

Freejets of Rotationally Relaxing Diatomic Gases

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Summary: Numerical solutions for the freejet expansion flow of rotationally relaxing diatomic gases have been obtained by the operator splitting method and two-step Lax-Wendroff scheme. Computational results of the translational and rotational temperature distributions are shown for four cases of the ratio of the characteristic times for the flow and rotational relaxation which characterizes a rotational nonequilibrium flow. The results show that as this ratio increases, the translational and rotational temperatures approach each other, and that for a large value of the ratio, both of the temperatures are similar to the equilibrium temperature.

1. INTRODUCTION

A freejet is such that a gas is expanded through an orifice into a vacuum chamber. According to Ashkenas and Sherman [1], the flow properties on the centerline of the freejet can be approximated by a source flow expansion. Also, they gave the expression of a Mach number on the freejet centerline for diatomic gases. The flow model given by Ashkenas and Sherman is only applied to the equilibrium freejets. However, the rapid decrease in the gas density in the freejet expansion is accompanied by insufficient molecular collisions for equilibrium between translational and rotational energy modes. Consequently, the difference between the translational and rotational temperatures is generated. In such a nonequilibrium state, the gas does not behave like diatomic molecule gases and also not like monatomic gases, and a Mach number cannot be specified. Therefore, Ashkenas and Sherman's flow model cannot be applied to nonequilibrium freejets any more. Hence, for the nonequilibrium freejets, it is required to directly solve the freejet equations including rotational relaxation.

With respect to the nonequilibrium freejets of rotationally relaxing gases, Labowski *et al.* made the flow field calculations in rotationally relaxing freejets by the method of characteristics [2]. Except for this work, the nonequilibrium freejets have been investigated by a source flow approximation [3, 4].

The present work is concerned with numerical calculations of a nonequilibrium freejet flowfield of rotationally relaxing diatomic gases. The freejet equations including rotational relaxation were solved by using the operator splitting method and the two-step Lax-Wendroff scheme.

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2. GOVERNING EQUATIONS

The following assumptions are introduced for the freejets of interest here: (1) the gas is composed of diatomic molecules, (2) the freejet is inviscid and axisymmetric, (3) the individual Boltzmann equilibrium state is established in translational and rotational energy modes.

Under the above assumptions, the unsteady flow equations including the rotational relaxation are written as, by using the coordinates system shown in Fig. 1,

$$\frac{\partial \rho}{\partial t} + \frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial r} + \frac{\rho v}{r} = 0, \quad (1)$$

$$\frac{\partial(\rho u)}{\partial t} + \frac{\partial(\rho u^2 + p)}{\partial x} + \frac{\partial(\rho uv)}{\partial r} + \frac{\rho uv}{r} = 0, \quad (2)$$

$$\frac{\partial(\rho v)}{\partial t} + \frac{\partial(\rho uv)}{\partial x} + \frac{\partial(\rho v^2 + p)}{\partial r} + \frac{\rho v^2}{r} = 0, \quad (3)$$

$$\frac{\partial e}{\partial t} + \frac{\partial[u(e+p)]}{\partial x} + \frac{\partial[v(e+p)]}{\partial r} + \frac{v(e+p)}{r} = 0, \quad (4)$$

$$\frac{\partial(\rho \dot{\epsilon}_r)}{\partial t} + \frac{\partial(\rho u \dot{\epsilon}_r)}{\partial x} + \frac{\partial(\rho v \dot{\epsilon}_r)}{\partial r} + \frac{\rho v \dot{\epsilon}_r}{r} = \rho \dot{\epsilon}_r, \quad (5)$$

with

$$e = \rho \left[\epsilon_t + \epsilon_r + \frac{1}{2} (u^2 + v^2) \right], \quad (6)$$

where t is the time, ρ the density, p the pressure, e the total energy per unit volume, ϵ_t the translational energy per unit mass, ϵ_r the rotational energy per unit mass, $\dot{\epsilon}_r$ the energy transfer rate per unit mass between the translational and rotational energy modes.

