Vortex Filament Simulation for the Crosslinking Rings

By

Akiko YAMASHITA* and Yuko OSHIMA**

(February 1, 1988)

Summary: When two vortex rings are arranged side by side coplanerly, they approach each other and join to one large ring due to their interaction. This process was numerically simulated using three dimensional vortex filament method. One vortex ring which has vorticity within the core was approximated by a bundle of seven vortex filament rings. Each pair of filaments were crosslinked in turn until a large vortex ring formed.

1. INTRODUCTION

Vortex has attracted many researchers due to its singularity in the laminar flow field, or as the fundamental element of the turbulent flow. In particular vortex rings were easily realized in laboratory and flow visualization researches showed various and fascinating phenomena caused by the mutual induced velocities, that is, passing-through of two coaxial vortex rings [*1*], bifurcation of an elliptic vortex ring [*2*], crosslinking of two vortex rings side by side [*3*]. Oshima and Noguchi [*4*] simulated numerically this crosslinking phenomenon successfully using three dimensional vortex filament method. Hot wire measurement of the flow field [*5*] gave the time dependent vorticity distribution of the process. And it was found that the crosslinking of vortex tube did not occur at one instant but the phenomenon proceeded gradually from the outside with complicated feature. To clear up the mechanism of the crosslinking of these vortex rings, 3-D vortex filament simulation was carried out numerically under the similar condition of the experiment. The core radius was set a half of the ring radius. This process is also supported by Navier-Stokes solution by Chen [*6*].

2. FLOW FIELD EQUATIONS AND MODELING

In three dimensional flow field, the velocity $\mathbf{u}(\mathrm{x}, \mathrm{t})$ induced by the vorticity $\omega(\mathbf{x}', \mathrm{t})$ is given as the solution of Poisson equation for vector potential.

$$\mathbf{u}(\mathbf{x}, \mathrm{t}) = -\frac{1}{4\pi}\int_{V'} \frac{(\mathbf{x}-\mathbf{x}')\times \omega(\mathbf{x}', \mathrm{t})}{|\mathbf{x}-\mathbf{x}'|^3}\, \mathrm{dV}', \tag{1}$$

where t is time and $\mathbf{x}$ and $\mathbf{x}'$ are position vectors. The prime is put to the characters for which ω is distributed. When the vorticity is confined to an isolated tubular region,

* The Institute of Space and Astronautical Science.
** Faculty of Science, Ochanomizu University.

that is, vortex filament, the volume integral is reduced to the line integral.

$$\mathbf{u}(\mathbf{x},\, t)=-\frac{\Gamma}{4\pi}\int\frac{[\mathbf{x}-\mathbf{r}(\xi')]\times\partial\mathbf{r}/\partial\xi'}{|\mathbf{x}-\mathbf{r}(\xi')|^3}\,d\xi', \tag{2}$$

where Γ is the circulation of the filament with diameter σ,

$$\Gamma=\omega\pi\sigma^2, \tag{3}$$

and $\mathbf{r}(\xi')$ is the space arc of the vortex filament parametered by the arc length ξ'. When x approaches to $\mathbf{r}(\xi')$, the integral diverges logarithmically. Rosenhead[6] applied the following approximation to avoid the singularity,

$$\mathbf{u}(\mathbf{x},\, t)=-\frac{\Gamma}{4\pi}\int\frac{[\mathbf{x}-\mathbf{r}(\xi')]\times\partial\mathbf{r}/\partial\xi'}{[|\mathbf{x}-\mathbf{r}(\xi')|^2+\mu^2]^{3/2}}\,d\xi'. \tag{4}$$

With the help of Moore's theoretical work[7], it was assumed that vorticity distribution within the core keeps constant and the value of μ was obtained as follows,

$$\mu=\sigma\exp(-0.75). \tag{5}$$

In the inviscid flow field, it can be considered that vortex filaments are transported as material entities, so the induced velocity for the i-th filament by the other filaments with subscript j was obtained as follows.

$$\frac{\partial\mathbf{r}_i}{\partial t}=-\sum_j\frac{\Gamma_j}{4\pi}\int\frac{[\mathbf{r}_i-\mathbf{r}_j(\xi')]\times\partial\mathbf{r}_j/\partial\xi'}{[|\mathbf{r}_i-\mathbf{r}_j(\xi')|^2+\mu^2]^{3/2}}\,d\xi', \tag{6}$$

Increment in number of the filaments enables to describe in more detail the transformation of the vortex ring, but the computation time increases at the rate of square of the number. Then seven filaments arranged at the vertices of hexagon and the center were employed as the least ones that could describe the phenomenon. The fourth-order Runge-Kutta formula was used for the integration forward in time of Eq. (6). In executing the calculation of the line integral, each filament was divided into 16 pieces of the same length and cubic spline was applied as the approximated curve between each two points. Every time step $\Delta t=0.01$, these node points were transported with the induced velocities calculated from Eq. (6). And after the transportation, 16 node points were renewed so as to keep the same order description. When a pair of vortex filaments were crosslinked, 32 node points were considered for calculation to the vortex filament with twice length for the same reason.

