Hypersonic Rarefied Flows Around a Circular Disk Perpendicular to the Stream

By

Kenichi NANBU*, Saburo IGARASHI* and Yasuo WATANABE*

(August 25, 1988)

Summary: Rarefied gas flows around a circular disk in a hypersonic stream are analysed by the direct simulation Monte Carlo method. The domain of calculation is fully three-dimensional. The rarefaction effect upon the flow field, the drag and heat transfer coefficients, and the recovery temperature is shown for Kn=0.1–20 at the wall to stagnation temperature ratios of 0.5 and 1. The drag coefficient is in good agreement with the experimental data for argon by Legge. The cell network proposed here can easily be applied to more general lifting flows.

1. INTRODUCTION

One of the most challenging problems in rarefied gas dynamics is the rarefied flow around the aeroassisted orbital transfer vehicle (AOTV). It is a three-dimensional flow of gas mixture with translational, rotational, vibrational, chemical, and radiative nonequilibrium. Only the direct simulation Monte Carlo (DSMC) method, together with a newborn supercomputer, may make it possible to compute such a complicated flow. Dogra, Moss and Simmonds [*1*] calculated an axisymmetric flow along this line by use of the DSMC method. The problem in adopting the DSMC method is that one must have recourse to many assumptions or models that have not yet been verified enough by experimental or theoretical studies. Our position on the problem is that such assumptions or models must be checked one by one for some flows which are simple enough to be treated by both the DSMC method and experiment.

Here is considered an axisymmetric hypersonic rarefied flow of monatomic gas perpendicular to a circular disk. This simple case was investigated experimentally by Legge [*2*] and theoretically by Hermina [*3*] who used the Bird method [*4*] together with a dubious assumption of weighting factor. In a near future we intend to extend the present work to the case when the disk has an angle of attack. Therefore, the network of cells used here is so devised as to be applicable to fully three-dimensional flows. The collision process in the DSMC method is treated by the modified Nanbu method [*5, 6*]. Only the assumption of diffuse reflection and the model of rigid sphere are employed in the present calculations.

* Institute of High Speed Mechanics, Tōhoku University.

2. Outline of Simulation Calculations

We consider a rarefied flow around a circular disk perpendicular to the hypersonic stream U_∞. Figure 1 shows the domain of calculation. It is a circular cylinder with radius R and length $(a+b)$. The radius of the disk is r_D. The cell network is as follows: first the domain is divided into N_z wafers with thickness Δz; next each wafer is subdivided by circles and radial lines as shown in Fig. 2. The radius r_k of the kth circle is given by

$$r_k=k\Delta r, \qquad (k=1, 2, \dots, N_r), \tag{1}$$

where Δr is the radius of the innermost circle. The annular region $r_{k-1}<r<r_k$ $(k=2, 3, \dots, N_r)$ is divided into N_k subregions where

$$N_k=3k, \qquad (k=2, 3, \dots, N_r). \tag{2}$$

It is to be noted that the innermost circle is not divided by radial lines. This network of cells is reasonable, since the maximum distance L between two molecules in a cell is of the same order for all cells. In fact, the distance L is

$$L=2\Delta r \qquad \text{for } k=1,$$

and

$$L=2r_k \sin(\Delta\phi/2) \qquad \text{for } k=2, 3, \dots, N_r,$$

where $\Delta\phi=2\pi/N_k$. Then we have

$$\frac{L}{2\Delta r}=k\sin\left(\frac{\pi}{3k}\right)<\frac{\pi}{3}(\cong 1.05). \tag{3}$$

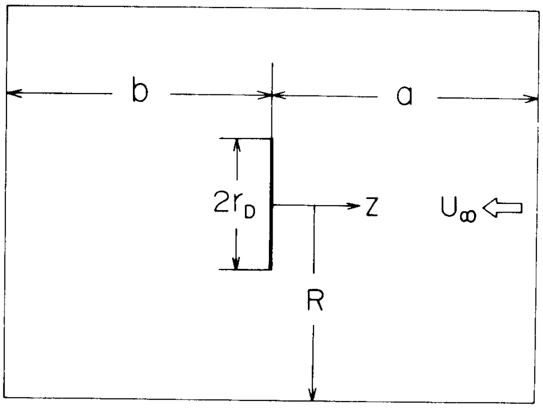


Fig. 1. Domain of calculation.

