Nutational Stability of a Satellite Equipped with an Active Magnetic Momentum Wheel

By

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Summary: This paper deals with an influence of the magnetically suspended rotor on the satellite motion; although cross-feedbacks in the magnetic bearing provide stability margin to the rotor gyroscopic motion, they may destabilize the satellite nutation. Stability analyses about the dynamical interaction between the satellite and the magnetically suspended rotor whose controller used three types of (proportional, integral and derivative) cross-feedbacks were performed. The results indicated that they had significant effects on the rotor and the satellite motion.

1. INTRODUCTION

Momentum wheel is one of the most widely utilized attitude control devices for 3-axis stabilized satellites. In order to meet the recent requirement for long lifetime and large angular momentum starage capacity a magnetic suspension technique has been developed [1], [2]. Additional advantage of the magnetically suspended momentum wheel is a vernier gimballing capability, which contributes to active nutation damping and fine attitude error correction by the feedback of the attitude signal to the gimbal angles. In spite of the attractive features, once the attitude information is not available the overall system including the the rotor and the satellite dynamics may become unstable.

One of the authors [3] has already investigated the gyroscopic stability of a magnetically suspended rotor with a fixed stator. However, the dynamic interaction between the rotor and the satellite was not considered. Heimbold [4] indicated that the rotor losses by the eddy current induced the satellite nutation and that the active magnetic bearing was able to compensate it. Lange [5] designed the magnetic bearing controller considering the satellite dynamics. On the other hand, to have a compatibility with the space application, a magnetic wheel with low power consumption has been required. In this paper, stability analyses on a model including phase shift at high frequency and integral compensations to eliminate stationary errors were performed. These factors have significant effects not only on the rotor motion but also satellite dynamics, which have not been investigated yet.

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2. Gyroscopic Stability of a Magnetic Bearing

Magnetic suspension applied to satellite momentum wheel is required not only to have small size and weight but also to consume little power. For that purpose, we made a test model of the magnetic momentum wheel controlled by eight PWM (pulse width moduration) power anplifiers. Fig. 1 shows the cross section of the wheel. The specifications are shown in Table 1.

The averaged efficiency of the power amplifiers is 70% at least. However, the frequency bandwidth is only 100Hz, while the rotor nutation speed at 10,000rpm is about 250Hz. It means that the phase shift at a high frequency may have a significant influence on the rotor motion. The narrow bandwidth also indicates that the gain has to be small for stable feedback control. In order to eliminate the stationary alignment errors due to the small stiffness, some integral compensations are required. However, that has influences on the rotor precession, which must be considered.

(1). Equations of Rotor Motion and Bearing Controller

Active magnetic bearing provides the rotor artificial stiffness and damping. Since the natural damping is very small compared with the artificial one, it is negligible for analizing the rotor motion. The equations of the gyroscopic motion of the spining rotor depicted in Fig. 2 are

$$\begin{aligned} J\ddot{\theta}_x + J_y \Omega \dot{\theta}_z &= u_x \\ J\ddot{\theta}_z - J_y \Omega \dot{\theta}_x &= u_z \end{aligned} \tag{1}$$

where θ_x, θ_z : gimbal angles
J_y : rotor moment of inertia about spin axis
J : rotor moment of inertia about transverse axis
Ω : spin rate
u_x, u_z : control torques.

An active magnetic bearing is a feedback system to stabilize the rotor motion by the use of position sensors and electro-magnetic actuators. The transfer functions of the controller used for the experiment are

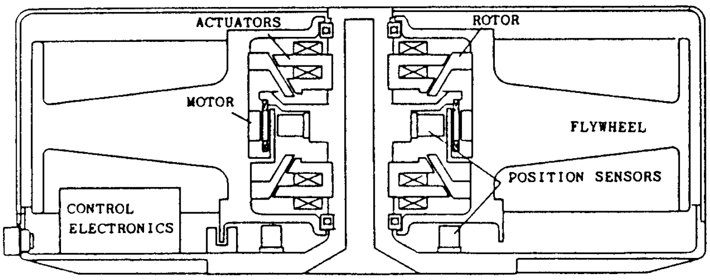

Fig. 1. Cross section of the magnetic momentum wheel.

