Numerical Simulation of Separated Flow around Two-Dimensional Wing Section by a Discrete Vortex Method

By

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Summary: Separated flows around pitching wing sections are simulated numerically by a discrete vortex method combined with a panal method. The potential flows around wing sections is expressed by vortex sheets and separated shear layers are expressed by discrete vortices.

In the calculation a separation point is determined by solving boundary layer equation. The strength of shed vortex is estimated using local velocity near separation point. Also modification for the estimation of pressure coefficients around wing section are proposed. The estimated pressure distributions show good agreements with experimental results. Also separated flows around pitching airfoils are simulated. A hysteresis of lift of airfoil at dynamic stall is obtained.

1. Introduction

One of approximate numerical methods for separatedd flows at high Reynolds number is a discrete vortex method. In the method separated shear layers shed from body surface are expressed as a row of discrete vortices. Recently a combination of the discrete vortex method and a panel method, in which potential flow around a body is expressed by a set of singular points distributed on the body surface, was proposed [*1*]. The present authors applied the method for the simulation of separated flows around square cylinders, trapezoidal cylinders and arc cylinders [*2*]. The results showed good agreements with experiments. And it is proved that the method is quite useful for the simulation of those flows.

In the previous calculations the separation points of those models could be specified a priori obvious and separated shear layers were shed from those points. However, for the bodies (e.g. wing section), whose separation points are not known a priori, the shedding points of separated shear layers should be determined in advance. The present authors have conducted the calculation of separated flows around wing sections. In the calculation a wing section is expressed by a set of linearly distributed vortex sheets and the separation points are determined by boundary layer calculations. Also separated shear layers are expressed as rows of discrete vortices.

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The separated flow around a wing section at a specified angle of attack are simulated properly and the results showed good agreements with experiments [3]. Also separated flows around sinusoidally pitching wing sections are simulated by the same method. The applicability of the methd for the calculation in those severe situations is examined. Also new procedures for the estimation of pressure coefficients are proposed. The calculated pressure distributions show good agreements with experimental results.

2. Analytical Method

Fig. 1 shows a schematic diagram of discrete vortex method combined with the panel method. The surface of the body is divided into N panels and a linear distribution of vorticity is assumed for each panel. The distribution of vorticity of the j-th panel is expressed as follows:

$$\gamma_{(s)} = \frac{\gamma_{j+1}+\gamma_j}{2} + \frac{\gamma_{j+1}-\gamma_j}{l_j}\, s \tag{1}$$

where s is a coordinate along the j-th panel and l_j is the length of the panel. The separated shear layers from the separation points (A and B) are expressed by rows of M discrete vortices as Γ_{A_k} (k=1, M) and Γ_{B_k} (k=1, M) respectively. The complex potential f of the flow field is expressed by the following form:

$$f = Ue^{-i\alpha}z + i\int_B \frac{\gamma(\varsigma)}{2\pi}\log(z-\varsigma)ds$$
$$+i\sum_{k=1}^{M}\left\{\frac{\Gamma_{A_k}}{2\pi}\log(z-z_{A_k})+\frac{\Gamma_{B_k}}{2\pi}\log(z-z_{B_k})\right\} \tag{2}$$

where U is the uniform flow velocity and α is the angle of attack. The discrete vortices expressing separated shear layers are shed from the separation points (A and B) on the body, and the circulation of each vortex is estimated using the velocity at the point assumed as the edge of the boundary layer near the separation point. Hence, unknown quantities in the flow field are the amounts of strength of the singular points

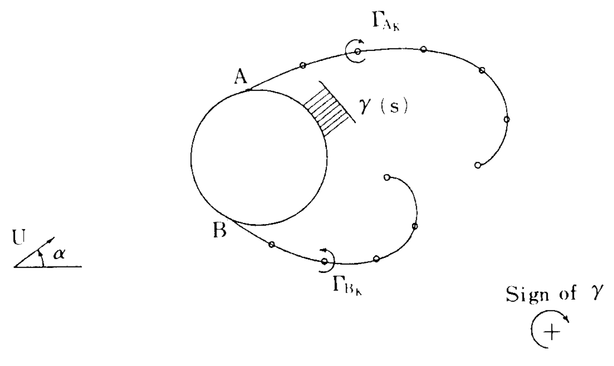

Fig. 1. Schematic diagram of discrete vortex method combined panel method.