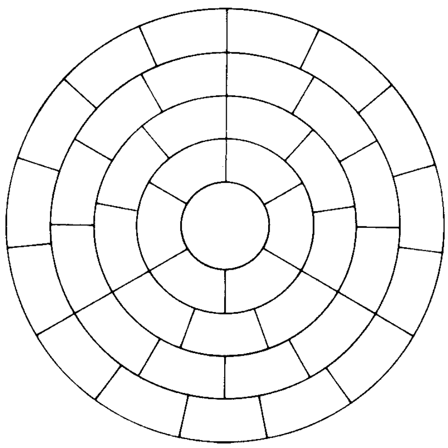


Fig. 2. Network of cells.

This means that $L/2\Delta r$ is between 1 and 1.05 for any cell. Next we consider the cell volume. The volume V_1 of the innermost cell is $V_1=\pi(\Delta r)^2\Delta z$ and the volume V_k of each of N_k cells in the annular region $r_{k-1}<r<r_k$ is given by

$$\frac{V_k}{V_1}=\frac{2k-1}{3k}, \qquad (k=2,\ 3,\ \ldots,\ N_r). \tag{4}$$

This ratio is between 1/2 and 2/3. The total number N_c of cells is

$$N_c=\left[\frac{3}{2}N_r(N_r+1)-2\right]N_z. \tag{5}$$

Our choice of the sizes of the cell and calculation domain is as follows,

$$R=3r_D,\ a=(2-6)r_D,\ b=(2-6)r_D,\ \Delta r=0.2r_D,\ \Delta z=(0.1-0.2)r_D.$$

In a typical case, N_c is 21480.

The simulation starts by putting molecules in the whole domain upstream of the disk. These molecules are given the velocity of the free stream. They have no peculiar velocity since the Mach number of the free stream is assumed to be infinitely large. As time goes on, fresh molecules come in across the upstream boundary $(z=a)$. Molecules which go out of the domain of calculation are eliminated. This is reasonable for hypersonic flows. Molecules which strike the disk are diffusely reflected with complete thermal accommodation to the wall temperature T_w. The intermolecular collision is simulated by the modified Nanbu method [*5*, *6*]. The outline of the method is as follows. The probability P_{ij} that molecule i collides with molecule j over the time interval $(t,\ t+\Delta t)$ is

$$P_{ij}=\frac{n}{N}\sigma_T g_{ij}\Delta t, \tag{6}$$

where N is the number of simulated molecules in a cell, n the number density, σ_T the total collision cross-section, g_{ij} the relative velocity of the collision pair at time t. Let us fix the attention on molecule i. The probability P_i that molecule i collides with others is

$$P_i=\sum_{j=1}^{N} P_{ij}.$$

The probability of no collision is $(1-P_i)$. Clearly, the summation of the two probabilities is unity, i.e.

$$1=(1-P_i)+P_i=\sum_{j=1}^{N}\left[\left(\frac{1}{N}-P_{ij}\right)+P_{ij}\right].$$

The probabilities $P_{i1}, P_{i2}, \ldots, P_{iN}$ are distributed as shown in Fig. 3, where the interval [0, 1] is divided into N equal segments. If a random fraction U_f between 0 and 1 lies in the jth segment, we have only to calculate P_{ij}. If $U_f<(j/N)-P_{ij}$, there is no collision, and otherwise there is a collision with molecule j. Equation (6) can be expressed in terms of dimensionless quantities as

$$P_{ij}=\frac{\hat{g}_{ij}\,\Delta\hat{t}}{2\sqrt{2}\,N_0 Kn}\cdot\frac{V_1}{V_k}, \tag{7}$$

where $\hat{g}_{ij}=g_{ij}/U_\infty$, $\Delta\hat{t}=\Delta t/(r_D/U_\infty)$, $Kn(=\lambda_\infty/2r_D)$ is the Knudsen number, and N_0 is the number of simulated molecules in a far upstream cell with volume V_1. Here λ_∞ denotes the mean free path in the free stream. In the simulation the values of Kn, $\Delta\hat{t}$ and N_0 are given as data. In a typical case our choice is $N_0=30$ and $\Delta\hat{t}=0.01$.

