Computational Aerodynamics of Supersonic Lifting Vehicles

By

P. I. CHUSHKIN* and G. P. VOSKRESENSKY**

(January 27, 1986)

Summary: A review of investigations on numerical modelling of supersonic three-dimensional steady flows about lifting vehicles and their elements carried out in the Soviet Union during the last five years is given. An inviscid gas both perfect and high-temperature one is considered. A number of characteristic examples is presented.

§I. INTRODUCTION

One of the major problems of computational aerodynamics is calculation of a flow-field and determination of forces, mometns and heat fluxes acting on an aerospace vehicle and its separate elements. A review of the applications of numerical methods for the calculation of three-dimensional steady flows of inviscid gas about supersonic lifting vehicles is given in this paper. The solution of the Euler equations gives important and often ample information about flows, because the effect of viscosity may be taken into acount with the help of the boundary layer theory. along with perfect gas, a real high-temperature gas is considered.

The investigations conducted in the Soviet Union during the last five years are discussed, although some earlier fundamental works are mentioned. Two other publications of the authors [*1*,*2*] are devoted to the review of such investigations in the previous years. The aim of the paper is to present the results of the applications of numerical methods to the calculation of concrete flows without touching upon purely computational aspects.

Three basic classes of methods are used for the numerical simulation of three-dimensional flows about bodies flying at supersonic velosity: the finite-difference net methods, the method of characteristics and the method of integral relations (including the method of lines). All these methods are used at present in various modifications, the finite-difference net methods being practised on a larger scale. In calculating subsonic regions of the flow the first two types of the methods are usually applied in their unsteady variants, when the time-dependent stationing principle is used. Then a bow shock wave is treated as strong discontinuity and embedded shocks are smeared. The applications of three above-mentioned types of numerical methods, developed or widely used in the Soviet Union, are considered in the paper.

* Computing Center of the USSR Acad. of Sci., Moscow.
** Keldysh Inst. of Appl. Mathem. of the USSR Acad. of Sci., Moscow.

§II. Explicit First-Order Method

This method which is an analogue of the well-known unsteady method of Godunov was worked out by Ivanov and Kraiko for computations of steady three-dimensional supersonic flows (see [*3*]). It is based on the explicit monotonic difference scheme. An interesting feature of its algorithm is the calculation of the interaction of two supersonic streams on the boundary of the neighbouring net cells. The method allows "through counting" of inner discontinuities and is effective for flows with complex structure. However it has restrictions on the integration step and it is difficult to extend it to nonequilibrium gas flows.

The given method is widely used for computing flows with complex structure. However it has restrictions on the integration step and it is difficult to extend it to nonequilibrium gas flows.

The given method is widely used for computing flows about both simple supersonic lifting bodies and configurations modelling complete aerospace vehicles. In the first case, a thin ellipitical cone at a large angle of attack [*4*], a wing with triangle airfoil and a curved leading edge [*5*], delta wings interfering with other aerodynamic elements [*6,7*], a combination of a circular semi-cone with a delta plate [*8*] were calculated. Rosin [*9*] investigated numerically and experimentally the flow about an axisymmetric body with cross-like fins at the Mach number $M_\infty=3$, various angles of attack α and roll angles γ. The calculation was carried out smearing all the embedded shocks produced by the fin panels. Disturbances from the fin panels did not reach the bow shock wave. The change of the normal force coefficient Cn and the pitch moment coefficient Cm (referred to the area of two fin panels) versus the angle of attack α are represented in Fig. 1 where these characteristics are compared with the experimental data (circles) for two roll angles.

Aukin and Tagirov [*10,11*] calculated a three-dimensional flow about two different supersonic airplane models. In the second case the airplane had nonaxisymmetric fuselage, two engine nacelles, swept wings and tail unit of zero thickness (Fig. 2). It was supposed that the nacelles had no inner ducts and the jet of elliptical cross-section was ejected out of the stern base of fuselage. A number of calculated subdomains with their own nets were introduced. The distribution of the pressure coefficient Cp along the

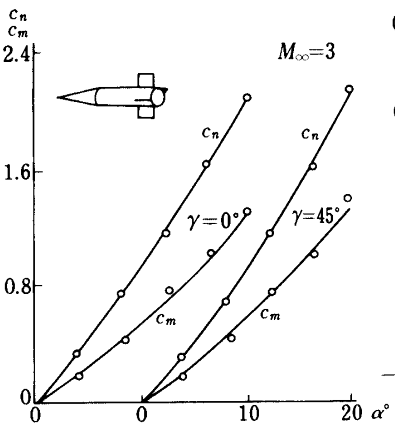

Fig. 1.

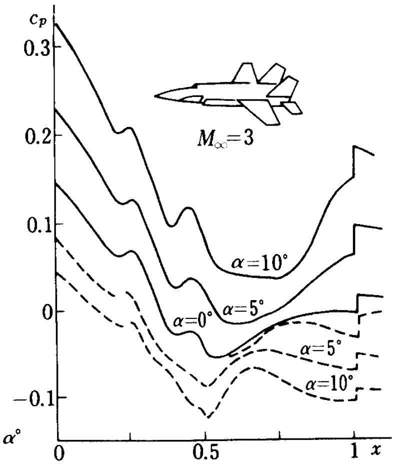

Fig. 2.