Table 1. Specification of the magnetic wheel

Total mass	5.2 kg
Size	314 mm (diameter)
Moment of inertia	
about spin axis	0.017 kgm^2
about transverse axis	0.011 kgm^2
Rotating speed	10,000 rpm
Angular momentum	18 Nms
Motor torque	0.05 Nm (max)

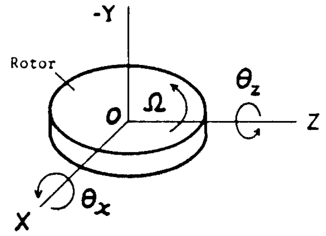

Fig. 2. Rotor reference coordicate frame.

$$
\begin{aligned}
u_x &= -\frac{1}{1+\tau_d s}\left[\left(K_1\frac{1+\tau_2 s}{1+\tau_1 s}+\frac{K_2}{s}\right)\theta_x-(sK_3-K_4)\theta_z\right] \\
u_z &= -\frac{1}{1+\tau_d s}\left[\left(K_1\frac{1+\tau_2 s}{1+\tau_1 s}+\frac{K_2}{s}\right)\theta_z+(sK_3-K_4)\theta_x\right]
\end{aligned}
\tag{2}
$$

where K_1 : proportional gain
K_2 : integral compensation gain
K_3 : derivative cross-feedback gain stabilizing nutation
K_4 : proportional cross-feedback gain stabilizing precession
τ_d : time constant of the power amplifier
τ_1 τ_2 : time constants of the lead-lag netework.
s : Laplace's operator

The gyroscopic characteristics of the rotor depends on its rotating speed. Applying the conventional PID controller ($K_3=K_4=0$) to the experiment model of magnetic momentum wheel displayed gyroscopic instabilities (both of nutation and of precession) at high spin rate [3]. Cross-feedbacks via K_3 and K_4 had effects to stabilize them. The root loci v.s. rotating speed is shown in Fig. 3 where dashed lines correspond to the case of conventional PID control and the solid lines correspond to the case of employing cross-feedbacks. It indicates that in the former case both roots of the nutation and the precession are unstable at high spin rate and that in the latter case both roots are stable over the speed of 10,000 rpm.