3. Results and Discussion

In order to clarify the rarefaction effect upon the flow field, the drag and heat transfer coefficients, and the recovery temperature, the numerical calculations are performed for the Knudsen numbers of 0.1–20 and the wall to stagnation temperature ratios T_w/T_0 of 0.5 and 1. Owing to the assumption of hypersonic flow, the present problem is governed by only two parameters Kn and T_w/T_0. There is no need to specify the Mach number of the free stream. Figure 4 shows the velocity profiles along the stagnation streamline for $T_w/T_0=1$ and 0.5. As the Knudsen number decreases, the gradient of the velocity profiles becomes very large, which means the formation of shock front. The wall temperature has little effect on the velocity profiles. In Fig. 5 are shown the density profiles along the stagnation streamline. A large increase in density occurs near the disk in the cold wall case. Figure 6 shows the temperature profiles along the stagnation streamline. It is seen that there appears a greater temperature jump for a larger Knudsen number. In Figs. 7 and 8 are shown the density and temperature profiles on the front surface of the disk as a function of the radial distance, respectively. At the edge of the disk the density and temperature suddenly drop due to the expansion of gas. In order to obtain more knowledge on the flow field, the density contours for $Kn=0.1$ and $T_w=T_0$ are given in Fig. 9. The compression and expansion regions can clearly be seen. Figure 10 represents the streamlines for the cold wall case $T_w/T_0=0.5$. In case of the smaller Knudsen number, there appears a flow towards the backside of the disk due to the higher frequency of molecular collisions.

Figure 11 shows the drag coefficient C_D for $T_w=T_0$ in comparison with the experimental data (denoted by a symbol ⊙) for argon by Legge [2]. Agreement is very good. The figure also includes the results for $T_w/T_0=0.5$. Here the drag coefficient C_D is defined by

$$C_D=\frac{D}{\frac{1}{2}\rho_\infty U_\infty^2 A}, \tag{8}$$

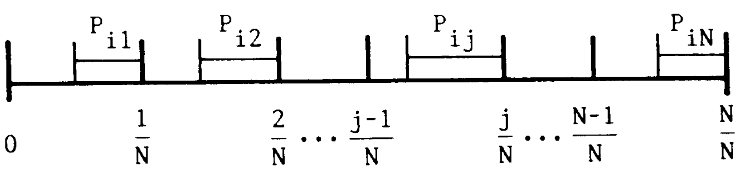


Fig. 3. The modified Nanbu method.