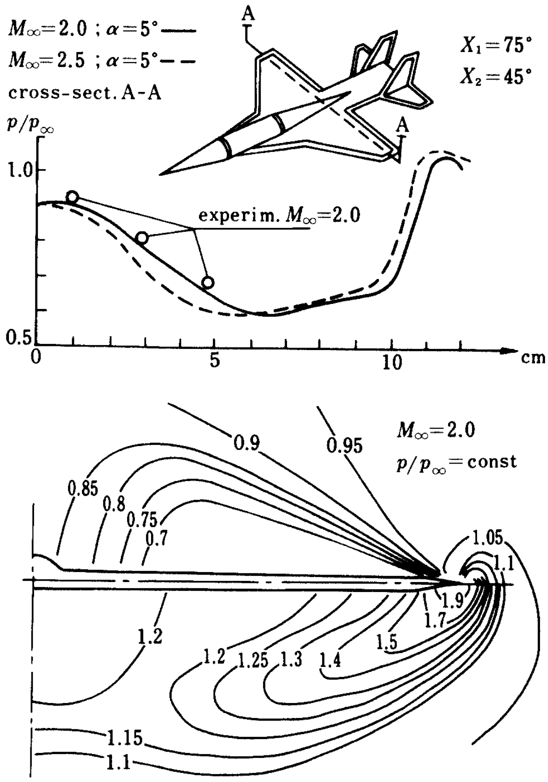

Fig. 3.

windward side (solid line) and the leeward side (dashed line) of the fuselage at the Mach number $M_\infty=2.3$ and a series of angles of attack is plotted in Fig. 2.

Another example of supersonic flow about a schematic airplane configuration is given in Fig. 3. The wings and the tail unit were represented by pointed plates, the fuselage had a sharp nose with an attached bow shock wave (embedded shocks were semared in the calculation). Gusev computed this case comparing the numerical results with the experiment carried out by Radzig and himself. The pressure distributions on the wing and the fuselage in the cross-section A—A are shown here for $M_\infty=2$ and 2.5 $\alpha=5°$. Experimental points are plotted for $M_\infty=2$. Besides, isobars in the disturbed flow region are drawn for the same cross-section.

Calculations of three-dimensional flows about complex bodies with subsonic regions are carried out with the help of the unsteady variant of the method. Mileshin computed the flow outside and inside the supersonic axisymmetric air intake with the central conical body in the regime of detached shock wave. The cases of zero [*12*] and nonzero angles of attack at the Mach number $M_\infty=2.48$ were studied. In Fig. 4 the air intake and the shock wave in the symmetry plane for different flow rate coefficients ψ at $\alpha=10°$ are shown. The sonic line for $\psi=0.773$ drawn by dashed line shows that the flow inside the air intake is subsonic. The contour of the shock wave has a triple point and this singularity was taken ito account in the calculations. Solving the problem a simpliied approach was used, in which the third independent variable (meridional angle) was eliminated with help of corresponding trigonometric approximations.

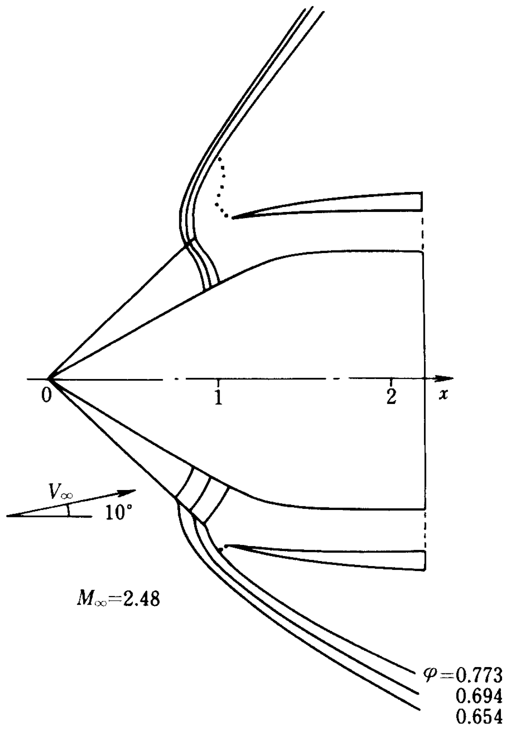

Fig. 4.

§III. Implicit Second-Oorder Method

This method for the computation of three-dimensional supersonic flows about smooth bodies was proposed by Babenko and Voskresensky [*13*] and then it was developed in [*14*]. Later, introducing the additional time variable Rusanov [*15*] and Babenko (see [*16*]) extended this method to three-dimensional mixed subsonic-supersonic flow about bluntnesses. Because the method is implicit along the marching direction, the iteration procedure is introduced into the algorithm. It consists of embedded cycles—two in steady and three in unsteady cases. The local boundary-value problem involved on the rays is solved with the help of one or another variant of the special sweep method, known in Russian literature as the "progonka" method. In calculating flows with smooth structure the efficiency of this method several times exceeds the efficiency of the explicit first-order method discussed above. Because of the approximation viscosity it allows to carry out "through counting" of weak embedded shocks (the use of a conservative form extends the possibilities of the method in this respect).

