# Markov Chain Monte Carlo Simulations and Binary Orbital Elements

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**ABSTRACT**

Recent simulations and observational data have been used to investigate the possibility of planet-planet scattering or Kozai oscillations to explain the formation of certain non-solar-like planetary systems. Markov Chain Monte Carlo (MCMC) simulations have benefits compared to standard Monte Carlo ones. With these advantages in mind, an advanced three dimensional MCMC simulator has been written to investigate the orbital solutions to the $\tau$ Boo system in order to better understand its possible formation history.

Over 890 extrasolar planets (or exoplanets) have been confirmed so far, with nearly 2000 more candidates awaiting ground-based follow-up, (Schneider et al. 2011; Batalha et al. 2013). Thanks to the large number of known planets, we are beginning to understand how planetary systems form and how abundant solar-like systems are. The first formation models assumed that planets formed on approximately coplanar and circular orbits, as we observe in our solar system today. However, as more planets were observed, it became clear that many (~17 %) are in close-in orbits, while others are eccentric or misaligned, (Nagasawa & Ida 2011). For these planets, new mechanisms were proposed to explain their existence. The two leading mechanisms are planet-planet scattering and Kozai oscillations, both of which involve a distant or ejected companion object.

In planet-planet scattering, the planetary system contains two planets with the larger one being slightly further out. Due to orbital interactions between these two, the outer planet is launched into a more distant and eccentric orbit where it then experiences dynamical friction with the planetary disk. This friction adds energy and angular momentum causing its pericenter to rise, (Chatterjee et al. 2011). Meanwhile, the inner planet will move inward and also see a raised eccentricity for some time until it is dissipated by tidal forces from the parent star. In the case of the 51 Pegasi system, if the tidal dissipation has not had the chance to fully circularize its orbit, the eccentric solution to the radial velocity curve can be explained using planet-planet scattering. The second body could possibly still be bound in a distant ($a > 20$ AU) and eccentric orbit outside the range to be easily detected, (Rasio & Ford 1996). Recent high-contrast efforts have found a class of planets matching this category with high-mass, distant and eccentric orbits from the parent star, providing further support to this mechanism, (Kalas et al. 2008; Marois et al. 2008).

For those systems where a planet hosting primary star also has a stellar mass companion, Kozai oscillations can play an important role in the system's evolution. Exchanges of energy and angular momentum between the stellar companion and planet induce an oscillation of their eccentricities and inclinations. When excited to a higher eccentricity, the planet would consequently also have closer periastron passages where tidal dissipation could damp the planet onto a short-period, nearly circular orbit, (Beust et al. 2012). This process is commonly referred to as Kozai migration. Now knowing that most stars form in multi-star systems, suggestions of the Kozai migration's possible role in evolution histories have become more common. This effect could possibly explain the orbit of $\tau$ Boo Ab, caused by the interactions with the very distant and eccentric companion star, $\tau$ Boo B. The orbit of $\tau$ Boo Ab is only ~3.3 days and has an eccentricity that is compatible with zero (Brogi et al. 2012; Rodler et al. 2012; Donati et al. 2008). With strongly constrained orbital parameters the equations of Ford et al. (2000) can be applied to compare the period of the Kozai oscillations to the ~2 Gyr age of $\tau$ Boo A, allowing for investigation if they could have played a role in the planet's formation.

Three previous investigations of $\tau$ Boo B's orbit estimated it to have a 390–2000 year period and an eccentricity of 0.42–0.91 (Hale 1994; Popović & Pavlović 1996; Roberts et al. 2011), see Figure 1. Even with 163 years of observations, $\tau$ Boo B has traversed less than half of the shortest proposed orbit. Investigations of such long-period systems require careful analyses, preferably with explicit priors and well-characterized posterior probability distributions. We therefore re-analyze the $\tau$ Boo system using Markov Chain Monte Carlo (MCMC).

One of the benefits of MCMC is that, in the limit of a long chain, its samples are drawn directly from the posterior probability distribution. A standard Monte Carlo simulation, by contrast, will draw many points in regions of parameter space with exceptionally low likelihood. MCMC can therefore be significantly more efficient at exploring the parameter space in spite of the fact that its samples are not independent. Because MCMC directly probes the posterior probability distribution, calculating 68 % and 95 % confidence intervals for each parameter also becomes trivial.

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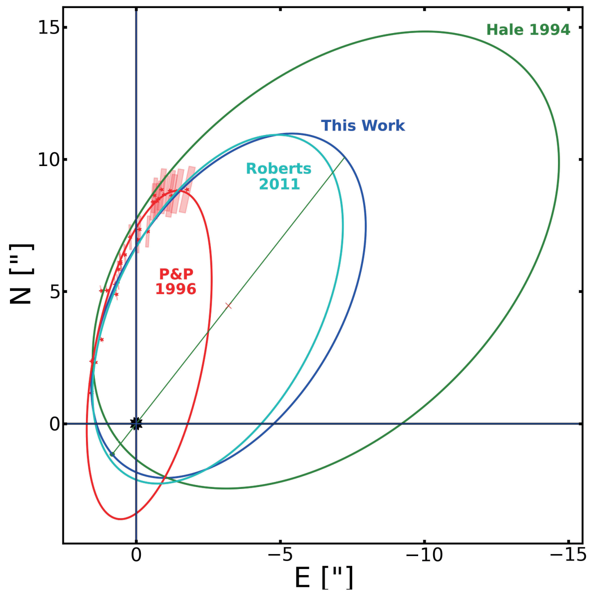


**Figure 1.** Three previously proposed orbits for $\tau$ Boo B (Hale 1994; Popović & Pavlović 1996; Roberts et al. 2011), together with our best-fit. The locations of $\tau$ Boo B are shown as the small stars, with their associated errors, and $\tau$ Boo A at the origin. The proposed orbits have not converged to a consensus solution, which is needed for a complete assessment of the Kozai formation scenario.