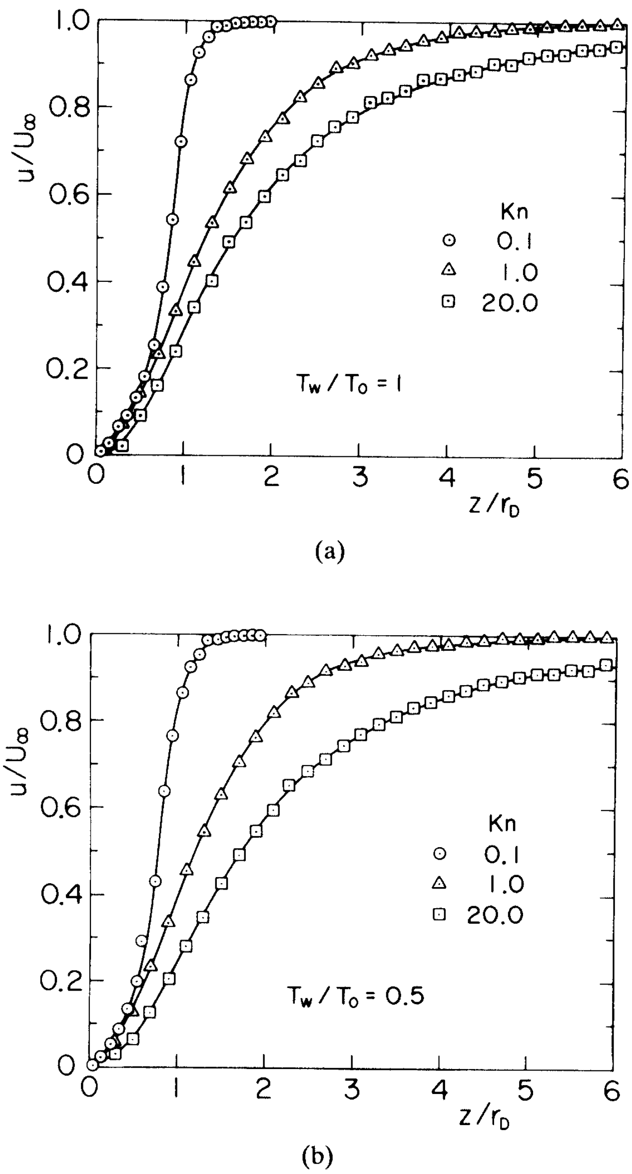
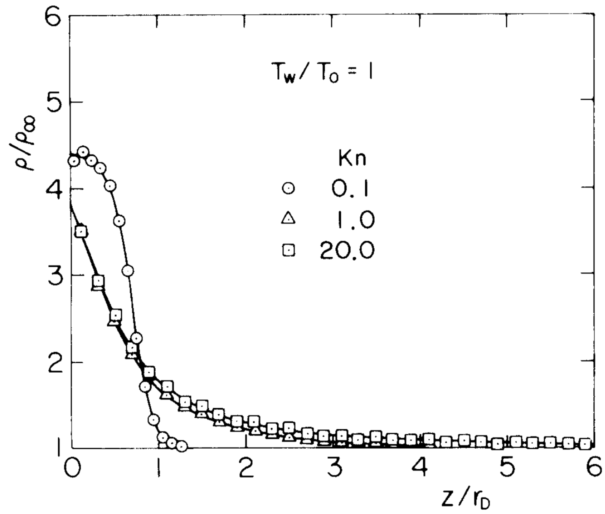
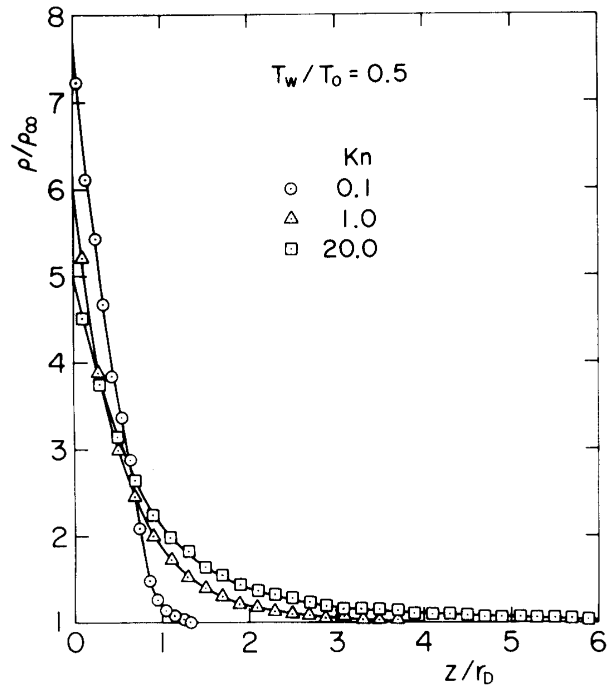


Fig. 4. Velocity profiles along the stagnation streamline. (a) $T_w/T_0=1$, (b) $T_w/T_0=0.5$.

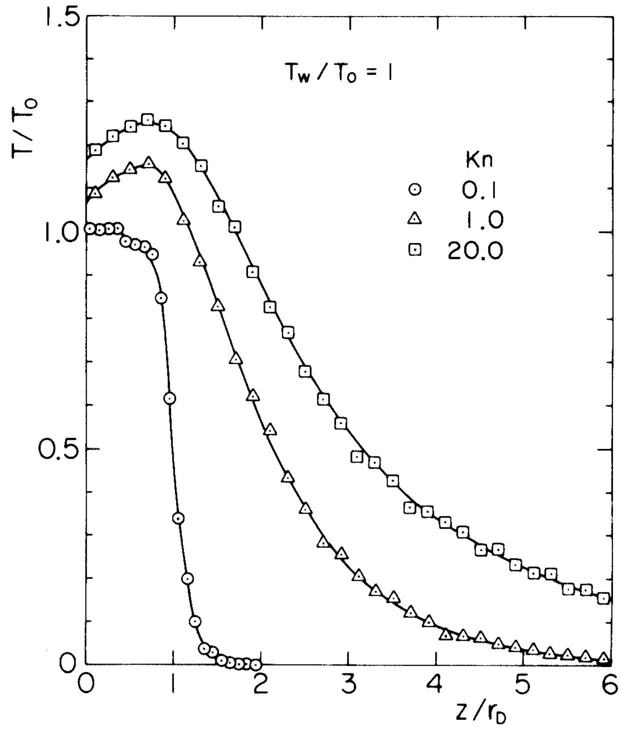


(a)