The method under consideration has wide applications (see the detailed reviews [*1,2*]). Calculatios of supersonic flows about various pointed bodies at angle of attack were carried out. The bodies were circular [*14*] and elliptical cones, different flat wings or wings with airfoils [*17*] (the tables of flow-fields were published for a number of cases), nonaxisymmetric nose parts of fuselages [*18*]. Supersonic regions on direct and inverse cones with different bluntnesses were calculated [*19,20*]. By the unsteady variant of the method, computations were carried out for mixed flows behind detached shock waves about pointed [*21*]and blunted [*22*] nose parts of bodies.

In case of blunted bodies, some authors used the simplified unsteady approach with

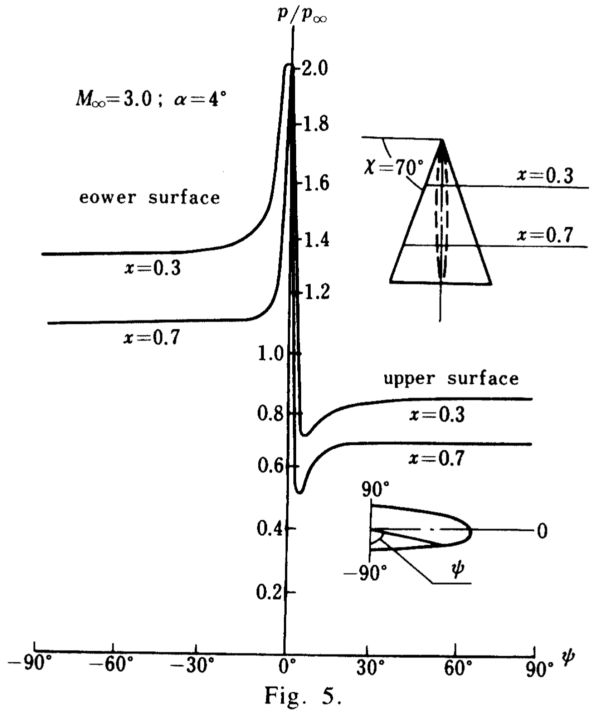

Fig. 5.

trigonometric approximations with respect to the meridional angle. In this way Minostsev and Savinov [*23*] investigated numerically the dynamics of subsonic regions appearing on the windward side of a cylinder with a spherical nose and a delta wing with blunted leading edges at large angles of attack ($\alpha \geq 40°$).

Fig. 5 represents the numerical results obtained by Voskresensky, Ivanov and Stebunov [*17*] for the supersonic flow about a delta wing with an airfoil and elliptical cross-section. The situation was considered when the bow shock wave was attached to the wing apex and was detached from the blunted leading edges. The pressure distributions on the lower surface (solid line) and the upper surface (dashed line) of the wing were plotted here for a number of cross-sections in the case of $M_\infty=3$, $\alpha=4°$, $\chi=70°$.

A more complex case of a flow about a wing of a rather general form with a shock wave detached both from the leading edge and from the wing apex was investigated by Voskresensky [*24–26*]. He worked out the algorithm for the solution of this problem. In Fig. 6 the wing and the shock wave calculated at $M_\infty=2$, $\alpha=5°$, are shown. Here the pressure distribution is also given along the chord in two cross-sections of the wing on its lower (solid line) and upper (dashed line) surfaces at two Mach numbers $M_\infty=2$, and 3.5, $\alpha=5°$.

Ivanova and Radvogin [*27*] applied the unsteady three-dimensional variant of the method for the calculation of a nonequilibrium high-temperature flow in a shock layer on a bluntness. An example of their computation of the flow about a nose part of the ellipsoid of revolution with axes a=b=1 m and c=2 m is presented in Fig. 7. The flow of air at the height H=71 km, $M_\infty=23$ and $\alpha=34°$ is considered. The contours of the shock wave and the sonic surface (dashed line) are shown in the symmetry plane. The temperature change T along some numbered rays between the body and the shockwave is

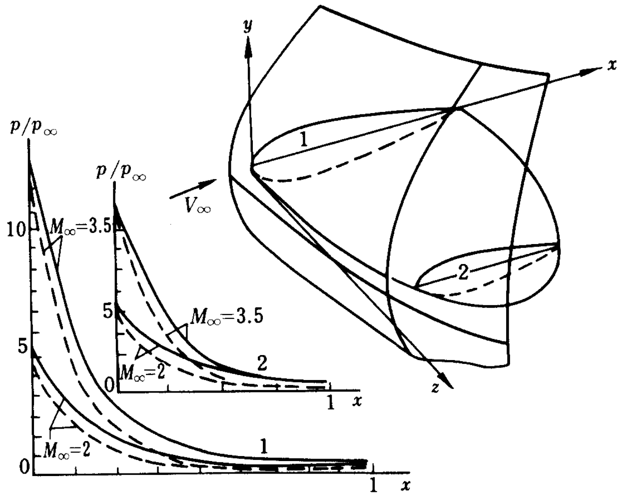

Fig. 6.

