, and when service is provided to the nearby areas such as east coast of USA, the elevation angle is lower than it is for Japan. The altitude of this orbit is lower than that of the 24-hour orbit, and launch costs will therefore be lower. For example, the required change in velocity ( $\Delta V$ ) from GEO transfer orbit to this 12-hour orbit is lower than it is for other orbits considered here, and so this orbit could be a candidate for a small satellite experiment launched as a piggy-back payload.

However perturbation forces are large, and fuel requirements for orbit maintenance for a longer mission period will be greater. The satellite lifetime will also be shorter, since the satellite will pass through the Van Allen belts.

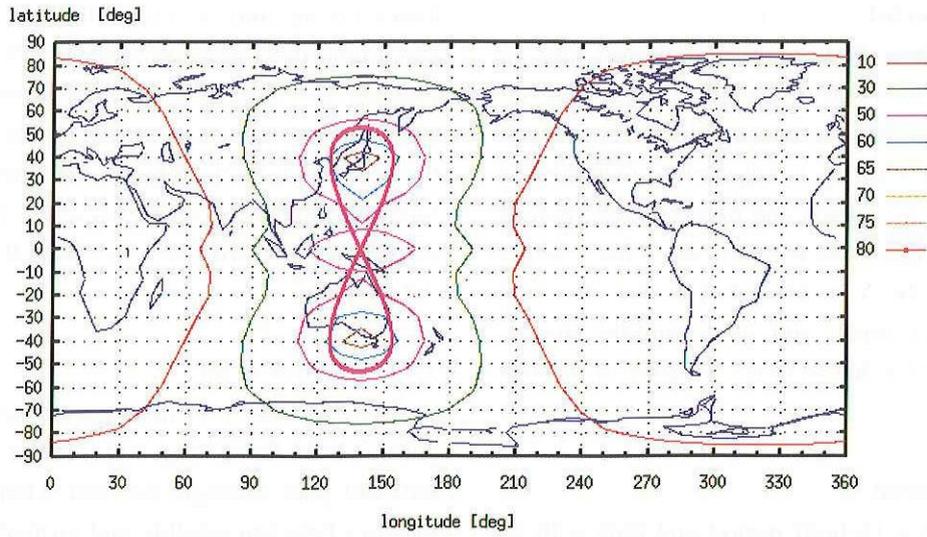


Fig. 7 ground track of 8-figure orbit and minimum elevation angle of 3-satellite constellation

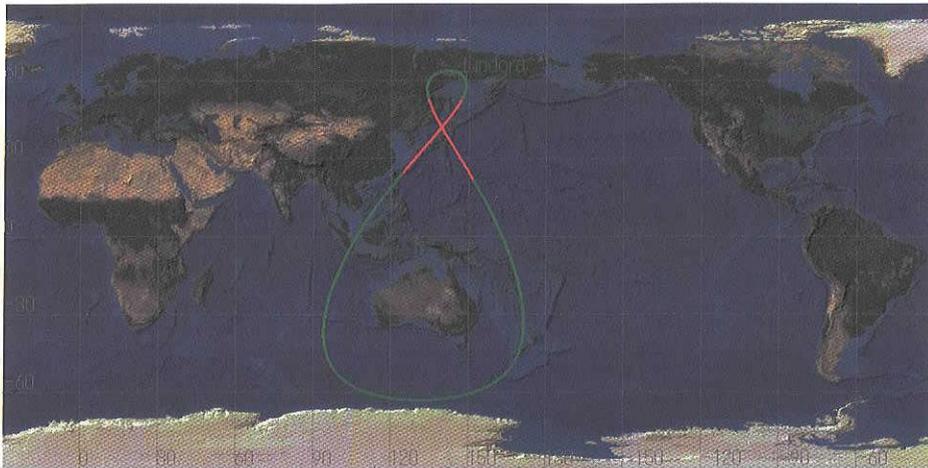


Fig. 8 ground track of a Tundra orbit

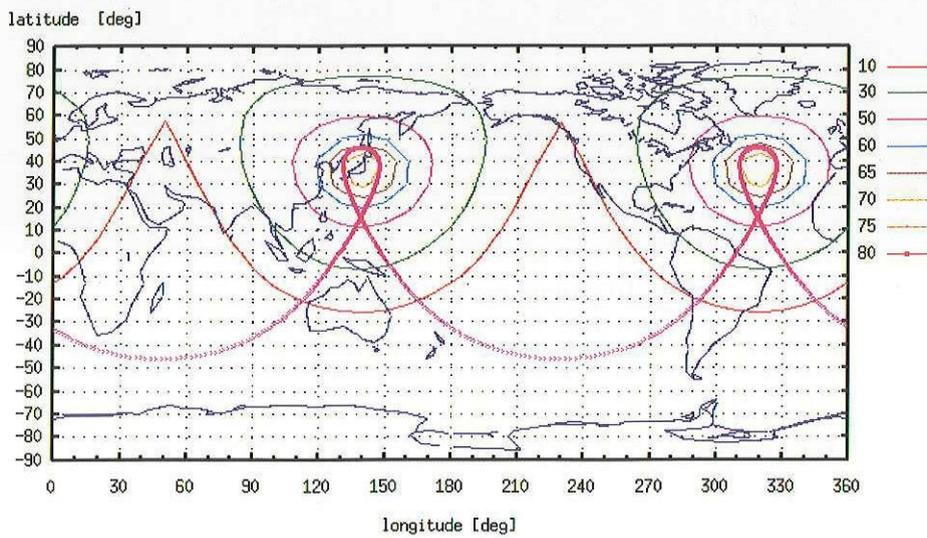


Fig. 9 ground track of 12-hour orbit and minimum elevation angle of 3-satellite constellation

### 3.5. Molniya orbit

An orbit with a 12-hour orbital period and an inclination of 63.44 deg, is called a Molniya orbit; (Fig. 10) the argument of perigee does not precess, as is the same as that of Tundra orbit. This orbit is commonly used by Russia and high-latitude regions. The lifetime of this orbit is short, since the satellite passes through the Van Allen belts. The elevation angle seen from Japan and other middle latitude regions is about 58 deg at most, because of its high inclination.