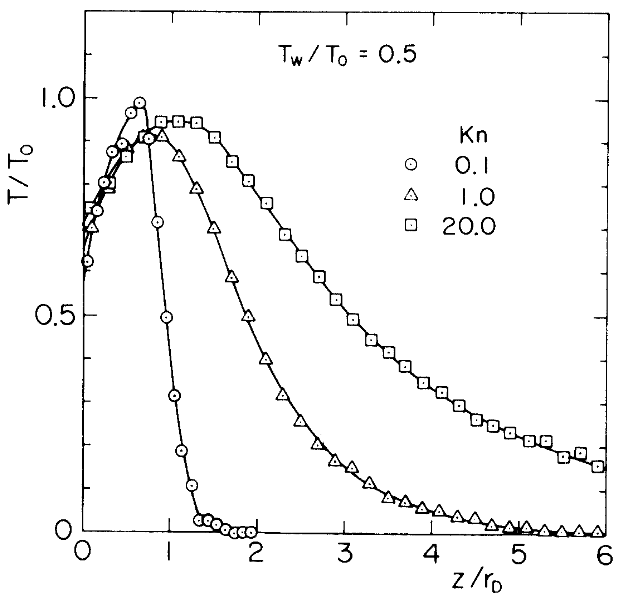


(b)

Fig. 5. Density profiles along the stagnation streamline. (a) $T_w/T_0=1$, (b) $T_w/T_0=0.5$.



(a)



(b)

Fig. 6. Temperature profiles along the stagnation streamline. (a) $T_w/T_0=1$, (b) $T_w/T_0=0.5$.

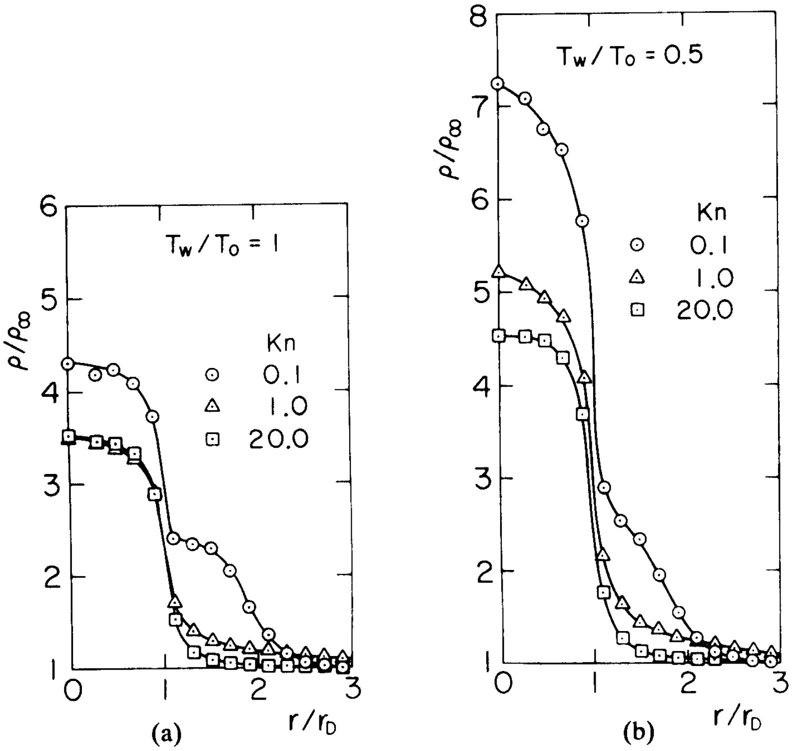


Fig. 7. Density profiles on the disk surface.
(a) $T_w/T_0=1$, (b) $T_w/T_0=0.5$.