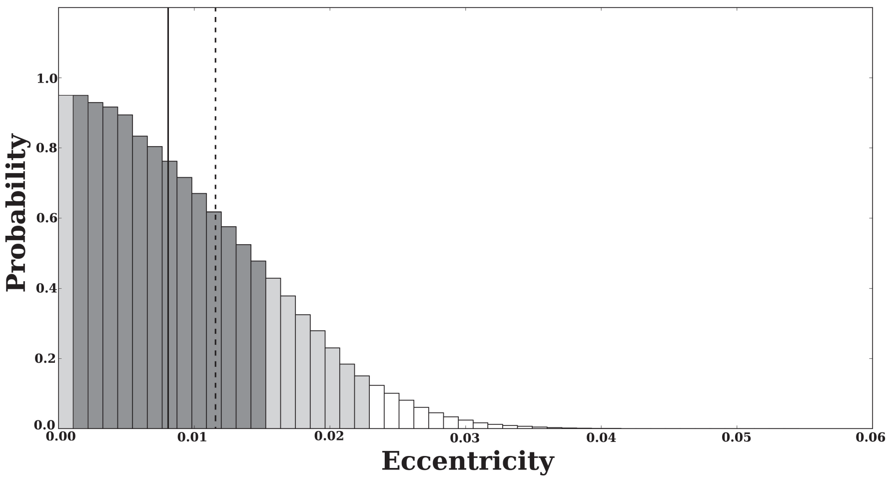


**Figure 2.** The posterior distribution of the eccentricity parameter from $\tau$ Boo Ab orbital fitting using our MCMC simulator. The black lines mark the median value, dashed line the best fit, dark grey bars are within the 68.3 % confidence region and light grey are within the 95 %.

A further advantage of well developed posterior distributions for the model parameters is to allow the investigation of values very close to the parameter's limits. For short period planets ($P \sim$ 1–20 days), the close proximity to the parent star causes strong tidal forces that will likely dampen any perturbations due to an eccentric orbit, (Husnoo et al. 2012). Owing to the hard minimum limit of zero on the eccentricity, $e$, it is impossible to prove that an orbit is circular, but rejection of any orbit over $e > 0.1$ to a 95 % degree of confidence is sufficient to show compatibility. In the case of $\tau$ Boo Ab, one of the most recent investigations into this planet's orbit demonstrated that it had an eccentricity compatible with zero by showing the parameter's posterior distribution had a strong trend towards zero (Brogi et al. 2012), compared to a previous solution of $e \sim 0.02$ by Butler et al. (2006). During the verification process of our own MCMC simulator, the $\tau$ Boo Ab orbit was found to contain 95 % of the solutions to $e < 0.022$ and a maximum value of 0.055, shown in Figure 2. We thereby also conclude that its orbit is circular and provide a plot of our fit in Figure 3.

Previous investigations of $\tau$ Boo B's orbit used only the positions measured from direct imaging data. However, radial velocity measurements of $\tau$ Boo A show a trend from $\tau$ Boo B superimposed on $\tau$ Boo Ab's 3.3 day orbit. We are taking advantage of these new data, along with more recent astrometry, in a full three-dimensional simulator. This analysis will improve our understanding of $\tau$ Boo B's orbit, and constrain models suggesting Kozai oscillations as the origin of the "hot

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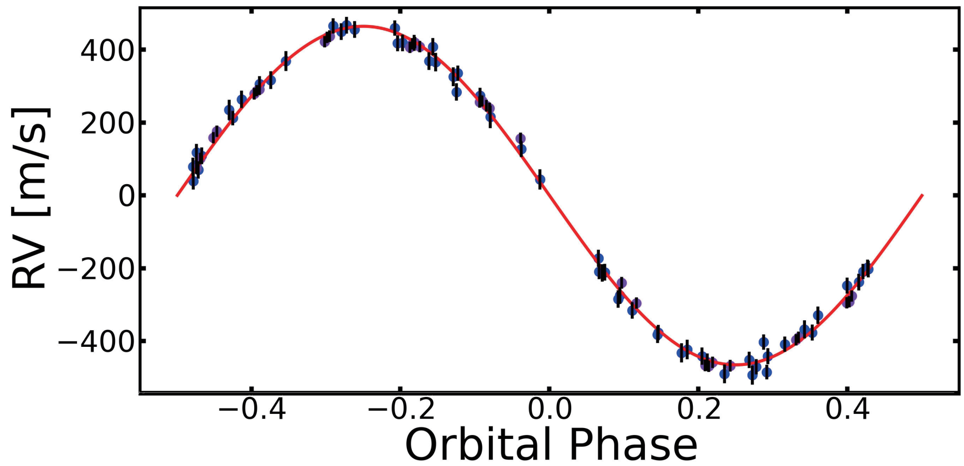


**Figure 3.** The radial velocity curve plot for our best fit circular solution for $\tau$ Boo Ab along with the post-1995 data from Butler et al. (2006) and Donati et al. (2008).

Jupiter" $\tau$ Boo Ab. Our initial results show a more eccentric orbit for $\tau$ Boo B than the most recent investigation, Roberts et al. (2011), with $e = 0.79 \pm 0.01$.

Our new constraints on the orbit of $\tau$ Boo B will allow dynamical calculations of plausible orbital histories of the system over its 2 Gyr life, and will confirm or reject the possibility of a Kozai origin.

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