on the surface of the body. In the calculation another unknown quantity ε, which represents the normal velocity at the mid-point of each panel, so called "control point", is introduced. Then the number of unknown becomes N+1. Those unknown quantities are obtained by solving the following N+1 simultaneous equations. Those are the boundary conditions requesting the normal velocity at the control point of each panel should be ε and the conservation of circulation:

(a) Boundary conditions:

$$\sum_{l=1}^{N} A_{jl}\gamma_l = \varepsilon + U\sin(\alpha-\delta_j) - \sum_{k=1}^{M} B_{jk}\Gamma_{A_k} - \sum_{k=1}^{M} C_{jk}\Gamma_{B_k} \quad (j=1, N) \tag{3}$$

(b) Conservation of circulation:

$$\int_B \gamma(\varsigma)ds = -\sum_{k=1}^{M}(\Gamma_{A_k}+\Gamma_{B_k}) \tag{4}$$

where δ_j is the angle of j-th panel.

In the actual calculation ε is small enough to be neglected and it could be regarded as dispersion of the error generated in the expression of the body by a number of panels. Vortices are shed into the flow field at the velocity induced by all the shed vortices and singularities on the body. The circulation of each vortex is estimated from the velocity at the point assumed as the edge of the boundary layer located near the separation point. The shedding vortices are convected by the Euler method as follows:

$$z_{W_k}(t+\Delta t) = z_{W_k}(t) + \bar{u}\cdot\Delta t \tag{5}$$

where Z_{W_k} is the position of k-th shed vortex, $\bar{u}$ is the complex velocity induced by all vortices and Δt is a time increment used for the convection of the shed vortices.

For the calculation of the flow around the pitching wing section, incident angle of the freestream is sinusoidally changed. Then the angle of attack of the wing section is given by following form:

$$\alpha = \alpha_0 + \alpha_1 \sin\omega t \tag{6}$$

where the angular frequency ω is related to the reduced frequency k by the following form:

$$k = \frac{\omega c}{2U_\infty} \quad \text{(c: chord length)} \tag{7}$$

In the estimation of pressure coefficient, two modifications are proposed. One is for the evaluation of the potential of the flow field and the other is for the correction of loss of the kinetic energy in the separated region.

In the first modification the pressure equation is expressed as:

$$\frac{\partial \Phi}{\partial t}+\frac{p}{\rho}+\frac{1}{2}U^2 = H\ (t) \tag{8}$$

Then the following relation could exist along a stream line connecting the stagnation point of a body and the infinite upstream point:

$$\frac{\partial \Phi}{\partial t}\Big|_\infty +\frac{p_\infty}{\rho}+\frac{1}{2}U_\infty^2 = \frac{p_0}{\rho}+\frac{\partial \Phi}{\partial t}\Big|_0 \tag{9}$$

and then the following relation is assumed in eq. (9).

$$\frac{p_\infty}{\rho}+\frac{1}{2}U_\infty^2 = \frac{p_0}{\rho} \tag{10}$$

In the second modification the term H (t) in eq. (8) is replaced with H (t)-ΔH, where ΔH is an equivalent of loss of the kinetic energy in the separated region.

3. Results and Discussion

Fig. 2 shows the model of a wing section used for the present calculation. The wing section is NACA 4412. As the separation point on the wing section is not obvious a priori, the separation point is determined by the boundary layer calculations. In the calculation the velocity distribution along the body surface is used for that at the edge of the boundary layer. And discrete vortices are shed from that point. The Thwaites' method [4] is used for the laminar boundary layer calculation and the Truckenbrodt's method [5] is used for the turbulent boundary layer calculation.

3.1 The flows around the fixed wing section

Fig. 3 shows the calculated flow pattern at angle of attack of 20 degrees. The shedding vortices are described by circular symbols and triangle symbols. Fig. 4 shows the stream line of the same result. Those figures show the separated flow with turbulent boundary layer separation about 50% of the chord. And the pressure distribution and the values of C_l and C_d are shown in Fig. 5. In the figure it is apparent that the calculated pressure distribution shows good agreement with that by experiment[6] including the separated region. Fig. 6 shows C_l and C_d with α. Compared with the experiments[7], the variation of C_l is more sensitive to angle of attack and the

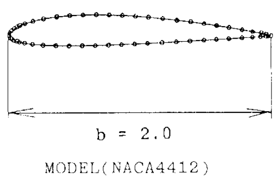

Fig. 2. Model used for the present calculation.