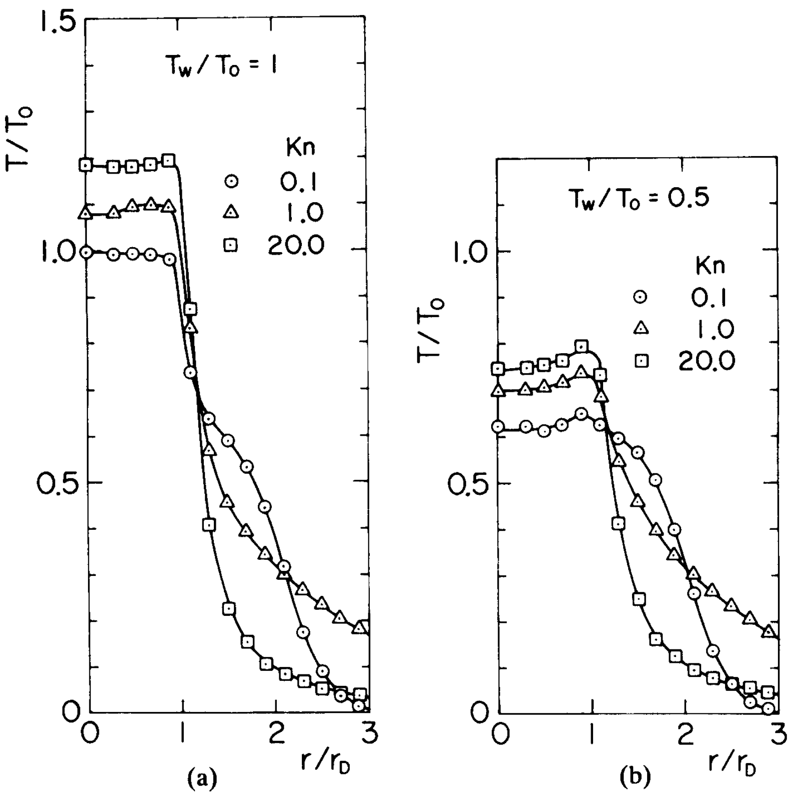


Fig. 8. Temperature profiles on the disk surface.
(a) $T_w/T_0=1$, (b) $T_w/T_0=0.5$.

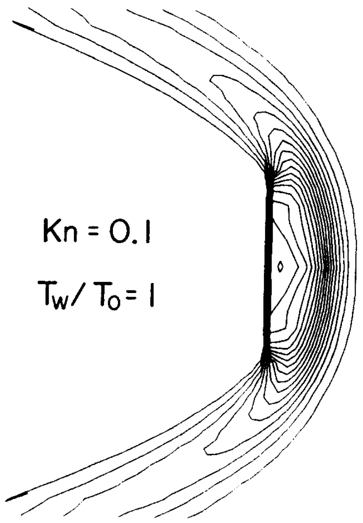


Fig. 9. Density contours for Kn=0.1 and T_w/T_0=1.