### 3.6. 16-hour orbit

An orbit with a 16-hour period and thus with 48 hours between repeating ground tracks has three service areas: Japan, and both areas where longitude is different from Japan by 120 deg (Fig. 5). These areas are over Europe and USA, that are densely-populated areas with high skyscrapers, and the service for these areas is also expected to be valuable. Five satellites are needed to maintain a 24-hour service but the cost per service area is

lower taking into account all three service areas. The minimum elevation angle is 73.6 deg when it consists of 5 satellites. The lifetime is medium, since the satellite passes through the higher ring of the Van Allen belts. When 6 satellites are used, the minimum elevation angle becomes 78.9 deg, and a satellite in this orbit will not pass through the Van Allen belts.

### 3.7. 48-hour orbit

An orbit with a 48-hour period needs to have its perigee over Japan (Fig. 11). A satellite in this orbit will not pass through the Van Allen belts but the distance between satellite and ground station will be long. The coverage duration seen Tokyo over 70-deg elevation angle is about 11.5 hours, or the minimum elevation angle during 8.5-hour coverage duration is 79.4 deg, and these are almost equal to those of 24-hour orbit. On the other hand, launch costs will be higher, and the required number of satellites will be six, and therefore this orbit is no better than a 24-hour orbit or other orbits.

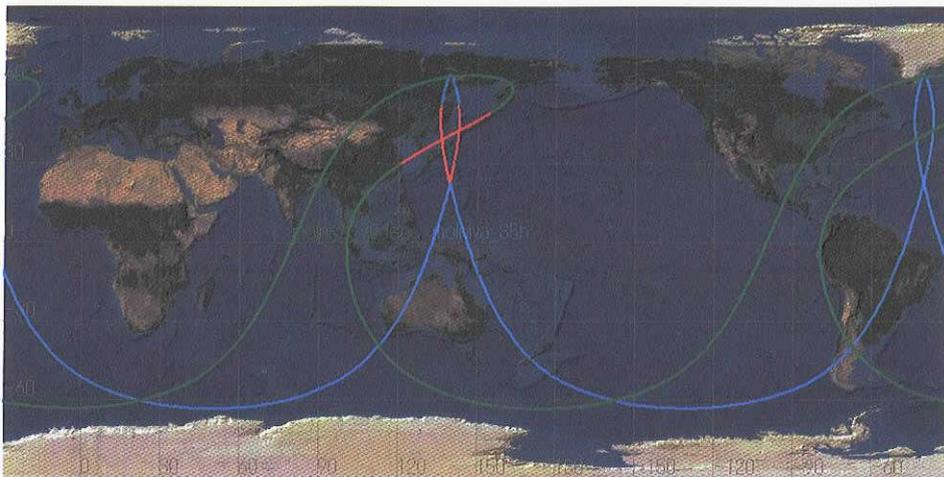


Fig. 10 ground track of Molniya orbit

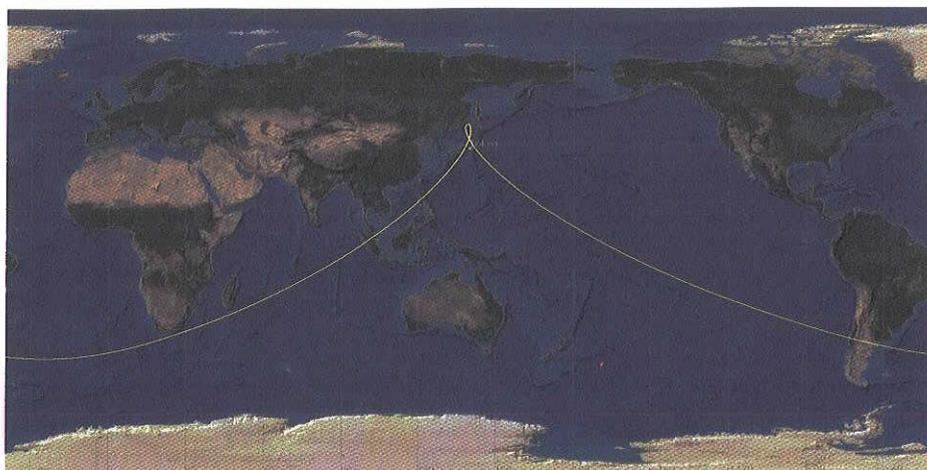


Fig. 11 ground track of 48-hour orbit

### 3.8. Others

The number of service areas is determined by the number of revolutions a satellite makes before its ground track repeats, and not only 2 (e.g. 12-hour orbit) and 3 (e.g. 16-hour orbit), but 5 (e.g. 9.6-hour orbit, 14.4-hour orbit) and 7 (e.g. 10.3-hour orbit, 13.7-hour orbit) are considered to be interesting, since they have another service areas in Europe or the east/west coast of the USA. Other service area numbers such as 4 (e.g. 6-hour orbit, 18-hour orbit) and 6 (e.g. 4-hour orbit and 20-hour orbit) are also possible, but when one service area is set over Japan, the other service areas lie over the ocean or depopulated areas. Furthermore, the required number of satellites is the same as or larger than that of 5 and 7 service area. From now, two orbits will be discussed, 9.6-hour as an example of five service areas, and 10.3-hour orbit as an example of seven service areas.

#### 3.8.1 9.6-hour orbit

A  $9.6(=48/5)$ -hour orbit can supply service not only to Japan but also other to four areas including Europe and the east coast of the USA (Fig. 12). As is seen in Fig. 12, the distance between each service area is shorter, thus any place in middle latitude regions has the elevation angle of over 50 deg. The

required number of satellites to maintain a 24-hour service is seven. A  $14.4(=72/5)$ -hour orbit also has five service areas with seven satellites.