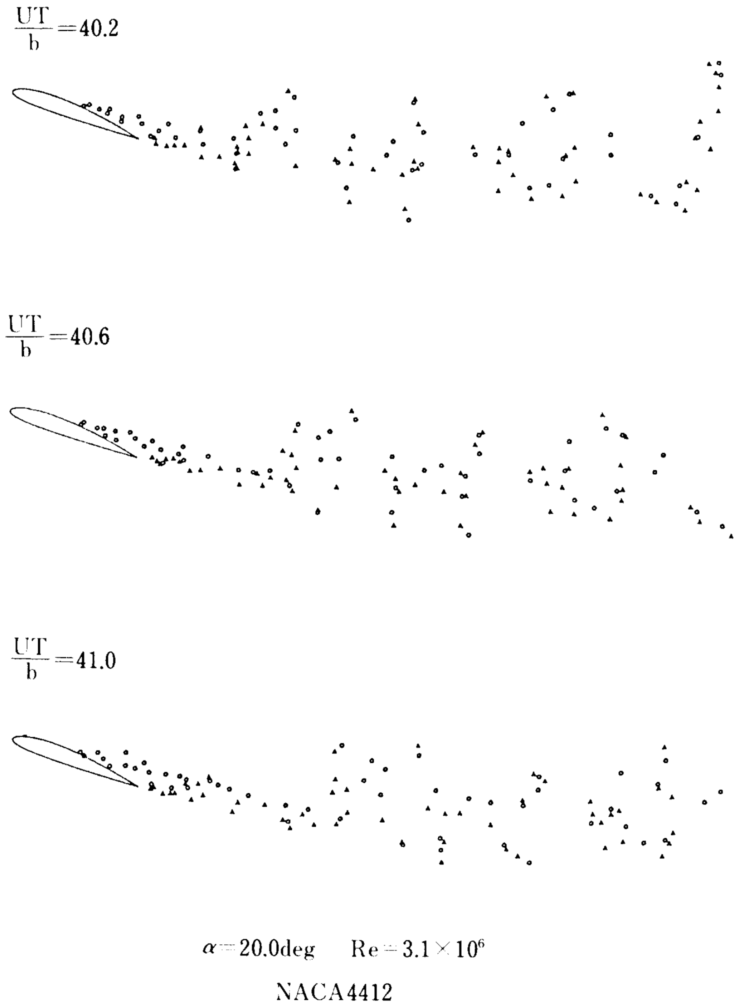

Fig. 3. Flow pattern (NACA4412, α=20 degrees).

maximum difference of C_l between the calculations and the experiments is about 14%. A good correspondence of C_l between the calculations and the experiments is obtained. Also the results of C_d show good correspondence with the experiments.

3.2 The flows around the pitching wing section

At first the calculated results under the condition of $\alpha=20°+5°\sin\omega t$ and the reduced frequency k of 0.20 is shown in Figs. 7–10. Fig. 7 shows the time histories of the circulations of shed vortices and the angle of attack. It shows that the circulations vary periodically with the angle of attack. Fig. 8 and Fig. 9 show the flow pattern and the stream line respectively. Figures show that the separation point, the state and the extent of the separated region are different both in α-increasing process ($\omega t=3.56\pi-4.33\pi$) and the α-decreasing process ($\omega t=4.71\pi-5.09\pi$) even at the same angle of attack in a cycle. Which is quite apparent when Fig. 8 (b) ($\omega t=3.95\pi$) and Fig. 8 (e) ($\omega t=5.09\pi$) are compared and Fig. 8 (c) ($\omega t=4.33\pi$) and Fig. 8 (d)