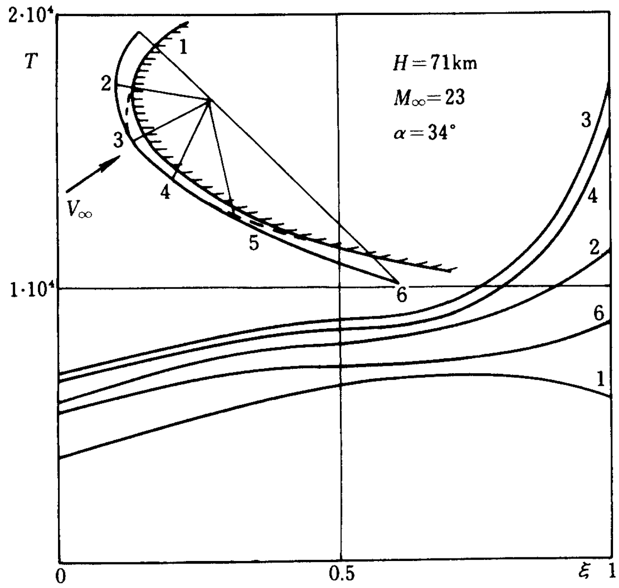

Fig. 7.

also given.

§IV. Splitting Methods

To solve multi-dimensional aerodynamical problems, splitting methods are successfully applied. They allow to reduce such problems to a sequence of simpler problems, which decreases the computation time. There are different kinds of the splitting. An analytical splitting is employed in the predictor-corrector scheme by MacCormack [*28*]. A geometrical splitting is the feature of the fraction step methods or the alternative directions method. A splitting with respect to physical processes is the basis of various methods of particles, e.g. the methods of large particles by Belotserkovskii and Davydov [*29*]. An implicit difference scheme which uses both geometrical and physical splitting was worked out by Kovenya, Tarnavskii and Yanenko (see [*30*]) who mainly intended it

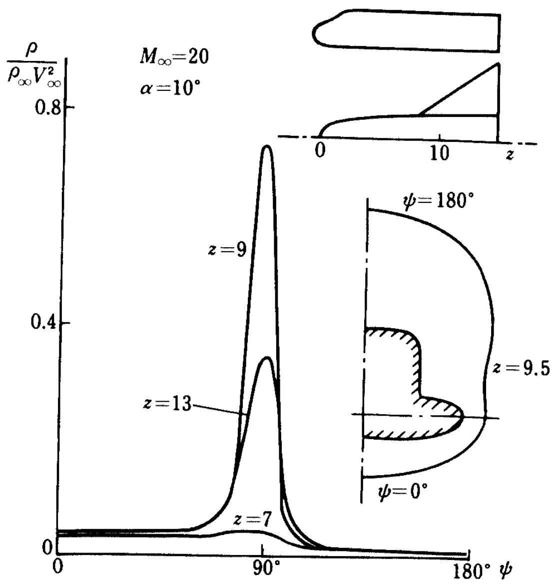

Fig. 8.

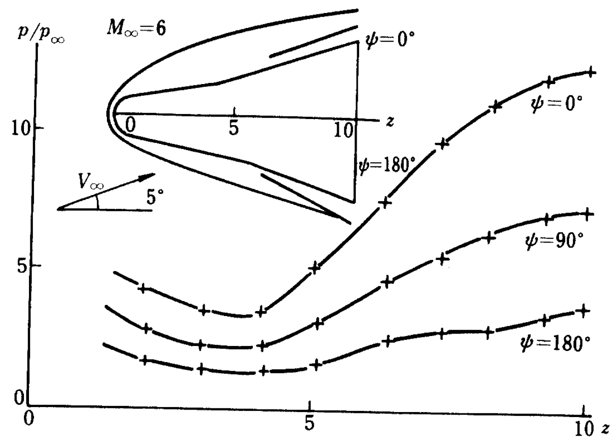

Fig. 9.

for viscous gas flows.

In the Soviet Union, considerable computations in the aerodynamics of supersonic lifting bodies were carried out with the help of MacCormack scheme [*28*] in original or modified forms. This scheme of the second-order accuracy was used by different authors both in the case of pointed bodies with curved axis [*31*], with tail stabilizing flare or with delta wings [*33–35*], and in the case of spherically blunted bodies with bielliptical cross-section [*36, 37*].

Calculations of flow about the configuration modelling a hypersonic vehicle were carried out in a number of papers [*38–40*]. The configuration represented a fuselage with spherical nose part, cockpit and swept wings with blunted leading edges. The cross-sections of the vehicle were simulated by a combination of two ellipses with axes varying along the body. Some numerical results by Galinskii and Timoshenko [*40*] are shown in Fig. 8. Here two projections of the body, the bow shock wave in the cross-section z=9.5 and the pressure distribution in three cross-sections of the body are given at $M_\infty=20$, $\alpha=10°$.

MacCormack [41] proposed another explicit-implicit second-order difference scheme for solving the unsteady two-dimensional Navier-Stokes equations. Pogorelov and Shevelev [42] developed an analogous hybrid two-step scheme with insignificant modifications of the algorithm for steady three-dimensional supersonic flows described by the Euler equations. Here, the explicit scheme with one-sided differences is used at the first step while the system of difference equations transformed to implicit form with block two-diagonal matrix is solved at the second step. The algorithm includes the splitting with respect to coordinate directions. Due to non-monotony of the scheme, the "through counting" of inner shocks requires a smoothing procedure. In regions with strong deceleration of the flow, the hybrid scheme proves to be more economical than the explicit scheme [28].