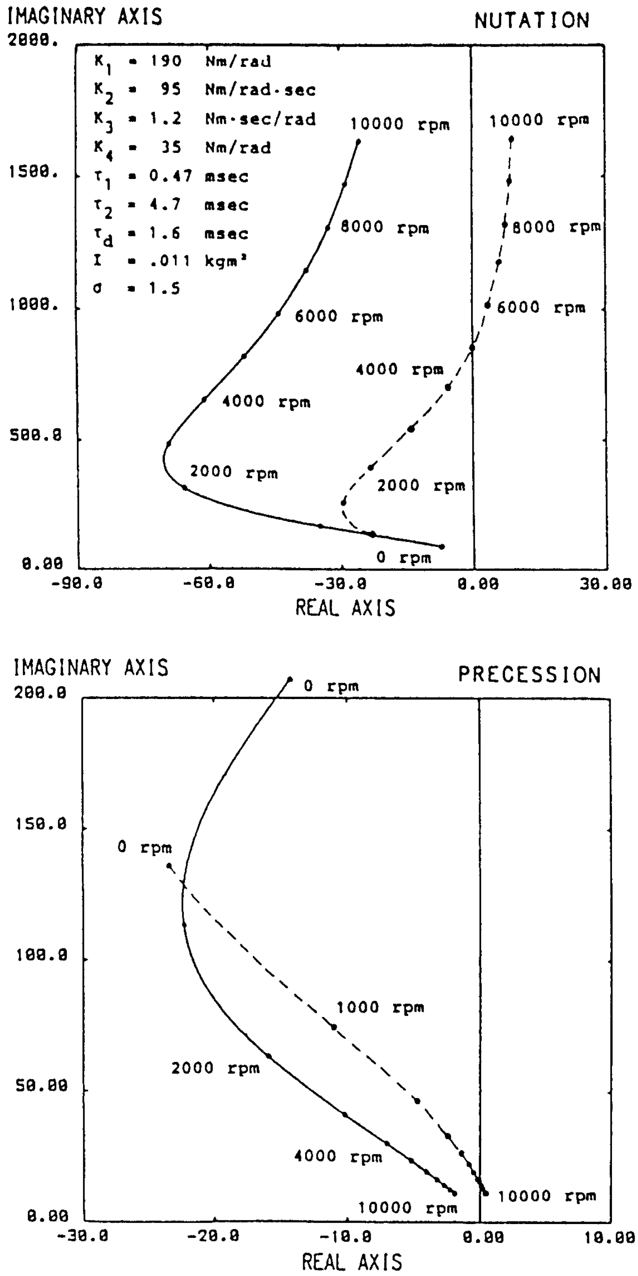

Fig. 3. Root loci of the rotor motion.

(2) Stability of the Rotor Nutation

Simplified equation of motion describing the nutational instability of the rotor influenced by the phase lag is written as

$$\left[s^2 - i\sigma\Omega s + \frac{k_1}{1+\tau_d s}\,\frac{1+\tau_2 s}{1+\tau_1 s}\right]\Theta = 0 \tag{3}$$

where $\Theta=\theta_x+i\theta_z$, $\sigma=J_y/J$ and $k_1=K_1/J$. The characteristic equation is

$$g(s)=\tau_d\tau_1 s^4+(\tau_1+\tau_d-i\sigma\Omega\tau_1\tau_d)s^3+[1-i\sigma\Omega(\tau_1+\tau_d)]s^2+(k_1\tau_2-i\sigma\Omega)s+k_1=0 \quad (4)$$

The eigenvalue corresponding to free nutation is known as $i\sigma\Omega$, which is slightly perturbed by the control torques. The approximate solution of the Eq. (4) may be obtained as

$$\mathrm{Re}[s]\simeq\mathrm{Re}\left[\frac{-g(i\sigma\Omega)}{\frac{dg}{ds}}\right]\simeq\frac{-k_1(\tau_2-\tau_1-\tau_d-\tau_1\tau_2\tau_d\sigma^2\Omega^2)}{(1+\tau_1^2\sigma^2\Omega^2)(1+\tau_d^2\sigma^2\Omega^2)} \quad (5)$$

It indicates that the rotor untation is unstable over the critical speed

$$\Omega_{cr.}=\frac{1}{\sigma}\sqrt{\frac{\tau_2-\tau_1-\tau_d}{\tau_2\tau_1\tau_d}} \quad (6)$$

(3) Stability of the Rotor Precession

Precession is considered as the rotation of the angular momentum vector. Since the precession rate is much smaller than the spin rate, the influences of the inertia and the phase lag on the rotor precession are negligible so that the simplified equation of motion may be given by

$$(k_1\tau_2-i\sigma\Omega)\ddot{\Theta}+(k_1-ik_4)\dot{\Theta}+k_2\Theta=0. \quad (7)$$

The characteristic root corresponding to precession is approzimated by

$$\mathrm{Re}[s]\simeq\frac{-k_1^2\tau_2+k_2/k_1\sigma^2\Omega^2-k_4\sigma\Omega}{\tau_2^2k_1^2+\sigma^2\Omega^2} \quad (8)$$

where $k_z=K_2/J$, $k_4=K_4/J$. Eq. (8) indicates that the precessional damping of the potor decreases with increasing spin rate and that the cross-feedback K_4 may compensate it.