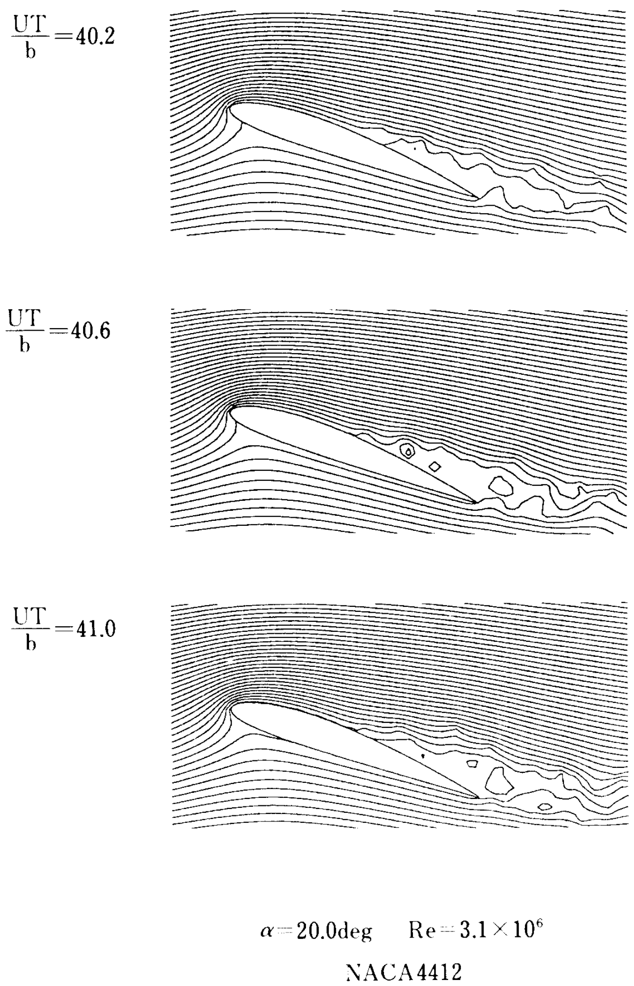

Fig. 4. Stream lines (NACA4412, α=20 degrees).

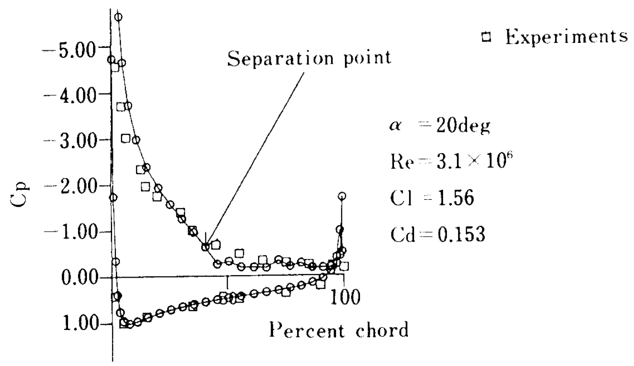

Fig. 5. Pressure distribution (NACA4412, α=20 degrees).

(ω t=4.71 π) are compared.

When α is decreased, the separation point is located more forward and the separation region over the wing section is more extensive and complex compared with α-increasing process. Fig. 9 shows there is extensive region of reversed flow and vortices over the wing section and the thickness of viscous layer is of the order of the

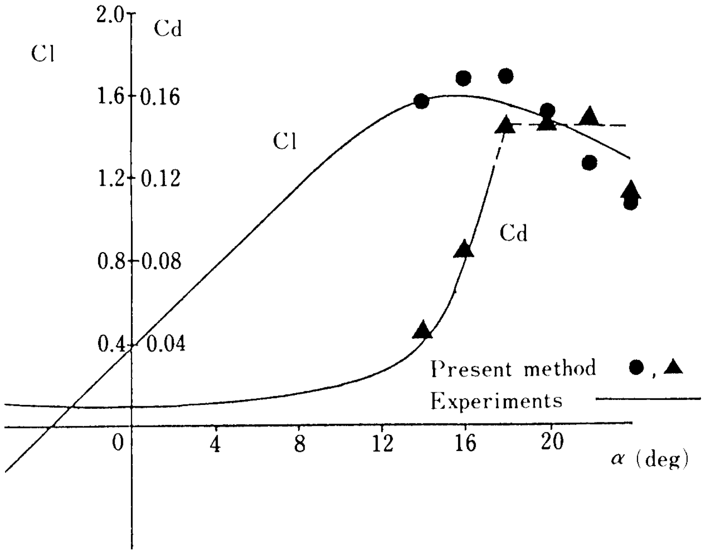

Fig. 6. Comparison of C_l and C_d curves with experiments.