3. Results and Discussions

Two equivalent vortex rings were arranged coplanerly along the x axis and across the y axis at t=0.00 as shown in Fig. 1-a, the radius of the ring was D and the distance between centers was 1.61D. The symmetric phenomenon is developed for both the x axis and the y axis. Then the computation was done for the half space $y>0$ in order to save CPU time. The simulation was attempted for two cases in which the core radius were 50% and 60% long for the ring radius. The former showed the clearer development and is reported in this paper. Figure 1 shows the time development of vortex filaments seen from four directions. Perspective view is shown at the upper left with positive mark as the origin. Top view is shown at the lower right with positive mark. Two side views are shown at the lower left and the upper right for the x and the y axial direction, respectively. Fig. 1-a shows the initial condition for which two bundles of seven filaments express two vortex rings. The vortex ring had the vorticity with which the fluid went up inside and down outside of each ring, and had the self induced velocity to go up. So the same rotation was given to each filament ring, and they moved up rotating around the central filament. The near parts of the z axis were decelerated due to the opposite vortex ring as shown in Fig. 1-b at $t=0.70$. At $t=1.40$ all the filaments gathered along the $y-z$ plane but not connected as shown in Fig. 1-c. They showed semicircles and both ends of the semicircular filaments had large curvatures, where the large velocities to stick out were induced. At $t=1.49$, the first pair of the filaments crossed at the two points. Then the calculation was returned to one time step before, these filaments were artificially cut and crosslinked at these points to make one large ring as shown in Fig. 1-d. In Figs. 1-d~f, only seven filaments that had formed right hand side vortex ring are shown so as to be identified easily. And it is also interesting that the crosslinked pair was the second lowest and not the lowest which was seemed to connect sooner. The second and third pairs of filaments were crosslinked at $t=1.55$ and 1.57, respectively. Figure 1-e shows that the fourth pair was crosslinked at $t=1.61$. The crosslinked filaments were hollowed near crosslinking points. The fifth and sixth pairs were crosslinked at $t=1.81$ and 1.82, respectively. At $t=1.85$ the last pair was crosslinked, then a large, distorted and long vortex ring formed completely as shown in Fig. 1-f. It should be noted that the vortex filament rings were crosslinked at two points outside apart from the central point in spite of intuitive expectation, and that it took a considerable time to complete crosslinking of all filaments. The calculation was proceeded to 2.00 time steps and thus filaments showed more distorted and complicated curves.

Figure 2 is the time dependent positions of filament cross sections corresponding to Fig. 1. The halves of the $x-z$ plane for positive x and $y-z$ plane for positive y are shown on the right-hand and the left-hand side of the figure, respectively. In Fig. 2-a, the cross section of the right-hand side vortex ring is shown and each filament is numbered, so it is not recognized any vortex filament in the $y-z$ plane at the initial stage. Six filaments surrounded the center one numbered as 1 and these seven filaments formed the vortex ring core, where size of the circle means the thickness of the filament. The diameter of the filament itself was determined from the condition that the volume of the filament keeps constant. In the unseen $x<0$ space another ring should move and transform symmetrically. As time going on, vortex filaments moved up rotating each other and also transfoming the cross section of the vortex ring core. The cross section area of the filaments itself changed according to the definition described above. In Fig. 2-b filaments approached to the $y-z$ plane, set in a vertical