3. Dynamics of The Satellite with a Magnetic Momentum Wheel

(1) Attitude Control using a Gimballed wheel

One of the most preferable features of a magnetic momentum wheel is vernier gimballing capability. Adjusting the gimbal angles according to the satellite attitude errors would precisely correct them and would effectively decrease the satellite nutation. For a satellite equipped with a gimballed momentum wheel as depicted in Fig. 4, the simplified equations of motion are written as follows.

$$\begin{aligned} I_x\dot{\phi}+h\psi&=-h\theta_z \\ I_z\dot{\psi}-h\phi&=h\theta_x \end{aligned} \quad (9)$$

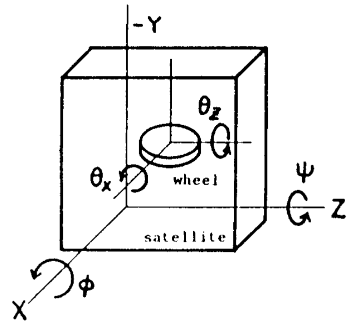

Fig. 4. Mathematical model of a satellite equipped with a gimballed wheel.

where I_x, I_z : moments of inertia of the satellite
ϕ, ψ : attitude angles
h : angular momentum.

The dynamics of the rotor is neglected since the speed of the precession is much faster than that of the attitude motion of the satellite. The active nutation control is performed by the law, for example,

$$\theta_x=0, \qquad \theta_z=G\phi. \tag{10}$$

Inserting Eqs. (10) to Eqs. (9) we find the characteristic equation

$$s^2+G\frac{h}{I}s+\left(\frac{h}{I}\right)^2=0 \tag{11}$$

where was assume $I=I_x=I_z$. Eq. (11) indicates that the damping factor of the nutation is a half of the feedback gain G. The large G provides the system sufficient damping.
(2) Stability of the Satellite Nutation without Attitude Feedback
While the active nutation suppression by a gimballed momentum wheel is effective, the attitude information is not always available. In such a case, the satellite nutations may be induced by the gyroscopic rotor motion under certatin condition in a long time. The eqations of the satellite motion considering the rotor dynamics are

$$\begin{aligned} I\ddot{\theta}&=-u_x \\ I\ddot{\psi}&=-u_z \\ J(\ddot{\phi}+\ddot{\theta}_x)+h(\dot{\psi}+\dot{\theta}_z)&=u_x \\ J(\ddot{\psi}+\ddot{\theta}_z)-h(\dot{\phi}+\dot{\theta}_x)&=u_z \end{aligned} \tag{12}$$

The bearing control torques are given by Eqs. (2). Eqs. (12) are reduced to Eqs. (13) by introducing the complex variables $\Phi=\phi+i\psi$ and $U=u_x+iu_z$.

$$
\begin{aligned}
&I\ddot{\Phi}=-U\\
&J(\ddot{\Phi}+\ddot{\Theta})-ih(\dot{\Phi}+\dot{\Theta})=U\\
&U=-\frac{1}{1+\tau_d s}\left[K_1\frac{1+\tau_2 s}{1+\tau_1 s}+\frac{K_2}{s}+i(sK_3-K_4)\right]\Theta
\end{aligned}
\tag{13}
$$

The characteristic equation is

$$f_1(s)=\sum_{i=0}^{6} a_i s^i=0 \tag{14}$$

where

$$
\begin{aligned}
&a_0=ihK_2\\
&a_1=IK_2-ih(K_1+K_2\tau_1-iK_4)\\
&a_2=I(K_1+K_2\tau_1-iK_4)-ih[K_1\tau_2+i(K_3-K_4\tau_1)]\\
&a_3=-ihI+K_3\tau_1 h+I[K_1\tau_2+i(K_3-K_4\tau_1)]\\
&a_4=IJ-iIh(\tau_d+\tau_1)+iIK_3\tau_1\\
&a_5=IJ(\tau_d+\tau_1)-ihI\tau_d\tau_1\\
&a_6=IJ\tau_d\tau_1
\end{aligned}
$$

In case of a satellite equipped with a momentum wheel supported by conventional ball bearings, the eigenvalue of the satellite nutation is (ih/I). The real part of the small deviation due to the magnetic suspension is estimated as

$$\mathrm{Re}[s]\simeq\mathrm{Re}\left[\frac{-f_1}{df_1/ds}\right]\simeq\frac{1}{\Delta}\left[-\frac{h^6}{I^3}(\tau_2-\tau_1-\tau_d)K_1+\frac{h^4}{I}K_2-\frac{\tau_d h^7}{I^4}K_3+\frac{h^5}{I^2}K_4\right] \tag{15}$$

where $\Delta=[df/ds]^2=I^2K_2^2+h^2K_1^2$. (16)