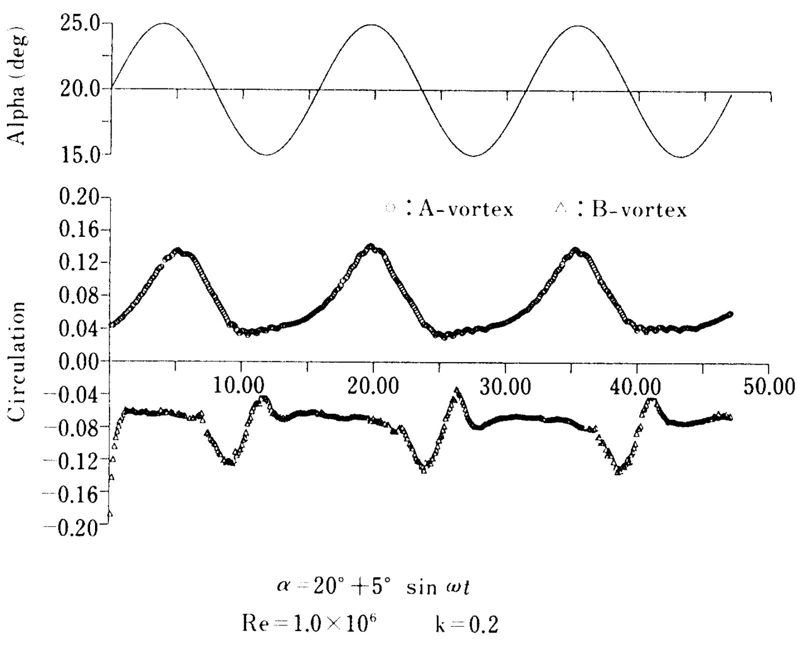

Fig. 7. Temporal changes of attack angle and strength of shed vortex of pitching wing section (NACA4412) (α=20°+5° sin ω t, k=0.2).

wing section chord during the α-decreasing process. Those are the features of the deep stall regime of the dynamic stall[8]. Fig. 10 shows the behavior of C_l versus α. Lift in the α-increasing process is greater than that in the α-decreasing process. And the maximum lift is greater than that for steady case. Also those features express the characteristics of the deep stall regime qualitatively.

Fig. 11 and 12 show the calculated result under the condition of α=16°+4° sin ω t

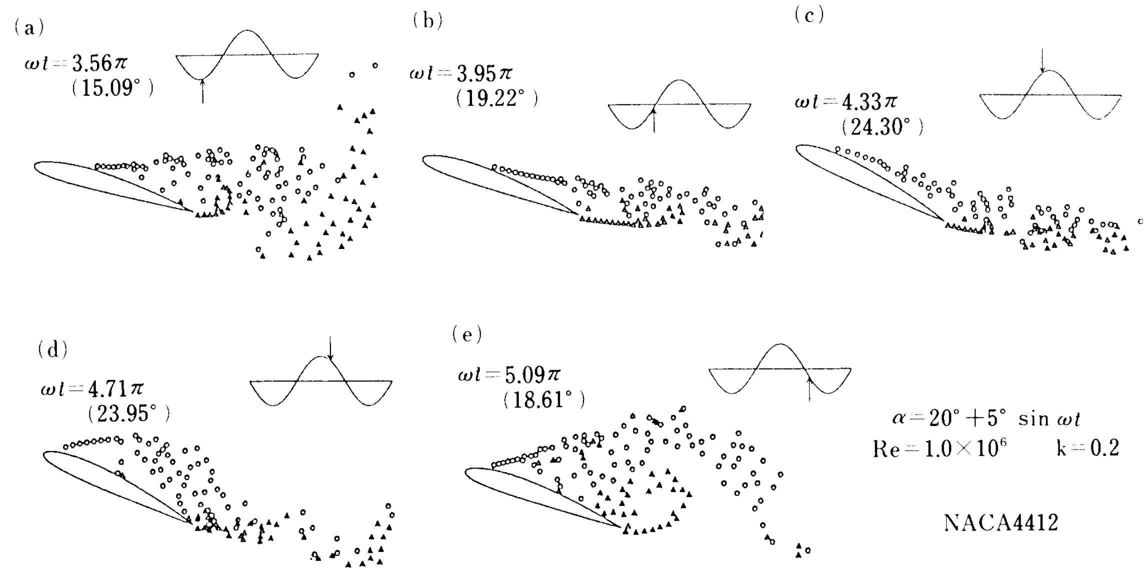

Fig. 8. Flow pattern around pitching wing section (NACA4412) (α=20°+5° sin ω t, k=0.2).