We assume the Boltzmann distribution in the translational and rotational energy modes. Furthermore, we treat a case when a rotational temperature is much higher than a characteristic temperature for rotation. Under these conditions, the translational and rotational energies can be related to a translational temperature T_t and a rotational temperature T_r as follows, respectively:

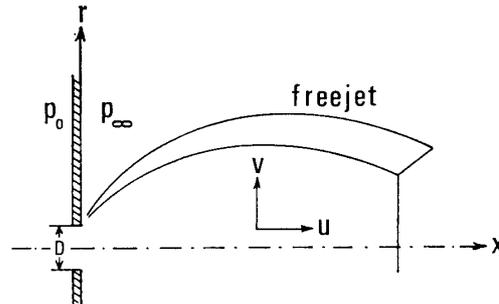


Fig. 1. Coordinates system for a freejet.

$$\varepsilon_t = \frac{3}{2} \frac{k_B}{m} T_t, \quad \varepsilon_r = \frac{k_B}{m} T_r, \quad (7)$$

where k_B is the Boltzmann constant and m the molecular mass. By using Eq. (7), the rotational energy equation (5) is rewritten as

$$\frac{\partial(\rho T_r)}{\partial t} + \frac{\partial(\rho u T_r)}{\partial x} + \frac{\partial(\rho v T_r)}{\partial r} + \frac{\rho v T_r}{r} = \rho \dot{T}_r. \quad (8)$$

The relaxation rate of the rotational temperature is governed by the following relation:

$$\dot{T}_r = (T_t - T_r) / \tau_r, \quad (9)$$

where τ_r is the relaxation time for rotation given by

$$\tau_r = Z_r / [4N\sigma^2(\pi k_B T_t / m)^{1/2}]. \quad (10)$$

In the above relation, Z_r is the collision number, N the number density, σ the effective hard-sphere diameter for a molecule. The pressure is related only to the translational temperature so that we have $p = \rho \tilde{R} T_t$ where \tilde{R} is the gas constant.

Except for x and r , the flow properties are nondimensionalized with respect to the properties at stagnation.

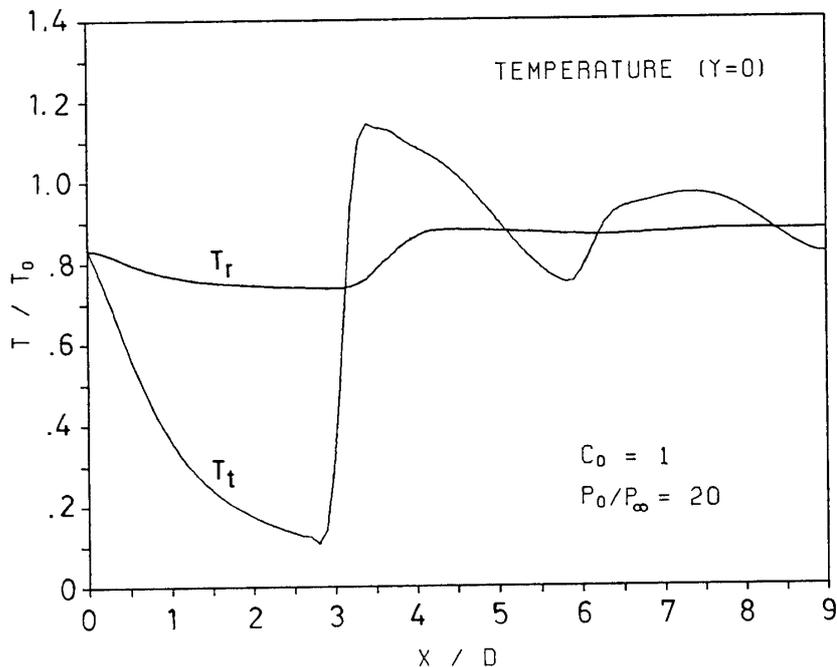
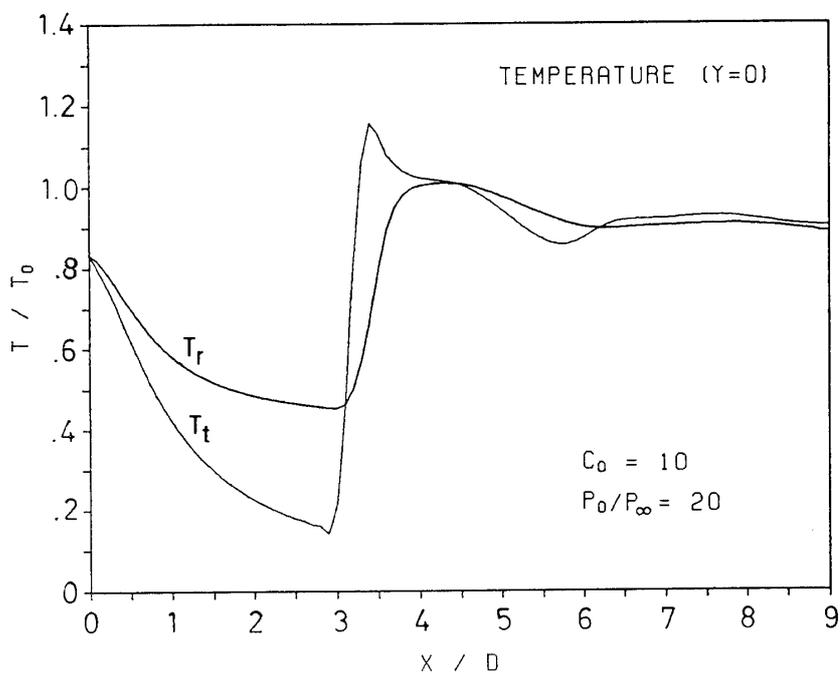
$$\begin{aligned} R &= \rho / \rho_0, & U &= u / a_0, & V &= v / a_0, & E &= e / \rho_0 a_0^2, & P &= p / p_0, \\ \Theta_t &= T_t / T_0, & \Theta_r &= T_r / T_0, & \tau &= a_0 t / D, & X &= x / D, & Y &= r / D, \end{aligned}$$