Since all the parameters are positive, the feedback gains K_1 and K_3 contribute to the stability of the satellite nutation while the gains K_2 and K_4 has the tendancy to destabilize it. The orders of the parameters are

$$
\begin{gathered}
K_1\sim 10^2,\quad K_2\sim 10^1,\quad K_3\sim 10^0,\quad K_4\sim 10^1\\
h\sim 10^1,\quad I\sim 10^2,\quad \tau_1,\ \tau_2,\ \tau_a\sim 10^{-3}.
\end{gathered}
$$

Then the orders of the each terms in eq. (15) are

$$
\begin{aligned}
&\text{1st term}\sim -10^{-7}\\
&\text{2nd term}\sim\ \ 10^{-2}\\
&\text{3rd term}\sim -10^{-10}\\
&\text{4th term}\sim\ \ 10^{-4}
\end{aligned}
\tag{17}
$$

These values imply that the satellite nutation is unstable since the 2nd term is positive and dominant. The integral compensation of the form Eq. (2), which is reffered as direct integral compensation hereafter, is not practical from a standpoint of satellite nutational stability.

On the other hand, the cross integral compensation also be applicable for eliminating the stationary errors. The transfer function of the controller is written as

$$U(s)=-\frac{1}{1+\tau_d s}\left[K_1\frac{1+\tau_2 s}{1+\tau_1 s}+i\left(sK_3-K_4+\frac{K_2'}{s}\right)\right]\Theta(s) \tag{18}$$

where the K_2' is the cross integral feedback gain.

Inserting Eq. (18) to Eq. (12) yields a characteristic equation

$$f_2(s)=\sum_{i=0}^{6} b_i s^i=0 \tag{19}$$

where

$$\begin{aligned}
b_0&=-hK_2'\\
b_1&=-iK_2'I-ih(K_1-i\tau_1K_2'-iK_4)\\
b_2&=I(K_1-i\tau_2K_2'-iK_4)+ih(\tau_2K_1+iK_3-i\tau_1K_4)\\
b_3&=-ihI+h\tau_1K_3+I(\tau_2K_1+iK_3-i\tau_1K_4)\\
b_4&=IJ-ihI(\tau_1+\tau_d)+iI\tau_1K_3\\
b_5&=IJ(\tau_1+\tau_d)-ihI\tau_1\tau_d\\
b_6&=IJ\tau_1\tau_d
\end{aligned}$$

The real part of the characteristic root of satellite nutation is approximately Eq. (20). The order of the terms are estimated as (21).

$$\mathrm{Re}[s]\approx\frac{1}{\Delta}\left[\frac{h^6}{I^3}(\tau_2-\tau_1-\tau_d)K_1+\frac{h^5\tau_d}{I^2}K_2-\frac{h^7\tau_d}{I^4}K_3+\frac{h^5}{I^2}K_4-\frac{h^8}{I^3}(\tau_d+\tau_1)\right] \tag{20}$$

$$\begin{aligned}
&\text{1st term}\sim-10^{-7}\\
&\text{2nd term}\sim\ 10^{-6}\\
&\text{3rd term}\sim-10^{-10}\\
&\text{4th term}\sim\ 10^{-4}\\
&\text{5th term}\sim-10^{-9}
\end{aligned} \tag{21}$$

Compared with Eq. (17), this result alleviates the nutational instability induced by the direct integral compensation. However, the satellite nutation is still unstable, in this stage, due to the cross proportional feedback via K_4. The rate cross-feedback via K_3 contributes in any case to the nutational stability of the satellite only a little.