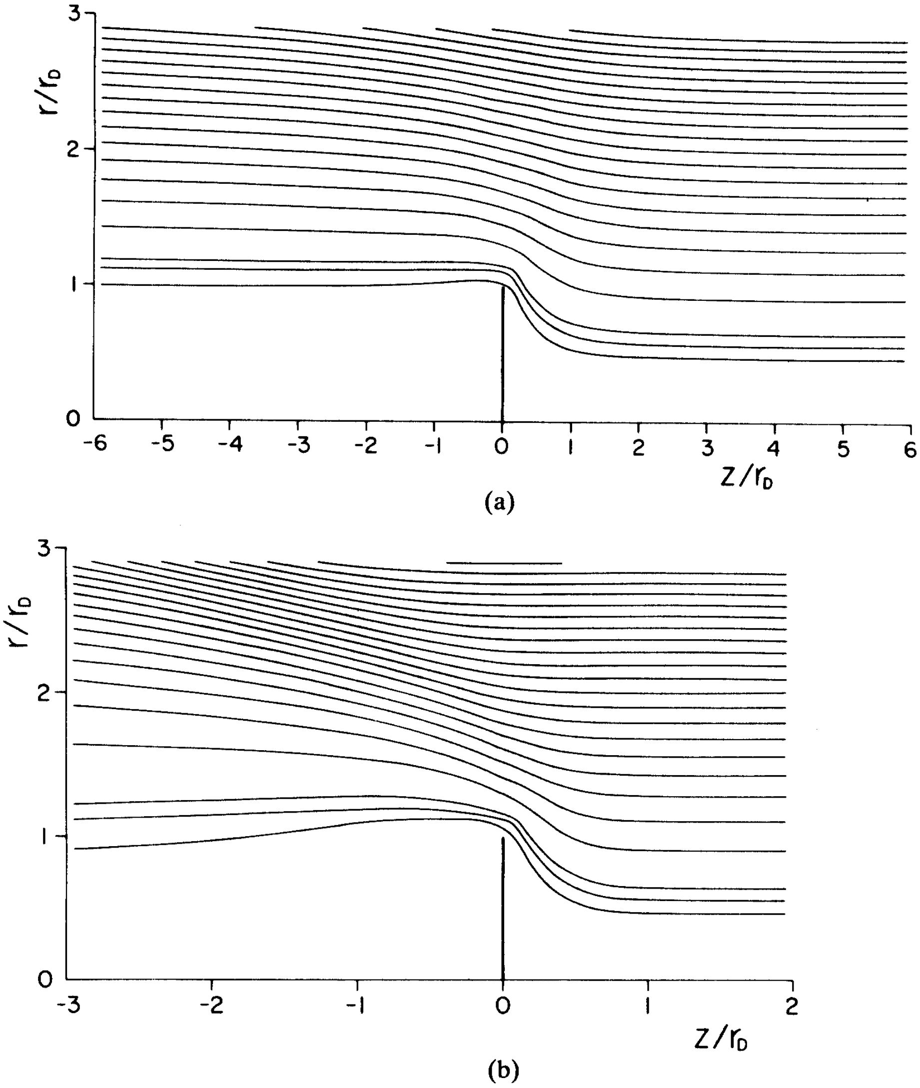


Fig. 10. Streamlines for T_w/T_0=0.5. (a) Kn=1.0, (b) Kn=0.1.

where D is the drag and $A = \pi r_D^2$. In the limits of the free molecular and continuum flows we have from [7]

$$C_D = 2 + \sqrt{\frac{\pi(\gamma-1)}{\gamma} \cdot \frac{T_w}{T_0}}, \qquad (Kn \gg 1), \tag{9a}$$

and

$$C_D = \frac{\gamma+3}{\gamma+1}, \qquad (Kn \ll 1), \tag{9b}$$

with γ (=5/3) the ratio of the specific heats. Equation (9b) is based on the modified Newtonian theory.

Figure 12 shows the heat transfer coefficient C_H defined by

$$C_H = \frac{\dot{Q}}{\frac{1}{2}\rho_\infty U_\infty^3 A}, \tag{10}$$

with $\dot{Q}$ the net heat transferred per unit time from gas to the disk. In case of the free molecular flow we have

$$C_H = 1 - \frac{\gamma+1}{2\gamma} \cdot \frac{T_w}{T_0}. \tag{11}$$

In Fig. 13 we show the recovery temperature T_r, that is, the wall temperature when the heat transfer to and from the disk is balanced ($\dot{Q}=0$). In the free molecular regime we have from Eq. (11)

$$\frac{T_r}{T_0} = \frac{2\gamma}{\gamma+1}. \tag{12}$$

The recovery temperature decreases with Kn and tends rather abruptly to a limiting value near $Kn=0.2$. A similar behavior is seen in the data of C_H for $T_w = T_0$ (Fig. 12).

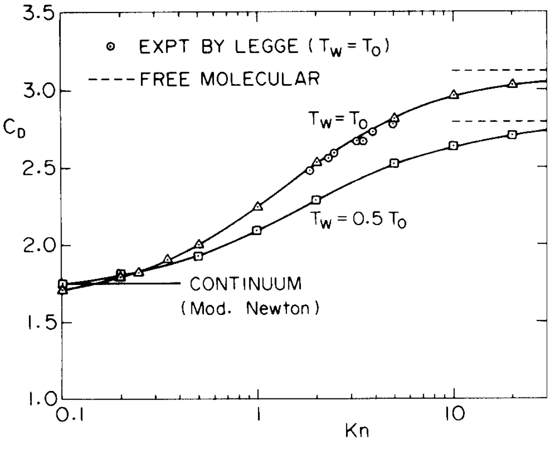


Fig. 11. Drag coefficient.