Using the explicit-implicit scheme Pogorelov and Shevelev [42] computed the supersonic flow about a spherically blunted double cone at $M_\infty=6$, $\alpha=5°$. The contours of the body, the bow and the inner shock waves in the symmetry plane are drawn in Fig. 9. The pressure distributions along three body generators are plotted here too. The corresponding data obtained with the help of the explicit MacCormack's scheme are shown by the crosses.

§V. Method Of Characteristics

The method of characteristics is expedient to calculate three-dimensional supersonic flows of not very complex structure. In this method owing to the application of characteristic compatibility relations, difference approximations are simplified, the dependence region of the solution is reasonably taken into account, computational algorithms for boundary points on the body and shock wave are natural. The possibilities of the method of characteristics in three-dimensional case widen essentially if a fixed net on reference planes along marching direction is introduced.

Among schemes with characteristic net of inverse type, in which the solution is advanced using reference planes, the bicharacteristic schemes were proposed—a tetrahedral one by Minostsev and a pentahedral one by Magomedov (see [43]). The schemes using characteristic lines on coordinate surfaces are especially simple. Here, the third variable connected with the transverse flow is eliminated in the governing system of equations by proper approximations. Such an effective implicit characteristic scheme of the second order of accuracy was built by Chushkin and Katskova [43,44]. These authors investigated three-dimensional supersonic flows about blunted cones and ducted bodies, including, in particular, complete consideration of nonequilibrium processes (dissociation, combustion).

Soviet scientists worked out the net-characteristic method firstly proposed by Magomedov and Kholodov [45] and then developed in a number of works (see [46]). Reference planes coordinates planes for marching variable) with a fixed net on them are taken in the method. Traces of characteristic surface on two other coordinate planes are considered and compatibility relations are represented in finite-difference form. At the points of intersection of these traces with the preceding reference plane, the interpolations using values of functions at fixed nodes are applied. All the scheme can be

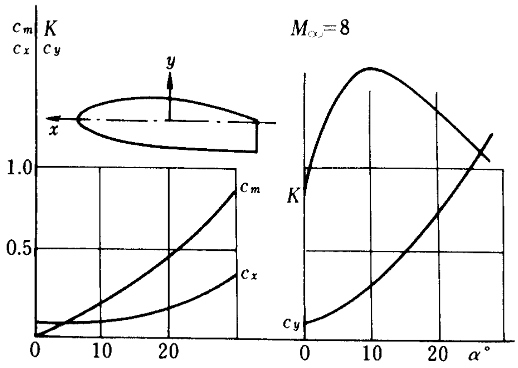

Fig. 10.

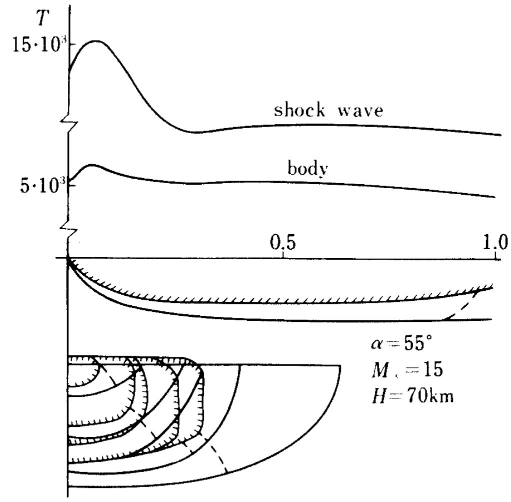

Fig. 11.

represented in the form usual for the finite-difference net methods, which explains the name "the net-characteristic method".

The method serves successfully for computations of three-dimensional supersonic flows about various bodies including blunted bodies, when the unsteady three-dimensional variant of the method is used. The method allows to carry out the "through counting" of weak inner shocks. A numerical example is given in Fig. 10, where aerodynamical coefficients of a lifting axisymmetric body with an inclined stern base are plotted at $M_\infty=8$.

An other example of the application of the net-characteristic method is presented in Fig. 11. Nikulin (see [*48*]) computed the flow past the windward region of a model fuselage-shaped body in nonequlibrium air freestream with the Mach number $M_\infty=15$ at height H=70 km at a very large angle of attack $\alpha=55°$. The contours of the shock wave and the sonic surface (dashed line) in the symmetry plane and in a number of cross-sections are drawn. The graphs of temperature behind the shock wave and along the body are also given for the windward side. As it is seen, at such a large angle of attack, the flow is subsonic in the considerable part of shock layer on the windward side, while the zone of very strong rarefaction arises on the leeward side.