(3) Precessional Stability due to the Cross Integral Compensation

The simplified characteristic equation of the rotor motion employing the cross integral compensation is written as Eq. (22) and the corresponding eigenvalue is approximated by Eq. (23).

$$(k_1\tau_2-i\sigma\Omega)s^2+(k_1-ik_4)s+ik_2=0 \tag{22}$$

$$\mathrm{Re}[s]\simeq\frac{-\tau_2 k_1^2+(k_2\tau_2-k_4)\,\sigma\Omega}{\tau_2^2k_1^2+\sigma^2\Omega^2} \tag{23}$$

Eq. (23) implies that the damping of the rotor precession decreases more moderately with increasing Ω than one of Eq. (8). In other words, the integral compensation has to be done in the form of cross feedback from standpoints both of rotor precession and of satellite nutation.

(4) Design Criteria

The satellite nutation is unstable as long as the transfer function of the controller is expressed by Eq. (18) or Eq. (2). A solution to this problem is employing negative K_4 under the condition that the bearing control is stable. Negative K_4 decrease the damping of rotor precession. The real part of the precessional eigenvalue must be negative. From Eq. (23), we get the criteria

$$\tau_2\left(K_2'-\frac{K_1^2}{h}\right)<K_4<0. \tag{24}$$

Eqs. (23), (24) indicate that the rotor precessional damping is to be derived via K_1 since negative K_4 decreases it. Large K_1 permits $-K_4$ to be large. That stabilizes satellite nutation. Employing neagative K_2' is impossible as an unstable root emerges.

(5) Example

A numerical example is shown in Fig. 4 where the damping factors of the rotor precession and the satellite nutation are plotted with varying K_4. The chart shows that the system is stable in the region $-6<K_4<0$.

The result is not satisfactory as the damping factor is rather small. The reason is that the proprortional gain is small comparared with the integral gain, whose values were determined without considering satellite dynamics. In order to derive better characteristics total optimization has to be performed.

4. Conclusions

The dynamic interaction between the satellite and the magnetically suspended rotor has been investigated. In order to save the power consumption, the bearing control electronics contained phase lag and integral compensation. Following results were derived. Proportional cross-feedbacks stabilized satellite nutation at the cost of decreased damping of rotor precession. Integral cross-feedbacks alleviated the satellite nutational instability. Derivative cross-feedbacks, which stabilized the rotor

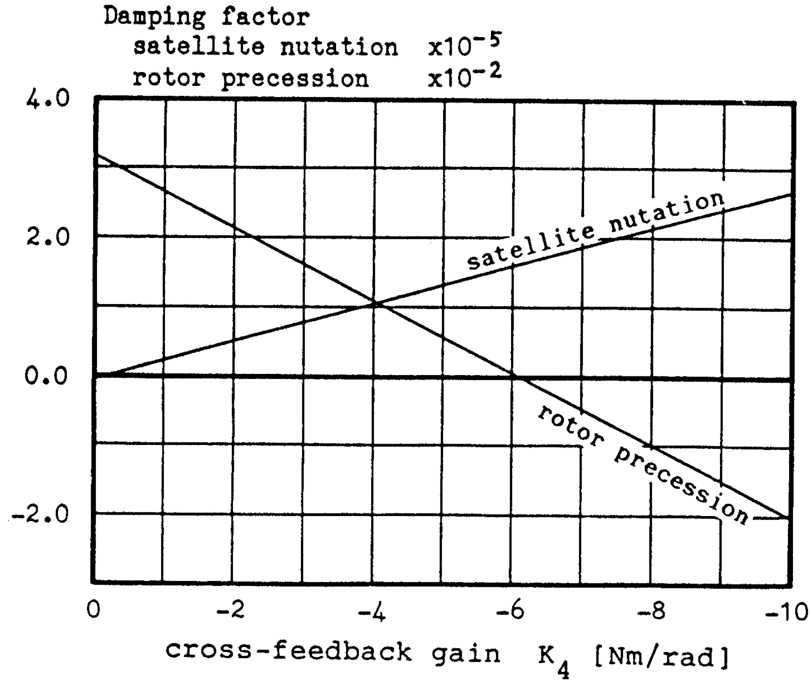

Fig. 5. Damping factor of the satellite and the rotor.

nutation, scarecely had any influences on a satellite motion.

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