where a is the speed of sound, D the orifice diameter and the subscript 0 denotes the stagnation condition. With these nondimensional properties, the governing equations (1) to (4) and (8) are rewritten as

$$\frac{\partial U}{\partial \tau} + \frac{\partial F}{\partial X} + \frac{\partial G}{\partial Y} + Z = 0,$$

with

$$\begin{aligned} U &= \begin{bmatrix} R \\ RU \\ RV \\ E \\ R\Theta_r \end{bmatrix}, & F &= \begin{bmatrix} RU \\ RU^2 + P/\gamma_0 \\ RUV \\ U(E + P/\gamma_0) \\ RU\Theta_r \end{bmatrix}, & G &= \begin{bmatrix} RV \\ RUV \\ RV^2 + P/\gamma_0 \\ V(E + P/\gamma_0) \\ RV\Theta_r \end{bmatrix} \\ Z &= \frac{1}{Y} \begin{bmatrix} RV \\ RUV \\ RV^2 \\ V(E + P/\gamma_0) \\ RV\Theta_r \end{bmatrix} - \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ c_0 R^2 \Theta_t^{1/2} (\Theta_t - \Theta_r) \end{bmatrix} \\ E &= R \left[\frac{3}{2} \frac{\Theta_t}{\gamma_0} + \frac{\Theta_r}{\gamma_0} + \frac{1}{2} (U^2 + V^2) \right], & P &= R\Theta_t, \end{aligned}$$

Fig. 4. Axial distributions of T_t and T_r for $c_0=1$.Fig. 5. Axial distributions of T_t and T_r for $c_0=10$.

of the temperatures approach each other. In the figure, the temperature distribution for an equilibrium freejet of diatomic gases is also depicted. When $c_0=100$ the translational temperature is very similar to the equilibrium temperature. Hence, we can consider that if c_0 approaches infinity, both of the translational and rotational temperatures will approach the equilibrium temperature.