The unsteady net-characteristic method extended to the case of a radiating gas in [*46*] was applied by Kostuzik and Rumynskii [*49*] to study a radiation heat transfer in a

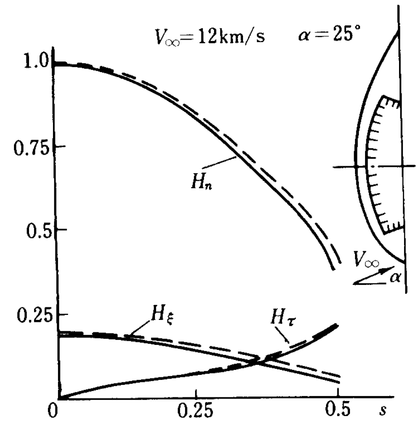

Fig. 12.

three-dimensional flow about various bluntnesses in a supersonic freestream air. Equilibrium state of a gas was assumed and spectral character of radiation was taken into account using ten-group model for absorption coefficient. Some numerical results are presented in Fig. 12 for two spherical sectors with the angle 60° and the radius R=1 m (solid line) and R=3 m (dashed line) at the freestream velocity V_∞=12 km/s and the angle of attack α=25°. Here it is shown how three radiation flux components—normal H_n and two tangent H_τ (along body generator) and H_ξ (normal to the latter)—vary along the body generator in the meridional plane ψ=90°. All these functions are referred to the value H_n at the body, apex from which the distance S is measured. The bow shock wave is also drawn in the symmetry plane.

§VI. Method Of Integral Relations

In the method of integral relations proposed by Dorodnicyn [*50*] the governing system of partial differential equations are reduced to some approximating system of ordinary differential equations. The method of straight lines is deduced from the generalized form of the mehod of integral relations as its particular case. The additional trigonometric approximations with respect to meridional angle are carried out for a three-dimensional flow about a body (Belotserkovskii, Chushkin [*51*]). The algorithm of the method is very simple and requires small computer memory. However, when a boundary-value problem of high order is considered for the approximating system, the efficiency of the method diminishes.

The method of integral relations was extensively applied to calculate a mixed flow behind a bow detached shock wave ahead of a blunted body [*52*], including high-temperature effects in a gas. Flows about supersonic lifting bodies (cone, delta wings, blunted and ducted bodies) were also computed with the help of this method (see the reviews [*1,2*]). The method allowed to solve successfully the three-dimensional problem of flow about bodies with a point or a very small bluntness, when a detached shock wave arises and the flow properties near a body apex change sharply. Chushkin [*53,54*] obtaied such a solution for circular and elliptic large-angle cones of finite length with a stern base, which were located at angle of attack in a supersonic freestream. Some results are

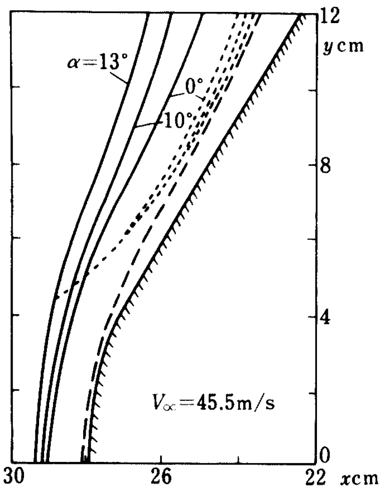

Fig. 13.

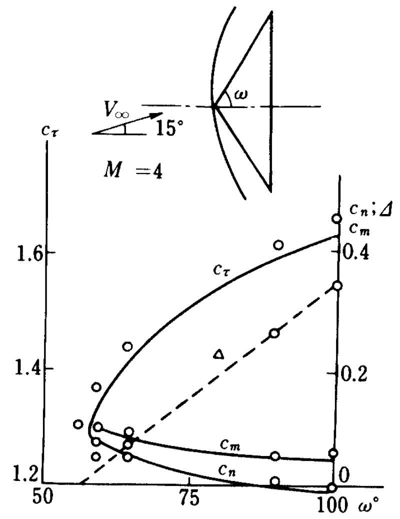

Fig. 14.

presented in Fig. 13 for cone-shaped bodies with different semi-apex angle ω at $M_\infty=4$, $\alpha=15°$ ($\varsigma=90°$ corresponds to a flat-faced cylinder and $\omega>90°$ corresponds to a concave nose). Here the aerodynamic coefficients C_τ, C_n, C_m, referred to the base area and the shock wave detachment distance at the body apex Δ referred to the base radius are plotted versus ω. The numerical solution is compared with the appropriate experiments by the authors of the work [*55*]. There is a good agreement of the results if one takes into consideration that the experimental values of C_τ are somewhat increased because of viscosity effect.

To solve two and three-dimensional problems of a mixed flow about a smooth bluntness, Telenin (see [*56*]) developed the method of straight lines. The method was used, in particular, for a spherical sector at an angle of attack in a supersonic freestream of perfect or nonequilibrium air (see the reviews [*1,2*]). Golomazov and Zyuzin [*57*] worked out another modification of the method of straight lines for such problems.

Using the latter method, Belotserkovskii, Golomazov and Shabalin [*58*] computed a flow about a blunted cone located in a supersonic equilibrium freestream when a gas was ejected uniformly from the body surface. The same two-layer flow model with ejection was used in [*59*] for evaporating bodies under complicated conditions when the ejection of vapours is caused by the radiation flux from shock layer, acting on the wall. Fig. 14 illustrates some results of this work for spherically blunted glasstextolit cone with semi-angle $\omega=60°$ in the free-stream of helium-hydrogen mixture with the velocity $V_\infty=45.5$ km/s and the density $\rho_\infty=0.192$ g/m^3. Here the flow-pattern on the windward side is drawn for a number of angles of attack. The figure shows the shock wave (solid line), the sonic line (dotted line) and the contact surface which bounds the region of vapours (dashed line). When the angle of attack increases to the value $\alpha=13°$, the sonic line touches the contact surface which almost does not depend on α. Small angles of attack also weakly influence heat fluxes on the body.