#### 3.8.2. 10.3-hour orbit

A 10.3 ( $72/7$ )-hour orbit has seven service areas (Fig. 13), and at any place in middle latitude regions, the elevation angle is over 60 deg. The required number of satellites to offer a 24-hour service is ten. A 13.7 ( $=96/7$ )-hour orbit has also seven service areas, with ten satellites.

### 3.9. Comparisons of each orbit

The number of service areas and the required number of satellites are decided by recurrent period, orbital period are shown in Table 1 and Table 2. Table 1 shows the coverage duration to maintain more than 70 deg elevation angles seen from Tokyo, and the minimum elevation angle during 8.5-hour coverage duration, and the orbital element at that time, for each orbit. A longer coverage duration or higher elevation angle is available optimal elliptic orbit for each orbital period, 8-figure, Tundra/Molniya, in that order.

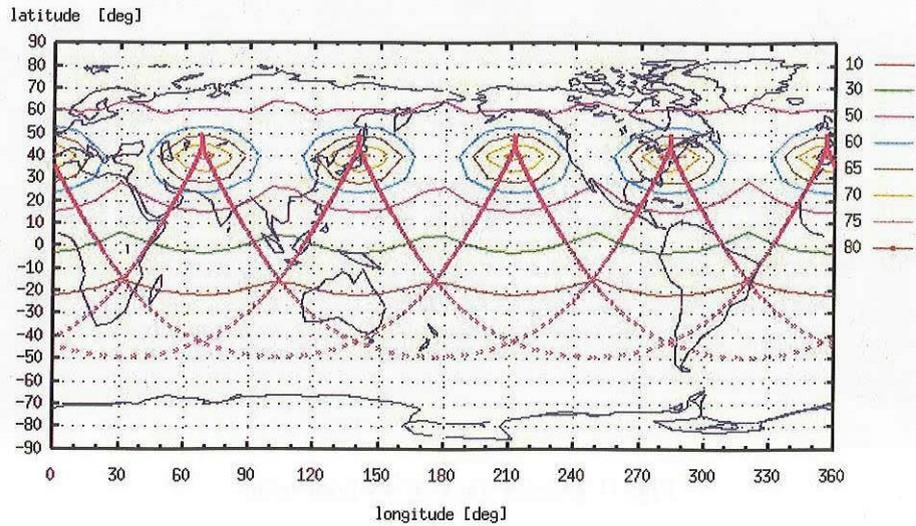


Fig. 12 ground track of 9.6-hour orbit and minimum elevation angle of 7-satellite constellation

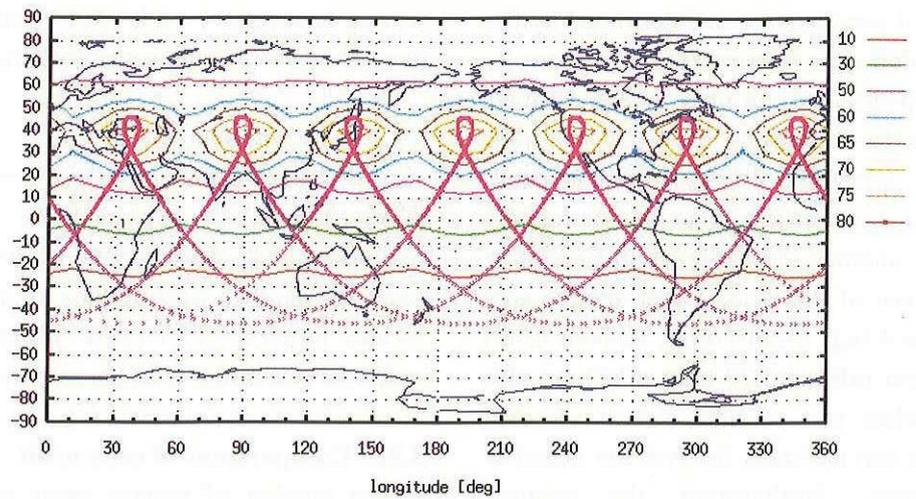


Fig. 13 ground track of 10.3-hour orbit and minimum elevation angle of 10-satellite constellation

Table 1 (1) Maximum coverage duration for a continuous elevation angle of >70 deg and orbital elements required to achieve them, and (2) minimum elevation angles during an 8.5-hour coverage duration and orbital elements required to achieve them.

	semi-major axis(km)	time elev. angle>70 deg in Tokyo						minimum elev. angle during 8.5-hour				
		time(h)	i(deg)	e	$\Omega$ (deg)	$\omega$ (deg)	elev. angle(deg)	i(deg)	e	$\Omega$ (deg)	$\omega$ (deg)	
12h orbit	Molniya	26600	3.5	63.4	0.75	73	315	58.1	63.4	0.75	133	270
	optimal	26600	10.2	52.0	0.75	133	270	78.0	46.0	0.69	131	270
16h orbit	optimal	32177	11.1	53.0	0.57	150	270	78.9	45.0	0.45	143	270
24h orbit	Tundra	42160	3.5	63.4	0.25	160	234	58.6	63.4	0.25	168	270
	8-figure	42160	8.4	52.0	0.00	139	270	69.6	53.0	0.00	140	270
	optimal	42160	11.3	53.0	0.26	169	270	79.3	45.0	0.18	159	273
48h orbit	optimal	66931	11.5	53.0	0.17	190	90	79.4	45.0	0.19	190	96

Table 2 The required number of satellites for each orbit.

		recurrent period					day hour
		1	2	3	4	5	
		24.0	48.0	72.0	96.0	120.0	
number of revolution(= num. of service area)	1	24.0 11.3 3	48.0 11.5 5	72.0 11~ 7~	96.0 11~ 9~	120.0 11~ 11~	
	2	12.0 10.2 3	24.0	36.0 11~ 7~	48.0	60.0 11~ 11~	
	3	8.0 5.2 6	16.0 11.1 5	24.0	32.0 11~ 9~	40.0 11~ 11~	
	4	6.0 2.67 12	12.0	18.0 11.18 7	24.0	30.0 11~ 11~	
	5	4.8 1.47 25	9.6 7.50 7	14.4 10.93 7	19.2 11.23 9	24.0	
	6	4.0 0.87 66	8.0	12.0	16.0	20.0 11~ 12~	
	7	3.4 0.57 361	6.9 3.78 15	10.3 8.08 10	13.7 10.75 10	17.1 10.90 12	
	8	3.0	6.0	9.0 6.60 12	12.0	15.0 10.95 12	

4. Merits and demerits

In this section all orbits are compared considering orbit perturbations, link budget, launching cost, and so on.