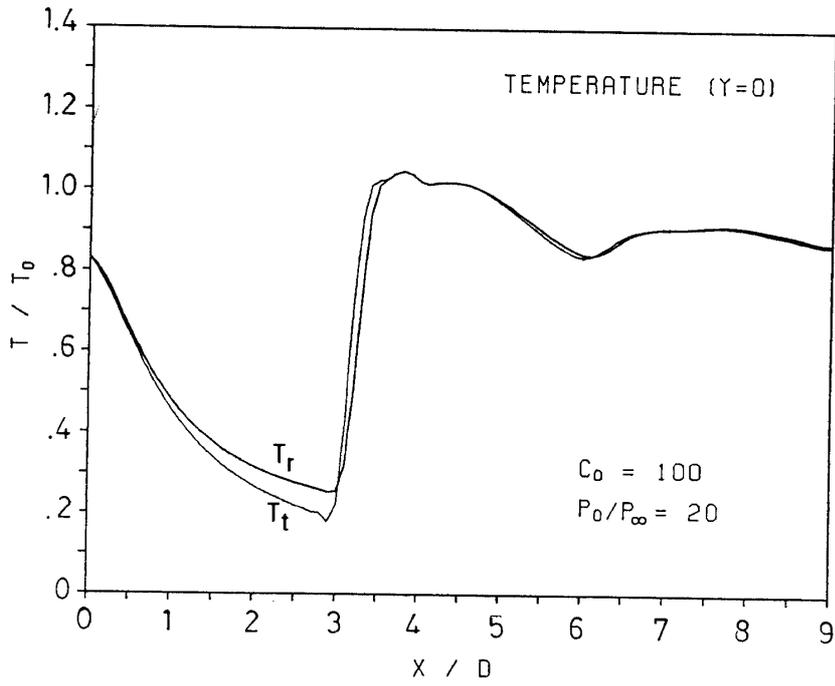


Fig. 6. Axial distributions of T_t and T_r for $c_0=100$.

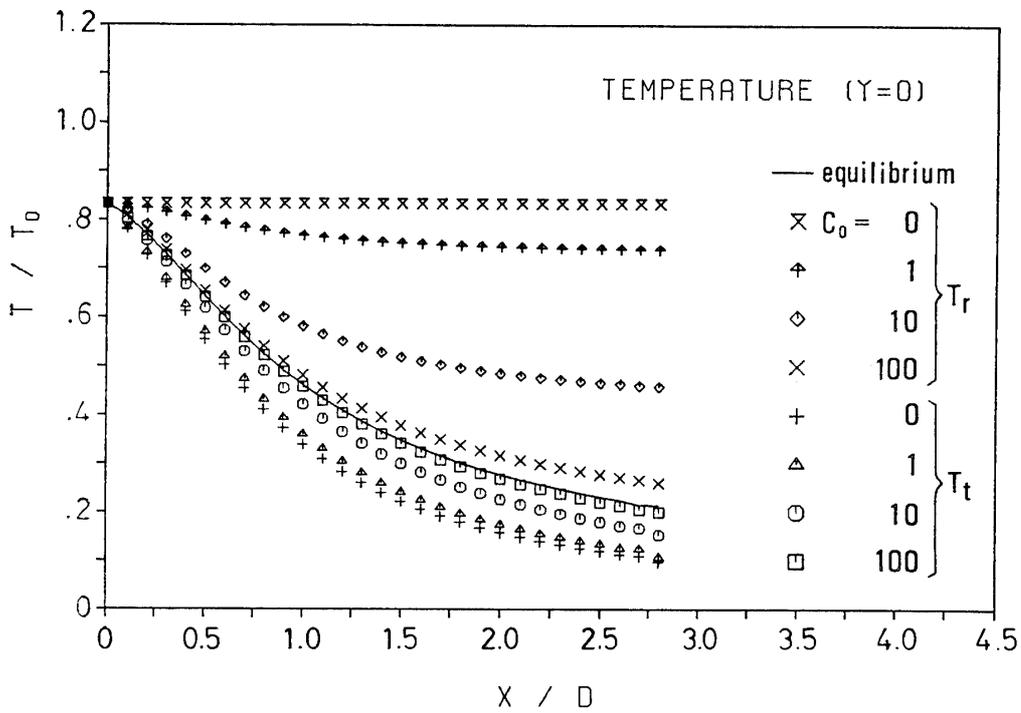


Fig. 7. Comparisons among T_t and T_r for various value of c_0 .

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