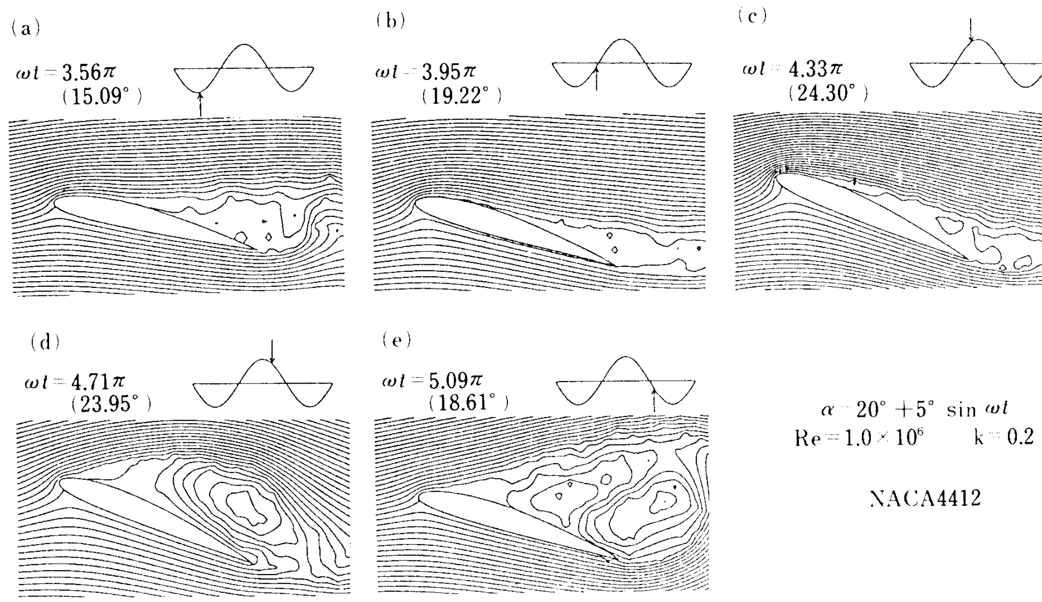

Fig. 9. Stream lines. (α=20°+5° sin ω t, k=0.2).

and the reduced frequency k of 0.20. The maximum angle of attack and amplitude are different from the previous calculation. Even in the α-decreasing process, the separated region over the wing section is comparatively small and simple. Also the state in the α-decreasing process is hardly different from that in the α-increasing process with exception that separation point is located more forward in α-decreasing process. The thickness of viscous layer is of the order of the wing section thickness and that is the feature of the light stall regime.

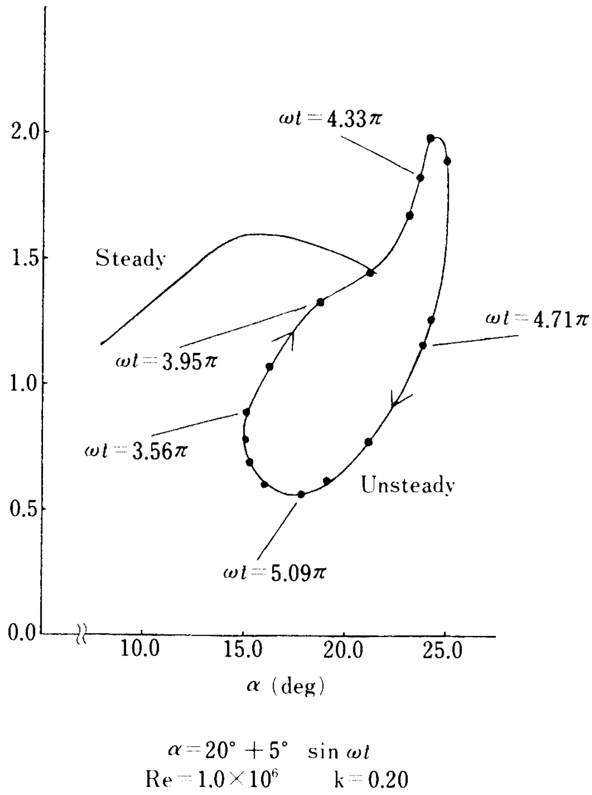

Fig. 10. Hysteresis of C_l of pitching wing section. (α=20°+5° sin ω t, k=0.2).