4.1. Orbit perturbations

Orbital elements vary with time, since perturbation forces arise from the non-spherical mass distribution of the Earth, gravitational forces from the Sun and the Moon and so on; orbit maintenance to correct the orbital elements is required. To evaluate the required ΔV to maintain a high elevation angle, numerical simulations have been performed using Gauss’s variational equations of motion:

$$\frac{da}{dt} = \frac{2}{na} (R \frac{ae}{\eta} \sin f + S \frac{a^2 \eta}{r})$$

$$\frac{de}{dt} = \frac{\eta}{na} \{R \sin f + S(\cos f + \cos u)\}$$

$$\frac{di}{dt} = \frac{r}{na^2 \eta} W \cos(f + \omega)$$

$$\frac{d\sigma^l}{dt} = -\frac{1}{na} (\frac{2r}{a} - \frac{\eta^2}{e} \cos f) R - \frac{\eta^2}{nae} (1 + \frac{r}{p}) S \sin f$$

$$\frac{d\omega}{dt} = \frac{\eta}{nae} \{-R \cos f + S(1 + \frac{r}{p}) \sin f\} - \frac{r \sin(f + \omega)}{na^2 \eta} W \cot i$$

$$\frac{d\Omega}{dt} = \frac{r \sin(f + \omega)}{na^2 \eta \sin i} W$$

$$\frac{d\rho}{dt} = n$$

$$\frac{d^2 \rho}{dt^2} = -\frac{3n}{2a} \frac{da}{dt}$$

where  $\frac{d\sigma^l}{dt} = \frac{d\sigma}{dt} + t \frac{dn}{dt}$ , R is the force in the orbit radial direction, W is the force in the orbit normal,

and  $S$  is the force perpendicular to  $R$  and  $W$ . Hereafter, non-spherical mass distribution of the Earth, the air drag, the gravitational force from the Sun and the Moon, and the solar pressure will be considered as perturbation forces.

#### 4.1.1. Non-spherical mass distribution of the Earth

The geo-potential  $U$  is written as follows:

$$U = \frac{Gm_e}{a_e} \sum_{n=1}^{\infty} \sum_{m=1}^n \left(\frac{a_e}{r}\right)^{n+1} P_n^m(\sin \phi) (C_{n,m} \cos m\psi + S_{n,m} \sin m\psi)$$

The force in the radial direction, the force in the latitudinal direction and longitudinal direction are:

$$\begin{aligned} f_r &= -\frac{\partial U}{\partial r} \\ &= -\frac{Gm_e}{a_e^2} \sum_{n=1}^{\infty} \sum_{m=1}^n (n+1) \left(\frac{a_e}{r}\right)^{n+2} P_n^m(\sin \phi) (C_{n,m} \cos m\psi + S_{n,m} \sin m\psi) \\ f_\phi &= -\frac{1}{r} \frac{\partial U}{\partial \phi} \\ &= -\frac{Gm_e}{a_e^2} \sum_{n=1}^{\infty} \sum_{m=1}^n \left(\frac{a_e}{r}\right)^{n+2} \frac{d(\sin \phi)}{d\phi} P_n^m(\sin \phi) (C_{n,m} \cos m\psi + S_{n,m} \sin m\psi) \\ f_\psi &= -\frac{1}{r \cos \phi} \frac{\partial U}{\partial \psi} \\ &= \frac{1}{\cos \phi} \frac{Gm_e}{a_e^2} \sum_{n=1}^{\infty} \sum_{m=1}^n \left(\frac{a_e}{r}\right)^{n+2} P_n^m(\sin \phi) m (C_{n,m} \sin m\psi - S_{n,m} \cos m\psi) \end{aligned}$$

Thus, by dividing this force into the above-mentioned direction and substituting them for Gauss's variational equation of motion, the perturbation can be calculated. EGM96 (Earth Gravitational Model) (10 \*10) was used to model the Earth's geo-potential field. To estimate the secular perturbation, the following equation can be used:

$$\begin{aligned} \frac{d\Omega}{dt} &= -\frac{3}{2} J_2 \sqrt{\mu_e} a_e^2 a^{-7/2} (\cos i) (1-e^2)^{-2} \\ \frac{d\omega}{dt} &= \frac{3}{4} J_2 \sqrt{\mu_e} a_e^2 a^{-7/2} (4-5 \sin^2 i) (1-e^2)^{-2} \end{aligned}$$

#### 4.1.2. Air Drag

The air drag can be modeled as

$$F_{air} = -\frac{1}{2} \rho C_D A v^2,$$

where  $\rho$  is the atmospheric density, and  $C_D$  is the

coefficient of drag,  $A$  is the satellite cross-sectional area, and  $v$  is the satellites velocity with respect to the atmosphere.

The acceleration by the air drag is in proportion to  $C_D A/m$ , where  $m$  is the mass of the satellite, and this term is modeled as a constant value of  $2.2*0.01 \text{ m}^2/\text{kg}$ . NRLMSISE-00 Model (NRL Mass Spectrometer, Incoherent Scatter Radar Extended Model) was used to model the atmospheric density.

#### 4.1.3. Third-body perturbations

The gravitational force of the Sun works as a tidal force, and the principal disturbing function is

$$R_s = \frac{Gm_s r^2}{r_s^3} P_2(\cos \theta) = Gm_s \frac{r}{r_s^3} \left(\frac{3}{2} \cos^2 \theta - \frac{1}{2}\right),$$

where  $m_s$  is the mass of the Sun,  $r_s$  is the radius of the Earth rotating around the Sun, and  $\theta$  is the angle between the Sun and the position vector of the satellite.