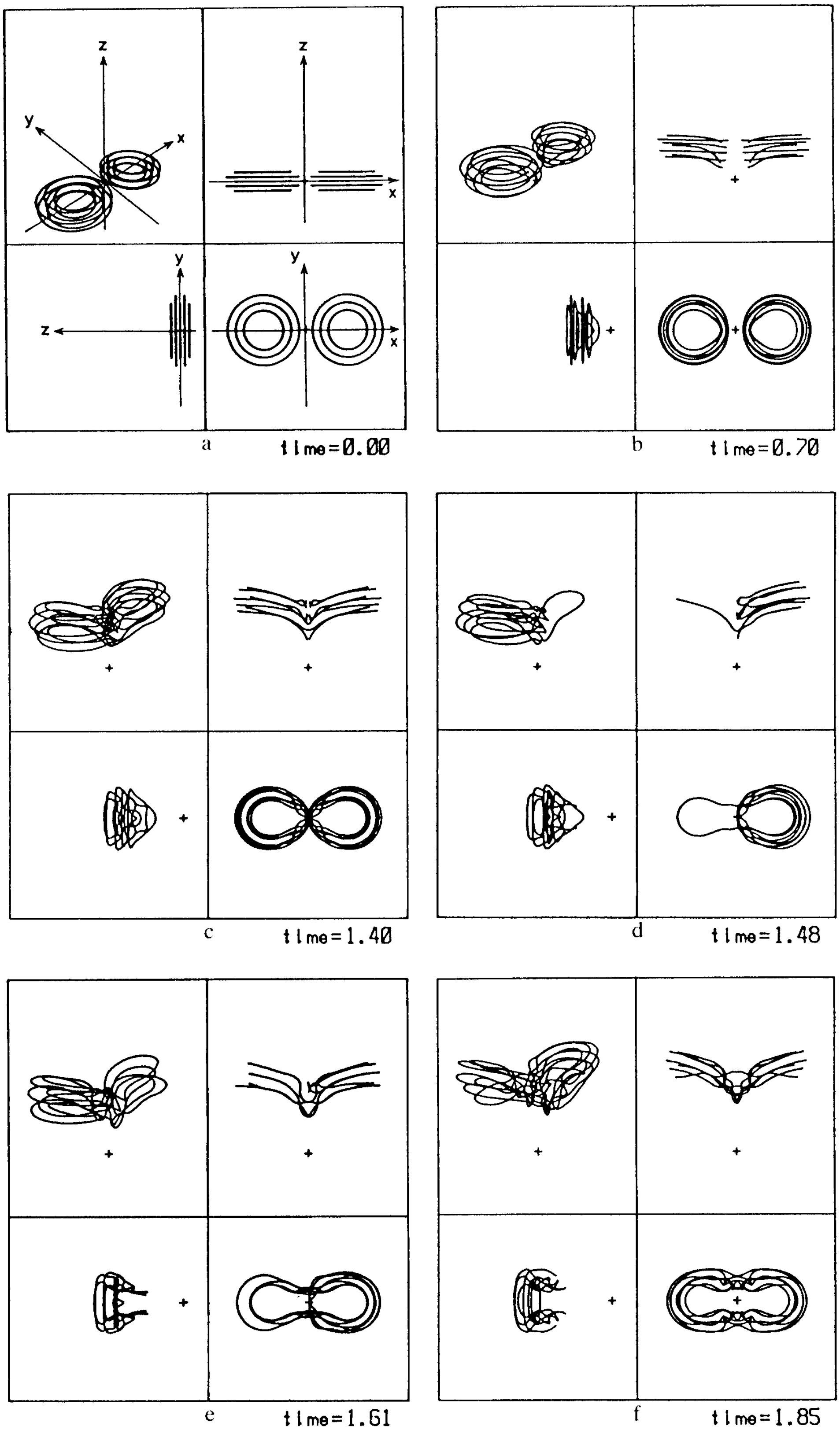

Fig. 1. Time development of vortex filaments.