§VII. Conclusion

The review of works published by Soviet authors during the last five years shows the achievements in the field of constructions and applications of numerical methods in the problem of three-dimensional flow about lifting vehicles in a supersonic freestream of an inviscid gas. The model, based on the Euler equations with the possible correction taking into account the viscous effect in the frame of the boundary-layer theory, is proved effective and economical. The finite-difference net methods, the method of characteristics and the method of integral relations (including the method of straight lines) are successfully used for the numerical solution of these equations. At the present time, two first types of the methods are more wide-spread.

Now the practical problem of calculations of three-dimensional flows about simple aerodynamic forms in a supersonic freestream of a pefect or a real high-temperature gas may be considered to be mainly solved. The tables of flow-fields are computed for a series of nose parts of aerospace vehicles (sharp cones, bluntnesses of various shape) and wings. There are computer codes for calculations of more general cases. Computer codes are also created for three-diemnsional flows about configurations modelling complete supersonic vehicles. Using these codes, it is possible to obtain numerical results which are sufficient for practice, although it requires very labour consuming calculations. The application of super-computers ensures the further progress for solving the problem of computational aerodynamics of supersonic lifting vehicles.

References[1)]

[*1*] Voskresensky G. P., Chushkin P. I.: Usp. Mekh., 1980, *2*, *No.* 2, *47_95*.
[*2*]* Chushkin P. I., Voskresensky G. P.: ICAS proc., 1980, 444–454.
[*3*] Godunov S. K., Zabrodin A. V., Ivanov M. Ya., Kraiko A. N., Prokopov G. P.: Numerical solution of multi-dimensional problems of gasynamics. Nauka, Moscow, 1976.
[*4*] Zarubin A. G.: Uch. zap. TsAGI, 1982, *13*, No. 2, 123–128.
[*5*] Kosykh A. P.: Uch. zap. TsAGAI, 1980, 11, No. 4, 129–138.
[*6*] Minailos A. N.: Uch. zap. TsAGI, 1980, 11, No. 5, 7–17.
[*7*] Minailos A. N.: Uch. zap. TsAGI, 1981, 12, No. 3, 30–38.
[*8*] Makarov V. E.: Izv. AN SSR. Mekh. Zhidk. i gaza, 1984, No. 2, 188–190.
[*9*] Rozin A. V. In: Aerodynamics of entry of bodies into planet atmospheres. Moscow Univ., 1983, 17–23.
[*10*] Aukin M. K., Tagirov R. K.: Chisl. met. mekh. splosh. sredy, 1980, 11, No. 6, 23–25.
[*11*] Aukin M. K., Tagirov R. K.: Uch. zap. TsAGI, 1982, *13*, No. 3, 136–141.
[*12*] Mileshin V. I., Tillyaeva N. I.: Uch. zap. TsAGI, 1982, *13*, No. 2, 135–141.
[*13*] Babenko K. I., Voskresensky G. P.: vychisl. mat. i mat. fiz., 1961, *1*, No. 6, 1051–1060.
[*14*] Babenko K. I., voskresensky G. P., Lyubimov A. N., Rusanov V. V.: Three-dimensional gas flow about smooth bodies. Nauka, Moscow, 1964.
[*15*] Rusanov V. V.: Zh. vychisl. mat. i mat. fiz., 1968, 8, No. 3, 616–633
[*16*] Babenko K. I. *et al.*: Unsteady flow of ideal gas about head part of blunted body. Inst. Appl. Math. USSR Acad. Sci., Moscow, 1969.
[*17*] Voskresensky G. P., Ivanov O. V., Stebunov V. A.: Prepr. No. 43. Keldysh Inst. Appl. Math. USSR Acad. Sci., Moscow, 1984.
[*18*] Voskresensky G. P., Ivanov O. V. Stebunov V. A.: Prepr. No. 67. Keldysh Inst. Appl. Math. USSR Acad. Sci., Moscow, 1981.
[*19*] Vishnevetskii S. L., Ivanov M. A.: Izv. AN SSSR. Mekh. zhidk. i gaza, 1983, No. 1, 144–149.
[*20*] Antonets A. V., Marinin V. P.: Izv. AN SSR. Mekh. zhidk. i gaza, 1981, No. 1, 178–183.
[*21*] Ivanova V. N., Radvogin Yu. B.: Prepr. No. 28. Keldysh Inst. Appl. Math. USSR Acad. Sci.,