These rates of change depend on the initial  $\Omega$ . The secular term can be calculated by taking the average over 1 year,

$$R_{s,sec} = \frac{Gm_s r^2}{r_s^3} \left\{ \frac{1}{8} \left(1 + \frac{3}{2} e^2\right) (3 \cos^2 \bar{i} - 1) + \frac{15}{16} e^2 \sin^2 \bar{i} \cos 2\bar{\omega} \right\}$$

thus,

$$\begin{aligned} \frac{d\bar{e}}{dt} &= \frac{Gm_s}{r_s^3} \frac{\eta}{n} \frac{15}{8} e \sin^2 \bar{i} \sin 2\bar{\omega} \\ \frac{d\bar{i}}{dt} &= -\frac{Gm_s}{r_s^3} \frac{\cot \bar{i}}{n\eta} \frac{15}{8} e^2 \sin^2 \bar{i} \sin 2\bar{\omega} \\ \frac{d\bar{\omega}}{dt} &= \frac{Gm_s}{r_s^3} \left\{ \frac{\eta}{n} \left( \frac{3}{8} (3 \cos^2 \bar{i} - 1) + \frac{15}{8} \sin^2 \bar{i} \cos 2\bar{\omega} \right) \right. \\ &\quad \left. + \frac{\cot \bar{i}}{n\eta} \left( \frac{3}{4} \left(1 + \frac{3}{2} e^2\right) \sin \bar{i} \cos \bar{i} - \frac{15}{8} e^2 \sin \bar{i} \cos \bar{i} \cos 2\bar{\omega} \right) \right\} \\ \frac{d\bar{\Omega}}{dt} &= \frac{Gm_s}{r_s^3} \frac{1}{na^2 \eta \sin \bar{i}} \left( -\frac{3}{4} \left(1 + \frac{3}{2} e^2\right) \sin \bar{i} \cos \bar{i} \right. \\ &\quad \left. + \frac{15}{8} e^2 \sin \bar{i} \cos \bar{i} \cos 2\bar{\omega} \right) \end{aligned}$$

where,  $\bar{a}, \bar{e}, \bar{i}, \bar{\omega}, \bar{\Omega}$ , are the ecliptic orbital

elements.

Fig. 14 shows that the change rates of  $i$ ,  $\omega$  and  $\Omega$  by the solar gravity for a 16-hour orbit depend on the initial  $\Omega$ . Thus the required  $\Delta V$  for maintaining  $i$ ,  $\omega$  and  $\Omega$  should be estimated for each worst case. The initial  $\Omega$  is set to be around 75 deg in the following example. The gravitational force of the Moon can be calculated as same as that of the Sun.

#### 4.1.4 The solar radiation pressure

The solar radiation pressure works in the opposite direction of solar vector, and it is modeled as

$$F_{air} = -\frac{\Phi}{C} C_R A,$$

where  $\Phi$  is the solar energy flux,  $C$  is the speed of light,  $C_R$  is the radiation force coefficient, and  $A$  is the satellite cross-sectional area.

#### 4.1.5. The variation in orbital elements

Fig. 15 shows the variation of the orbital elements of 16-hour orbit as one example. The orbital elements vary because of the perturbation forces described above. Particularly, as a result of the  $J_2$  effect and the gravity of the Sun and the Moon,  $\Omega$  will increase and  $\omega$  will decrease in orbits of inclination less than 63.44 deg. Fig. 16 and Fig. 17 show the minimum elevation angle for a 16-hour orbit during 8.5-hour coverage duration in Tokyo, for each orbital elements  $i$  and  $e$  (Fig. 16), and  $\Omega'$  and  $\omega$  (Fig. 17). It is shown that the dominant elements for maintaining a high elevation angle are  $\omega$  and  $\Omega$ . Here, Fig. 17 shows that if  $\Omega'$  decreases while  $\omega$  increases, the elevation angle changes only gradually. Thus, it has small effect on maintaining high elevation angle for couple of years although  $\Omega'$  and  $\omega$  will vary because of the  $J_2$  effect and so on.

Fig. 18 shows the ground track of an optimal 16-hour orbit at a certain time, and again one year later. Although the shape of ground track has changed, the minimum elevation angle changes from 78.9 to 75.5 deg.

It is also possible to compensate these variations by on-board thruster control. Argument of perigee,  $\omega$  can be maintained by giving the thrust normal to the orbital plane around apogee. A Chemical

propulsion appears to be unfeasible, since the required  $\Delta V$  is about 600 m/s/year for a 16-hour orbit and the required mass of the propellant is about one third of the satellite for a year. On the other hand, when a highly-efficient propulsion system, such as an ion engine, is used, the required mass ratio is sufficiently small. Assuming that an ion engine has a specific impulse of 3,500 sec, the required propellant mass for the same case is only about 2%. Fig. 19 shows the ground track of an orbit at a certain time, and one year later with low thrust normal to the orbital plane for some time passing nearby apogee and it was found that  $\omega$  could be controlled. The RAAN,  $\Omega$ , was not maintained at the same time, but by changing the orbital altitude, it is possible to maintain a high elevation angle. In this case, when the altitude is increased by about 1 km, then a high elevation angle is maintained (Fig. 20).