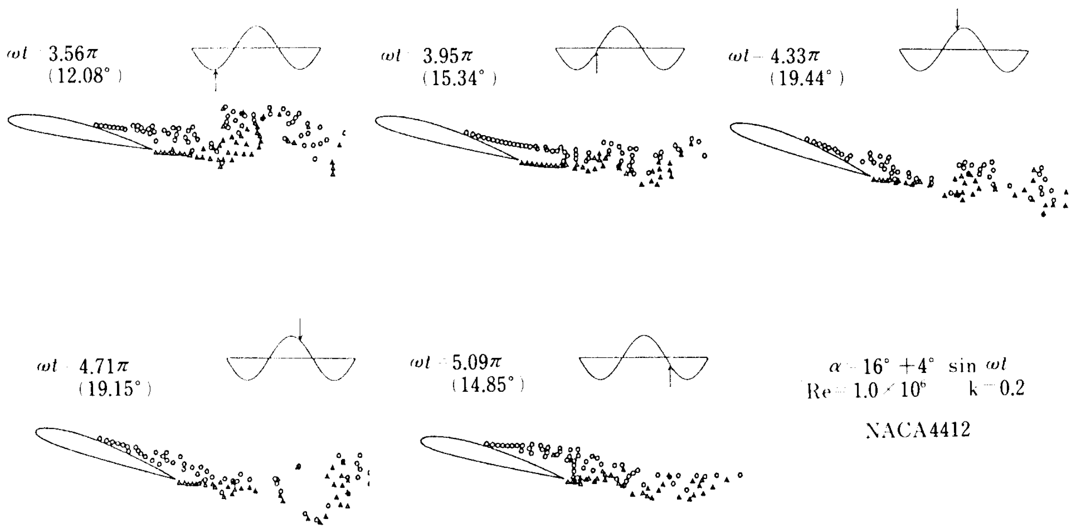

Fig. 11. Flow pattern around pitching wing section (NACA4412) (α=16°+4° sin ω t, k=0.2).

4. Conclusion

The flows around wing sections have been calculated by the method that the body is expressed by a set of linearly distributed vortex sheets and the separated shear layer is

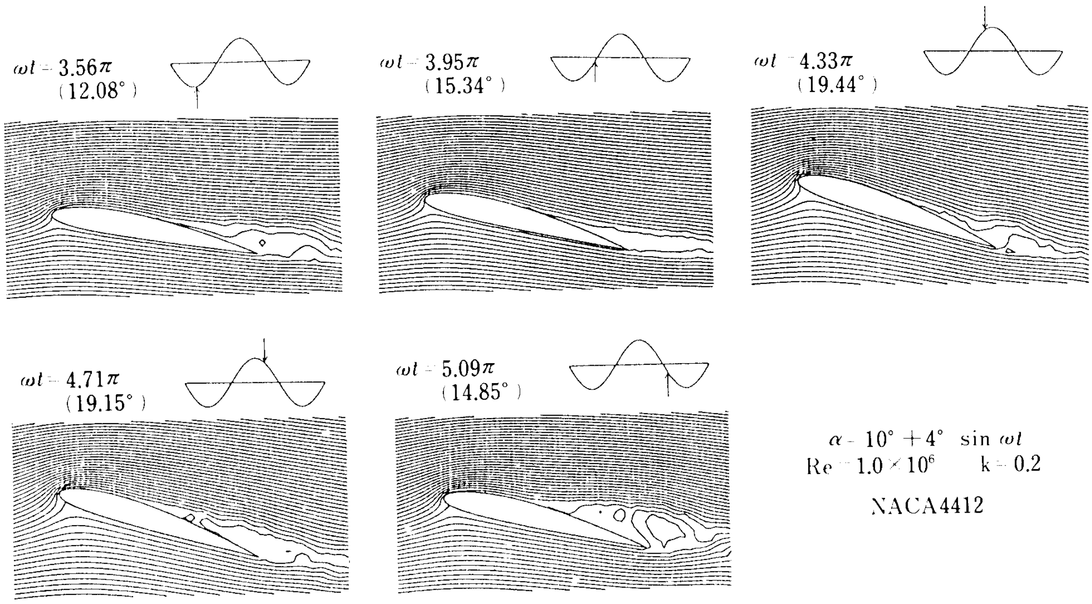

Fig. 12. Stream lines. ($\alpha=16°+4° \sin \omega t$, k=0.2).

expressed by a row of the discrete vortices. And new procedures for the estimation of the pressure coefficient are proposed. The calculated C_p, C_l and C_d show good agreements with experimental results. The results prove that the method is quite useful for the simulation of separated flows around wing sections. Also separated flows around the pitching wing sections are simulated by the same method. The results prove that the method is quite useful even for those complicated flow fields.

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