Moscow, 1981.
[*22*] Lyubimov a. N., Rusanov V. V.: Flows about blunt bodies, pt. 1, 2. Nauka, Moscow, 1970.
[*23*] Minostsev V. B., Savinov K. G. *In*: Aerodynamics of entry of bodies into planet atmospheres. Moscow Univ., 1983, 5–11.
[*24*]* Voskresensky G. P.: Comput. Meth. Appl. Mech. and Eng., 1979, No. 19, 257–275.
[*25*]* Voskresensky G. P.: Comput. Meth. Appl. Mech. and Eng., 1983, No. 38, 45–61.
[*26*] Voskresensky G. P.: Zh. vychisl. mat. i mat. fiz., 1984, *24*, No. 6, 900–915.
[*27*] Ivanova V. N., Radvogin Yu. B.: Prepr. No. 82. keldysh Inst. Appl. Math. USSR Acad. Sci., 1981.
[*28*]* MacCormack R. W.: AIAA Pap. No. 364, 1969.
[*29*] Belotserkovskii O. M., Davydov Yu. M.: Method of large particles in gasdynamics. Nauka, Moscow, 1982.
[*30*] Kovenya V. M., Yanenko N. N. Splitting method in problems of gasdynamics. Nauka, Novosibirsk, 1981.
[*31*] Volkov V. F.: Chisl. met. mekh. splosh. sredy, 1982, 13, No. 5.
[*32*] Volkov V. F. *In*: Questions of working out and exploitation of packets of applied programmes. Novosibirsk, 1981, 98–112.
[*33*] Lobanovskii Yu. I.: Uch. zap. TsAGI, 1980, II, No. 6, 58–65.
[*34*] Lobanovskii Yu. I.: Chisl. met. mekh. splosh. sredy, 1984, 15, No. 3, 85–97.
[*35*] Kotenev V. P., Sakharov V. I., Tirskii G. A. *In*: Gasdynamics of noneqilibrium processes.
[*36*] Pogorelov N. V., Shevelev Yu. D.: Chisl. met. mekh. splosh. sredy, 1981, *12*, No. 6, 65–79.
[*37*] Sakharov V. I., Kotenev V. P., Tirskii G. A. *In*: Hypersonic three-dimensional flows with physical-chemical transformations. Moscow Univ., 1981, 106–112.
[*38*] Sakharov V. I., Shevelev Yu. D.: Izv. AN SSR. Mekh. zhidk. i gaza, 1980, No. 4, 172–175.
[*39*] Pogorelov N. V., Shevelev Yu. D. *In*: Numerical methods in dynamics of viscous fluid. Novosibirsk, 1983, 262–268.
[*40*] Galinskii V. P., Timoshenko V. I.: Space investigations in Ukraine, 1981, No. 15, 89–92.
[*41*]* Mac Cormack R. W.: AIAA Pap., No. 110, 1981.
[*42*] Pogorelov N. V., Shevelev Yu. D. Zh. vychisl. mat. i mat. fiz., 1985 *25*, No. 9.
[*43*]* Chushkin P. I. *In*: Progress in Aeronaut. Sci., vol. 9. Pergamon Press, Oxford, London, 1968, 41–122.
[*44*] Katskova O. N., Chushkin P. I.: Zh. vychisl. mat. i mat. fiz., 1968, 8, No. 5, 1049–1062.
[*45*] Magomedov K. M., Kholodov A. S.: Zh. vychisl. mat. i mat. fiz., 1969, 9, No. 2, 373–386.
[*46*] Belotserkovskii O. M. *et al.*: Numerical investigations of modern problems of gasdynamics. Nauka, Moscow, 1974.
[*47*] Reshetin A. G., Lychkin E. N., Kotov V. M., Schelkalov A. N.: Chisl. met. mekh. splosh. sredy, 1980, *11, No. 6, 110_122.*
[*48*] Antonets A. V., Nikulin A. N. *In*: Scientific lectures on aviation and astronautics. Nauka, Moscow, 1981, 257.
[*49*] Kostuzik A. A., Rumynskii A. N.: Zh. vychisl. mat. i mat. fiz., 1984, *24*, No. 3, 435–441.
[*50*] Dorodnicyn A. A. *In*: Proc. of 3–d All Soviet Math. Congress, vol. 3. USSR Acad. Sci., Moscow, 1958, 447–453.
[*51*] Belotserkovskii O. M., Chushkin P. I.: Zh. vychisl. mat. i mat. fiz., 1962, *2*, No. 5, 731–759.
[*52*] Belotserkovskii O. M. *et al.*: Flow about blunted bodies in supersonic stream of gas. Theoretical and experimental investigations 2–d ed. Comp. Center of USSR Acad. Sci., Moscow, 1967.
[*53*] Chushkin P. I.: Zh. vychisl. mat. i mat. fiz., 1980, 20, No. 6, 1525–1535.
[*54*]* Chushkin P. I. Arch. mech. stosow., 1984, 36, No. 2, 219–232.
[*55*] Amarantova I. I., Bukovshin v. G., Shustov V. I.: Izv. AN SSR. Mekh. zhidk. i gaza, 1978, No. 2, 195–198.
[*56*] Telenin G. F., Tinyakov G. P.: Dokl. AN SSSR, 1964, 154, No. 5, 1056–1058.
[*57*] Golomazov M. M., Zyuzin A. P.: Zh. vychisl. mat. i mat. fiz., 1975, *15*, No. 5, 1349–1355.
[*58*] Belotserkovskii O. M., Golomazov M. M., Shabalin A. V.: Zh. vychisl. mat i mat. fiz., 1981, *21*, No. 4, 1018–1030.
[*59*] Golomazov M. M., Shabalin A. V.: Izv. AN SSSR. Mekh. zhidk. i gaza, 1984, No. 1, 132–137.

1) All the publications are in Russian, except the references marked by asterisk.