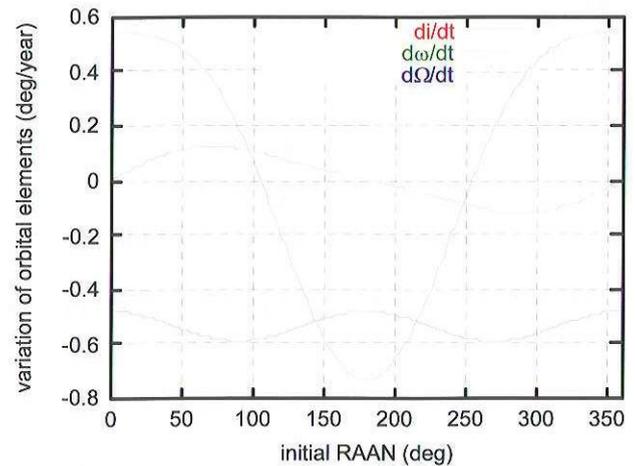
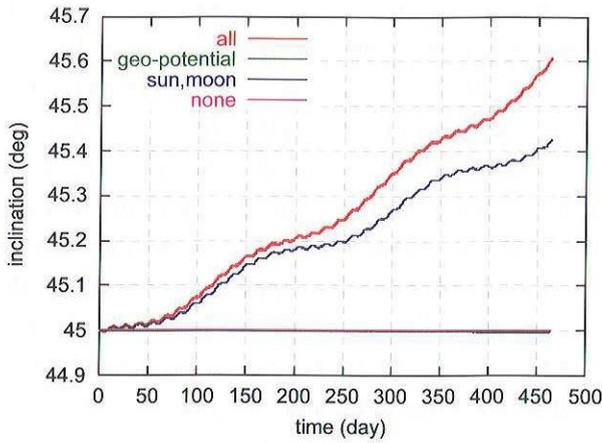
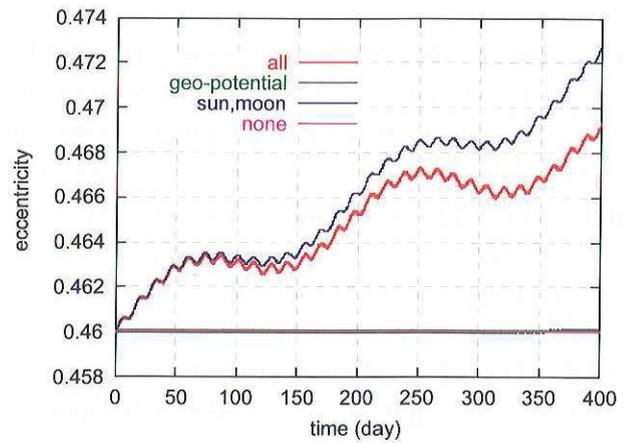


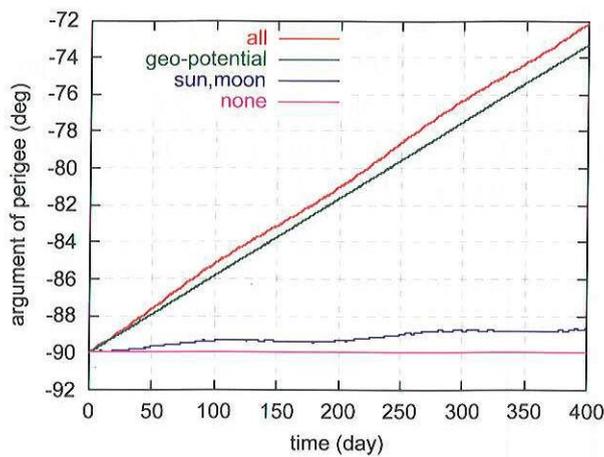
Fig. 14 The dependency of initial RAAN,  $\Omega$  by the gravity of the Sun.



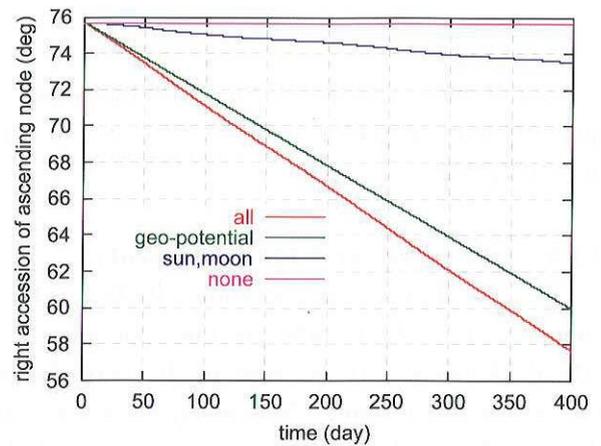
variation in inclination (deg)



variation in eccentricity



variation in arg. of perigee (deg)



variation in RAAN (deg)

Fig. 15 variations in each orbital element by perturbation force

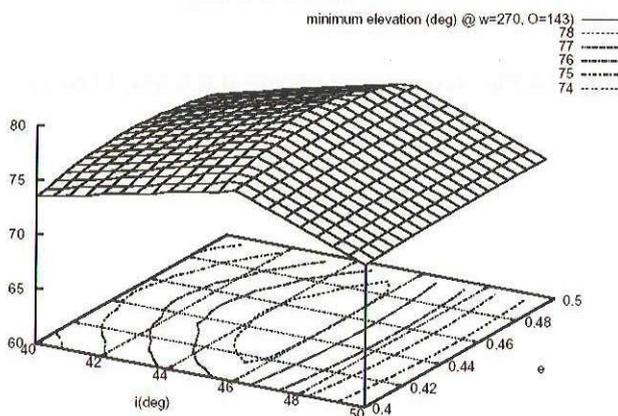


Fig. 16 The dependency of the minimum elevation angle for  $i$  (deg) and  $e$

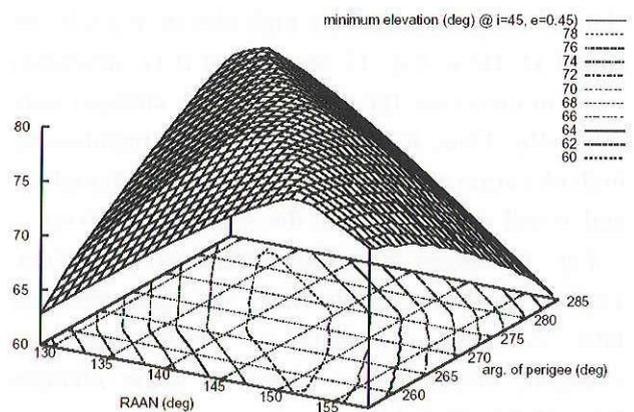


Fig. 17 The dependency of the minimum elevation angle for  $\Omega$  (deg) and  $\omega$  (deg)