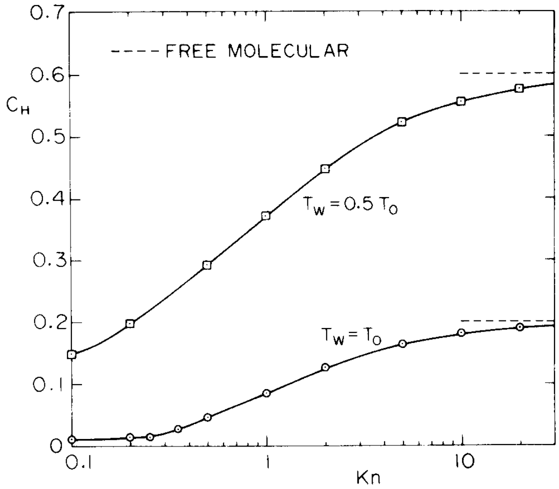


Fig. 12. Heat transfer coefficient.

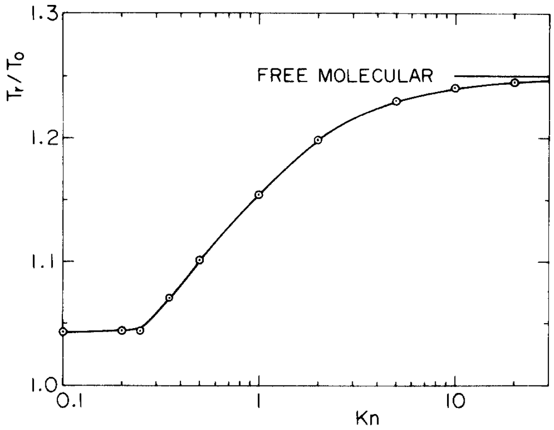


Fig. 13. Recovery temperature.

The numerical calculations were carried out by making use of the supercomputer SX-1 at the Computing Center of Tōhoku University.

This work was partly supported by the Institute of Space and Astronautical Science through the Joint Research "Aeroassisted Orbital Transfer Vehicle" (chief, Prof. H. Oguchi). It was also partly supported by the Grant-in-Aid for Scientific Research from the Ministry of Education as the Special Research Project "The Computational Fluid Dynamics".

References

[1] V. K. Dogra, J. N. Moss, and A. L. Simmonds, Direct Simulation of Aerothermal Loads for an Aeroassist Flight Experiment Vehicle, AIAA 22nd Thermophysics Conference, AIAA-87-1546 (1987).

[2] H. Legge, Wiederstandsmessungen an parallel angeströmten Platten und senkrecht angeströmten Scheiben in Überschallfreistrahlen grosser Verdünnung, Deutsche Luft- und Raumfahrt Forschungsbericht 73–17 (1973).

[3] W. L. Hermina, Monte Carlo Simulation of Transitional Flow around Simple Shaped Bodies, *Proc. 15th Int. Symp. Rarefied Gas Dynamics* (B. G. Teubner, Stuttgart, 1986), Vol. 1, pp. 452–460.

[4] G. A. Bird, *Molecular Gas Dynamics,* Clarendon Press, Oxford, 1976, pp. 118–132.

[5] K. Nanbu, Theoretical Basis of the Direct Simulation Monte Carlo Method, *Proc. 15th Int. Symp. Rarefied Gas Dynamics* (B. G. Teubner, Stuttgart, 1986), Vol. 1, pp. 369–383.

[6] R. Illner and H. Neunzert, On Simulation Methods for the Boltzmann Equation, *Transport Theory and Statistical Physics,* Vol. 16 (1987), Nos. 2 & 3, pp. 141–154.

[7] G. Koppenwallner and H. Legge, Drag of Bodies in Rarefied Hypersonic Flow, *Thermophysical Aspects of Re-Entry Flows,* (eds., J. N. Moss and C. D. Scott), AIAA Inc., 1986, pp. 44–59.