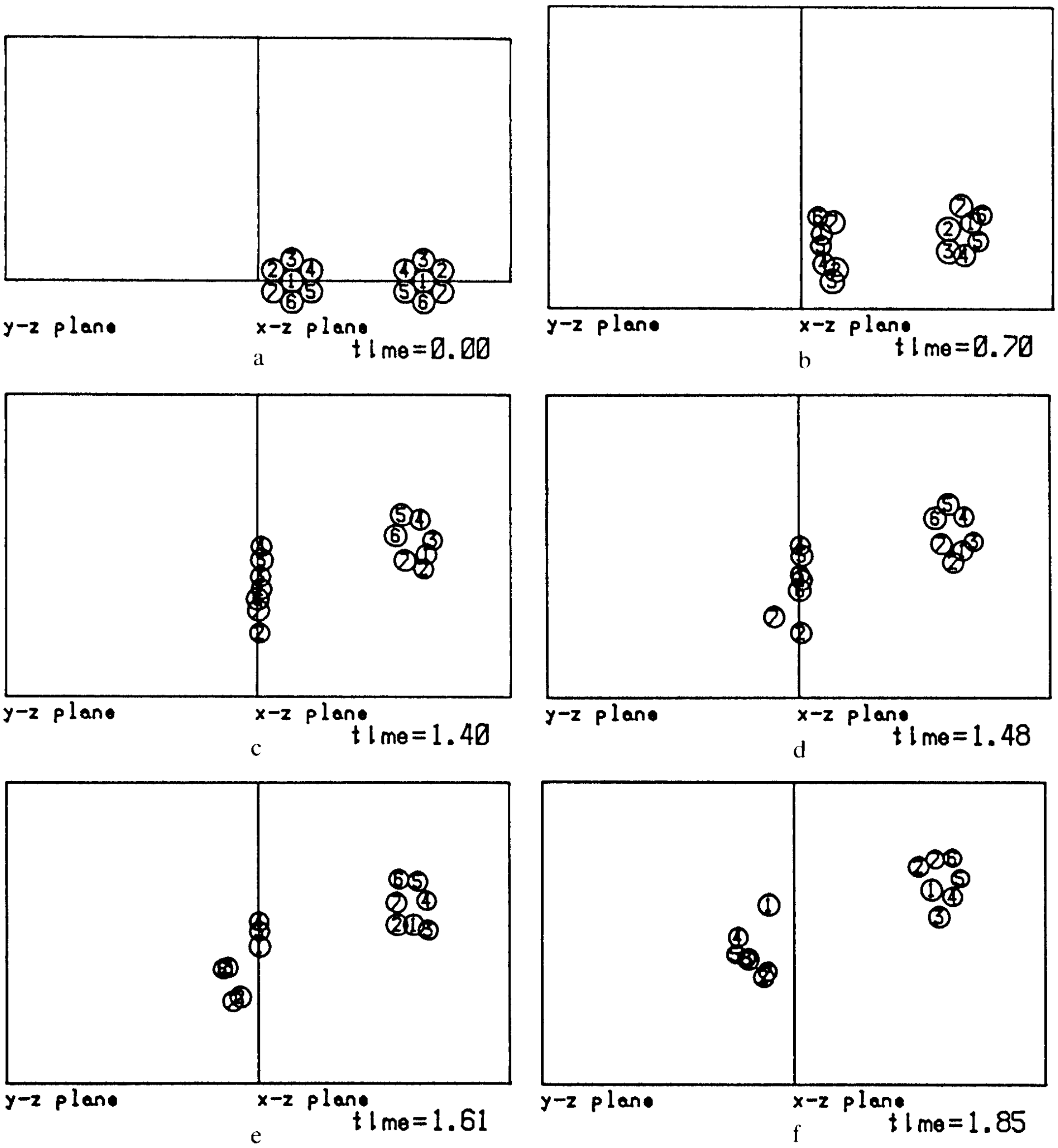

Fig. 2. Cross sections on the x−z plane and the y−z plane.

line. For the filament numbered 2, the right-hand side cross section went up vortex ring. On the other hand the left-hand side one suffered the stronger interaction from the opposite vortex ring and remained lower. Overlapping of the filaments means increment of the vorticity and did not influence the numerical integration, in which the vorticity was assumed to be concentrated on the center line. Fig. 2-c shows that filaments were alomost on the z axis in a line at t=1.40. As stated before, the first pair of filaments was crosslinked to one and in the y−z plane the cross section appeared at t=1.48 in Fig. 2-d. It is clearly seen that the crosslinked points were apart from the z axis and lower than the cross section of the filaments in the x−z plane. The topological conditions were not equal in the x−z plane and in the y−z plane. It means that new large vortex was formed with complicated filament arrangement.

4. Conclusion

Two vortex rings arranged side by side approach each other and transform symmetrically because of the mutual induced velocities. And they join to one large ring taking time gradually from the outside of the rings. This process was numerically simulated using three dimensional vortex filaments method. The vortex ring which has the vorticity within the core was approximated by a bundle of seven vortex filament rings. Vortex rings approached each other and transformed to semicircles symmetrically. Both ends of semicircles had large curvature and the fast velocities to stick out were induced there. Each pair of filaments connected and were crosslinked at these points in turn until a large and distorted vortex ring formed. Simulation agreed well with the time dependent vorticity measurement using hot wire and numerical N−S solution. The vortex filament model may give a good approach to understand the crosslinking phenomenon.

References

[1] Y. Oshima, and S. Asaka: Interaction of two vortex rings moving along a common axis of symmetriy, J. Phys. Soc. Jpn., **38**, 271, (1975).
[2] Y. Oshima, N. Izutsu, K. Oshima, and AKMF Hussain: Bifurcation of an elliptic vortex ring, Fluid Dyn. Research, **3**, (1988), (in print).
]3] Y. Oshima, and S. Asaka: Interaction of two vortex rings along parallel axes in air, J. Phys. Soc. Jpn., **42**, 708 (1977).
[4] Y. Oshima, T. Noguchi, and K. Oshima: Numerical study of interaction of two vortex rings, Fluid Dyn. Research, **1**, 215 (1986).
[5] Y. Oshima, and N. Izutsu: Crosslinking of two vortex rings side by side, ISAS Rep. S. P. 6, 17 (1988).
[6] H. L. Chen, and K. Oshima: Numerical analysis of reconnection of vortex rings, ISAS Rep. S. P. 6, 1 (1988).
[7] L. Rosenhead: The spread of vorticity in the wake behind a cylinder, Proc. R. Soc. London Ser. A, **127**, 590 (1930).
[8] D. W. Moore: Finite amplitude waves on aircraft trailing vortices, Aeronaut. Quart. **23**, 307 (1972).