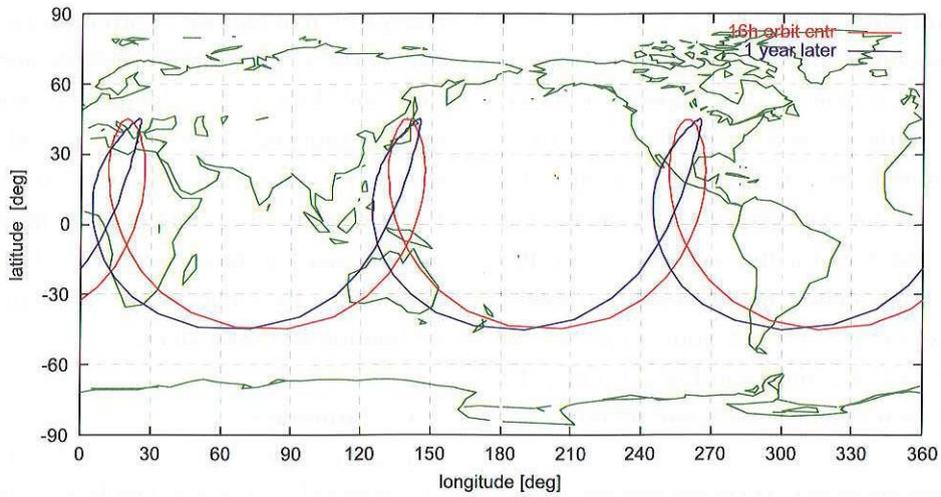


Fig. 18 Ground tracks of 16h orbit and its 1 year later without keeping menueover

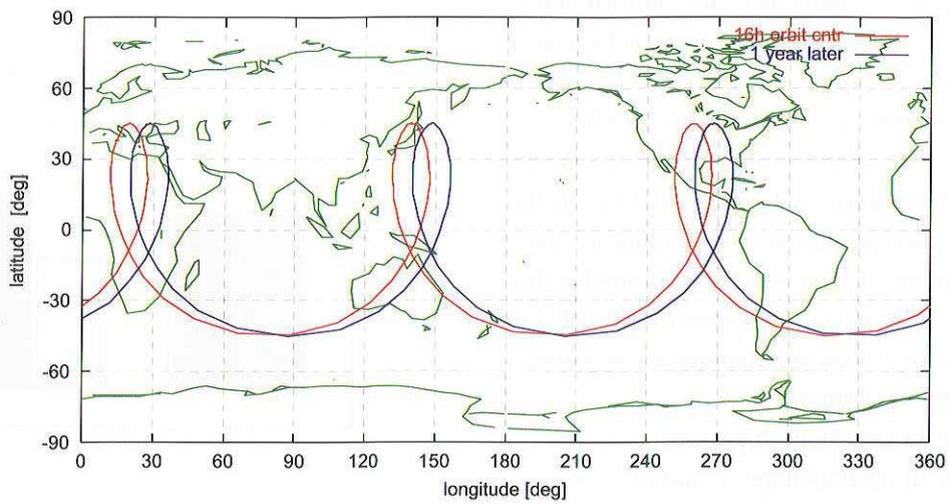


Fig. 19 Ground tracks of 16h orbit and its 1 year later with control. Although Argument of perigee is maintained with control, high elevation angle is not maintained since RAAN is not controlled.

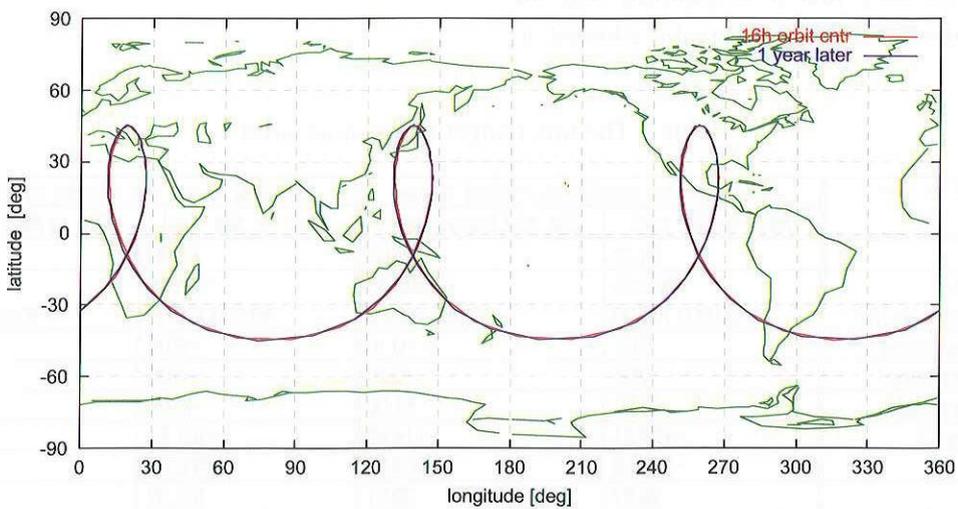


Fig. 20 Ground tracks of 16h orbit and its 1 year later with control. Initial semi-major-axis is set to be higher by 1 km to maintain high elevation angle. Argument of perigee is maintained with control.

**4.2. Communication link**

The communication link was calculated for a small satellite experiment. A data transmission rate between the ambulance and medical center of 6 Mbps was required. By assuming a an antenna diameter of 0.4 m for the ambulance, 2 m for the small satellite, and 5 m for the medical center, the link margin was calculated for each orbit. Table 3 shows the link budget for 16-hour orbit as an example. The results are shown in Fig. 21. Only the 48-hour orbit does not have a sufficient margin.

**4.3. ΔV for transfer from GTO to the mission orbits**

In the demonstration phase of this system, a small, piggyback satellite will be used. The opportunities for such piggyback launches are unfortunately limited, and the orbit of delivery cannot be freely chosen. Therefore a piggyback ride to GTO seems the best starting point from where the satellite can be transferred to its desired orbit. In the first place the initial orbit is assumed to be a GTO with the same orbital parameters as the Japanese laser ranging equipment (LRE) satellite nominal orbit, which are typical for GTO. To transfer from GTO to the target orbit, the following changes are required:

- inclination change,  $i$
- change in argument of perigee
- change in orbital shape ( $a$  and  $e$ )

Correct positioning of the satellite and its orbit in terms of RAAN,  $\Omega$ , is not considered here, since this is dependent on the number of satellites in the constellation. To save fuel it is assumed that the argument of the perigee is passively allowed to

change by making use of orbital perturbation effects due to the  $J_2$  term and third-body attractions by the Sun and Moon. To change the above elements, several sequences are possible, and the minimum required  $\Delta V$  for each orbit was calculated (Table 4). For all orbits, the sequence for minimizing  $\Delta V$  is waiting until  $\omega$  has changed by 90 deg, and then providing the necessary  $\Delta V$  to change the inclination and orbit shape.

**4.4. Summary**

Finally, the above-considered orbits are summarized in Table 5 The link budget for 16-hour orbit. The most appropriate orbit should be chosen on the basis of these merits and demerits.

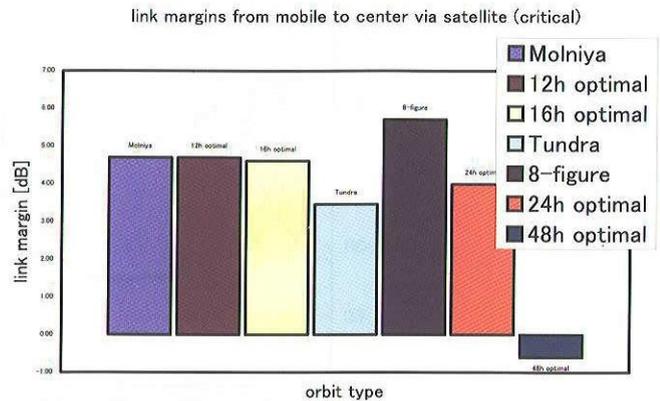


Fig. 21 The link margin for each orbit

Table 3 The link budget for 16-hour orbit

<i>16h optimal</i>	Ambulance via satellite to medical center		Medical center via satellite to mobile	
	<i>Mob. to Sat. (up)</i>	<i>Sat. to Center (down)</i>	<i>Cent. to Sat. (up)</i>	<i>Sat. to Mob. (down)</i>
EIRP	36.29	40.03	61.00	40.03
Path loss	-202.68	-205.45	-205.45	-205.45
apogee altitude [m]	40,600,700.00	40,600,700.00	40,600,700.00	40,600,700.00
atm. attenuation	-0.05	-0.10	-0.05	-0.10
rain attenuation	-0.05	-0.10	-0.05	-0.10
Re. antenna gain	40.27	51.00	43.04	29.06
Re. power C	-126.21	-114.62	-101.51	-136.56
No	-202.58	-203.83	-202.58	-203.83
C/No	76.37	89.21	101.07	67.27
requ. C/No	71.54	71.54	59.60	59.60
total C/No	76.15		67.27	
Margin	4.61		7.67	

Table 5 Comparison of each orbit

		semi-major axis(km)	inclination(deg)	eccentricity	perigee (km)	apogee (km)	d $\Omega$ (deg/year)	d $\omega$ (deg/year)	link budget	Van Allen belts	dV
12h orbit	Molniya	26600	63.44	0.75	272	40172	-60.5	0.9	○	×	○
	optimal	26600	46	0.69	1868	38576	-64.9	65.2	○	×	○
16h orbit	optimal	32177	45	0.45	11319	40279	-15.9	16.6	○	△	○
24h orbit	Tundra	42160	63.44	0.25	25242	46322	-4.2	-6.7	○	○	△
	8-figure	42160	53	0	35782	35782	-4.2	2.0	○	○	△
	optimal	42160	45	0.18	28193	43371	-5.3	5.9	○	○	△
48h orbit	optimal	66931	45	0.19	47836	73270	-2.0	5.7	×	○	△

Table 4 minimum  $\Delta V$  to transfer GTO to each orbit

		minimum $\Delta V$ (m/s)	#days for $90^\circ \Delta \omega$
12h orbit	Molniya	937	151.0
	optimal	559	151.0
16h orbit	optimal	922	151.0
24h orbit	Tundra	1647	151.0
	8-figure	1744	0.0
	optimal	1402	151.0
48h orbit	optimal	1524	151.0

### 5. Conclusion

Candidate orbits for the remote medical treatment utilizing communication satellites have been investigated. In this paper, several types of orbits such as 24-hour orbits, figure-eight orbit, Tundra orbit, 16-hour orbit, 12h-orbit, Molniya orbit, or other interesting orbits are compared, and their merits and demerits have been discussed considering orbit perturbations, link budget and launching cost. The orbit should be chosen on the basis of these merits and demerits; for a demonstration phase, 12-h orbit or smaller orbits are suitable which will use a small satellite with small cost and shorter mission period. For a real system, 16-hour orbit or 24-hour orbit, figure-eight orbit are considered suitable.

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### Reference

- [1] A. Nakajima, K. Ohmori, T. Kumagai, N. Nagaoka, and I. Nakajima, "Conceptual Study on Small Satellite for Mobile Communication",
- [2] I. Nakajima, Y. Yagi, Y. Zhao, N. Hamamoto, A. Nakajima, "Cost Comparison between HEO Satellites and 3G Wireless Mobile Phones for Video Transmission from Ambulances"
- [3] 河本, 熊谷, 大森, 中島, 「医療ミッション用高仰角軌道の評価」第45回宇宙科学技術連合講演会 2001
- [4] 木村, 田中, 「準天頂衛星システムの軌道保持」第44回宇宙科学技術連合講演会 pp.1337-1342, 2000
- [5] 木下, 「天体と軌道の力学」、東京大学出版会
- [6] J. Wertz and W. Larson, "Space Mission Analysis and Design"



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